Assignment 12: Review of Selected Topics

Due: May 13, 2022 (Friday) **Marks: 15**

1 Normal Modes

(a) Consider the system shown in Figure (1) that represents a CO_2 molecule. Find the ratio of the two normal mode frequencies. The masses of oxygen and carbon are taken to be 16 units and 12 units respectively. **[5]**

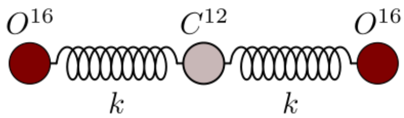

Figure 1: A schematic of a CO_2 molecule.

2 Fourier Series

(a) Consider the function $f(x) = \sin(\pi x)\sin(3\pi x)$. Find the Fourier series coefficients, i.e. find the set of coefficients a_0, a_n, and b_n such that: **[3]**

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2\pi n x) + \sum_{n=1}^{\infty} b_n \sin(2\pi n x) \tag{1}$$

3 Diffraction

(a) Write down the aperture function $A(x)$ for a double slit in terms of the slit-width w and the slit separation a. Both slits can be considered to be identical, and you can define the function over piece-wise intervals. (It might help to first draw out the function.) **[2]**

(b) Now, choosing $w = 1$ and $a = 2$, and using the fact that the diffraction pattern obtained is proportional to the Fourier Transform of the aperture function, find the diffraction pattern due to this aperture. In other words, if C is some proportionality constant and $k_x = kx/r$, find **[3]**

$$\mathscr{F}(k_x) = C\int_{-\infty}^{\infty} A(x')e^{ik_x x'}\mathrm{d}x', \tag{2}$$

(c) We will now try to calculate C. If I_0 is the intensity of the incident light, argue that **[2]**

$$\int_{-\infty}^{\infty} \mathscr{F}(k_x)^2 \mathrm{d}k_x = I_0. \tag{3}$$

Use the above relation to compute C. You may use the following integral identity:

$$\int_{-\infty}^{\infty} \frac{\sin(nk)\sin(mk)}{k^2}\mathrm{d}k = \left(\frac{|m+n| - |m-n|}{2}\right)\pi \tag{4}$$