The Normal Matrix

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The gl_NormalMatrix is present in many vertex shaders. In here some light is shed on what is this matrix and what is it for. This section was inspired by the excellent book by Eric Lengyel "Mathematics for 3D Game Programming and Computer Graphics".

Many computations are done in eye space. This has to do with the fact that lighting is commonly performed in this space, otherwise eye position dependent effects, such as specular lights would be harder to implement.

Hence we need a way to transform the normal into eye space. To transform a vertex to eye space we can write:

```
	vertexEyeSpace = gl_ModelViewMatrix * gl_Vertex;
```

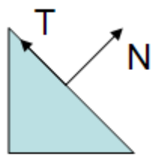
So why can't we just do the same with a normal vector? A normal is a vector of 3 floats and the modelview matrix is 4×4. Secondly, since the normal is a vector, we only want to transform its orientation. The region of the modelview matrix that contains the orientation is the top left 3×3 submatrix. So why not multiply the normal by this submatrix?

This could be easily achieved with the following code:

```
	normalEyeSpace = vec3(gl_ModelViewMatrix * vec4(gl_Normal,0.0));
```

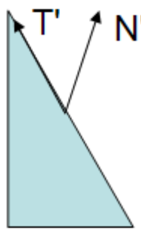
So, *gl_NormalMatrix* is just a shortcut to simplify code writing or to optimize it? No, not really. The above line of code will work in some circumstances but not all.

Lets have a look at a potential problem:



;

In the above figure we see a triangle, with a normal and a tangent vectors. The following figure shows what happens when the *modelview* matrix contains a non-uniform scale.



Note: if the scale was uniform, then the direction of the normal would have been preserved, The length would have been affected but this can be easily fixed with a normalization.

In the above figure the *Modelview* matrix was applied to all the vertices as well as to the normal and the result is clearly wrong: the transformed normal is no longer perpendicular to the surface.

We know that a vector can be expressed as the difference between two points. Considering the tangent vector, it can be computed as the difference between the two vertices of the triangle's edge. If P_1 and P_2 are the vertices that define the edge we know that:

$$T = P_2 - P_1$$

Considering that a vector can be written as a four component tuple with the last component set to zero, we can multiply both sides of the equality with the *Modelview* matrix

$$T * Modelview = (P_2 - P_1) * Modelview$$

This results in

$$T * Modelview = P_2 * Modelview - P_1 * Modelview$$
$$T' = P_2' - P_1'$$

As P'_1 and P'_2 are the vertices of the transformed triangle, T' remains tangent to the edge of the triangle. Hence, the *Modelview* preserves tangents, yet it does not preserve normals.

Considering the same approach used for vector *T*, we can find two points Q_1 and Q_2 such that

$$N = Q_2 - Q_1$$

The main issue is that the a vector defined through the transformed points, $Q_2' - Q_1'$, does not necessarily remain normal, as shown in the figures above. The normal vector is not defined as a difference between two points, as the tangent vector, it is defined as a vector which is perpendicular to a surface.

So now we know that we can't apply the *Modelview* in all cases to transform the normal vector. The question is then, what matrix should we apply?

Consider a 3×3 matrix *G*, and lets see how this matrix could be computed to properly transform the normal vectors.

We know that, prior to the matrix transformation *T.N* = 0, since the vectors are by definition perpendicular. We also know that after the transformation *N'.T'* must remain equal to zero, since they must remain perpendicular to each other. *T* can be multiplied safely by the upper left 3×3 submatrix of the modelview (*T* is a vector, hence the *w* component is zero), let's call this submatrix *M*.

Let's assume that the matrix *G* is the correct matrix to transform the normal vector. *T*. Hence the following equation:

$$N'.T' = (GN).(MT) = 0$$

The dot product can be transformed into a product of vectors, therefore:

$$(GN).(MT) = (GN)^T * (MT)$$

Note that the transpose of the first vector must be considered since this is required to multiply the vectors. We also know that the transpose of a multiplication is the multiplication of the transposes, hence:

$$(GN)^T(MT) = N^TG^TMT$$

We started by stating that the dot product between *N* and *T* was zero, so if

$$G^TM = I$$

then we have

$$N'.T' = N.T = 0$$

Which is exactly what we want. So we can compute *G* based on *M*.

$$G^TM = I \Longleftrightarrow G = (M^{-1})^T$$

Therefore the correct matrix to transform the normal is the transpose of the inverse of the *M* matrix. OpenGL computes this for us in the *gl_NormalMatrix*.

In the beginning of this section it was stated that using the *Modelview* matrix would work in some cases. Whenever the 3×3 upper left submatrix of the *Modelview* is orthogonal we have:

$$M^{-1} = M^T \Longrightarrow G = M$$

This is because with an orthogonal matrix, the transpose is the same as the inverse. So what is an orthogonal matrix? An orthogonal matrix is a matrix where all columns/rows are unit length, and are mutually perpendicular. This implies that when two vectors are multiplied by such a matrix, the angle between them after transformation by an orthogonal matrix is the same as prior to that transformation. Simply put the transformation preserves the angle relation between vectors, hence transformed normals remain perpendicular to tangents! Furthermore it preserves the length of the vectors as well.

So when can we be sure that *M* is orthogonal? When we limit our geometric operations to rotations and translations, i.e. when in the OpenGL application we only use *glRotate* and *glTranslate* and not *glScale*. These operations guarantee that *M* is orthogonal. Note: *gluLookAt* also creates an orthogonal matrix!