Project Report for Intro to AI HW3

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1 Boolean

1.1 All True

We can write the probability of all 5 Boolean variables being simultaneously true can be written as:
$P(A) * P(B) * P(C) * P(D|A \wedge B) * P(E|B \wedge C)$
0.2*0.5*0.8*0.1*0.3 = 0.0024

1.2 All False

We can write the probability of all 5 Boolean variables being simultaneously false can be written as:
$P(\neg A) * P(\neg B) * P(\neg C) * P(\neg D|\neg A \wedge \neg B) * P(\neg E|\neg B \wedge \neg C)$
0.8*0.5*0.2*0.1*0.8 = 0.0064

1.3 A False Given B-E True

The probability that A is false given that the four other variables are all known to be true can be written as:
$\frac{P(\neg A)*P(B)*P(C)*P(D|\neg A \wedge B)*P(E|B \wedge C)}{P(A,B,C,D,E)+[P(\neg A)*P(B)*P(\neg A)*P()*P(E|\neg A \wedge B)*P(E|B \wedge C)]}$
$\frac{0.8*0.5*0.8*0.6*0.3}{0.0024+[0.8*0.5*0.8*0.6*0.3]} = 0.96$

2 Natural Disaster - John Mary

2.1 Probability Calculation

To calculate $P(Burglary|JohnCalls = true, MaryCalls = true)$ we can use variable elimination after we have expand out the probability. For clarity and simplicity, we will shorten $P(Burglary) = P(B), P(Earthquake) = P(E), P(JohnCalls) = P(J), P(MaryCalls) = P(M)$.
We can then get $P(B|J = true, M = true)$ as:

$\hookrightarrow \alpha P(B) \Sigma_E P(E) \Sigma_A P(A|B,E)P(J|A)P(M|A)$

From there, we are simply adding the branches of the enumeration tree that represents the equation above. (In slide 26/63 of lecture) This can be be rewritten as:

$\hookrightarrow \alpha P(B)P(E)[P(A|B,E)P(J|A)P(M|A) + P(\neg A|B,E)P(J|\neg A)P(M|\neg A)] + P(\neg E)[P(A|B,\neg E)P(J|A)P(M|A) + P(\neg A|B,\neg E)P(J|\neg A)P(M|\neg A)]$

The respective equation with probabilities subbed in is:
$\hookrightarrow \alpha[(0.001)[(0.002)(0.95)(0.90)(0.70)+(0.05)(0.05)(0.01)]]+(0.998)[(0.94)(0.90)(0.70)+(0.06)(0.05)(0.01)]$
$\hookrightarrow \alpha(0.001)(0.001197 + 0.5910156)$
$\hookrightarrow \alpha(0.001)(0.5922126)$
$\hookrightarrow \alpha(0.00059221)$
With that we can compute the other half needed to find the alpha values.
To do that, we expand $P(!Burglary|JohnCalls = true, MaryCalls = true)$

and perform variable elimination in the same manner of the previous part. We get the following expression:

$\hookrightarrow \alpha P(!B)\Sigma_E P(E)\Sigma_A P(A|\neg B, E)P(J|A)P(M|A)$

Can be expanded to:

$\hookrightarrow \alpha P(\neg B)P(E)[P(A|\neg B, E)P(J|A)P(M|A) + P\neg !A|\neg B, E)P(J|\neg A)P(M|\neg A)] + P(\neg E)[P(A|\neg B, \neg E)P(J|A)P(M|A) + P(\neg A|\neg B, \neg E)P(J|\neg A)P(M|\neg A)]$

Substituting respective probabilities:
$\hookrightarrow \alpha[(0.999)[(0.002)(0.29)(0.90)(0.70) + (0.71)(0.05)(0.01)]] + (0.998)[(0.001)(0.90)(0.70) + (0.999)(0.05)(0.01)]$
$\hookrightarrow \alpha(0.999)(0.000366 + 0.00112774)$
$\hookrightarrow \alpha(0.999)(0.001491374)$
$\hookrightarrow \alpha(0.001489882626)$

We can now find the alpha values using:

$\alpha = 1/(0.0005922 + 0.001489)$
$\hookrightarrow = 480.490202$
Furthermore, 1 - 0.28452 = 0.71546. This checks out with the solution in our book and lecture slides.
$\alpha =< .28454, 0.71546 >$

2.2 Bayesian Network Chain

The time complexity for enumeration of this specific problem can be calculated by determining the total number of variables to be sum as well as how many times each term will be multiplied. In this case, for, P($X_1|X_2$ = true), there is n - 2 variables to sum and n - 1 times for each term to be multiplied. Therefore, the total time complexity for computing this probability using enumeration is O($n2^n$).

The time complexity for variable elimination of this specific problem can be calculated by determining, again, the total number of summations. In this problem, we know that there are two terms, therefore, there are two additions meaning n-2 summations in total. The total running time/time complexity for this specific problem is then just O(n).

3 Interplanetary Search

3.1 Filtering

Probability Distribution for Day 3:
P(A) = 0, P(B) = 0.2, P(C) = 0.8, P(D) = P(E) = P(F) = 0
P(X3 |hot1, cold2, cold3) determined from using evidence from X1 and X2
P(X1 |hot1, cold2, cold3) : P(A) = 1, P(B) = P(C) = P(D) = P(E) = P(F) = 0

P(X2 |hot1, cold2, cold3) : P(A) = 0, P(B) = 1, P(C) = P(D) = P(E) = P(F) = 0
P(X3 |hot1, cold2) : $\alpha_3 P(e3|X3)\Sigma_{x2} P(X3|X2)[\alpha_2 \Sigma_{x1} P(X2|X1)P(X1|E1)]$

3.2 Smoothing

Probability Distribution for Day 2: P(A) = 0, P(B) = 1, P(C) = P(D) = P(E) = P(F) = 0
P(X2 — hot1, cold2, cold3) determined from using evidence from X1
P(X1 — hot1, cold2, cold3) : P(A) = 1, P(B) = P(C) = P(D) = P(E) = P(F) = 0
P(X2 — hot1, cold2, cold3) : P(A) = 0, P(B) = 1, P(C) = P(D) = P(E) = P(F) = 0
$\alpha_2 \Sigma_{x1} P(X2|X1)P(X1|E1)$

3.3 Most Likely Explanation

The rover must have been at A on day 1, because that is where it started based on evidence, this makes sense, additionally, because it reported that it was hot on that day. For day 2, the rover must have moved in order for it to report cold, so that means it must be on B. For day 3, there is an 80% chance that it has successfully moved east one mile to C. There is a 20% chance that it was unable to move and is still on B, but that is fairly unlikely.

3.4 Prediction

In the case of being P(hot4, hot5, cold6 |hot1, cold2, cold3), we already know that the distribution is 0.2 for B and 0.8 for C. We are now finding P(hot4 |hot1, cold2, cold3), then P(hot5 |hot4), and finally, P(cold6 |hot5). In order to record hot4, we need to be on C for day 3, because the rover must move onto D. This is 0.8 from the probability distribution for day 3. In order to record two hot days in a row, the rover has encountered the 0.2 chance to stay in the same place once it has arrived on D. Finally, to record a hot to cold change, the rover must have moved a complete mile east to E, which is a 0.8 chance. In order to find the probability that all this happened, we do (0.8 * 0.2 * 0.8) to find there is a 0.128 (12.8%) chance for P(hot4, hot5, cold6 |hot1, cold2, cold3).

3.5 Prediction

3.5.1 Day 4

Probability Distribution for Day 4: P(A) = 0, P(B) = 0.04, P(C) = 0.32, P(D) = 0.64, P(E) = P(F) = 0
$P(X4|hot1, cold2, cold3) = \Sigma_{x3} P(X4|X3)P(X3|hot1, cold2, cold3)$
P(B) = (0.2 * 0.2)
P(C) = (0.8 * 0.2) + (0.2 * 0.8) (order can be different)
P(D) = (0.8 * 0.8)

3.5.2 Day 5

Probability Distribution for Day 5: P(A) = 0, P(B) = 0.008, P(C) = 0.096, P(D) = 0.384, P(E) = 0.512, P(F) = 0
$P(X5|hot1, cold2, cold3) = \Sigma_{x4} P(X5|X4)P(X4|hot1, cold2, cold3)$
P(B) = (0.2 * 0.2 * 0.2)
P(C) = (0.8 * 0.2 * 0.2) + (0.2 * 0.8 * 0.2) + (0.2 * 0.2 * 0.8)
P(D) = (0.2 * 0.8 * 0.8) + (0.8 * 0.2 * 0.8) + (0.8 * 0.8 * 0.2)
P(E) = (0.8 * 0.8 * 0.8)

4 Fraud Detection

4.1 Bayesian Network

Refer to Figure 1 at the end of the document.

4.2 Probability Queries

4.2.1 Prior Probability

We need to find $P(Fraud)$
$P(Fraud|Trav) = P(Trav|Fraud)P(Fraud)/P(Trav)$
$P(Fraud) = P(Fraud|Trav) * P(Trav)/P(Trav|Fraud)$
We don't know $P(Trav|Fraud)$. We must use joint probability distributions to find this probability.
$P(Trav = true|Fraud) = \alpha\Sigma_{FP}\Sigma_{IP}\Sigma_{CRP}\Sigma_{OC}P(Trav = true, Fraud, FP, IP, CRP, OC)$
$= \alpha\Sigma_{FP}\Sigma_{IP}\Sigma_{CRP}\Sigma_{OC}P(Fraud|Trav)P(FP|Fraud = true, Trav)P(IP|Fraud = true, OC)P(OC)P(CRP|OC)$

$F1(FP) = [P(FP|Fraud = true, Trav = true); P(\neg FP|Fraud = true, Trav = true)]$
$F2(IP, OC) = [P(IP|Fraud = true, OC), P(\neg IP|Fraud = true, OC); P(IP|Fraud = true, \neg OC), P(\neg IP|Fraud = true, \neg OC)]$
$F3(OC) = [P(OC); P(\neg OC)]$
$F4(CRP, OC) = [P(CRP|OC), P(\neg CRP|OC); P(CRP|), P(\neg CRP|\neg OC)]$

$P(Trav = true|Fraud) = \alpha P(Fraud|Trav) * \Sigma_{FP}F1(FP)X\Sigma_{IP}\Sigma_{OC}F2(IP, OC)XF3(OC)\Sigma_{CRP}F4(CRP, C$

$P(Trav|Fraud) = 0.1163$
$P(Fraud) = P(Fraud|Trav) * P(Trav)/P(Trav|Fraud) = 0.01$ *0.05/0.1163 = 0.0043
$P(Fraud) = 0.0043$

4.2.2 Probability Given Evidence

$P(Fraud = true|FP, \neg IP, CRP)$
$= \alpha\Sigma_{Trav}\Sigma_{OC}P(Fraud = true, FP, \neg IP, CRP, Trav, OC)$
$= \alpha\Sigma_{Trav}\Sigma_{OC}P(Fraud = true|Trav)P(FP|Fraud = true, Trav)P(\neg IP|Fraud = true, OC)P(OC)P(CRP|OC) * P(Trav)$
$= \alpha P(Fraud = true|Trav = true) * P(Trav = true) * P(FP|Fraud = true, Trav = true)P(\neg IP|Fraud = true, OC = true)P(OC = true)P(CRP|OC = true)$

$+\alpha P(Fraud = true|Trav = false) * P(Trav = false) * P(FP|Fraud = true, Trav = false)P(\neg IP|Fraud = true, OC = true)P(OC = true)P(CRP|OC = true)$
$+\alpha P(Fraud = true|Trav = true) * P(Trav = true) * P(FP|Fraud = true, Trav = true)P(\neg IP|Fraud = true, OC = false)P(OC = false)P(CRP|OC = false)$
$+\alpha P(Fraud = true|Trav = false) * P(Trav = false) * P(FP|Fraud = true, Trav = false)P(\neg IP|Fraud = true, OC = false)P(OC = false)P(CRP|OC = false)$

$= 0.01*0.9*0.05*(1-0.02)*0.75*0.1\alpha+0.004*0.1*(1-0.02)*0.95*0.75*0.1\alpha+ 0.01*0.9*(1-0.011)*0.05*0.25*0.001\alpha+0.004*0.1*(1-0.011)*0.95*0.001\alpha$
$= 6.14920825 * 10^{-5} = 0.00000614920825\alpha$

$P(Fraud = False, FP, \neg IP, CRP) = \alpha P(Fraud = False|Trav = true) * P(Trav = true)*P(FP|Fraud = False, Trav = true)P(\neg IP|Fraud = False, OC = true)P(OC = true)P(CRP|OC = true) + \alpha P(Fraud = False|Trav = false) * P(Trav = False) * P(FP|Fraud = False, Trav = false)P(\neg IP|Fraud = False, OC = true)P(OC = true)P(CRP|OC = true)+\alpha P(Fraud = False|Trav = true) * P(Trav = true) * P(FP|Fraud = False, Trav = true)P(\neg IP|Fraud = False, OC = false)P(OC = false)P(CRP|OC = false) + \alpha P(Fraud = False|Trav = false) * P(Trav = False) * P(FP|Fraud = False, Trav = false)P(\neg IP|Fraud = False, OC = false)P(OC = false)P(CRP|OC = false)$

$= 0.99*0.9*0.05*(1-0.01)*0.75*0.1\alpha+0.996*0.01*(1-0.01)*0.95*0.75* 0.1\alpha+0.99*0.9*(1-0.001)*0.05*0.25*0.001\alpha+0.996*0.01*(1-0.001)* 0.95*0.25*0.001\alpha$
$= 0.004023880497\alpha$

$\alpha = 1/(P(Fraud = True, FP, \neg IP, CRP)+P(Fraud = False, FP, \neg IP, CRP))$
$= 1/(0.00000614920825 + 0.004023880497)$
$= 244.7757164$
$P(Fraud = True, FP, \neg IP, CRP) = 0.00000614920825\alpha = 0.001505$

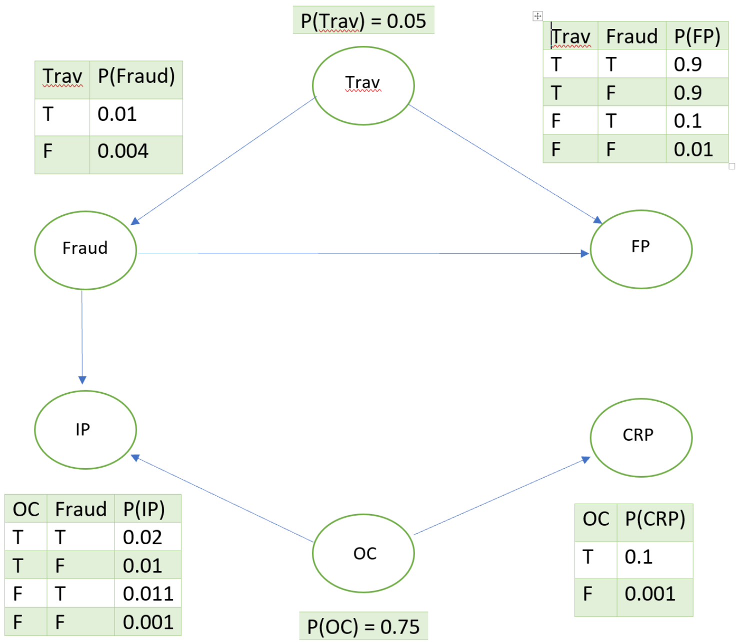

Figure 1: Bayesian Network