Quantum Nondemolition

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A Quantum Nondemolition (QND) measurement, in which an observable is monitored by a sequence of back action evading measurements, was initially proposed in order to overcome the standard quantum limit in the detection of gravitational waves [1]. Full demonstration of QND measurements has been achieved in Quantum Optics [26], using optical nonlinearities to couple a probe to the system QND observable. Using atomic interferometry, the non-destructive detection of a single photon in a cavity has recently been demonstrated [40]. This review focuses on the development of QND measurement theory and charts the progress in realising QND measurements in Quantum Optics.

Historical Introduction

The concept of a Quantum Nondemolition (QND) measurement was introduced by Braginsky *et al.*[1] in relation to attempts at gravitational wave detection. The existence of gravitational waves is predicted by Einstein's theory of General Relativity. Rotating neutron stars and binary stars are the primary candidates for the emission of periodic gravitational waves with frequencies of approximately 1kHz [2]. Although gravitational waves reaching detectors are highly classical (many quanta of excitation), they interact very weakly with matter.

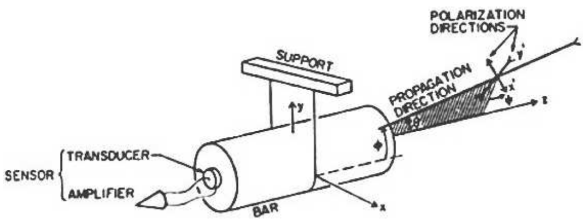

Figure 1, Schematic diagram of a resonant-bar detector [2]

From 1960s to 80s, most experimental effort was focused on developing the 'Weber resonant bar detector'. This detector consists of a large, heavy, solid bar with mechanical oscillations driven by gravitational waves. In the design shown below (Figure 1) a transducer and amplifier converts these oscillations into an electrical signal. Gravitational waves incident on a 10-ton Weber bar would be expected to produce a displacement of the order of 10^{-19}cm [2].

Braginsky was concerned with the ultimate fundamental quantum limits to the sensitivity of such a detector. Clearly successful detection would require that the experimental noise, including the quantum uncertainty, be less than the displacement caused by the gravitational wave.

The fundamental mode of the bar can be regarded as a simple harmonic oscillator (SHO). Standard electronic methods monitor both the amplitude and phase of the oscillator simultaneously - which is equivalent to measuring both amplitude and phase quadratures, X_1 and X_2. However, the Heisenberg Uncertainty Principle forbids precise simultaneous measurement of noncommuting observables:

$$1) \qquad \Delta X_1 \Delta X_2 \geq \frac{\hbar}{2mw}$$

Thus there is an ultimate limit on the accuracy with which such a measurement scheme could monitor variations in the oscillator's energy, for example, in order to detect an external classical force. This is known as the standard quantum limit (SQL) and is common to conventional repeated measurement strategies [3].

$$2) \qquad \Delta E_{SQL} \approx \hbar w \sqrt{n}$$

Braginsky *et al.* [1] realised that sensitivity better than the SQL could be achieved if the measurement extracts information only on one specially chosen observable, and nothing on its conjugate variable. They concluded that 'non-destructive recording' of the *n*-quantum state of an oscillator is possible in principle – introducing the notion of a QND measurement.

Braginsky *et al.* suggested a measurement scheme using an electron beam as a probe interacting with an L-C circuit (the SHO). However, Unruh [4] subsequently showed this particular measurement scheme to be flawed – due to the linear coupling between the probe and the oscillator system. Unruh proved that quadratic (or higher) coupling was necessary when measuring the number of quanta in a mode to avoid back-action. Caves *et al.* [5] showed that it should also be possible to perform a QND measurement on the amplitude quadrature, X_1 – measuring it precisely and continuously.

Definition

Unruh [6] and Caves *et al.* [7] proceeded to develop the QND theoretical formalism. Caves *et al.* [7] defined a QND measurement as:

'A QND measurement of an observable $\hat{A}$ is a sequence of precise measurements such that the result of each measurement (after the first) is completely predictable (in the absence of an external classical force) from the result of the preceding measurement. If an observable (in principle) can be measured in this way, we call it a QND observable.'

We shall follow the Caves derivation of the criterion for a QND observable, before considering a QND measurement scheme. We shall work in the Heisenberg picture of Quantum Mechanics rather than the more familiar Schrödinger picture [8]. In this formulation the state vectors are essentially fixed but the operators evolve according to the equation of motion 3) - equivalent to the time-dependent Schrödinger equation in the Schrödinger picture.

$$3) \qquad \frac{d\hat{A}}{dt} = \tfrac{i}{\hbar}\left[\hat{H},\hat{A}\right] + \frac{\partial \hat{A}}{\partial t}$$

QND observable criterion

We wish to find the condition for $\hat{A}$ to be a QND observable and consider the general case where $\hat{A}$ has degenerate eigenstates:

$$4) \qquad \hat{A}(t_i)\left|A_i,\alpha\right\rangle = A_i\left|A_i,\alpha\right\rangle$$

QND generally presumes that nothing is known about the state of the system before measurement. The first measurement is made at time t_0, resulting in the collapse of the wavefunction into the state, $\left|\Psi(t_0)\right\rangle$ - an arbitrary linear combination of the eigenstates corresponding to the measured eigenvalue, A_0:

$$5) \qquad \left|\Psi(t_0)\right\rangle = \sum_\alpha c_\alpha \left|A_0,\alpha\right\rangle$$

$$6) \qquad \hat{A}(t_o)\left|\Psi(t_0)\right\rangle = A_0\left|\Psi(t_0)\right\rangle$$

The system now evolves according to the equation of motion 3), until the next measurement. In the Heisenberg picture, this means that state of the system does not change. If the result, A_1, of the second measurement at time t_1 is to be completely predictable, then all of the states $\left|A_0,\alpha\right\rangle$ must also be eigenstates of $\hat{A}(t_1)$ sharing the eigenvalue $A_1 = f_1(A_0)$. Hence:

$$7) \qquad \hat{A}(t_1)\left|\Psi(t_0)\right\rangle = A_1\left|\Psi(t_0)\right\rangle$$

$$8) \qquad \hat{A}(t_1)\left|A_0,\alpha\right\rangle = f_1(A_0)\left|A_0,\alpha\right\rangle$$

This condition must hold for all possible values of A_0. The operator $\hat{A}(t_0)$ must therefore satisfy:

$$9) \qquad \hat{A}_1 = f_1\left[\hat{A}_0\right]$$

We extend this to a sequence of measurements at time *t*:

$$10) \qquad \hat{A}(t) = f\left[\hat{A}(t_0);t,t_0\right]$$

If $\hat{A}$ satisfies condition 10) at all times t then we call it a *continuous* QND observable; if $\hat{A}$ satisfies this condition only at particular times $t = t_k$ then we call it a *stroboscopic* QND observable. All constants of motion are necessarily continuous QND observables.

A similar criteria, 11), was independently derived by Unruh [6] that is necessary but not sufficient for Cave's QND observable condition, 10).

$$11) \qquad \Rightarrow \left[\ \hat{A}(t),\hat{A}(t_0)\ \right] = 0$$

Additionally, Unruh [6] assumed no explicit time-dependence of $\hat{A}$ in his derivation. This was noted by Caves *et al.* [7] as a narrowing of the definition. Caves *et al.* theoretical paper is generally regarded as the standard reference.

For a SHO, position and momentum are *stroboscopic* QND observables. Therefore in a QND measurement scheme either observable would have to be measured arbitrarily quickly, in intervals of half the period.

$$12) \quad \left[\ \hat{x}(t), \hat{x}(t+\tau)\ \right] = \frac{i\hbar}{w} \sin w\tau$$

$$13) \quad \left[\ \hat{p}(t), \hat{p}(t+\tau)\ \right] = i\hbar w \sin w\tau$$

Examples of *continuous* QND observables for the SHO are the number of quanta, $\hat{n}$, and the quadrature components, $\hat{X}_1$ and $\hat{X}_2$.

General QND measurement scheme

We now consider how a QND measurement may be realized practically. The key property of a QND measurement is repeatability [7]. Quantum measurements in which the system interacts directly with the classical measuring device are called *direct* measurements. In Quantum Optics, for example, conventional photon counting techniques absorb quanta – they are destructive.

Therefore we must generally perform an *indirect* measurement on the QND observable, $\hat{A}$, of the system. An indirect measurement is a two-step process. In the first step the system interacts with a probe that has been prepared in some known initial state. The second step is a direct measurement of a particular observable, $\hat{B}$, of the probe [9].

The total Hamiltonian for the system-probe may be expressed as:

$$14) \quad \hat{H} = \hat{H}_S + \hat{H}_P + \hat{H}_I$$

The Hamiltonians of the system, probe and their interaction are represented by $\hat{H}_S$, $\hat{H}_P$, $\hat{H}_I$ respectively. $\hat{A}$ must not be affected during the interaction with the probe:

$$15) \quad \left[\ \hat{A}, \hat{H}_I\ \right] = 0$$

This is the back-action evasion (BAE) criterion for $\hat{A}$ and ensures that 10) holds in the presence of the interaction with the probe [10]. (Although in the broader Caves definition of QND measurement this is allowable as long as the interaction is predictable [11].) For effective system-to-probe coupling the evolution of $\hat{B}$ must be dependent on $\hat{A}$ [11]:

$$16) \quad \frac{\partial \hat{H}_I}{\partial \hat{A}} \neq 0$$

$$17) \quad \left[\ \hat{B}, \hat{H}_I\ \right] \neq 0$$

Quantum Optics

Although first proposed for mechanical SHOs, QND measurement schemes have so far only been realised in Quantum Optics. Coupling between the signal and probe is generally achieved using quadratic or higher optical nonlinearities in the medium:

$$18) \quad P_i = \varepsilon_0 \left\{ \chi^{(1)}_{\mathrm{ij}} E_j + \chi^{(2)}_{\mathrm{ijk}} E_j E_k + \chi^{(3)}_{\mathrm{ijkl}} E_j E_k E_l + \ldots \right\}$$

Coupling via the second-order nonlinear susceptibility, $\chi^{(2)}$, is weak but generally adds small excess noise to the output light beams. Third-order optical nonlinearities, $\chi^{(3)}$, are usually accompanied by significant excess noise from the nonlinear medium, but have extremely large values in resonant atomic media and can operate with very small optical power [12].

Before reviewing proposed schemes and experimental progress, we introduce criteria for evaluating the results.

Characterisation of QND Measurements

Imoto *et al.* [13] first addressed the issue of nonideal QND measurements in considering the effects of a lossy Kerr medium. Clearly with attenuation, the number of photons is no longer a perfect QND observable. They proposed that results should be compared against the ideal lossless beamsplitter and a photon counter with unity quantum efficiency.

Holland *et al.* [14] introduced a set of correlation functions to evaluate results from attempts at QND experiments. They considered a general QND measurement of the field quadratures. The incident signal field X_1 - the QND observable of the 'system' - is strongly coupled to a probe field on which a subsequent readout measurement of the quadrature phase, Y_2, is made (Figure 2).

Figure 2, General QND measurement of signal field quadrature [14]

The ideal QND measurement would satisfy:

$$19) \qquad X_1^{out} = X_1^{in}$$

$$20) \qquad Y_2^{out} = GX_1^{in} + Y_2^{in}$$

where G defines the 'QND gain'.

The correlation between the signal input and probe output measures the strength of coupling from signal to probe.

$$21) \qquad C_{X_1^{in}Y_2^{out}} = \frac{\left|\left\langle X_1^{in}Y_2^{out}\right\rangle - \left\langle X_1^{in}\right\rangle\left\langle Y_2^{out}\right\rangle\right|}{\sqrt{V_{X_1^{in}}V_{Y_2^{out}}}}$$

The correlation between the signal input and signal output measures degradation of the signal from back action.

$$22) \qquad C_{X_1^{in}X_1^{out}} = \frac{\left|\left\langle X_1^{in}X_1^{out}\right\rangle - \left\langle X_1^{in}\right\rangle\left\langle X_1^{out}\right\rangle\right|}{\sqrt{V_{X_1^{in}}V_{X_1^{out}}}}$$

The conditional variance measures the effectiveness of the scheme in placing the signal beam into a well-defined eigenstate.

$$23) \qquad V\left(X_1^{out}\middle|Y_2^{out}\right) = V_{X_1^{out}}\left(1 - C^2_{X_1^{out}Y_2^{out}}\right)$$

Grangier *et al.* [15] realised that the variance $V_{X_1^{in}}$ is not easily obtained in an experiment, since it requires the knowledge of the fluctuations in the input, which would in turn require another QND device for unperturbed measurements! Grangier *et al.* proposed measuring the transfer coefficients:

$$24) \qquad T_p = \frac{SNR_P^{out}}{SNR_S^{in}}$$

$$25) \qquad T_S = \frac{SNR_S^{out}}{SNR_S^{in}}$$

$$26) \qquad SNR_P^{out} = \frac{\left\langle Y_2^{out}\right\rangle^2}{V_{Y_2^{out}}} \quad \text{etc…}$$

T_P and T_S describe the transfer of the signal to noise ratio (SNR) from the signal input to the probe output and the signal output respectively. The beamsplitter/photon-detector would have $T_P + T_S = 1$ and unity conditional variance. The ideal QND measurement, 19)-20), would have $T_P = T_S = 1$, and zero conditional variance. 'True QND measurements' [16] lie within the region:

$$27) \qquad 1 < T_P + T_S \le 1$$

$$28) \qquad V\left(X_1^{out}\middle|Y_2^{out}\right) < 1$$

Figure 3 shows selected results from the QND measurement schemes reviewed below.

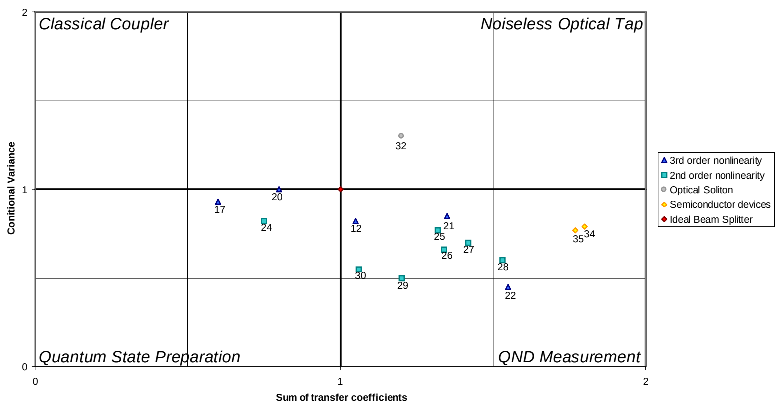

Figure 3, Characterisation of nonideal QND measurements (based on [33,41]**)**

Experiments using $\chi^{(3)}$ nonlinearities

Unruh [4] had shown that for QND measurements of $\hat{n}$ the coupling must be at least quadratic (with respect to the different $\hat{n}$ operators). Milburn *et al.* [10] proceeded to consider a QND measurement with coupling from a four wave mixing interaction:

$$29) \qquad \hat{H}_I = \hbar\kappa\hat{a}^+\hat{a}\hat{b}^+\hat{b}$$

$\hat{a}$ and $\hat{b}$ represent the annihilation operators for the signal and probe modes respectively; $\hat{n} = \hat{a}^+\hat{a}$ is the QND observable of the system; κ, the coupling constant.

However, Milburn *et al.* were still concerned with mechanical SHOs for detecting gravitational waves. It was left to Imoto *et al.* to transform this into the optical domain.

Imoto *et al.* [11] proposed a scheme to measure the photon number (QND observable) of a travelling signal wave via the phase of a travelling probe wave using the cross Kerr effect. The interaction Hamiltonian takes the same form as 29) with the coupling constant, κ, proportional to the third-order nonlinear susceptibility, $\chi^{(3)}$. The change in refractive index of the Kerr medium is proportional to the optical intensity of the signal wave. Thus, the photon number of the signal wave can be determined from the phase shift in the probe, measured using a Mach-Zehnder interferometer (Figure 4).

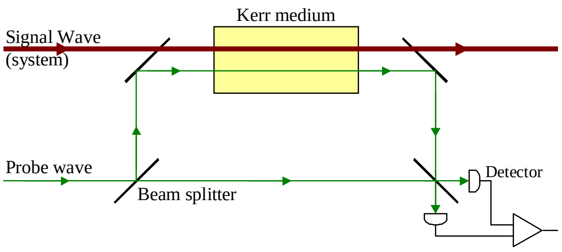

Figure 4, QND measurement of photon number using the Kerr effect [11]

The third-order optical nonlinearity, $\chi^{(3)}$, is defined not only for the Kerr effect but also for every process in which four photons are emitted or absorbed. We are considering the ideal cross Kerr effect in Equation 29), ignoring the unwanted self-modulation of the phase caused by the signal and probe waves.

Initial attempts used optical fibres as a nonresonant Kerr medium for travelling waves. Levenson *et al.* [17] observed sufficient a correlation between the QND measurement and subsequent destructive detection of the observable, satisfying the Quantum State Preparation criterion for sub-shot-noise, 28). However the Kerr effect is relatively weak in optical fibres, and the transfer from signal to probe observable was insufficient for the result to constitute a QND measurement. Imoto *et al.* [18] achieved similar results.

Improved results were obtained by Levenson *et al.* [19] using Raman resonances in calcite to enhance the third-order optical nonlinearity. Detuning is necessary to prevent loss/noise due to absorption/emission processes. Spontaneous absorption, Broullion scattering, Doppler broadening and dispersion degrade the signal-to-noise ratio. The coupling has to be sufficiently strong to compensate for these effects in resonant Kerr media.

Grangier *et al.* [20] used two-photon absorption resonances to enhance the third-order optical nonlinearity. The signal laser beam was coupled into a resonant cavity with sodium atoms inside. The probe laser beam, tuned to a second cascaded atomic resonance picks up a modulation of the phase introduced by the intensity fluctuations of the signal beam. Improved results with this scheme were achieved by Poizat *et al.* [21]. Roch *et al.* [22] used laser cooled rubidium atoms to reduce the noise due to Doppler broadening. Their results remain the closest to an ideal QND measurement achieved to date (Figure 3).

Experiments using $\chi^{(2)}$ nonlinearities

Yurke [23] proposed a scheme to measure the quadrature component, X_1, employing the parametric amplification and splitting in a nonlinear type-II crystal – an Optical Parametric Amplifier (OPA). The coupling takes the form:

$$30) \qquad \hat{H}_I = \hbar\kappa\hat{X}_1\hat{Y}_1$$

La Porta *et. al* [24] were the first to realise such an experiment, using a potassium trihydrogen phosphate (KTP) crystal (Figure 5). The signal input beam is a linearly polarised pulsed wave; the probe (meter) input is the vacuum field. The signal and vacuum field undergo a mixing interaction using half-wave plates, after which a mixture of the signal and probe fields propogate along each of the ordinary and orthogonal extraordinary axes of the KTP crystal, pumped by a pulsed intense classical field. After this amplification step the fields then pass through a second half-wave plate and are separated using a polarisation beam splitter.

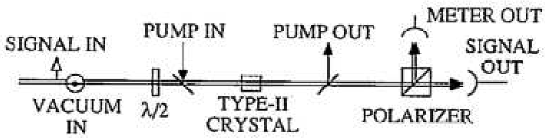

Figure 5, Measurement of quadrature via Parametric down conversion [24]

Optical losses and a low parametric gain prevented full achievement of the QND criteria, 27)-28). QND measurements were achieved by Levenson *et. al.* [25] and Bencheikh *et al.* [26] using better materials with smaller losses. Further improvements were made by Pereira *et al.* [27] with the OPA in a cavity configuration to enhance parametric gain.

The QND measurement criteria, 27)-28), omit the repeatability property emphasised by Caves *et al.* [7] – "once is not enough!" In this sense, a full demonstration of a QND measurement must consist of a sequence of BAE measurements not just the single BAE devices reviewed so far. Bencheikh *et al.* [26,28] were the first to address this issue, operating two independent OPAs in series. Although the second measurement was not quite as good as the first, both consecutive results fulfilled the QND measurement criteria.

Schiller *et al.* [29] employed a squeezed vacuum state – reducing the partition noise which enters – so that the probe and signal output beams are quantum correlated. They also used also a $LiNbO_3T$ crystal instead, and performed consecutive measurements to fully demonstrate QND repeatability. Bruckmeier *et al.* [30] used continuous waves instead of pulsed signal/probe waves, and demonstrated reliable, 36 hour stable operation. These features are important for future application of QND techniques to precision experiments.

Optical Solitons

Haus *et al.* [31] proposed QND measurements of the photon number of optical fibre solitons. Optical solitons propogate without changing shape or losing energy by maintaining a balance of self-phase-modulation from the fibre's Kerr nonlinearity and the negative group-velocity dispersion. Brillouin scattering and associated noise can be minimised by using short duration solitons.

When two solitons with different wavelengths and velocities collide, their respective phases and centre positions are shifted according to their photon number and momentum. Optical fibre losses are very small – solitons can travel and interact over many meters, giving rise to large effective nonlienar interactions. A QND measurement of the photon number of a signal soliton is performed by monitoring the phase shift of a second probe soliton after collision. This was carried out by Friberg *et al.* [32].

Semiconductor devices

Experiments using semiconductor emitters and receivers suggest that the QND measurement criteria 27)-28) are not complete [33]. Two results [34,35] included on the graph might not be regarded as QND devices however they satisfy the QND measurement criteria. In these experiments a photodiode directly measures the intensity of the light, destroying the signal beam in the process. A light emitting diode simultaneously "recreates" the quantum statistics of the intensity of a light beam. The experiment demonstrates photon number amplification, duplication, and information tapping without the introduction of significant noise. However, the square of one quadrature is measured, whilst the information about the other quadrature is completely destroyed.

Recent Developments:

Brune *et al.* [36] proposed measuring the photon number of a cavity mode by the phase shift in a probe atomic beam (Figure 6).

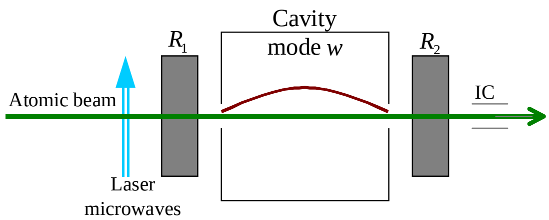

Figure 6, QND setup for measuring the photon number in a cavity [36]

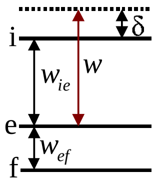

Figure 7, Ryberg levels e, f and i [36]

The transition i→*f* is forbidden (Figure 7). The cavity field, detuned by δ from the *e*→*i* atomic transition, shifts the *e* (and *i*) by an amount proportional to the number of photons in the cavity field mode, $\hat{n}$. The detuning, δ, is large enough to make stimulated absorption/emission improbable, but such that level *f* is unperturbed. Hence the interaction is nonresonant, enabling small photon numbers to be measured.

The dispersive energy shift is detected using the Ramsey method of separated oscillatory fields [37]. The atoms are initially prepared into Ryberg state *e*; they then interact in the first field zone R_1 with a microwave field tuned at frequency w_{ef}, and are thus prepared in a coherent superposition of levels *e* and *f*. The atom crosses the cavity, and a phase shift is introduced in the amplitude of the state *e*. Finally after passing zone R_2 the atom is detected in state *e* or *f* by a field ionisation counter. The probability of detecting an atom in level *e* or *f* results from quantum interference between the two paths in which the atom crosses the cavity in either *e* or *f*. The probability is a periodic function of the phase shift - resulting in Ramsey fringes [38]. Repeated measurements result in the collapse of the photon distribution in the field to a number state [39].

Laser cooling the atomic beam can be used to produce large phase shifts per photon. The advantage of dispersive atom-field coupling (no absorption) is that very small number of photons (down to *n*=0) could be continuously detected without back action on the photon number. In comparison, single-photon resolution is unrealistic with QND schemes using the Kerr-type effect ($\chi^{(3)}$) or parametric amplification ($\chi^{(2)}$) due to photon absorption/noise and weak coupling respectively.

Nogues *et al.* [40] performed a restricted version of this QND measurement scheme in which the cavity mode was limited to 0 or 1 photon occupancy. They successfully detected the presence of a single microwave photon in a resonator without the absorption of the photon. This is important for the development of quantum logic gates and multi-electron entanglement.

Conclusion

QND measurement strategies are important because they allow one to circumvent the SQL inherent to standard repeated measurement techniques. From a theoretical viewpoint, they are the most fundamental type of quantum measurement, free of any non-fundamental uncertainties. The continued development of QND methods on the engineering level undoubtedly promises a qualitative improvement of sensitivity in many experiments.

QND measurements have been fully demonstrated in the optical domain, within the criteria 27)-28) adopted. QND detection of photons will find applications in new generations of communication devices [42,43] and has implications for the development of a quantum computer.

In contrast to the optic experiments we have reviewed, the quantum noise regime of a mechanical oscillator has not yet been reached in experiments. It is a formidable experimental challenge to reach the quantum limit in low frequency mechanical experiments, due to the effect of the environment and other sources of noise [44].

Gravitational wave detection, which inspired the QND measurement principle has yet to be achieved [45]. Hopes now rest with the LIGO project which measures the displacement of free masses using laser interferometry [46]. Increasing the interferometer's arm length (to 4km!) reduces the SQL of this non-QND measurement [47]. Results from LIGO are due in 2002.

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