Envy-Free Prices

Notation:

c_j is a single campaign

I_j is a a positive integer value denoting the number of impressions to be fulfilled by a campaign c_j

R_j is a positive integer value representing the reward attained by a campaign c_j if it is entirely fulfilled.

C $= \{c_1, c_2, \ldots c_m\}$ is a set of campaigns. We will use the shorthand $m = |C|$.

u_i is a user class

N_i is a positive integer value representing the number of users in a user class u_i

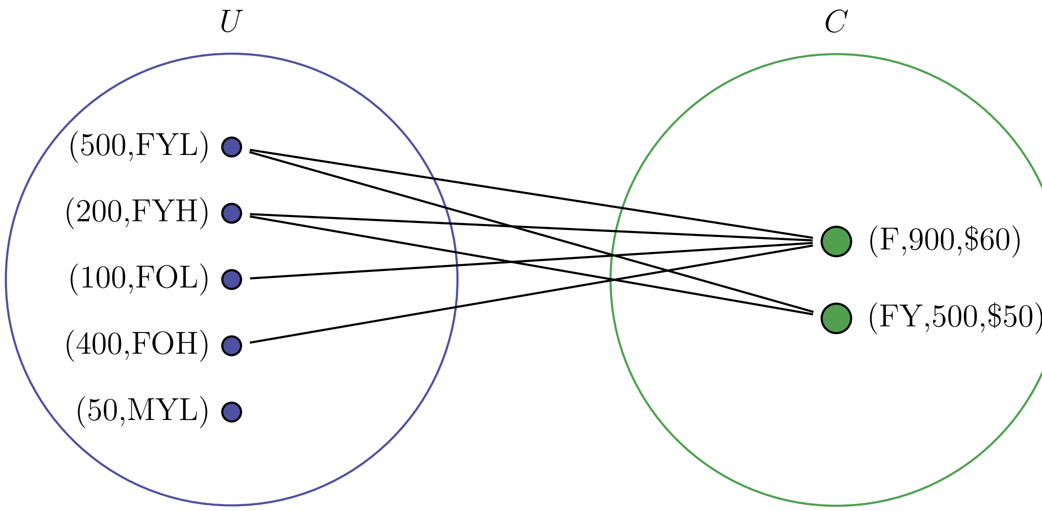
U $= \{u_1, u_2, \ldots, u_n\}$ is a set of user classes. We will use the shorthand $n = |U|$.

M is a market segment denoted by a string from the alphabet $\{F, M, Y, O, H, L\}$ satisfying $(F + M)^{0,1}(Y + O)^{0,1}(H + L)^{0,1}$.

E is a set of undirected edges between U and C.

Definitions:

1. **Campaign:** a campaign c_j is a triplet (M_j, I_j, R_j).
2. **User class:** a user class u_i is a pair (M_i, N_i) where M_i is a combination of precisely 3 letters, i.e., $(F + M)^1(Y + O)^1(H + L)^1$
3. **Market:** a market is a bipartite graph (U, C, E) with partitions U and C and edges E. An edge $(u_i, c_j) \in E$ exists between user class $u_i \in U$ and campaign $c_j \in C$ if and only if market segment M_j from c_j is a substring of M_i from u_i. For example:



In the previous example we have 2 campaigns: $c_1 = (F, 900, 60)$ and $c_2 = (FY, 500, 50)$, and 5 user classes: $u_1 = (FYL, 500), u_2 = (FYH, 200), u_3 = (FOL, 100), u_4 = (FOH, 400)$ and $u_5 = (MYL, 50)$. Campaign c_1 is connected to users classes u_1, u_2, u_3 and u_4 because F is a sub-string of their respective market segments. Contrast this with campaign c_2 which is connected only to user classes u_1 and u_2 since these are the only user classes with market segments for which FY is a sub-string. Finally, note that user u_5 is not connected with any campaign because there isn't any campaign with a market that is a sub-string of MYL.

4. **Allocation:** an allocation A is a labeling $w(u_i, c_j)$ of E by positive integers that represents how many of N_i is allocated to campaign c_j. An allocation can be represented as a matrix.

5. **Allocation Matrix:** a matrix X is a representation of an allocation. An entry x_{ij} of an allocation matrix X is a positive integer value that represents how many of N_i from u_i are allocated to campaign c_j. As a convention $\forall i, j$: if $(u_i, c_j) \notin E$ then $x_{ij} = 0$

   For example, the following matrix is an allocation for the previous example market:

$$\begin{array}{c c} & \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} & \begin{pmatrix} 100 & 400 \\ 100 & 100 \\ 100 & 0 \\ 400 & 0 \\ 0 & 0 \end{pmatrix} \end{array}$$

6. **Feasible Allocation:** an allocation matrix X is feasible if

$$\forall i : \sum_{j=1}^{m} x_{ij} \leq N_i$$

7. **Campaign Reward:** Given a feasible allocation X, campaign c_j produces reward R_j only if it is completely satisfied, i.e., if $\sum_{i=1}^{n} x_{ij} \geq I_j$.

8. **Reward for an allocation:** a feasible allocation X produces a reward $R(X) = \sum_{j=1}^{m} R_j$

9. **Efficient Allocation:** a feasible allocation X is efficient if it maximizes rewards over all feasible allocations. If we let F be the set of all feasible allocations, then X is efficient if and only if:
$$X \in \arg\max_{X' \in F} \{R(X')\}$$

10. **Bundle:** the bundle received by campaign c_j is the j-th column of the allocation matrix, denoted as x_{*j}

11. **Price mapping:** a mapping $P : U \mapsto \mathbb{R}^+$ is a mapping from user classes to prices. It represents the price per one unit of user class. We will use the shorthand $P_i \equiv P(i)$.

Efficient Allocation is NP-hard

Finding an efficient allocation is the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{j=1}^{m} R_j y_j \\ \text{subject to} \quad & \forall u_i : \sum_{j=1}^{m} x_{ij} \leq N_i \end{aligned}$$

where y_j is the following indicator function:

$$y_j = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_{ij} \geq I_j \\ 0 & \text{otherwise} \end{cases}$$

<u>Proof:</u> We want to reduce the following version of set cover to the problem of finding an efficient allocation:

<u>Set Cover Problem:</u>

Inputs: Universe $U = \{u_1, u_2, \ldots, u_n\}$

Subsets $S_1, S_2, \ldots, S_k \subseteq U$

Costs $c_1, c_2, \ldots, c_k$

Goal: Find a set $I \subseteq \{1, 2, \ldots, m\}$ that minimizes $\sum_{i \in I} c_i$, such that $\bigcup_{i \in I} S_i = U$

Integer linear program for Efficient Allocations

$$\begin{aligned}
&\text{maximize} && \sum_{j=1}^{m} R_j y_j, \text{where } y_j \in \{0,1\} \\
&\text{subject to} && (1)\ \forall u_i : \sum_{j=1}^{m} x_{ij} \le N_i \\
& && (2)\ \forall u_i, c_j : \text{If } (u_i, c_j) \notin E \text{ then } x_{ij} = 0 \\
& && (3)\ \forall c_j : y_j \le \frac{1}{I_j} \sum_{i=1}^{n} x_{ij} \le 1
\end{aligned}$$

Defining Envy-Free prices

Given a feasible allocation X, an envy-free price set $\{P_1, P_2, \ldots, P_n\}$ is an assignment of prices to user classes so that every campaign is happy with the bundle it receives.

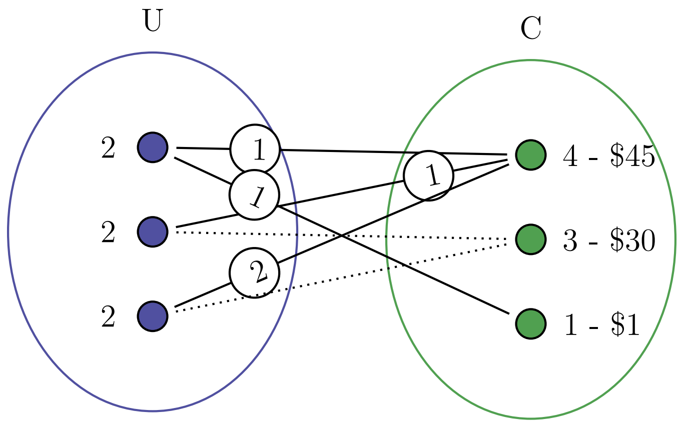
Mathematically, a set of prices $\{P_1, P_2, \ldots, P_n\}$ is envy-free if and only if, given a feasible allocation X, for every campaign c_j:

$$x_{*j} \in \arg\max_{X' \in F} \{v_j(x'_{*j}) - p_j(x'_{*j})\}$$

where $v_j(x'_{*j}) = \begin{cases} R_j & \text{if } \sum_{i=1}^{n} x'_{ij} \ge I_j \\ 0 & \text{otherwise} \end{cases}$ and $p_j(x'_{*j}) = \sum_{i=1}^{n} x'_{ij} P_i$.

Envy-Free prices do not exists in general

For a given allocation there might not be a set of envy-free prices. For example:



In this example the maximum rewards attainable for all campaigns is $46 which is possible only by satisfying campaigns c_1 and c_3. For the efficient allocation, note that campaign c_2 cannot be satisfied because doing so would mean that c_1 could not be satisfied, but c_1 has a greater reward than c_2. An efficient allocation in this case is given by the following matrix:

$$\begin{array}{c c} & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \end{array} & \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} \end{array}$$

An envy-free price set does not exists in this case. Consider the price P_1 that has to be set for u_1. This price has to be $0 < P_1 < 1$ in order for c_3 to make a profit. Next, $P_1 = P_2$ or otherwise c_1 wish it could have gotten more from either u_1 or u_2. Also, $0 < P_3 < 21.5$ or else c_1 does not make a profit. Under these conditions c_2 is envy since it could have gotten two from u_2 at a price of at most 1 and one from u_3 at a price of at most 21.5, for a bundle that satisfies it at a price of at most 23.5 and thus, make a profit.

Necessary and Sufficient conditions for Envy-Free prices (when they exists)

Given an allocation, consider the following conditions as candidates for sufficient and necessary conditions for Envy-Free prices. Note that these conditions apply only to satisfied campaigns. In what follows $(u_i, c_j) \in E$.

1. Price indifference: If $0 < x_{ij} < N_i$ and $0 < x_{kj} < N_k$ then $P_i = P_k$.

   If you are allocated SOME of two markets (but not ALL and not NONE), then the prices in those two markets must be the same

2. Dominance: If $x_{ij} = N_i$ and $0 < x_{kj} < N_k$, then $P_i \leq P_k$.

   If you are allocated ALL of a market, then the price in that market can be less than the price in other markets where you are allocated SOME (or NONE, by transitivity)

3. Preference: If $x_{ij} = 0$ and $0 < x_{kj} < N_k$, then $P_i \geq P_k$

   If you are allocated SOME of a market, then the price in that market can be less than the price in other markets where you are allocated NONE

Compact Condition

The next condition imply all others (note $(u_k, c_j) \in E$):

$$\forall u_i, c_j : \text{ If } x_{ij} > 0 \text{ then } [\forall u_k : \text{ If } x_{kj} < N_k \text{ then } P_i \leq P_k]$$

In all markets where you are allocated NOT NOTHING in one market i, and NOT EVERYTHING in another market k, then the price in i is less than or equal to the price in k.

This condition implies all three conditions above.

Proof: Suppose the Compact Condition holds.

Price indifference: Suposse (a) $0 < x_{ij} < N_i$ and (b) $0 < x_{kj} < N_k$. Apply the compact condition in two ways:

(1) by (a) $0 < x_{ij}$ compact condition implies $\forall u_k$: If $x_{kj} < N_k$ then $P_i \leq P_k$. By (b) we get that $P_i \leq P_k$

(2) by (b) $0 < x_{kj}$ compact condition implies $\forall u_i$: If $x_{ij} < N_i$ then $P_k \leq P_i$. By (a) we get that $P_k \leq P_i$

(1) and (2) imply that $P_i = P_j$

Dominance: Suppose (a) $x_{ij} = N_i$ and (b) $0 < x_{kj} < N_k$. Since N_i is a positive integer we have that (a) implies $x_{ij} > 0$ which together with the compact condition and (b) implies $P_i \leq P_k$

Preference: Suppose (a) $x_{ij} = 0$ and (b) $0 < x_{kj} < N_k$. Since N_i is a positive integer (a) implies $x_{ij} = 0 < N_i$ which together with with the compact condition and (b) implies $P_k \leq P_i$

Conditions A and B

$$A \coloneqq \forall c_j : \text{ If } x_{*j} > \mathbf{0} \text{ then } \sum_{i=1}^{n} x_{ij} P_i < R_j$$

If you are allocated SOMETHING, then total price of all goods/users allocated to you cannot exceed your value

$$B \coloneqq \forall c_j : \text{ If } x_{*j} = \mathbf{0} \text{ then } \forall X' \in F : \left[\text{If } \sum_{i=1}^{n} x'_{ij} \geq I_j \text{ then } \sum_{i=1}^{n} x'_{ij} P_i > R_j \right]$$

If you are allocated NOTHING, then total price of all your feasible bundles must exceed your value.

Heuristic

Checking condition B is probably hard in general. Here is a heuristic we can use:

Algorithm 1 Heuristic to check envy-free-ness for campaign c_j

procedure ENVY-FREE(MARKET M, CAMPAIGN c_j)
  construct priority queue $U_j := \{\langle u_i, P_i \rangle : (u_i, c_j) \in E\}$ priority is P_i in ascending order
  bundle := 0, cost := 0
  while bundle $< I_j$ and U not empty **do**
    $u_i = U$.pop
    bundle = bundle + $u_i.N_i$
    cost = cost + $(u_i.P_i) * (u_i.N_i)$
  end while
  if bundle $\geq I_j$ and cost $<$ cost for c_j current bundle **then**
    return false
  else
    return true
  end if
end procedure

The worst case performance of this heuristic is linear in the size of the user set, i.e., $O(n)$.

Conditions A, B and Compact Condition are sufficient for envy-free-ness.

Proof: Let (U, C, E) be a market, X be a feasible allocation and $\{P_1, P_2, \ldots, P_n\}$ be a set of prices for U such that conditions A, B and compact condition are true. Suppose that a set of envy-free prices exists. Consider the allocation to c_j from X. There are two cases: (1) $x_{*j} = \mathbf{0}$ or (2) $x_{*j} > \mathbf{0}$.

(1) Suppose $x_{*j} = \mathbf{0}$. Consider a feasible allocation X' such that $x'_{*j} > \mathbf{0}$ which means that c_j can be satisfied under X', i.e., $\sum_{i=1}^{n} x'_{ij} \geq I_j$. Condition B implies that

$$\sum_{i=1}^{n} x'_{ij} P_i > R_j \iff p_j(x'_{*j}) > v_j(x'_{*j}) \iff v_j(x'_{*j}) - p_j(x'_{*j}) < 0$$

which means that the maximum value of $v_j(x'_{*j}) - p_j(x'_{*j})$ over all feasible allocations is 0. Since $x_{*j} = \mathbf{0}$ implies $v_j(x_{*j}) - p_j(x_{*j}) = 0$, we have that $x_{*j} \in \arg\max_{X' \in F}\{v_j(x'_{*j}) - p_j(x'_{*j})\}$

(2) Suppose $x_{*j} > \mathbf{0}$. Condition A implies that

$$\sum_{i=1}^{n} x_{ij} P_i < R_j \iff p_j(x_{*j}) < v_j(x_{*j}) \iff v_j(x_{*j}) - p_j(x_{*j}) > 0$$

In particular this means that $\max_{X' \in F}\{v_j(x'_{*j}) - p_j(x'_{*j})\} > 0$. (*)
Now, suppose for a contradiction that

$$x_{*j} \notin \arg\max_{X' \in F}\{v_j(x'_{*j}) - p_j(x'_{*j})\}$$

By assumption there exists a feasible allocation $Y \in F$ such that

$$y_{*j} \in \arg\max_{X' \in F}\{v_j(x'_{*j}) - p_j(x'_{*j})\}$$

Note that $y_{*j} > \mathbf{0}$ since c_j is satisfiable. If Y satisfies condition A and compact condition, then we are done. Otherwise, suppose it does not satifies either condition A or compact condition.

If Y does not satisfy condition A, then $y_{*j} > \mathbf{0} \implies \sum_{i=1}^{n} y_{ij} P_i > R_j \iff p_j(y_{*j}) > v_j(y_{*j}) \iff v_j(y_{*j}) - p_j(y_{*j}) < 0$ contradicting (*).

If Y does not satisfy the compact condition, then there exists u_k such that $y_{kj} < N_k$ and $P_i > P_k$. But then Y is not a maximizer since we can construct a better bundle (see Appendix).

Single user-impression market

Definition: single user-impression market

A single user-impression market is a market where $\forall i : N_i = 1$ and $\forall j : I_j = 1$.

In this case condition B reduces to the following linear condition:

$$B \coloneqq \forall c_j : \text{ If } x_{*j} = \mathbf{0} \text{ then } \forall u_i : \text{ If } (u_i, c_j) \in E \text{ then } P_i \geq R_j$$

Theorem

Given an allocation for a single user-impression market, a set of envy-free prices always exists.

Proof: Let R_{low} be the lowest reward of any allocated campaign. Set all prices of allocated users to R_{low} and the price of non-allocated users to the highest reward over all campaigns. This is an envy-free price assignment because (1) any non-allocated campaign would have a reward less than any of the prices and thus it would be unsatisfiable, and (2) any allocated campaign would have a reward at least as big as R_{low} and thus, it would be satisfied with no envy since all prices are the same. □

Envy-free price ranges for efficient allocation in single user-impression market

Let R_{low} be the lowest reward of any allocated campaign and let R_{high} be the highest reward of any non-allocated campaign. Either $R_{low} \geq R_{high}$ or not. Let us look at each of these cases:

1. Suppose $R_{low} \geq R_{high}$.
2. Otherwise, $R_{low} < R_{high}$, means that there exists a campaign c_{high} with no allocation but reward R_{high} that is bigger than the reward of some allocated campaigns. In this case there exists at least one other allocated campaign c_k with access to the same users as R_{high} and such that $R_k > R_{high}$, or otherwise the efficient allocation would have allocated to c_{high} instead of c_k. Set the price of the user u allocated to c_k to be in the range $(R_{high}, R_k]$ so that c_{high} do not wish it had been allocated from u.

Appendix

Necessity

All three conditions, preference, dominance and indifference, are necessary

The following proof is written specifically for Preference, but the same idea holds for Price indifference and Dominance.

Proof: Let (U, C, E) be a market, X be an allocation and $\{P_1, P_2, \ldots, P_n\}$ be a set of envy-free prices for X. Consider users $u_i, u_k \in U$ and campaign $c_j \in C$ such that $(u_i, c_j) \in E$ and $x_{ij} = 0$ and $x_{kj} > 0$. Suppose for a contradiction that $P_i < P_k$, then c_j is not happy with the bundle x_{*j} it received since the bundle x'_{*j}, constructed as follow, is better: interchange one user class u_i with one of user class u_k, i.e. $x'_{ij} = 1$, $x'_{kj} = x_{kj} - 1$, and otherwise select the rest from the original bundle: $x'_{li} = x_{li}$.
Now, bundle x'_{*j} contains the same number of user classes as bundle x_{*j}, so if x_{*j} satisfies c_j then so does x'_{*j}, thus, $v_j(x_{*j}) = v_j\left(x'_{*j}\right) = R_j$. However, the price of the new bundle is less than the price of the original bundle since:

$$\begin{aligned}
p_j\left(x'_{*j}\right) = \sum_{t=1}^{n} x'_{tj} P_t \quad &= \quad x'_{1j}P_1 + \cdots + x'_{kj}P_k + \cdots + x'_{ij}P_i + \cdots + x'_{nj}P_n \\
&= \quad x_{1j}P_1 + \cdots + (x_{kj} - 1)P_k + \cdots + P_i + \cdots + x_{nj}P_n \\
&= \quad x_{1j}P_1 + \cdots + x_{kj}P_k - P_k \cdots + P_i + \cdots + x_{nj}P_n \\
&< \quad x_{1j}P_1 + \cdots + x_{kj}P_k + \cdots + 0P_i + \cdots + x_{nj}P_n \qquad \text{since } \mathrm{P}_i < \mathrm{P}_k \\
&= \quad \sum_{t=1}^{n} x_{tj} P_t \\
&= \quad p_j(x_{*j})
\end{aligned}$$

So, $p_j\left(x'_{*j}\right) < p_j(x_{*j})$. Since the rewards are the same for each bundle it follows that c_j is better off (maximizes profit) using bundle x'_{*j}. This contradicts our assumption that $\{P_1, P_2, \ldots, P_n\}$ is a set of envy-free prices. $\square$

Sufficiency

None of the conditions are sufficient in isolation:

Preference is not sufficient

Counterexample: Consider the following market (U, C, E), with $U = (u_1 = (F, 12), u_2 = (FY, 1))$ and $C = (c_1 = (F, 2, 100), c_2 = (FY, 2, 10))$, and the following efficient allocation:

$$\begin{array}{cc} & \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \end{array} & \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \end{array}$$

The set of prices $\{P_1 = 12, P_2 = 1\}$ satisfy the preference condition:

1. c_1 is such that $(u_1, c_1) \in E$ and $x_{11} = 0$ and $x_{21} = 2 > 0$ and $P_1 > P_2$
2. c_2 there is no u_k such that $x_{k2} > 0$, so preference is trivially satisfied.

However, campaign c_2 is not envy-free since it could have made a profit by acquiring two of u_2 Hence, preference is not a sufficient condition for envy-free prices.

Indifference is not sufficient

Counterexample: Consider the following market (U, C, E), with $U = (u_1 = (FYL, 2), u_2 = (FYH, 2)), u_3 = (MYH, 1))$ and $C = (c_1 = (FYL, 1, 10), c_2 = (Y, 2, 10), c_3 = (FYH, 1, 10)), c_4 = (MYH, 1, 3))$, and the following efficient allocation:

$$\begin{array}{c c} & \begin{matrix} c_1 & c_2 & c_3 & c_4 \end{matrix} \\ \begin{matrix} u_1 \\ u_2 \\ u_3 \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The set of prices $\{P_1 = 2, P_2 = 2, P_1 = 1\}$ satisfy the indifference condition. Note that we need to check only c_2 since it is the only one that satisfies the conditions of indifference.

Campaign c_2 is such that $0 < x_{12} = 1 < N_1 = 2$ and $0 < x_{22} = 1 < N_2 = 2$ and $P_1 = P_2$. However, this campaign is not envy-free since it could have acquired one of u_3 for a cheaper price. Hence, indifference is not a sufficient condition for envy-free prices.

Dominance is not sufficient

Counterexample: Consider the following market (U, C, E), with $U = (u_1 = (F, 2), u_2 = (FY, 1))$ and $C = (c_1 = (F, 2, 100), c_2 = (FY, 1, 10))$, and the following efficient allocation:

$$\begin{array}{c c} & \begin{matrix} c_1 & c_2 \end{matrix} \\ \begin{matrix} u_1 \\ u_2 \end{matrix} & \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \end{array}$$

The set of prices $\{P_1 = 12, P_2 = 9\}$ satisfy the dominance condition:

1. c_1 is such that $x_{21} = N_2$ and $x_{11} = 1 > 0$ and $P_2 < P_1$
2. c_2 there is no u_k such that $x_{k2} = N_k$, so dominance is trivially satisfied.

However, campaign c_2 is not envy-free since it could have made a profit by acquiring one of u_2 Hence, dominance is not a sufficient condition for envy-free prices.

Compact Condition

This condition is necessary. The idea of the proof is the same as the proof for Preference: Let (U, C, E) be a market, X be an allocation and $\{P_1, P_2, \ldots, P_n\}$ be a set of envy-free prices for X. Suppose for a contradiction that the Compact Condition does not hold. Then, using the negation of this condition:

$$\exists u_i, c_j : x_{ij} > 0 \text{ and } [\exists u_k : (u_k, c_j) \in E \text{ and } x_{kj} < N_k \text{ but } P_i > P_k]$$

construct a bundle for c_j that is better than the current bundle thereby contradicting envy-freeness.

This condition is not sufficient.
Counterexample: Consider the following market (U, C, E), with $U = (u_1 = (F, 2), u_2 = (FY, 1))$ and $C = (c_1 = (F, 2, 100), c_2 = (FY, 1, 10))$, and the following efficient allocation:

$$\begin{array}{cc} & \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \end{array} & \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \end{array}$$

The set of prices $\{P_1 = 12, P_2 = 9\}$ trivially satisfy the compact condition. However, campaign c_2 is not envy-free since it could have made a profit by acquiring one of u_2 Hence, the compact condition is not a sufficient condition for envy-free prices.

Conditions ALL and NONE

1. ALL: $\forall c_j, u_i :$ If $x_{ij} = N_i$ then $[\forall u_k :$ if $0 < x_{kj} < N_k$ then $P_i \leq P_k]$.
2. NONE: $\forall c_j, u_i :$ If $0 < x_{ij} < N_i$ then $[\forall u_k :$ if $(u_k, c_j) \in E$ and $x_{kj} = 0$ then $P_i \leq P_k]$.

In plain English these two conditions state:

1. ALL: the price of any market where you get all of the supply should be less than or equal to the price of any market where you get strictly less than all of the supply.
2. NONE: the price of any market where you get some of the supply should be less than or equal to the price of any market where you get none of the supply.

These conditions are not sufficient: Consider the following market (U, C, E), with $U = (u_1 = (FYL, 2), u_2 = (FYH, 1)), u_3 = (MYH, 1), u_4 = (MYL, 1))$ and $C = (c_1 = (FY, 2, 100), c_2 =$

$(YH, 2, 10), c_3 = (MY, 2, 5)))$, and the following efficient allocation:

$$\begin{array}{cc} & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

The set of prices $\{P_1 = 10, P_2 = 1, P_3 = 1, P_4 = 2\}$ satisfy both ALL and NONE. However, campaign c_2 is not envy-free since it could have made a profit by acquiring one of u_2 and one of u_3.