



# SYNC or Swim

## A Particle Model of the Interaction within Fish Schools

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The coordination of events to operate a system in unison.  
Some examples:

- Circadian Rhythms

The coordination of events to operate a system in unison.  
Some examples:

- Animal Swarming



The coordination of events to operate a system in unison.  
Some examples:

- Human Imitation (Memes/Trends)





The coordination of events to operate a system in unison.  
Some examples:

- Round of Applause

# Example - Human Grouping



Rules to Follow:

- **Walk** slowly toward center of the group.
- **Slow down** if you're within two feet of another person.
- **Stop** if you are within one foot of another person.

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- The coordinated behavior of animals of the same species and the emergent properties that arise.



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- For mathematical purposes, consider a swarm as an emergent behavior with no central coordination.





- Learning C/CUDA
- Applying mathematical models to real life phenomenon
- How will environmental factors affect the animal aggregate
- How animal aggregates will affect the environment

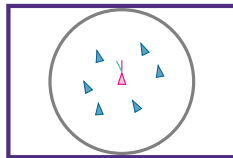


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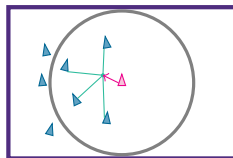
- **Alignment**





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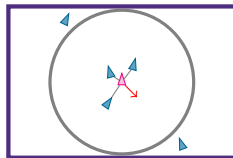
- **Alignment**
- **Cohesion**





Our model represents each fish adhering to the following three rules:

- **Alignment**
- **Cohesion**
- **Separation**





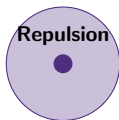
- Lagrangian Algorithm



- Metric distance model



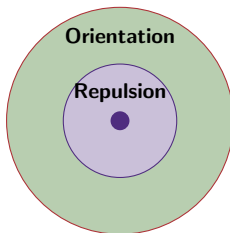
- Metric distance model





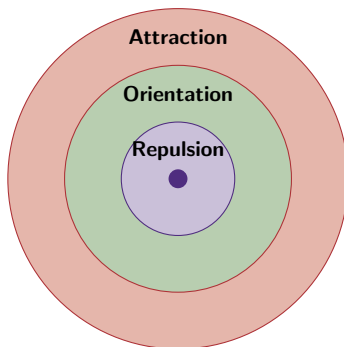


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# Attractive and Repulsive Forces Between Fish



- Attraction between a fish  $i$  and neighbor  $j$  :

$$A = C_a \frac{p_j - p_i}{d^2}$$



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- Repulsion between fish  $i$  and neighbor  $j$ :

$$R = -C_r \frac{p_j - p_i}{d^4}$$

# Attractive and Repulsive Forces Between Fish



$$A = C_a \frac{p_j - p_i}{d^2}$$

$$R = -C_r \frac{p_j - p_i}{d^4}$$

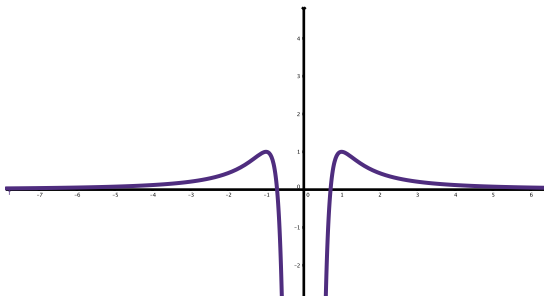
- Overall Attraction:

$$F_A = C_a \frac{p_j - p_i}{d^2} - C_r \frac{p_j - p_i}{d^4}$$

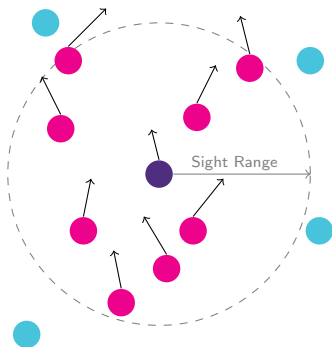
# The Attraction and Repulsion Coefficients



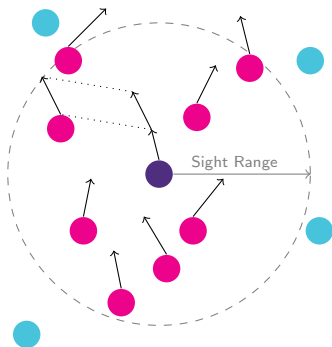
$$f(x) = \frac{2}{x^2} - \frac{1}{x^4}$$



# Directional Alignment of Fish $i$

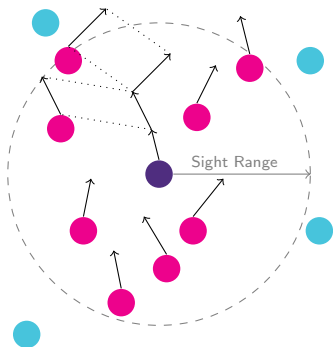


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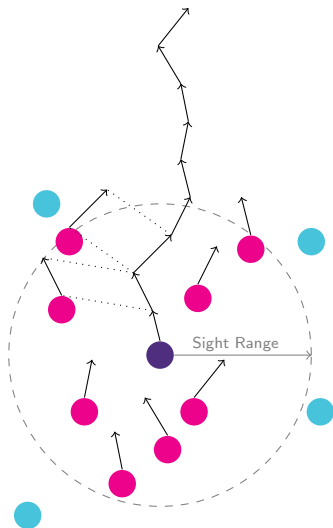




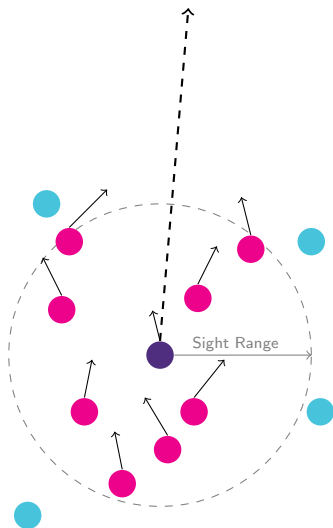
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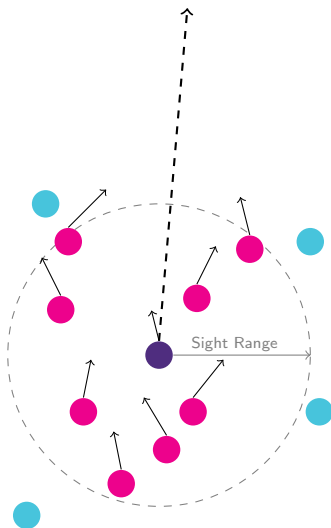
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# Directional Alignment of Fish $i$



$$F_{D_i} = \sum_{j=1}^N \frac{v_j}{||p_i - p_j||}$$



# Total Force on Fish $i$ From All Neighbors



$$F_{i_N} = \sum_{j=1}^N \left( W_a \left( C_a \frac{p_j - p_i}{d^2} - C_r \frac{p_j - p_i}{d^4} \right) + W_d \left( \frac{v_j}{\|p_i - p_j\|} \right) \right) \quad (1)$$

# Total Force on Fish $i$ From All Neighbors



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And, acceleration of each fish is the same as our force (taking each mass to be 1).

# Calculating Forces, Velocities, and Positions



At every timestep, the following calculations occur for each particle (let's call it particle  $i$ ):

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At every timestep, the following calculations occur for each particle (let's call it particle  $i$ ):

- 1 Calculate  $\|p_i - p_j\|$ .



# Calculating Forces, Velocities, and Positions



At every timestep, the following calculations occur for each particle (let's call it particle  $i$ ):

- 2 If the  $\|p_i - p_j\| < \text{SIGHT}$ , use (1) to determine the force between particle  $j$  and particle  $i$ , and sum forces over all particles within SIGHT of particle  $i$  ( $F_{iN}$ ).

# Calculating Forces, Velocities, and Positions



At every timestep, the following calculations occur for each particle (let's call it particle  $i$ ):

- 3 Use  $F_{iN}$  calculated above to update particle  $i$ 's velocity as follows:

$$v_i = v_i + F_{iN} \cdot dt$$

# Calculating Forces, Velocities, and Positions



At every timestep, the following calculations occur for each particle (let's call it particle  $i$ ):

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- 4 And update particle  $i$ 's position using:

$$p_i = p_i + v_i \cdot dt$$



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# Where Do We Go From Here?



- Add initial conditions for species-specific parameters
  - Density of swarms, how they behave towards targets and obstacles, etc.
- Move calculations from CPU to GPU to speed up calculation time



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Thank you to Dr. Wyatt and the Particle Modelling Lab for their time and resources.

## QUESTIONS?