PHY202:

Classical Mechanics and Electromagnetism

Undergraduate Physics Laboratory

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Experiment 1

Kater's Pendulum

Objectives

1. To find the local value of acceleration due to gravity using a specially designed device called Kater's pendulum.

Introduction

Kater's pendulum is a compound pendulum that is used to measure the local value of acceleration due to gravity.

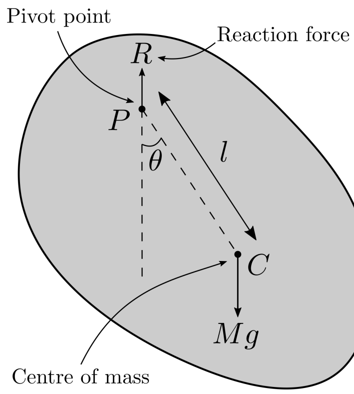

Figure 1.1: Schematic of a compound pendulum oscillating about a pivot P

A compound pendulum is an extended object whose mass is distributed throughout its body. The time period for small oscillations of such a pendulum is given by

$$T = 2\pi\sqrt{\frac{I_l}{mgl}} \tag{1.1}$$

where I_l is the moment of inertia of the body about the axis of rotation (at l), g is acceleration due to gravity, M is the total mass of the body, and l is the distance between the pivot point and the centre of mass. (You should try deriving Equation (1.1) using the torque balance equation.)

Let I_{CM} be the moment of inertia of a body about an axis passing through its centre of mass. By the parallel axis theorem, moment of inertia about any axis parallel to this is

$$I_l = I_{\text{CM}} + ml^2 \tag{1.2}$$

where l is the distance from the centre of mass. I_{CM} is usually simply written as mk^2, where k is a quantity known as the radius of gyration, and which depends on the configuration of mass and the distance from the parallel axis through the centre of mass, about which the body is rotating.

$$T = 2\pi\sqrt{\frac{k^2 + l^2}{gl}} \tag{1.3}$$

Question: Write Equation (1.3) as a quadratic equation in l (for a given period T). What do the roots of this equation correspond to?

From this, it should be clear that there are – in principle – an infinite number of points of suspension on a compound pendulum which have the same time period, all of which are at the same distance l from the centre of mass. You can thus imagine drawing a circle of radius l around the centre of mass, and every point on this circle would have the same time period T_l. What is less intuitive, however, is that there is a *second* circle, of radius k^2/l, such that all points suspended a distance of k^2/l from the centre of mass *also* have the same time period T_l. There are thus two circles of "conjugate" points about which the time period is the same.

Question: Using Equation (1.3), show that a point of suspension at a distance d from the centre of mass, and a point of suspension at a distance k^2/l from the centre of mass have the same time period of oscillation.

Question: Now, imagine a one-dimensional rod. Show that there are strictly four points of suspension which produce the same time period of oscillation:

$$l, \quad -l, \quad \frac{k^2}{l}, \quad -\frac{k^2}{l}. \tag{1.4}$$

Do all of them have to lie on the pendulum? Show that as one point of suspension approach the centre of mass, the other recedes from it.

Rearranging Equation (1.1),

$$g = \frac{4\pi^2}{T^2}\frac{I_l}{ml} \tag{1.5}$$

To get the value of g from Equation (1.5), one has to know its moment of inertia accurately, which requires an accurate calculation of the body's mass distribution. Furthermore, using the formula as it stands requires us to know the position of the centre of mass precisely, since l is measured with respect to the centre of mass. Doing this, however, is not easy. Kater's pendulum ingeniously sidesteps this this problem through its design by depending primarily on the sum of two distances from the centre of mass in opposite directions, with the points between which distance is measured being the easily located knife-edges.

Kater's Pendulum consists of a thick rod (mass M) with two knife edges and two unequal masses, M_1 and M_2. These masses are adjustable i.e. they can be moved along the rod.

Theory

Consider oscillations of the Kater's pendulum – shown in Figure (1.2) – in two different configurations:

1. One where the pendulum is suspended about knife edge A, at a distance l_A from the centre of mass, with some period of oscillation T_A, and
2. One where the pendulum is suspended about knife edge B, at a distance l_B from the centre of mass, with some period of oscillation T_B.

In each of the above cases, we can use Equations (1.2) and (1.1) to find the time periods:

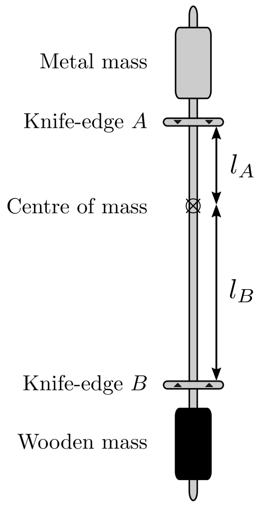

Figure 1.2: Schematic of the Kater's pendulum setup.

$$T_A = 2\pi\sqrt{\frac{I_{CM} + m\,l_A^2}{m\,g\,l_A}}$$
$$T_B = 2\pi\sqrt{\frac{I_{CM} + m\,l_B^2}{m\,g\,l_B}}$$

where m is the total mass of the pendulum.

We can eliminate I_{CM} from the above equations, to get

$$\frac{1}{g} = \frac{1}{4\pi^2}\left(\frac{T_A^2 + T_B^2}{2(l_A + l_B)} + \frac{T_A^2 - T_B^2}{2(l_A - l_B)}\right)$$

Now, we have a formula for g which does not depend on the mass distribution of the pendulum. However, in the second term, we need to find $l_A - l_B$, which requires knowledge of the location of the centre of mass which is also not easy to get. We would thus like to eliminate this second term. If we are able to make $T_A = T_B$, we can eliminate the second term, which depends on the position of the centre of mass. And we know that that is a possibility because for any compound pendulum, there are two distances with the same oscillation period. The asymmetric mass distribution pushes the centre of mass away from the geometrical centre of the pendulum, and allows two conjugate points of suspension to be accessible.

Question: Can you show why in this case $T_A = T_B$, but $l_A \neq l_B$?

Thus, when we find the pivot points with same time periods, $T_A = T_B = T$, we have the following condition:

$$g = 4\pi^2 \left(\frac{l_A + l_B}{T^2} \right)$$

which provides us an accurate measurement of the acceleration due to gravity g which does not require knowing either the mass distribution or the location of the centre of mass.

Question: In the above formula, $l_A + l_B$ seems like the sum of two separate lengths measured with respect to the centre of mass. Experimentally, is this truly the sum of two lengths?

Experimental Setup

Apparatus

1. Kater's Pendulum
2. Stopwatch
3. Markers

Warnings

- The pendulum system is very heavy, make sure you do not drop it on your (or rather, anyone's) feet.
- The pendulum can be a spear. Make sure you carry it such that it does not hurt anyone.
- Make sure the knife edges are tightened (using a pair of pliers) to ensure that they do not slide during oscillations.
- The wooden weight has a tendency of slipping even when the screw is tightened by a pair of pliers. Hence, you might want to mark the position of the wooden block to keep track of it in case it moves.

Procedure

Part A

In this part of the experiment, you will use the rod of the Kater's pendulum as a compound pendulum, and try to find four points that have the same time period T.

1. Take the rod of the pendulum and a single knife-edge. Measure the centre of mass of the rod using a heavy wedge.

   Question: What is the radius of gyration k of a rod of mass M? Assume the rod to be cylindrical; you will need to calculate its moment of inertia, and then use the definition of the radius of gyration.

2. Attach the knife-edge at one end of the rod, at some distance l from the centre of the rod, and measure the time-period for 10 oscillations.
3. Change l by moving the knife-edge from one of the rod to the other, and measure the time period for 10 oscillations in each case.

   Ensure that the centre of mass is always below the knife-edge or the pendulum will topple over. You can do this by flipping the knife-edge as you cross the centre of mass.

4. Plot a graph of T vs. l, and show that there are four points which have the same time period.

   Question: Why do we not need to take the moment of inertia of the knife-edge into account in this case?

Part B

In this part of the experiment, you will use Kater's pendulum to determine the acceleration due to gravity g.

1. Mark the centre of the rod and fix the masses at some fixed distance d from this centre.
2. Fix both the knife-edges some distance apart (call this l) on either side of the centre of the rod.
3. By moving both masses with respect to their knife edges (i.e. the configuration of the system), change the centre of mass of the system. For each such position, measure T_A and T_B for 10 oscillations, and make a table of $T_A - T_B$, making sure you keep track of the sign.
4. You will notice that $T_A - T_B$ will change from positive to negative, thus indicating the configuration for which $T_A = T_B$. (Make sure you note down both $l_A + l_B$ and $l_A - l_B$.)
5. Move to this configuration, and measure T_A and T_B again for 10 oscillations. If the difference is significant, make small adjustments until T_A and T_B are as close to each other as you can make them. One you have decided on your final configuration, measure T_A and T_B for 100 oscillations, making sure you count your oscillations carefully.

   Tip

   If you make a mistake in counting the oscillations, this will show up in a comparison of the time periods obtained from 10 oscillations and those obtained from 100 oscillations.

Part C

In this part of the experiment, you will perform a detailed error analysis of your result.

Question: Which of the above measured parameters would you use in the error analysis?

There is a reason that the error analysis of this experiment takes up an entire part of the experiment. It is extremely involved, please make sure you pay attention to doing it well.

Experiment 2

Air Track

Objectives

1. Studying motion in one dimension and Newton's Laws.

Background

Newton's Laws

Newton's second law famously states that

$$\mathbf{a} = \left(\frac{1}{m}\right)\mathbf{F} \tag{2.1}$$

Question: You are probably a little uncomfortable about the way Equation (2.1) has been written. Can you explain why this way is – in some sense – superior to the common $\mathbf{F} = m\mathbf{a}$?

Question: Consider an object of mass m acted on by a constant force F_0. Show by solving the differential equation that is Newton's second law, that the object's velocity and position vary as:

$$v(t) = v(0) + \left(\frac{F_0}{m}\right)t \tag{2.2}$$

$$x(t) = x(0) + v(0)t + \frac{1}{2}\left(\frac{F_0}{m}\right)t^2 \tag{2.3}$$

From the above equations, it should be clear that if a body is given an initial velocity, and no external force is applied to it, it should continue to move at that velocity. In other words, its position should vary linearly with time.

However, this clearly contradicts our everyday experience. Imagine placing your book on the table and giving it an initial velocity by pushing it and letting go: the book will – at best – move a little and come to rest. This does not, of course, contradict Newton's Law. The surface provides a force to the object that opposes its motion. Frictional forces are notoriously difficult to take into account, and so if we want to study simple one dimensional systems, it would be helpful to minimise them as far as possible. However, if you imagine an object in outer space, it is not difficult to see that giving it a push will cause it to move at a constant velocity.

It is not possible for us to conduct this experiment in outer space. However, what essentially distinguishes the environment in outer space from the table-top is that in the former there is no friction. A device that almost eliminates friction if the air track, which simulates a low-friction environment by creating a 'cushion' of air on which an object (called a "rider") may float. (More on this in the description below.) To test the above equations, we need to impose a constant force on the rider. Consider setting the Air Track up as shown in Figure (**??**). Connect a mass m to the rider (of mass M) with an inextensible string. When this is released, the only horizontal force on the rider comes from the tension on the string, and so

$$Ma = T \tag{2.4}$$

The hanging mass m feels both gravity (downwards) and tension T (upwards).

Question: Can you argue that the two blocks must accelerate at the same rate a? What assumptions do you need to make for this?

$$ma = mg - T \tag{2.5}$$

Question: Show that:

$$g = a\left(\frac{M+m}{m}\right) \tag{2.6}$$

Thus, by measuring the acceleration, one can measure the acceleration due to gravity g.

Conservation of momentum

When such a low-friction environment ensures that (almost) no external forces are acting on the object, Equation (2.1) tells us that $m \times v -$ constant. When we have a single object, this just tells us that – when there are no external forces acting on the object – its momentum stays constant. If the mass of the object is constant, this is just a statement that the velocity remains constant.

Such 'momentum conservation' applies to a single object, but it's a lot more interesting to look at a situation with at least two interacting objects. Consider two objects moving at constant velocities (say v_1 and v_2) which collide. From Newton's Third Law, the force of the first on the second (say, F_{12}) is equal and opposite to the force of the second on the first (F_{21}). Thus:

$$F_{12} = -F_{21} \tag{2.7}$$

If we consider that the forces act over a small time interval Δt, it follows that the impulses experienced by the two objects are also equal in magnitude and opposite in direction, i.e.

$$\begin{aligned} F_{12}\Delta t &= -F_{21}\Delta t \\ m_1\Delta v_1 &= -m_2\Delta v_2 \end{aligned} \tag{2.8}$$

where in the last case we have simply used the definition of the acceleration. Thus, momentum is transferred from one object to the other.

Question: In the above case, show that the velocities before and after the collision are related by:

$$m_1 v_1^{\text{before}} + m_2 v_2^{\text{before}} = m_1 v_1^{\text{after}} + m_2 v_2^{\text{after}} \tag{2.9}$$

Elastic and inelastic collisions

It may have struck you that in the earlier case we seemed to imply that after the collision the objects moved away at different velocities, and thus remained *distinct* (consider two billiard balls colliding). However, from real world experience you have often seen another case: collisions where the two objects 'stick' together, and thus move at together at the same velocity after the collision (consider two lumps of clay thrown at each other). Consider both these cases: since we could engineer it such that the objects (billiard balls or lumps of clay) have the same masses and the same velocities, and since we know that the results after the collisions are clearly different, there must be some way to distinguish between them.

You will show – in *Classical Mechanics* – that the distinction between such collisions is to do with another conserved quantity: energy. In particular, the kinetic energy (or the energy that an object possesses by virtue of its motion) is given by

$$\text{K.E.} = \frac{1}{2}mv^2$$

There are thus two types of collisions:

1. **Perfectly elastic:** in which both momentum and kinetic energy are conserved. In other words, if two masses m_1 and m_2 and velocities v_1 and v_2 collide elastically and move away with velocities v_1' and v_2', then:

$$\begin{aligned} m_1 v_1 + m_2 v_2 &= m_1 v_1' + m_2 v_2' \\ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 &= \frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2 \end{aligned} \tag{2.10}$$

2. **Perfectly inelastic:** in which momentum is conserved, but kinetic energy is not conserved. In other words, if the masses stick together to form a mass of $m_1 + m_2$ moving at some velocity V, then:

$$\begin{aligned} m_1 v_1 + m_2 v_2 &= (m_1 + m_2) V \\ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 &\neq \frac{1}{2} (m_1 + m_2) V^2 \end{aligned} \tag{2.11}$$

It is important to realise that while *kinetic* energy is not conserved, this does not mean *energy* is not conserved. Since energy can take different forms, some of the initial kinetic energy may go into deforming the objects, or producing heat. In general, most collisions are inelastic (though not *perfectly* inelastic), and some energy is always lost to the environment.

Question: Perfectly elastic collisions generally only occur at the atomic scales. The example of two billiard balls is not a perfectly elastic collision. Where do you think the lost kinetic energy goes?

Question: Show that in perfectly inelastic collisions above, the 'loss' in kinetic energy is can be written as

$$\text{Loss in K.E.} = \frac{1}{2} \mu \left(v_2 - v_1\right)^2 \tag{2.12}$$

Where you need to determine μ (called the *reduced mass* of the body). What do you think this means? *Hint:* Imagine placing yourself in the reference frame of the centre of mass of the two objects.

Experimental Setup

Apparatus

1. Air track
2. Blower

3. Assorted accessories (shown in Figure (**??**))
4. Two Vernier photo-gates and associated cables
5. Retort stands

Description

The Air Track

The air track, as explained, can be used as a low-friction environment for kinematics experiments. Compressed air is injected into the cavity beneath the track. Since this cavity is sealed, the air can only escape through the small holes on the track. This provides a force strong enough to lift the riders, drastically lowering the contact friction.

Connect the hose from the blower to the air track and turn it on. The speed of the blower (and thus the pressure on the rider) can be controlled by the knob on the blower.

The first step is to level the track, which you will do with a spirit level by adjusting its legs. A good way to test if the track is levelled and the pressure is sufficient is simply to place one of the riders on it and observe its behaviour. If the rider floats gently above the track and shows no preference for the right or left end, your air track is ready to use.

Tip

Do not be tempted to keep the blower at maximum: too much pressure will cause more air to escape near the inlet than elsewhere, which will cause turbulence and destabilise experiments. On the other hand, using the bare minimum pressure requires you to be sure that the track is completely free of scratches or dirt on its surface. Try to find a middle ground before you start.

Using the air track: accessories

End attachments

The steel springs and pulley given with the setup may be attached to the ends of the air track. The springs can be used to launch the riders, or to allow them to bounce back nearly elastically. The pulley may be placed on one end to change the direction of an applied force from vertical to horizontal.

Riders

The air track has three different types of riders, on top of which the different interrupter cards and prongs may be added when working with the photo-gates. The riders can be distinguished by their mass and size:

1. **Short riders** ($\sim$ 200g) with attachments on either side to add masses. (Make sure that the masses are added symmetrically on both sides.) The hole on the top may be used to attach the photo-gate flags to the rider.
2. **Long rider** ($\sim$ 200g) along the top of which the short interrupter cards may be slotted.
3. **Long rider** ($\sim$ 400g) along the top of which the longer interrupter cards may be slotted.

Buffers and counterweights

The riders have two holes on their edges, at different heights, to which different buffers may be attached.

> Whenever a buffer is added on one side, a counter-weight or plug must be added on the other side, to avoid the air track leaning to one end. If this happens, there will be an unbalanced horizontal force which will make the rider move in the direction it leans.

1. **Rubber-band bumpers** may be connected to the riders to simulate nearly elastic collisions. In order to do this, the bumpers will each need to be aligned at 45° to the riders in opposite directions, such that the elastic bands are perpendicular to each other, ensuring that they have plenty of room to stretch.
2. **Magnetic buffers** may also be connected for nearly elastic collisions. As they act over a distance, no physical contact is made, and thus no energy can be lost to heat or sound.
3. **Brass buffers** may be used as counterweights when the magnetic buffers are used for elastic collisions, but can also be used for inelastic collisions, as they lose energy as heat and sound when they collide. In this case, take care to add the magnets on the other side, as counterweights.
4. **Velcro buffers** may be used to simulate perfectly inelastic collisions, as they will force the vehicles to stick together after impact. When these are used, appropriately sized plastic buffers should be used as counterweights.
5. **Plastic buffers** are generally used as counterweights, but are suitable for inelastic collisions as well, provided the Velcro buffers are used as counterweights.

The photo-gates

Photo-gates allow for extremely accurate timing of events within physics experiments. While we will use them to analyse motion on the air track, they could also be used for studying free fall, pendulum periods, the speed of a rolling object, among other things.

The photo-gates work by changing the state of their output signal whenever an object comes in the way of their infrared sensors, which allows us to measure the time interval Δt the object takes to pass the gate. If the object is sufficiently small – and has a known length Δx – then we can find a good approximation of its instantaneous velocity:

$$v \approx \frac{\Delta x}{\Delta t}$$

If the instantaneous acceleration is required, an interrupter with two prongs may be used. By measuring the change in velocity between these prongs Δv and the change in time Δt, the instantaneous acceleration may be approximated:

$$a \approx \frac{\Delta v}{\Delta t}$$

You have two types of photo-gates, Vernier and Osaw. The Vernier photo-gates will be used with the *LabQuest* interface and its data can be collected and analysed using the *LoggerPro 3* software. Each gate has an input port so multiple gates can be connected in a daisy-chain configuration. With the *LabQuest* interface and Vernier gates, up to four gates going to a single interface channel. The Vernier photo-gate also has a laser gate mode, which requires the addition of a common pen laser, which is directed into the laser port. The laser may be some distance from the gate, so that you can measure the speed of larger objects.

Tutorials can be found online,[1] as can the manual. Make sure you go through it before collecting data to make sure your data is useful.

Warnings

- The air track must be level: first use a spirit level to level the track roughly, then place a rider at different points on the track and adjust the legs until it shows no preference for the left or right end of the track.
- Make sure the track's surface is clean, and none of the holes are plugged by dirt.
- Ensure that the rider never tilts towards one direction. If this happens, you have either placed the weights asymmetrically or forgotten the counterweights.
- If the rider wobbles, the air pressure is too high.

[1] https://www.vernier.com/training/videos/play/?video=201

- The collisions need to be as gentle as possible. A large source of uncertainty is the transfer of energy to the air track and table.

Inherent errors: The riders are supported by a thin film of air approximately 0.01cm thick. This film exerts a viscous drag on moving surfaces which should – in theory – cause the rider to gradually slow down forever. However, practically, the rider oscillates around a position midway between the air holes. An air track with a practically infinite number of air holes would eliminate this problem, but this is not reasonable. Increasing the thickness of the film of air would reduce the coefficient of friction, but at the expense of the rider's stability. Another potential fail point is the straightness of the airtrack. While an ideal air track should not sag, our tube is till heavy enough to sag slightly under the force of gravity.

Procedure

Part A

In this part you will attempt to verify Newton's first law.

1. Set up the metal springs at either end of the air track. Placing the rider at one end of the air track, gently push it against the spring and let go.
2. Measure the velocity of the rider using the photo-gates at two different points in its trajectories, and measure its velocity at both points.
3. Repeat this for different initial velocities.

Question: Assuming the steel spring to be ideal, show that the velocity of the rider is directly proportional to its initial displacement of the spring.

Part B

In this part you will attempt to verify Newton's second law.

1. Remove one of the steel springs and connect the pulley. Pass a long string across the pulley and connect it to the rider at one end, and a slotted mass at the other.
2. With the rider at one end, drop different masses and measure the change in velocity between two points along the riders trajectory to measure the acceleration.
3. Plot a graph to calculate the acceleration due to gravity.

Part C

In this part, you will attempt to verify Newton's third law.

1. Put the steel spring back at the end of the air track, and attach two buffers (and counterweights) to the riders to simulate elastic collisions.
2. Place the photo-gates such that the initial and final velocities of both the riders may be determined.
3. Collide the two riders elastically using different initial conditions. If you require to launch the riders at the same speeds, use the steel springs as done in **Part A**.
4. Replace the elastic buffers and with brass or Velcro buffers (and counterweights) to simulate inelastic collisions.
5. In each case, calculate the loss in kinetic energy and compare it to the theoretically expected value.

Experiment 3

Free Fall

Objectives

1. To determine the acceleration due to gravity experienced by a freely falling object.
2. To study the dependence of terminal velocity on mass.

Introduction

The dynamics of simple classical systems are given by Newton's second law:

$$\mathbf{a} = \frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = \left(\frac{1}{m}\right)\mathbf{F} \tag{3.1}$$

In the theory course on *Classical Mechanics*, you will learn how to solve the above equation for a variety of situations. In this experiment, we will attempt to arrive at $x(t)$ experimentally and compare it with the theoretical solutions to Equation (3.1).

Theory

In the first part of the experiment you will be dealing with objects that fall under gravity. In a vacuum, such objects would only experience a gravitational force mg.

Question: Starting from Equation (3.1), write out the differential equation the object's centre of mass satisfies. Solve it to show that its velocities and positions satisfy the following equations:

$$\begin{aligned} v &= u - gt \\ y &= y_0 + ut - \frac{1}{2}gt^2 \end{aligned} \tag{3.2}$$

in an appropriately chosen coordinate system.

Question: Sketch the graphs of y, v, and a, one below the other, as a function of time.

Introducing damping

However this is slightly unrealistic, as air resistance also contributes to a damping force F_d which opposes the motion of the object. Therefore, the force acting on the object is

$$\mathbf{F} = m\mathbf{g} + \mathbf{F_d}$$

In the second part, you will be considering the problem of falling cupcake liners which experience a non-negligible air drag force that opposes their free fall.

In general, damping forces are related to the velocity of the object. Eventually, as the velocity increases, a critical 'terminal' velocity is reached where the forces balance out and the object stops accelerating and continues to move at this constant velocity v_t.

In most introductory mechanics courses, you are asked to consider damping forces that are proportional to v rather than v^2. It turns out that this form of fluid 'drag' only works when the velocities are sufficiently small and the fluid has enough time to flow 'around' the object.[1]

In our case, however, the fluid is simply pushed out of the way by the object. Let us say that the object has some cross-sectional area A in the direction of its motion. In a small instant of time Δt, it pushes on a mass of air $m_{\text{air}} = \rho_{\text{air}} A v \Delta t$. If this air is made to move with the same velocity as the object, its final momentum is thus $m_{\text{air}} v$. Thus, it experiences a total increase in momentum of $\rho_{\text{air}} A v^2 \Delta t$. By Newton's third law, the object must experience a total *decrease* in momentum by the same amount. However, in our analysis we have not taken into the account the different shapes of objects. This information is usually incorporated using a dimensionless quantity close to 1, called the *drag coefficient* C.

Thus, we can see that

$$F_{\text{d}} = -C\rho_{\text{air}} A v^2 = -\alpha v^2 \tag{3.3}$$

and so, using what we know so far,

[1] This is called *laminar* flow.

$$v_t = \sqrt{\frac{mg}{C\rho_{\text{air}}A}} \tag{3.4}$$

We will suppose an air-resistance force that is proportional to v^2, i.e.

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - \alpha v^2$$

It can be shown that the solution to this equation is:

$$v(t) = v_t \tanh\left(\frac{g}{v_t}t\right) \tag{3.5}$$

Question: Argue on physical grounds that $\alpha > 0$.

Question: The terminal velocity v_t is attained when the right-hand side of the above equation is zero. Find v_t, and then rewrite the equation as shown below, and solve it to get Equation (3.5):

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g\left(1 - \left(\frac{v}{v_t}\right)^2\right)$$

Plot this function.

Question: Calculate $y(t)$ using $v(t)$.

Question: If, instead of v^2, we had a damping force that depended on v, what would $v(t)$ look like?

Experimental Setup

Apparatus

1) Camera (Nikon D5700 or GoPro Hero 5)
2) A tripod with appropriate attachments to hold the cameras.
3) Tracker Video Analysis (Software, available from `http://physlets.org/tracker/`)
4) Collection of small spheres of different materials
5) A set of cupcake liners
6) A set of small magnets

Description

In this experiment, you will analyse videos using Tracker Video Analysis and extract physical information from them. In **Part A**, you will study a freely falling sphere's position and velocity as a function of time. In **Part B** you will drop cupcake liners with different masses and study the variation of terminal velocity with mass.

Software

Tracker Video Analysis

Tracker is a free video analysis and modelling tool built on the Open Source Physics (OSP) Java framework. It is designed specifically to be used in physics education, and is extremely versatile. You can download the latest version of Tracker online (`http://physlets.org/tracker/`).

A video is basically a collection of images recorded at a fixed number of frames-per-second (fps). For example, if a video is shot at 60fps, this means that the interval between any two frames is 1/60 seconds. Tracker works by breaking the video into these individual frames and 'tracking' a pre-defined object's position through them. As a result, you will have a collection of points x_i at different times t_i which represent the object's position. If the fps of the video is sufficiently large, $x_i(t_i)$ is a good approximation of the function $x(t)$, the instantaneous position of the object.

To measure an object's position in a video you will need to do two things:

1. Define a coordinate system, including an origin and x and y axes.
2. Define a *scale*, or standard unit of length in the video, so that the software knows how far the object has moved in a given time interval. This can be done by having an object of known length, say a metre scale, in the frame. This is called **calibration**.

The software measures the position of where you clicked in units of pixels and then uses the calibration and your definition of the coordinate system to convert this position in pixels to a position in meters (or whatever units are used in the calibration).

Question: Do you also need to calibrate the software so that it knows how long a time interval is? Why?

Question: It's very important that the calibration instrument, like a metre scale, is in the same plane as the object's motion. If the scale is closer to the camera or further from the camera than the object you are studying, then your measurement of position will be inaccurate. Can you explain why this is the case?

Given this introduction, you are now ready to import a simple video and begin analysis.

1. **Importing a video:** Go to `Video` → `Import`, and select your video file. After importing, you can go to `Video` → `Properties` and find out more information about the file.

2. **Applying filters:** Often, your videos will require some editing before you begin analysing them. You can do this by going to `Video` → `Filters` → `New`. The most common filter you will use is the `Rotate` filter.

   > **Tip**
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   > Once you have started tracking the position of a particle, you cannot adjust these filters without starting over from scratch.

3. **Video controls:** These are located at the bottom of the video pane. Click on each of the icons to see what they are used for. You can also control the video with the arrow keys.

   > **Tip**
   >
   > Usually, you will only be interested in a small section of the video. You can set the start and end frame by dragging the little triangles under the slider.

4. **Defining a coordinate system:** Once you have imported the video and edited it, you can define your coordinate system. Click the `Axes` icon ( ) on the top of the window, and drag the coordinate system to place your origin at a convenient location. If you click the x-axis and drag, you can also rotate the coordinate system. Click the `Axes` icon again to hide them.

5. **Calibration:** Now you must calibrate distances measured in the video. Click the `Calibration Tools` icon ( ) and choose (say) a calibration stick. Keeping the `SHIFT` key pressed, click on two points that are separated by a known distance. Set the length between them by clicking the number that appears above them. Click the Calibration Tools icon again to hide it.

   > **Tip**
   >
   > You can use real-world units here (like 'cm') but you may have to do it more than once so that the software realises you haven't made a mistake.

6. **Creating a point mass:** Click the `Create` button and select `Point Mass`. An object titled `Mass A` will be created in a different pane, and a new x vs. t graph will appear on the right. You can now change the mass of this object, again in real world units.

7. **Tracking your object:** Once `Mass A` has been created, you can hold down the `SHIFT` key and click an object in the video pane to note down its position, and then manually go through all the frames marking the position of your object. However, Tracker allows for an **Auto-tracking** option which is very useful. Click on `Mass A` → `Autotracker`. An `Autotracker` pane will open out. Now, holding down the `CTRL` and `SHIFT` keys, click on the object you want to track automatically. The point you clicked should now be surrounded by a solid red circle (which defines the template that Tracker will search for in the next frame) and a dotted red square (which is the area within which the object will be searched for in the next frame). You can then click the `Search` button and watch Autotracker do all the work.

Tracker works by looking for a 'Template' image in every successive frame. It is remarkably accurate, but not infallible. One way to make your life easier is to use plain backgrounds and brightly coloured objects that show a very clear contrast against the background.

Tracker also allows for advanced Data Analysis. Go to `View` → `Analyze` to play around with your data. Alternatively, you could just copy your data into Microsoft Excel or Python.

Procedural Instructions

Part A

1. Choose an appropriately massive sphere so that the damping force may be effectively neglected.
2. Set up the camera on the tripod using the appropriate attachments, and place it at a suitable distance from the sphere will be dropped.
3. Make sure the sphere's complete trajectory is visible in the screen. Make note of a reference point to be used as the origin of your coordinate system on Tracker. Also arrange for a calibration stick. These should not be changed between releases.
4. Record a video of the sphere's fall and track it using Tracker. You will notice that it becomes harder to discern the sphere's centre as it moves faster; think of a way to approximate its position.
5. After analysing one video, collect a sufficient number of data sets and try to find the value of acceleration due to gravity g using this method.

Part B

1. The cupcake liners – having a broad base – will be acted on by a force due to air drag. Try to drop a liner from a height and you will see that after a point its velocity is more or less constant.
2. You may use the magnets provided to increase the mass of the liner (find a way to estimate their mass).[2]
3. Plot a graph of position versus time. Then plot a graph of velocity versus time. Try to fit the velocity graph to an equation of the form:[3]

$$v(t) = v_t \tanh\left(\frac{g}{v_t} t\right)$$

and estimate the constants v_t and g.

[2] We have a limited supply of both, so please make sure you don't damage either of them!

[3] This function is easy to fit if you use Python. If you choose to use Excel, try to use a different technique to find v_t.

4. Repeat this for a large range of different masses by adding magnets to the liner, until terminal velocity is no longer reached in the distance provided.

5. Plot the variation of terminal velocity with mass, and attempt to fit this to some power law. i.e. if $v_t \propto m^p$, find p.

Warnings

- Before leaving the lab, make sure you place the battery on charge (if it needs charging), and return the SD card to the camera.
- Make sure there is no warping of the image by the camera.
- Always analyse a few videos immediately after you take them. You will most likely realise how to improve the videos you have taken during analysis.
- You may adjust the RAM size on Tracker, to improve computing speed.

Experiment 4

Electromagnetic Damping

Objectives

1. To study the effects of electromagnetic damping.

Introduction

In this experiment we will study the effects of electromagnetic damping on a rotating metal disc.

The flywheel

Consider an aluminium disk mounted on a horizontal axle around which a cord is wound, as shown in Figure (4.1). This cord is attached to a slotted mass, which is allowed to fall, rotating the disc in the process.

For rotating objects, Newton's Second Law may be generalised to

$$\tau = I\alpha \tag{4.1}$$

where τ is the net external torque on the object, I its moment of inertia, and α the angular acceleration.

As the mass begins to fall, it will to accelerate, spinning the disk. It experiences two forces: gravity and the tension from the string, and thus accelerates at a rate $a < g$:

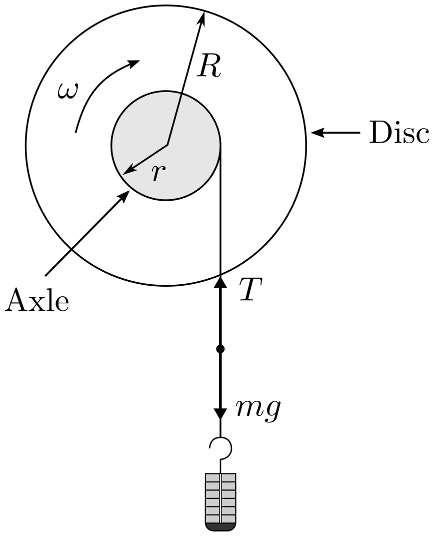


Figure 4.1: Side view of the flywheel assembly: the falling mass causes the disc to accelerate.

$$ma = mg - T \quad \Longrightarrow \quad T = m(g - a) \tag{4.2}$$

Consider the cord being wound around an axle of radius r. This means that the acceleration at the rim of the axle (i.e. at radius r) is the same as the acceleration of the mass. Thus

$$\alpha = \frac{a}{r} \tag{4.3}$$

Question: The above situation is highly idealised, as it does not take into account friction. In practice, the disk will also experience a 'frictional' torque τ_f which opposes its motion. Show, by balancing the torques on an axle of radius r, that

$$I\alpha = Tr - \tau_f. \tag{4.4}$$

Question: Using Equations (4.3) and (4.2), show that

$$a = g\left(\frac{mr^2}{I + mr^2}\right) - \left(\frac{\tau_f r}{I + mr^2}\right). \tag{4.5}$$

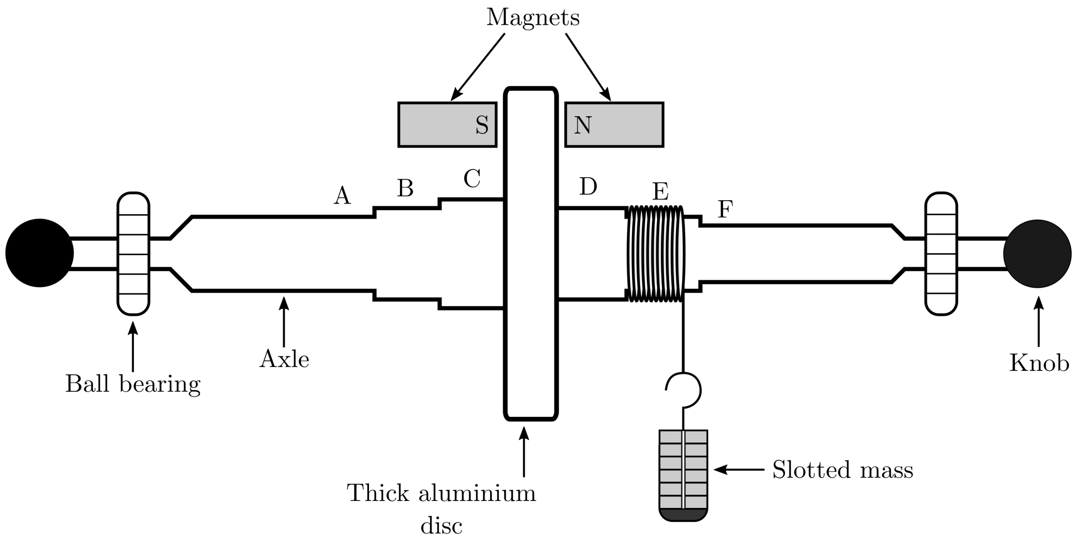


Figure 4.2: Schematic of the disc and magnet assembly: the different axles (A–F) are distinguished by their radii. The disc eventually rotates at a constant angular velocity because of the damping force due to the magnets.

Introducing damping

The predominant damping in this set-up is due to electromagnetic damping. To understand how this occurs, let us focus on the electrons in the disc, which, because the disc is a conductor, can move freely. When an electron passes through the magnetic field, it experiences a Lorentz force $\mathbf{F} = e\mathbf{v} \times \mathbf{B}$. In this set-up, if we consider a point between the pole-pieces of the magnet, the magnetic field is perpendicular to the disc and the velocity of the electron is along the azimuthal direction. Thus the Lorentz force is along the radial direction. This force, acting on the electrons, causes them to move systematically in the radial direction. If the electrons were all pushed radially and could pile up at either the axle or the rim, there would be electric field developed, which would grow until its effect exactly counteracted that of the magnetic field. But what we have here is a conductor that extends outward from the immediate vicinity of the pole pieces. Thus, the electrons moved by the Lorentz return to their original positions by a more circuitous route; i.e. closed, spread out currents are set up, some it flowing through the region between the pole pieces, and some of it flowing outside that region. (Currents set up like this in conductors are called eddy currents.)

Now let us focus on the current as it moves through the pole pieces. In these currents the charges move radially. A charge moving radially through the magnetic field experiences a force in the azimuthal direction. Note the three-step process: first, because the rotating conductor was carrying along the electrons with it, they were moving azimuthally, resulting in a radial Lorentz force; second, this radial Loretnz force causes a radial current, giving a radial component to the velocity of the electrons, and causing a current; third, the radially moving electrons in the current experience a Lorentz force in the azimuthal direction.

The question now arises: in which azimuthal direction does the Lorentz force on the radial current act? The direction can be established by carefully examining the direction of the field etc, but it can also be established from a more general principle. The azimuthal force must act such it it either helps the motion of the disk or hinders it. Suppose that the force acted such as to help the motion; then, a little push or fluctuation would start the process, and, with the help of the additional Lorentz force, would cause the wheel to turn faster, leading to a runaway situation that violates the conservation of energy. Thus the only possibility that can be realised in nature is that the Loretnz force acts such as to hinder the motion of the wheel, i.e. it is a damping force.

Thus, the full torque balance on the disc is given by:

$$I\alpha = Tr - \tau_f - \tau_B \tag{4.6}$$

where τ_B is the opposing magnetic torque that these currents produce. We will now try to calculate τ_B with an extremely simplified model.

Calculating the magnetic torque

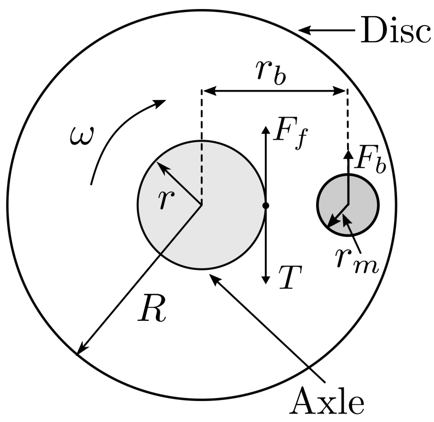


Figure 4.3: Side view of the setup: the axle of the disc experiences a frictional force F_f and the tension T due to the string. A circular region of radius r_m – at a distance r_b from the axle – experiences a damping force due to a nearly uniform B.

The magnetic field is confined to a small circular region $r_m \ll R$, the radius of the disc (as shown in Figure (4.3)), and is uniform over the region, i.e. over a distance of $2r_m$. Let us now consider a thin conducting path along the diameter of this region, of length $2r_m$. It moves at a velocity $v = r_b\omega$, where r_b is the distance from the centre of the circular region to the axis of rotation.

The emf E_b induced is given by

$$\mathcal{E}_b = (r_b\omega) \times B \times (2r_m) \tag{4.7}$$

This emf $\mathcal{E}_b$ will give rise to a current i (the eddy current) in a closed conducting path within the disc. Let us assume that this path has some resistance R^*.[1] Thus, the current is given by

$$i = \frac{\mathcal{E}_b}{R^*} = \frac{(r_b\omega) \times B \times (2r_m)}{R^*} \tag{4.8}$$

These currents will in turn experience a force F_b given by

$$F_b = Bi(2r_m) = \frac{4r_m^2(\omega)r_bB^2}{R^*} = \left(\frac{4r_m^2 r_b}{R^*}\right)\omega B^2 \tag{4.9}$$

This force is responsible for the magnetic torque at r, τ_B.

$$\tau_b = r_bF_b = \left(\frac{4r_m^2 r_b^2}{R^*}\right)\omega B^2 = C'\omega B^2 \tag{4.10}$$

Now, in this derivation, we considered a conducting path going through the circular region of radius r_m. However, not all paths will have this length or even be radially directed. The effect of taking all these paths into consideration would be to given an average factor C instead of C'. However, the dependence on ω and B will remain unchanged. Thus, we can write the total torque as

$$I\alpha = m(g-a)r - \tau_f - C\omega B^2 \tag{4.11}$$

As the retarding torque depends proportionately on the angular velocity of the disc, the disc will begin to slow until a steady state is obtained, i.e. until ω reaches some constant 'terminal' value ω_t. As a result, the mass will drop at some constant terminal velocity v_t. In this case, $\alpha = 0$ and $a = 0$.

Question: Show that

$$v_t = \omega_t r = \left(\frac{gr^2}{CB^2}\right)\left(m - \frac{\tau_f}{gr}\right). \tag{4.12}$$

In this experiment, we will study the terminal velocity of the falling mass and its dependence on the mass and magnetic field.

[1] This is a huge simplification: the reason eddy currents are so hard to model is because there are many different paths within the conductor of varying resistances.

Experimental Setup

Apparatus

1. A flywheel disc assembly, mounted on the wall,
2. Two pairs of magnets with different pole strengths,
3. A set of slotted masses,
4. A set of acrylic plates of different thicknesses,
5. Metre scales and tapes,
6. A spirit level.

Procedure

Part A

In this part, you will verify Equation (4.5) and determine the moment of inertia of the flywheel.

1. Choose an appropriate axle and measure its diameter and note it down. Choose an appropriate length of cord and attach it to the axle.
2. Attach the cord to a small slotted mass, and wind it carefully on the axle.
3. Allow the mass to drop gently, taking care that it does not oscillate as it falls.
4. Use a camera to take a video of the falling mass and analyse it to find its acceleration.
5. Once you have analysed your data for one video, repeat this procedure for different masses.
6. Plot an appropriate graph between the a and m, and calculate the moment of inertia and frictional torque of the disc.

> **Question:** You will find that you have to plot a graph of the form
>
> $$a = g\left(\frac{mr^2}{I + mr^2}\right) - \left(\frac{\tau_f r}{I + mr^2}\right)$$
>
> Show, using an appropriate approximation, that this relation reduces to:
>
> $$a \approx m\left(\frac{gr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right). \tag{4.13}$$
>
> How would you find I and τ_f?

Part B

In this part of the experiment, you will introduce electromagnetic damping to this system, which will make the mass fall at a terminal velocity v_t which depends on the parameters of the problem.

1. Insert a pair of magnets in their holders, and place them on the assembly.
2. Make sure that the magnets are equally spaced from the disc. In order to do this, bring them as close to the disc as possible, and note down the readings on the screw gauges on either side. Using this as your reference, rotate the screw gauge by the same amount on either side.
3. Repeat the procedure given in Part A. In this case, the disc should finish by rotating at a constant angular velocity due to the electromagnetic damping.
4. Plot a graph between the terminal velocity and the mass of the object, and show that it satisfies Equation (4.12).

Part C

1. Vary the distances between the magnets and measure the magnetic field exactly between the magnets using a Gauss meter. Plot a graph of this magnetic field as you vary the magnets' separation. From this graph, you should know what the magnetic field should be at the centre of the disk as you vary the separation between the magnets.
2. Once this is done, keep the magnets some known distance apart and calculate the terminal velocity for a fixed value of mass.
3. Without changing any of the parameters in the system (including the mass), change the separation between the magnets (effectively varying B) and calculate the terminal velocity. Repeat the above procedure for different separations between the magnets. Find how the terminal velocity v_t varies with B.

Warnings

- Make sure the mass falls vertically, without oscillations.
- The knobs on either end of the disc should be used to wind or unwind the cord.
- Wind the cord on the axle carefully, so that the turns do not cross or overlap. Successive turns should touch each other, and should not extend beyond the length or diameter of the axle chosen.

Experiment 5

Equipotential Curves

Objectives

1. To study equipotential curves in two dimensional electrostatics.

Introduction

Electric fields and potentials

You may remember from school that any configuration of static electric charges induces an electric field in space. Such an electric field is a vector quantity that is a function of position, i.e. it has both a magnitude and direction at every point in space. Consider, for example, a point charge q at the origin, whose electric field is well known from Coulomb's law:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

The above equation tells you that the electric field vectors point radially outward[1] from the charge and that their magnitudes fall as the reciprocal of the square of the distance from it.

When one has a collection of several charges, the electric field obeys the Principle of Superposition. In other words, the resulting electric field is the vector sum of all the contributions of the individual parts. This makes it possible to (theoretically) calculate the field produced by any collection of charges. Practically, however, this is very hard to do even for simple charge configurations, because of the vector nature of the field. This also results in the electric field being difficult to grasp or

[1] Or inward, if the charge is negative.

visualise. There is, however. a very simple way to arrive at this vector field, using a concept known as the *electric potential* V.

Maxwell's equations completely describe a static electric field through its divergence and curl:

$$\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{E} &= \rho/\epsilon_0 \\ \boldsymbol{\nabla} \times \mathbf{E} &= 0 \end{aligned} \tag{5.1}$$

Question: Using the second equation above, show that this implies that every static electric field can be expressed as the gradient of a *scalar* field, called the electric potential V. i.e. show that

$$\mathbf{E} = -\boldsymbol{\nabla} V$$

When the region has no net charge accumulating, $\rho = 0$.

Question: If the charge density is zero in a region, show that the potential satisfies *Laplace's Equation*:

$$\nabla^2 V = 0 \tag{5.2}$$

Being a scalar, V just requires a single number at every point, which makes it much easier to work with. The gradient of a scalar function is a vector whose x, y, and z components are the rate at which the function changes along x, y, and z, respectively. A consequence of this is that the gradient vector, at any position, points along the direction in which function changes fastest and its magnitude is the rate of change of the function along this direction. Thus, the potential contains all the information that the field does.

Here is a simple way to visualise this: imagine connecting the points at which the potential has the same value. This is rather like connecting the points on a hill that are at the same height.[2] If this is done for a series of equally spaced values, what results is a contour diagram, each contour representing a single value of the potential. These contours are called equipotential curves.[3] It is easy to see that where the contours are closely spaced the potential changes rapidly and where they are widely spaced the potential changes slowly in space. Given how the potential is defined (as a gradient), it should be easy to see that the magnitude of the electric field is larger in the first case than in the second case. Another important rule about these contour lines and the electric field is that field lines always meet contour lines at right angles. This is because the **E** field always points in the direction where V is changing most quickly. It makes sense, then, that the direction perpendicular to the **E** field is the direction where V is not changing at all. Since this direction must be along the equipotential surface by definition, the electric field is locally perpendicular to the equipotential surface.

[2] Or 'gravitational' potential!

[3] In general, in three dimensions, the points which have the same potential lie on a two-dimensional surface, known as an equipotential surface.

Question: What are the equipotential surfaces for the point charge we discussed earlier? Consider three equipotential surfaces that are equally spaced in potential. Are they spaced equally **in space**? If yes, why? If no, how do they change? How would this change if the charge was negative instead of positive?

Question: Now imagine an infinite wire of charge lying along the x-axis. What would the equipotential surfaces be in this case?

Suppose now you have a region of space enclosed by two equipotential surfaces (say, two concentric spheres), and you are interested in the potential at all points between them. Since the potential satisfies Laplace's Equation (Equation (5.2)), if we solve it with the appropriate value at the boundaries, we will have the potential everywhere in between. Thus, equipotential surfaces are the locus of points where the solution to Laplace's Equation has the same value.

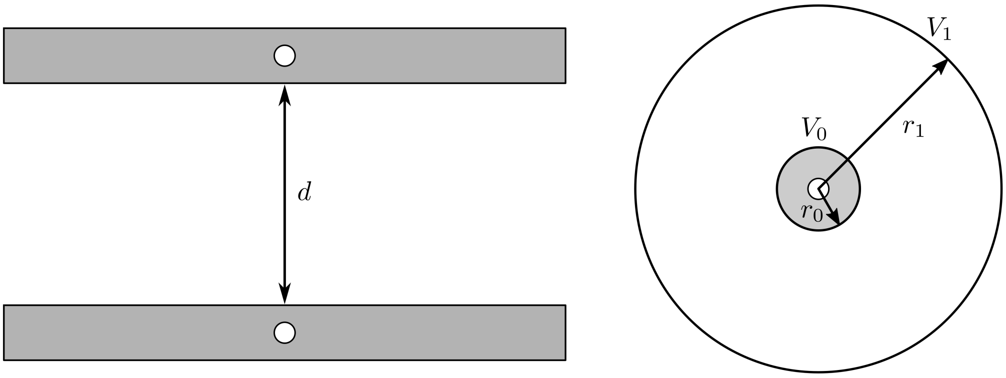
Conductors

A conductor is a material with the property that the charge in it is free to move around. A material which does not have this property (i.e. one in which the charges are stationary) is called an insulator. In practice, almost all metals are good conductors and most other substances are insulators. In static equilibrium has the property that the electric field is zero everywhere inside. The reason is simple: if it weren't zero somewhere, the charge there would move in the direction of the field, so it wouldn't be in static equilibrium. One important consequence of this is that all points within a conductor are at the same potential. This is because a difference in potential would imply an electric field, which would push the electrons in the conductor in such a way as to reduce the field; this continues until the electrons in the interior or the conductor are pushed to the surface. And, on the surface, they are moved around until the component of the field along it vanishes. Thus, the surface of a conductor is an equipotential surface.

Since the conductors are equipotential surfaces, in the region between them the potential obeys Laplace's Equation. We can use this and the symmetries of some configurations to solve for V.

Some interesting configurations

In this experiment, you will be given some metal electrodes and asked to find the curves of constant potential. You will be asked routinely to solve such problems in electrostatics in the course of Electricity and Magnetism you have this semester. Let us try to do this for some simple configurations. We will be working in two dimensions of space (the surface of the water).



(a) "Infinite" parallel bars (b) Concentric cylinders

Figure 5.1: Two solvable electrostatic configurations showing different symmetries.

The parallel plate capacitor

Consider first two parallel bars as shown in Figure (5.1a). Laplace's equation can be written in Cartesian coordinates as

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

However, since the plates are infinitely long, it should be clear to you that there can be no variation in the potential along x (as the plates are infinite, moving a finite distance to the right or left would not change anything). Thus, the equation reduces to

$$\frac{\mathrm{d}^2 V}{\mathrm{d}y^2} = 0$$

Question: Suppose one plate is at some potential V_0 and the other is grounded (i.e. at potential 0), and they are both separated by some distance d. Show that the solution to this equation is

$$V(y) = V_0 \frac{y}{d}. \tag{5.3}$$

Two concentric cylinders

Consider now that you have two concentric cylinders, as shown in Figure (5.1b) The inner cylinder (of radius r_0) is at some potential V_0 and the outer cylinder (of radius r_1) is grounded. In this case, it is better to solve the problem in cylindrical coordinates. You can easily show that Laplace's equation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}=0$$

Just as before, we can ignore the θ term by symmetry (if we rotate the system by any angle, nothing seems to change). i.e.

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}V}{\mathrm{d}r}\right)=0 \tag{5.4}$$

Question: Show in this case that Laplace's equation is satisfied by a function of the form:

$$V(r)=a\ln\left(\frac{r}{r_0}\right)+b.$$

Question: By using the fact that when $r=r_0, V=V_0$, and when $r=r_1\neq r_0, V=V_1$, show that:

$$V(r)=\frac{(V_1-V_0)\ln\left(\frac{r}{r_0}\right)}{\ln\left(\frac{r_1}{r_0}\right)}+V_0. \tag{5.5}$$

Experimental Setup

Apparatus

1) An acrylic tray with electrolytic medium (water)
2) An AC power supply with a voltage stabilizer
3) A set of metallic electrodes
4) A digital multimeter
5) A pointed probe attached to an $x-y$ stage
6) A set of connecting cords
7) A set of measuring scales

Description

In this experiment you will study the electric potential produced by sets of metallic electrodes of different shapes and sizes, held at a fixed voltage. To plot the equipotential curves, the metallic electrodes are kept in an acrylic tray such that three-fourths of their height is submersed within an electrolyte (tap-water will do) which is used as an electrolytic tank. An AC potential difference is maintained between the electrodes, which causes a distribution of potential in the electrolyte. By measuring the potentials at different points, one can identify the coordinates of the points of equal potential and consequently plot the equipotential curves on a graph sheet.

Tip

This experiment can be performed either with AC or DC voltage from the provided power supply. However, while using DC voltage, electrolysis is found to occur, and the aluminimum electrodes begin to get eaten away. As a result, it is advisable to use AC voltage from the power supply.

Procedural Instructions

Begin by taking two graph sheets and drawing the same coordinate system on them. You will stick one of these sheets on the bottom of the acrylic tray and use it as a reference. You will then mark out the potentials at different points on the other sheet, using the first as a reference.

Part A

1. Place the two long electrodes parallel to the x-axis symmetrically about the origin, as shown in Figure (5.1a).
2. A potential difference is established across the electrodes by connecting them to the AC power supply.
3. Using the multimeter, test the potential difference between the electrodes in volts. If it is found to be significantly different from the value you are supplying, ask a TF or instructor for help.

   Question: What should the equipotential surfaces look like in this case?

4. Using the same ground as a reference, connect the positive wire of the multimeter to the probe and qualitatively measure the change in the potential from one end of the acrylic tray to the other.
5. Measure the voltage at an arbitrary point and then find multiple points with the same potential thus drawing out one of the equipotential curves.
6. Choosing an appropriate increment in voltage, find the equipotential curves for different values of the potential. Verify Equation (5.3) in the appropriate limit.

   Question: Do you see Equation (5.3) being satisfied everywhere? What could be the reason for this?

Part B

1. Set up the electrodes as shown in Figure (5.1b). Maintain the inner cylinder at a higher voltage V_0 and ground the outer cylinder.

   > **Question:** What should the equipotential surfaces look like in this case?

2. As before, choose an appropriate increment in voltage and find the equipotential curves for different values of the potential.
3. Use this to verify Equation (5.5).

Part C

Explore the equipotential curves for one configuration of your choice.

You can visit following online applet: `falstad.com/emstatic/` and play around with the various different configurations to see how the field lines and contour lines are produced by different distributions of charges.

Warnings

- Arrange the electrodes and keep them stable and undisturbed throughout the measurements. You can use some tape to keep them fixed.
- Make sure the uninsulated tip of the measuring probe just grazes the surface of the water.
- The graph paper attached below the tray and the one used for marking the points should have same coordinates and precision in scale.

Experiment 6

Electronics circuits

Objectives

1. To study the response of an RC circuit as a Low Pass Filter (LPF), High Pass Filter (HPF), Integrator and Differentiator.
2. To study the response of an RLC circuit both in series and in parallel combination.

Introduction

What is resistance?

Resistance measures the opposition to a flow of current. It is present in all conductors. When the alternating current goes through a resistance, a voltage drop is produced that is in-phase with the current. Resistance is mathematically symbolised by the letter "R" and is measured in the unit of ohms.

What is reactance?

Reactance is an opposition to a change in current. It is mostly notable in capacitors and inductors. When the alternating current goes through a pure reactance, a voltage drop is produced that is 90° out of phase with the current. Reactance is mathematically symbolised by the letter "X" and is measured in the unit of ohms.

What is impedance?

The opposition offered by an electronic component to AC or DC current is known as impedance (Z). Impedance is a phasor quantity that can be represented by two independent scalar quantities:

resistance and reactance. It is mathematically symbolised by the letter "Z" and is measured in the unit of ohms, in complex form.

Perfect resistors possess resistance, but not reactance. Perfect inductors and perfect capacitors possess reactance but no resistance. While resistance, R remains constant with frequency, reactance, X varies with frequency.

The impedance phase angle for any component is the phase shift between the voltage across that component and current through that component. For a perfect resistor, the voltage drop and current are always in phase with each other, and so the impedance angle of a resistor is said to be 0°. For a perfect inductor, voltage drop always leads current by 90°, and so an inductor's impedance phase angle is said to be +90°. For a perfect capacitor, voltage drop always lags current by 90°, and so a capacitor's impedance phase angle is said to be −90°.

What is *Phase*?
It is the angular measurement of the sine wave which specifies the exact position of the wave.
What is *Instantaneous Value*?
It is the value of the sine wave at any instant of the cycle. This value changes with the waveform.

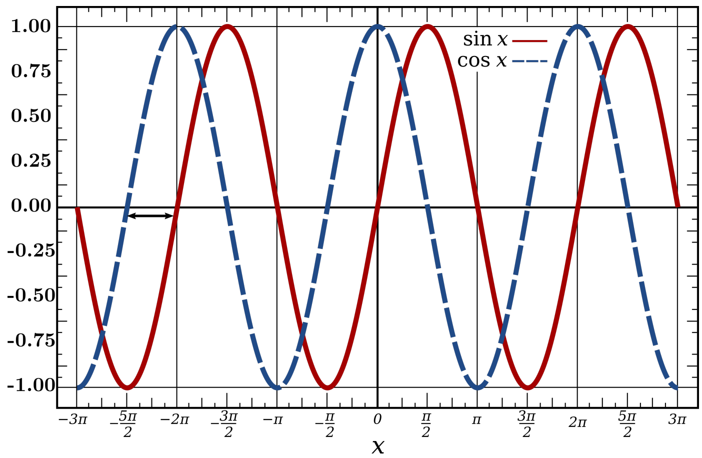


Figure 6.1: Phase difference between a sine wave and a cosine wave

$$Z = R \pm iX$$
$$|Z| = \sqrt{R^2 + X^2}$$

Capacitive reactance, (X_c) is large at low frequencies and small at high frequencies. In case of DC signal which is zero frequency (f = 0Hz), X_c is infinite, which means that capacitors acts as an open circuit.

$$X_c = \frac{1}{2\pi f C}$$

Inductive reactance, (X_L) is small at low frequencies and large at high frequencies. For steady DC signal, X_L is zero which means that inductors acts as a short circuit for DC.

$$X_L = 2\pi f L$$

Response of R-C circuit to D.C. signal

Let a DC voltage V be applied to a series R-C circuit as shown in fig.6.2.

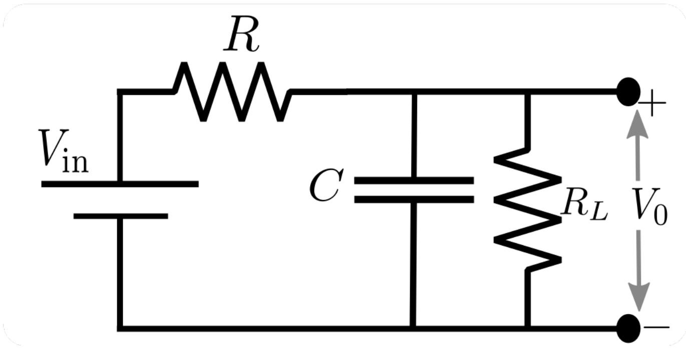


Figure 6.2: RC circuit with DC power supply

By Kirchoff's Voltage law we have,

$$RI + \frac{1}{C}\int I dt = V$$

Differentiating both side of the above equation w.r.t. time we get,

$$R\frac{\mathrm{d}I}{\mathrm{d}t} + \frac{I}{C} = 0$$
$$or, \frac{\mathrm{d}I}{\mathrm{d}t} + \frac{I}{RC} = 0$$

This is a homogeneous differential equation and by solving it we get,

$$I = I_c = Ke^{-\frac{t}{RC}}$$

On application of the DC voltage and assuming there is no initial charge across the capacitor, the capacitor will act as a short circuit and will produce no voltage. Hence, from the above equation we get,

$$K = \frac{V}{R}$$

Therefore,

$$I = \frac{V}{R} e^{-\frac{t}{RC}}$$

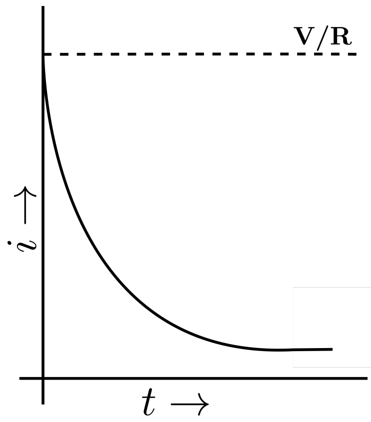


Figure 6.3: Charging current in RC circuit with DC power supply

Therefore charging current is an exponentially decaying current whose plot is shown in fig.6.3. Using this equation we can obtain,

$$V_R = IR = Ve^{-\frac{t}{RC}} \tag{6.1}$$

and

$$\begin{aligned} V_C &= \frac{1}{C}\int I dt = \frac{1}{C}\int_0^t \frac{V}{R} e^{-\frac{t}{RC}} dt \\ &= V(1 - e^{-\frac{t}{RC}}) \end{aligned} \tag{6.2}$$

From 6.1 and 6.2 we can easily conclude that V_R is a decaying function while V_C is an exponentially increasing function. Thus resistor acts as a charge dissipating element whereas capacitor is a non-dissipating element in the circuit. The steady state voltage across the capacitor is V volts.

Question: Find out the current in the circuit when the switch is off and study the nature of the curve.

R-C circuit as an Integrator and Low-Pass Filter

Now suppose an alternating sinusoidal signal is applied to a resistor and a capacitor connected in series, as shown in fig.6.4.

It works as an integrator for some range of frequencies. Each R-C pair has a certain cutoff frequency which is determined by the time constant RC of the circuit. The cutoff frequency is given by,

$$f_c = \frac{1}{2\pi RC}$$

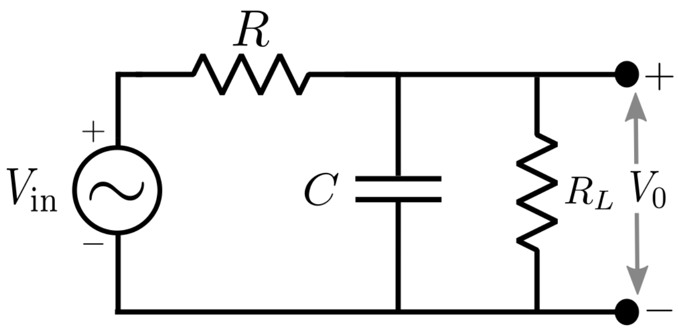


Figure 6.4: RC circuit acting as Low-pass filter

At frequencies higher than f_c, the circuit will act as an integrator. This is because, the capacitor is gradually charging (integrating) the current flowing through it. For lower frequencies the capacitor saturates relatively faster and the effect of the circuit as an integrator is not visible.

The instantaneous current through the Capacitor is,

$$I_C = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{6.3}$$

and ,

$$Q = CV_C \tag{6.4}$$

By substituting 6.4 in 6.3 we get,

$$I_C = C\frac{\mathrm{d}V_C}{\mathrm{d}t} \tag{6.5}$$

Now, from Ohms Law,

$$I = \frac{V_{\text{in}}}{R} \tag{6.6}$$

And since, $R \gg X_c$, $V_{\text{in}} = V_R$

$$I = \frac{V_R}{R} = C\frac{\mathrm{d}V_C}{\mathrm{d}t} \tag{6.7}$$

$$\Rightarrow V_C = V_o = \frac{1}{C}\int I dt \tag{6.8}$$

Using Equation 6

$$V_o = \frac{1}{RC}\int V_{\text{in}} dt \tag{6.9}$$

Hence, clearly, the output voltage of this circuit is directly proportional to the integral of the input voltage

A Low Pass Filter is a circuit that passes signals with a frequency lower than the cut-off frequency (f_c) and attenuates those with frequencies higher than the cut-off frequency. A simple passive low pass filter can be easily constructed using the same circuit as shown in fig.6.4, where the input signal is V_{in} and the output, V_o is measured across the capacitor only.

In the above circuit, at low frequency when $f_c \rightarrow 0, X_c \rightarrow \infty$ then the capacitor acts as an open circuit. Thus signals with low frequency can easily be measured across the load at the output. At high frequency when $f_c \rightarrow \infty, X_c \rightarrow 0$ then the capacitor acts as a shorted path and allows the current to easily pass to the ground.

R-C circuit as a Differentiator and a High-Pass Filter

The position of the resistor and the capacitor are now interchanged in the circuit, as shown in fig.6.5 and the output is now measured across the resistance only, parallel to the load.

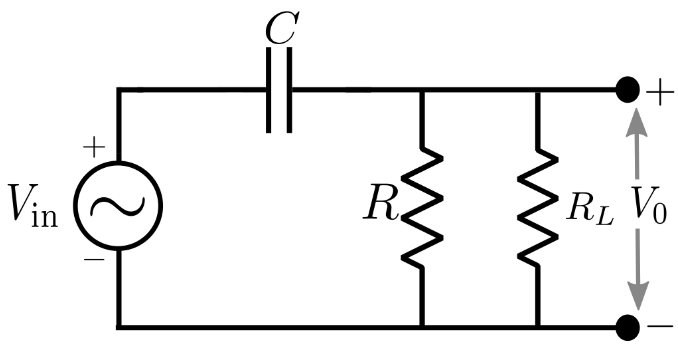


Figure 6.5: RC circuit acting as High-pass filter

In such a circuit as shown in fig.6.5, the total voltage drop is the sum of individual voltage drops,

$$V_{\text{in}} = V_C + V_R \tag{6.10}$$

$$\Rightarrow V_{\text{in}} = \frac{Q}{C} + R\frac{\mathrm{d}Q}{\mathrm{d}t} \tag{6.11}$$

since, C and R are both small,

$$V_{\text{in}} = \frac{Q}{C} \tag{6.12}$$

Rearranging, and differentiating w.r.t time,

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = C\frac{\mathrm{d}V_I}{\mathrm{d}t} \tag{6.13}$$

since, the output is taken against the resistor,

$$V_o = R\frac{\mathrm{d}Q}{\mathrm{d}t} \tag{6.14}$$

Substituting 16 in 15,

$$V_o = RC\frac{\mathrm{d}V_I}{\mathrm{d}t} \tag{6.15}$$

Hence, we show that the output voltage across the resistor is directly proportional to the differential of the input voltage.

The High Pass Filter is the exact opposite of the low-pass filter. It attenuates the signals with frequencies lower than the cut-off frequency (f_c) and passes the ones with frequencies higher than f_c. In this circuit, at low frequency when $f_c \to 0, X_c \to \infty$ then the capacitor provides high impedance. Thus current faces high resistance to flow to the load at the output. At high frequency when $f_c \to \infty, X_c \to 0$ then the capacitor acts as a shorted path and provides very small impedance to the current. Thus we can easily measure signals with high frequency across the output.

The cut-off frequency point is also known as the **-3dB** point. At this point if the amplitude of the output signal is compared to that of the input signal, we will find that the output signal is $\frac{1}{\sqrt{2}}$ of the input signal.

The formula of Calculating gain of a LPF and HPF is shown in 6.16 and 6.17 respectively.

$$\text{Gain (in dB)} = -20\log\left(\frac{V_o}{V_{\text{in}}}\right) \tag{6.16}$$

$$\text{Gain (in dB)} = +20\log\left(\frac{V_o}{V_{\text{in}}}\right) \tag{6.17}$$

After f_c the rate of increment of the responses of the circuit becomes -20dB/Decade for LPF and +20dB/Decade for HPF. If we calculate the increase per octave it will be $\pm 6dB$ respectively.

R-L-C series resonance circuit

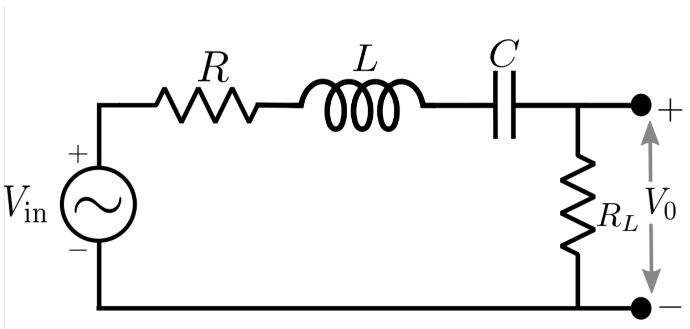


Figure 6.6: Series RLC resonance circuit

In a series RLC circuit as shown in fig.5, the current through the circuit is,

$$I = \frac{V}{Z}$$

where Z represents the equivalent impedance in the circuit.

$$\begin{aligned} Z = &\quad R + iwL + \frac{1}{iwC} \\ = &\quad R + iwL - \frac{i}{wC} \\ = &\quad R + i(X_L - X_C) \end{aligned}$$

where $X_L = wL$ and $X_C = \frac{1}{wC}$.

Thus the net reactance is $X = X_L - X_C$. Therefore,

$$I = \frac{V}{R + i(X_L - X_C)} = \frac{V}{R + iX}$$

Resonance condition in series RLC circuit is obtained when $X_L = X_C$ i.e. when $X_L = X_C, Z = R + i0 = R$. Then,

$$I_o = \frac{V}{Z} = \frac{V}{R}$$

Clearly higher current will flow through the circuit at resonance condition as impedance is minimum at this point.
The expression of frequency at resonance can be obtained by equating X_L and X_C which gives us,

$$\begin{aligned} X_L &= X_C \\ w_o L &= \frac{1}{w_o C} \\ w_o &= \frac{1}{\sqrt{LC}} \\ f_o &= \frac{1}{2\pi\sqrt{LC}} Hz \end{aligned}$$

Furthermore, since the input voltage is the same regardless of the frequency, the voltage drop across the resistor is maximum at resonance.

Question: What is Q-factor?

Question: What will be the output response if DC input signal is applied to the series LCR circuit?

Phase Shift Around Resonance

The capacitor and the inductor both affect the phase of current and voltage of the input wave. However, this affect is opposite in capacitor and in resistor.

In a capacitor, it takes time for the voltage across the two plates to build up and the current flows only till a voltage difference exists between them.

$$Q = CV$$

and

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$\Rightarrow I = C\frac{\mathrm{d}V}{\mathrm{d}t}$$

if $V(t) = \sin(wt)$,

$$I = I = C\frac{\mathrm{d}V}{\mathrm{d}t} = C(\cos(wt)) = C(\sin(wt + 90))$$

Therefore, in a capacitor, the current leads the voltage.

In an inductor, we simply have a coiled wire. Thus, a voltage difference appears as soon as it is applied. However, it takes time for current to start flowing through the inductor, because of the fact that it first converts electrical energy to magnetic energy.

$$V_L = L\frac{\mathrm{d}I}{\mathrm{d}t} \tag{6.18}$$

and if source voltage is $V_S = \sin(wt)$, then I= $\frac{\sin(wt)}{R}$

hence,

$$V_L = \frac{L}{R}\frac{\mathrm{d}\sin(wt)}{\mathrm{d}t} \tag{6.19}$$

$$\Rightarrow V_L = \frac{L}{R}\cos(wt) = \frac{L}{R}\sin(wt + 90) \tag{6.20}$$

Hence, the voltage leads over the current in this case.

We can also use "Lissajous figure" to visualise the phase difference between two waveforms. This is done by plotting two sinusoidal waves against each other. When a single straight line is observed, it implies that the two waves are perfectly in phase. When a circle is observed, it implies that the two waves are out of phase. Using this, we can easily calculate the phase difference of the two waves by,

$$\sin^{-1}\frac{a}{b}$$

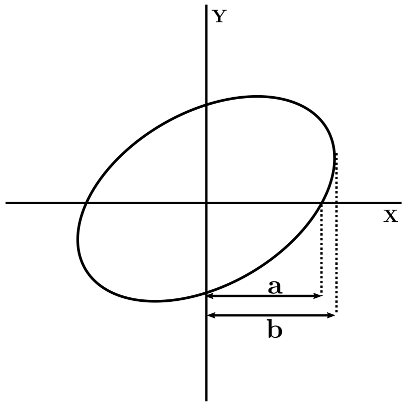


Figure 6.7: Lissajous figure

R-L-C parallel resonance circuit

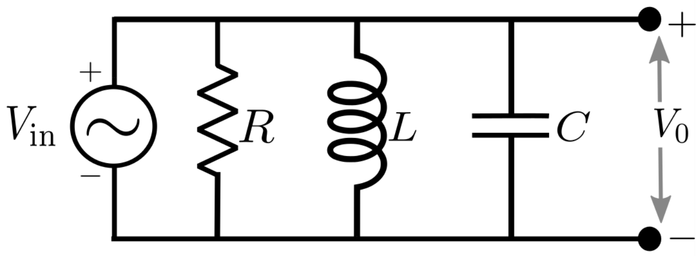


Figure 6.8: Parallel RLC resonance circuit

Similar to the series LCR circuit, a parallel LCR circuit can also be designed as shown in fig.6.8 which shows a resonance property, when $X_C = X_L$. In the above parallel RLC circuit, we can see that the input voltage is common to all three components while the supply current I_{in} consists of three parts, each of which has different value. The current flowing through the resistor is denoted by I_R, the current flowing through the inductor is denoted by I_L and the current through the capacitor is denoted by I_C. Therefore we have,

$$
\begin{aligned}
I_{in}^2 &= I_R^2 + (I_L - I_C)^2 \\
I_{in} &= \sqrt{I_R^2 + (I_L - I_C)^2} \\
\frac{V}{Z} &= \sqrt{\frac{V}{R}^2 + \Big(\frac{V}{X_L} - \frac{V}{X_C}\Big)^2}
\end{aligned}
$$

Then the impedance of a parallel LCR circuit is given by:

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}}$$

when $X_C = X_L$, Z= R (which is the minimum value it can take) i.e. the Capacitor and the Inductor form an open circuit. Thus the effective circuit becomes purely resistive.

Experimental Setup

Apparatus

1) A resistor
2) A capacitor
3) An inductor
4) A breadboard
5) A signal generator
6) A digital multimeter
7) A digital storage oscilloscope

Procedural Instructions

Part A: Low-Pass Filter and High-Pass Filter

1. In **Part A** connect a resistor and a capacitor in series as shown in fig.6.4 and connect another load resistor (R_L) parallel to the capacitor, across which the output will be measured.
2. Connect the signal generator at the input of the circuit and a digital storage oscillator (DSO) across the output in order to measure the change in output waveform.
3. Also calculate the cut-off frequency with the help of the resistor and capacitor's values.
4. Apply a sinusoidal waveform at the input with a desired amplitude and frequency.
5. Now, vary the frequency from $0Hz$ to $2f_cHz$ and study the changes in the output waveform.
6. Similarly make the connections for a High-Pass Filter by interchanging the positions of the resistor and the capacitor, as shown in fig.6.5.
7. Observe and note the change in output waveform with the help of a DSO.

Part B: Integrator and Differentiator

1. Instead of feeding sinusoidal waveform to the Low-Pass Filter circuit, let us feed a Square Wave signal to its input.
2. We will observe a triangular waveform at the output. Hence it behaves as an integrator.
3. To observe the characteristics of a differentiator we will use the same circuit as of a High-Pass filter.
4. Now if we feed a square wave input signal to a High-Pass filter we will get short duration pulse or spikes at the output.

Part C: Series and parallel RLC

1. Connect a resistor, an inductor and a capacitor in series with a function generator at the input, as shown in fig.6.6.
2. Keep the amplitude of the input waveform fixed and vary its frequency in order to measure the optimum bandwidth of the filter.
3. To observe the change in output waveform a DSO is connected at the output.
4. Plot a graph accordingly as shown in fig.6.9.
5. We will see a peak near the calculated f_c and a change in slope at the lower and upper cut-off frequencies, where the gain is $\frac{1}{\sqrt{2}}$ times.
6. Note down the lower and the upper cut-off frequencies which will help to calculate the followings:

$$\begin{aligned} Bandwidth &= (f_2 - f_1) \\ Q - factor &= \frac{f_c}{Bandwidth} \end{aligned}$$

7. Similarly make the connections for parallel RLC resonant circuit, as shown in fig.6.8 and observe the resonance at f_c.
8. After plotting the graph we will observe a minimum at f_c as shown in fig.6.10. Note the graph looks exactly opposite to that obtained for series RLC circuit.

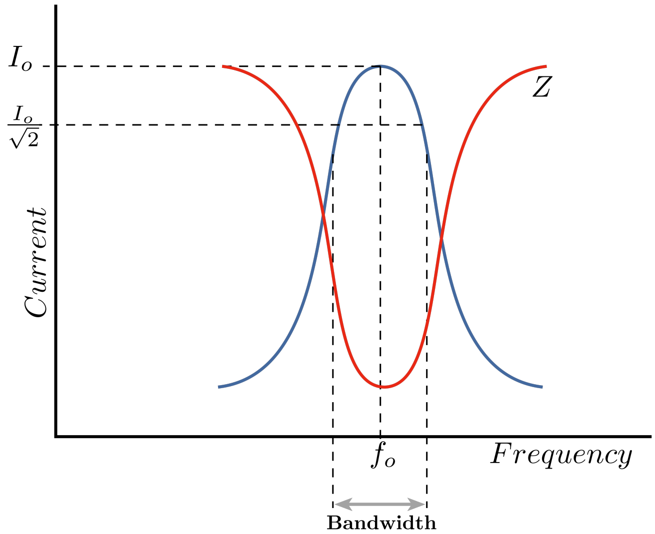


Figure 6.9: Current and impedance variation with frequency in series resonant circuit

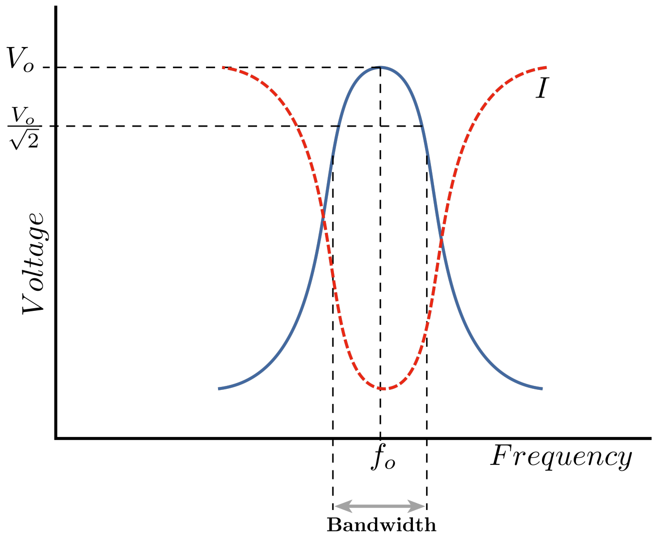


Figure 6.10: Voltage and current variation with frequency in parallel resonant circuit

Warnings

- Check the value of each of the component used before making the circuit and calculate the cut-off frequency theoretically.
- Change the frequency in small steps near the cut-off frequency in order to observe the change

in slope properly.

- Keep the amplitude of the sine wave fixed to a certain value while changing the frequency.
- Handle the CRO carefully and note the change in horizontal or vertical position knob.