VEHICLE ROUTING OPTIMIZATION @ DPDHL

With an Introduction to Linear Programming

Forecast & Vehicle Routing @ DPDHL Meetup
Bonn, 2018-02-01

Data Analytics Center of Excellence

Deutsche Post DHL Group

Agenda

- Introduction to (Integer) Linear Programming
- Current Optimization Topics @ DPDHL
- Optimizing Vehicle Routing for Mail and Parcel
- Outlook

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What's a Linear Program?

One Linear Program

$$\begin{aligned} \textbf{max.} \quad & c^T x \\ \textbf{s.t.} \quad & Ax \le b \\ & x \ge 0 \end{aligned}$$

Objective Function, linear in *decision variables* x.

Constraints.

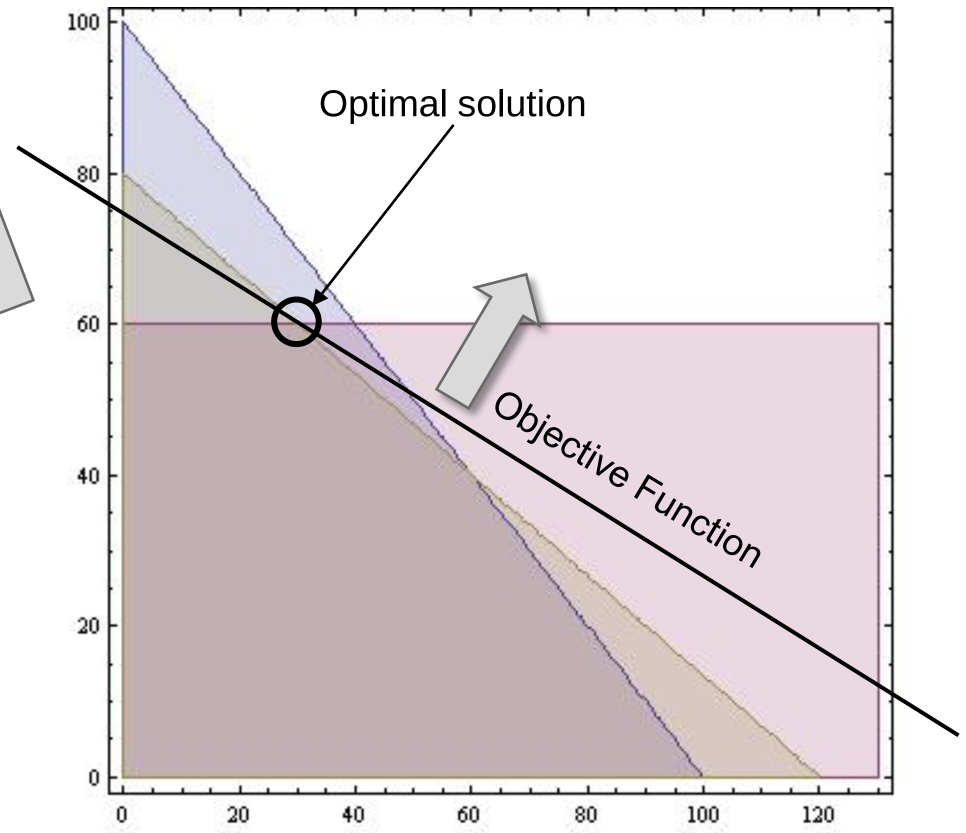
By convention, all decision variables are ≥ 0.

What's a Linear Program?

$$\textbf{max.} \quad c^T x$$

$$\textbf{s.t.} \quad Ax \leq b$$

$$x \geq 0$$



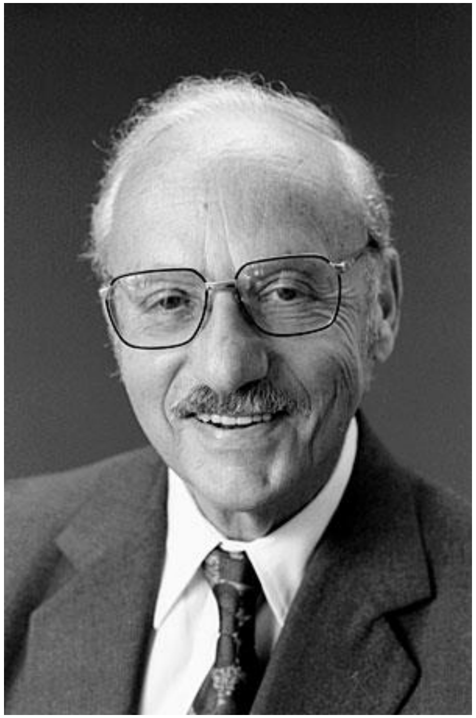
The Simplex Algorithm

The Simplex Algorithm

- Exact algorithm to solve linear problems (motivated mostly by military applications).
- Computationally efficient for most problems.
- Pioneered by George B. Danzig (~1947)
- Considered to be among the greatest algorithms of the 20th century!

Today's Use

- Linear problems with tens of thousands of decision variables are routinely solved.
- Important building block to solve more complex problem classes (i.e. Integer Programs)



[Picture: Stanford University]

George B. Dantzig (1914 – 2005)

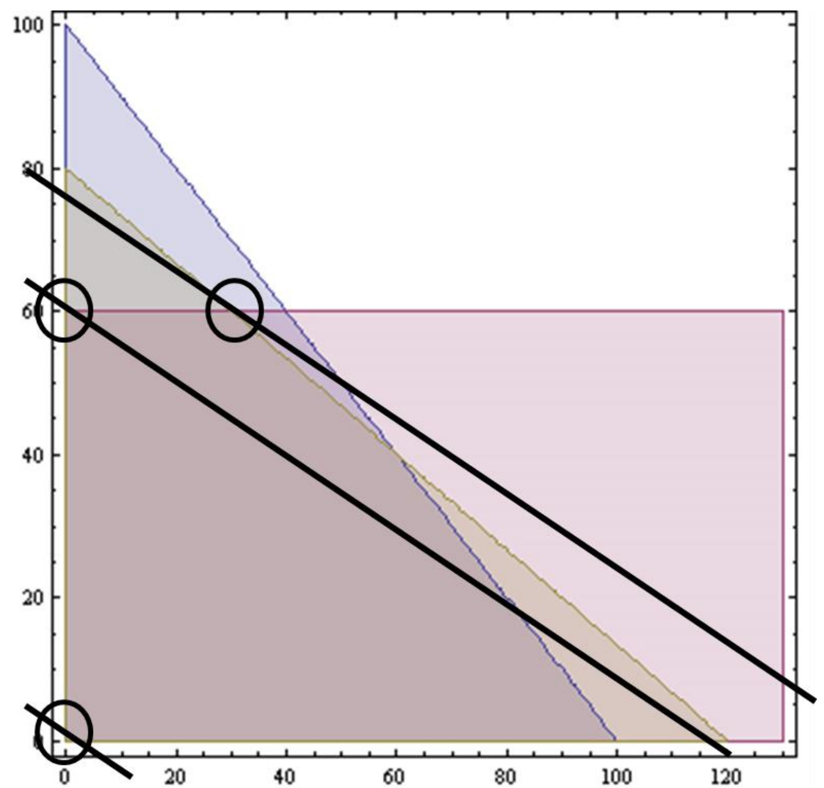
If you can state your problem as a linear program, it can be solved fast! There are good open source implementations. Excel has a simplex solver!

An Intuition for the Simplex Algorithm in 4 Steps

Intuition

1. Every linear constraint is a *hyperplane* in the solution space.
2. The space that satisfies all constraints is convex.
3. Optimal solutions can be found on the boundaries and in particular on the "corners" of such feasible solution space (called "Basic Feasible Solutions").
4. If we jump from one corner to another always increasing the target function, we will (eventually) find the optimum.
   - This happens by changing the set of constraints met at equality (pivoting).

A *hyperplane* is an $n-1$ dimensional object that cuts an n dimensional space in two subspaces.

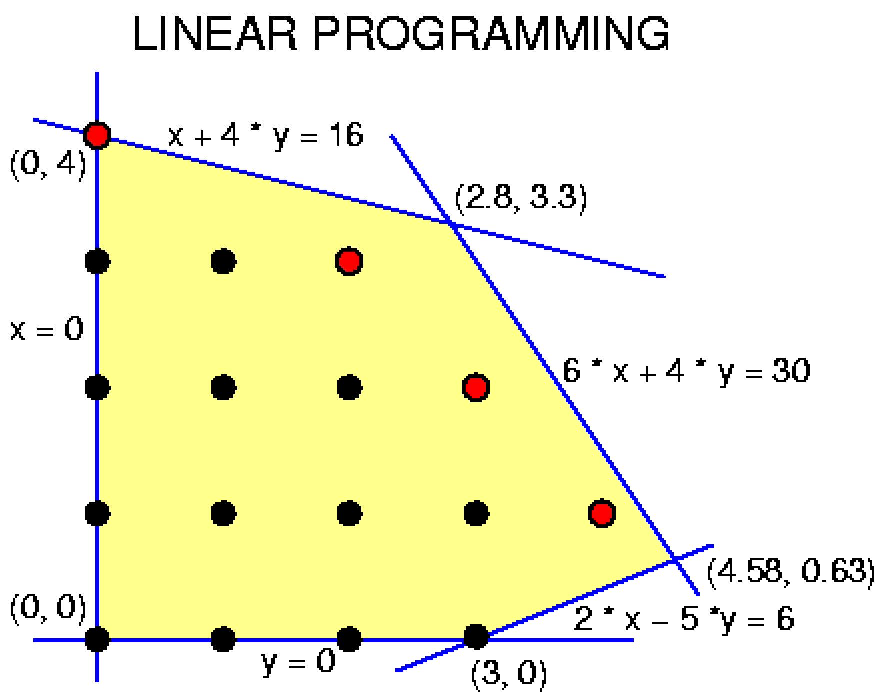


What you can model with a Linear Program

- Constrained mixing problems (e.g. optimal combination of products to maximize profits, given capacity/material constraints)
- Distribution planning: Which factories should supply which warehouses if capacities are limited and transport cost are linear in volume shipped?
- Tactical to strategic capacity planning: When should we create/reduce additional capacity if doing so induces a cost and we have to meet a set capacity demand?
- …

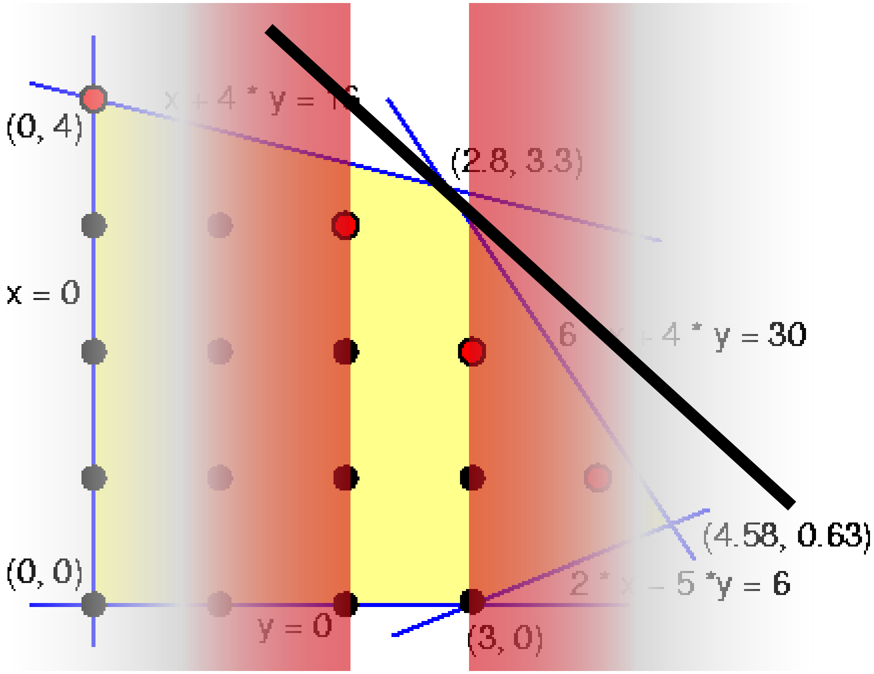
Integer Linear Programming (ILP)

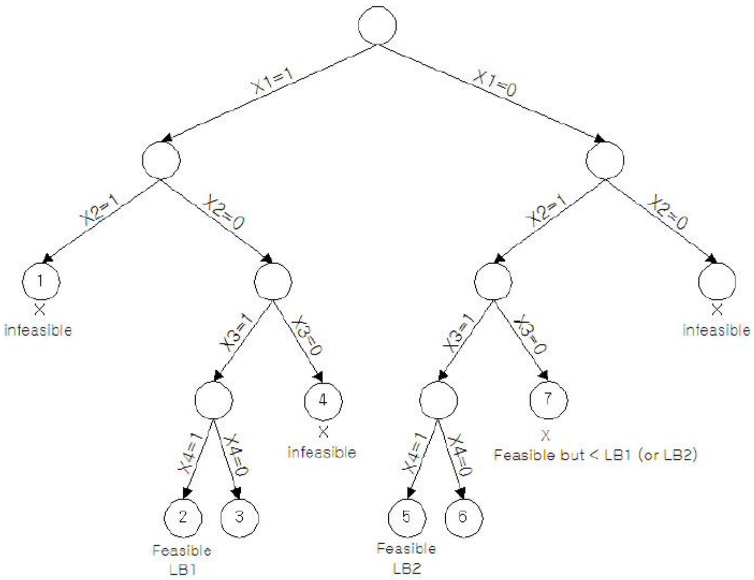
$$\begin{aligned} &\text{maximize} && \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ &\text{subject to} && A\mathbf{x} \le \mathbf{b}, \\ & && \mathbf{x} \ge \mathbf{0}, \\ &\boxed{\text{and} \qquad \mathbf{x} \in \mathbb{Z}^n,} \end{aligned}$$



As a result of the integer constraints, “corners” of the continuous solution space no longer necessarily represent feasible solutions. Time to solve Integer Programs can grow exponentially with solution variables!

One Exact Approach for Solving Integer Programs: Branch & Bound





Worst case number of sub-problems grows exponentially. Deciding which variable to choose for branching and how to proceed (depth first or breadth first) means solvers for IPs may vary greatly in performance. A good solver (like IBM CPLEX or Gurobi) can cost ~150k€!

Source of Figure: Rosenberger et al. "The generalized weapon target assignment problem"

What you can model with an Integer Program

- Binary decisions (e.g. open a warehouse at a location)
- “If-Then-Else“ conditions.
- Step objective functions.
- Fixed cost (e.g. for every truck/warehouse/… independent of usage)
- Explicit assignments (e.g. order to worker, parcel to truck, etc.)
- …

The Vehicle Routing with Time Windows (VRPTW) Problem

We need to consider:

- That every customer is visited (exactly once).
- That vehicles are not loaded beyond capacity.
- That time windows at customers are observed.
- That we vehicles cannot appear/disappear.

Model

$$\min \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk}$$

subject to,

$$\sum_{k \in V} \sum_{j \in N} x_{ijk} = 1 \quad \forall i \in C$$

$$\sum_{i \in C} d_i \sum_{j \in N} x_{ijk} \leqslant q \quad \forall k \in V$$

$$\sum_{j \in N} x_{0jk} = 1, \quad \forall k \in V$$

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0, \quad \forall h \in C, \quad \forall k \in V$$

$$\sum_{i \in N} x_{i,n+1,k} = 1, \quad \forall k \in V$$

$$s_{ik} + t_{ij} - K(1 - x_{ijk}) \leqslant s_{jk}, \quad \forall i,j \in N, \quad \forall k \in V$$

$$a_i \leqslant s_{ik} \leqslant b_i, \forall i \in N, \quad \forall k \in V$$

$$x_{ijk} \in \{0,1\}, \forall i,j \in N, \quad \forall k \in V$$

Minimize distance travelled.

Visit each customer exactly once.

Observe vehicle capacity.

Vehicles must leave the depot (technically).

A vehicle arriving at a customer must leave again.

Vehicles must return to the depot (technically).

Driving between locations takes time.

All customers must be served within their time interval.

El-Sheberny, N. A.: “Vehicle Routing with time windows: An overview of exact, heuristic and metaheuristic methods” in Journ
131, doi:10.1016/j.jksus.2010.03.002

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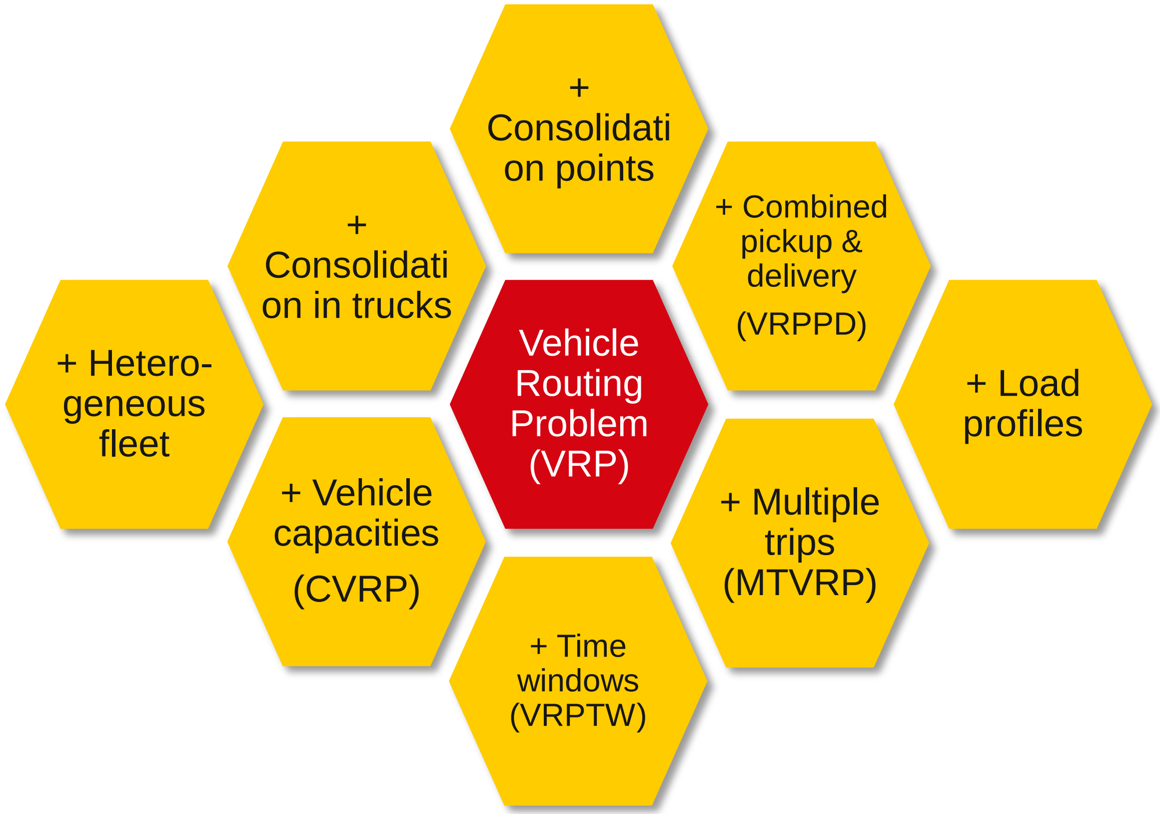
Known Standard Problems



We have a capacitated, multi-trip, heterogeneous fleet Vehicle Routing Problem with time-windows, pickup & delivery and consolidation points or CMTHFVRPTWPDC (!?).

Our Possible Approaches

Challenges	Descriptions
Exact Methods	• Mixed Integer (Linear) Programming • Constraint Programming • Column Generation

Neighborhood Search & Metaheuristic Toolkits	• Solving with a general heuristic based on Optimization models
	    • LocalSolver
	    • Google OR-Tools
	    • jsprit
	    • …

Own Dedicated Heuristics	• Higher Investment • Allows us to tackle further problems: Live scheduling, fault tolerant routes, driving restrictions

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THANK YOU