**Powell, M.S. , Weerasekara, N.S. , Cole, S., LaRoche, R.D. , Favier, J. DEM modeling of liner evolution and its influence on grinding rate in ball mills. Minerals Engineering 24 (2011), 341-351**

This paper is from EDEM group. It describes with detail the EDEM model, and how the slippage condition is modeled as , so .

Also presents a wear model.

**Tuley, R., Danby, M., Shrimpton, J., Palmer, M. On the optimal numerical time integration for Lagrangian DEM within implicit flow solvers. Computers and Chemical Engineering 34 (2010), 886-899.**

For powder flows in which a fluid particle coupling exists, there can be defined micro-scale, meso-scale and macro-scale techniques for numerically predict its behavior.

It covers aspects of numerical integration methods and errors associated with them.

The test case is a binary collision between two particles, one initially stationary and the other with a certain velocity.

It mentions to Kruggel-Emden 2007, and 2008 as the basis for their model.

**Xu, B.H., Yu, A.B. Numerical simulation of the gas-solid flow in a fluidized bed by combining discrete particle method with computational fluid dynamics. Chemical Engineering Science 52 (1997), 2785-2809.**

Here as in the original Cundall and Strack paper, clearly states that the slipping condition is:

**Geng, Y., Che, D. An extended DEM-CFD model for char combustion in a bubbling fluidized bed combustor of inert sand. Chemical Engineering Science 66 (2011), 207-219.**

Also, they cite and uses the model as in Cundall and Strack, Tsuji et al 1993, and Xu and Yu 1997.

**Siiriä, S., Yliruusi, J. DEM simulation of influence of parameters on energy and degree of mixing. Particuology 9 (2011), 406-413.**

It refers to another paper by the same authors in 2007. Also refers to the review made by Zhu, Zhou, Yang & Yu 2007 to introduce different methods for simulating powder behavior including the DEM method.

**Zhu, H.P., Zhou, Z.Y., Yang, R.Y. Discrete particle simulation of particulate systems: Theoretical developments. Chemical Engineering Science 62 (2007), 3378-3396.**

A general review about DEM and CFDEM. It includes review about non-contact forces (van der Waals, and electrostatic forces), liquid bridge between particles, as well as different contact forces models. They include the Langston method as an alternative to a full Hertz-Mindlin model.

They use a Langston modified model, but as it appears in the table 1, the slipping is handled in an equivalent way as Cundall and Strack although in a very different form (it’s a hysteretic method).

**Siiriä, S., Yliruusi, J. Particle packing simulations based on Newtonian mechanics. Powder Technology 174 (2007), 82-92.**

They use some kind of a hysteretic model in normal direction. It is given by: . When particles approach, but as <0, then adopts a value less than unity. So the hysteretic behavior occurs instantaneously.

They introduce the energy balance equation, and consider the energy error. First the total energy inside the simulation is , gravitational potential, plus kinetic (linear and rotational), and spring potential energy terms. There is also the Energy lost (dissipated) in the collisions , and the error associated to numerical and model inaccuracies, so the energy is not conserved. Then is

**Maw, N., Barber, J.R., Fawcett, J.N. The oblique impact of elastic spheres. Wear 38 (1976), 101-114**

This paper handles non-collinear impact of spheres. Mindlin and Deresevicz stated the theory of non-linear solution for tangential forces, in the same way than Hertz did for the normal forces. This solution is based in normal and tangential displacements independently applied (first one, and then the other). Nevertheless it’s very important to state that always there is some kind of slipping. When particles are separating always some dissipation of energy due to micro slipping occurs. As the particles separate, the area of contact is diminishing, and then in an infinitesimal area the normal force is approaching to zero, so the slipping occurs. In this way, before the particles lose contact, the accumulated tangential strain is released with frictional slippage. Three regimes exist: i) initial stick, and then micro-slipping; ii) initial gross slip, then stick, and then micro-slipping; iii) gross slip during whole collision.

**Maw, N., Barber, J.R., Fawcett, J.N. The rebound of elastic bodies in oblique impact. Mech. Res. Comm. 4 (1977) , 17-22**

Normal frequency is different from Tangential frequency. Here the definition for is given

**Maw, N., Barber, J.R., Fawcett, J.N. The Role of Elastic Tangential compliance in Oblique Impact. Journal of Lubrication Technology 103 (1981), 101-114**

Elastic tangential compliance. The model has 5 hypothesis.

1. Area of contact << D
2. Static coefficient of friccion equal to the dynamic coefficient of friction
3. The material has a linear response
4. Hertz quasi-static regime
5. Tangential traction axysimmetric

It’s shown that the behavior of steel and rubber is mostly the same, but the curves are displaced and with different values.

**Cundall, P.A., Strack, O.D.L. A discrete numerical model for granular assemblies. Géotechnique 29 (1979), 47-65.**

Basic paper in which the DEM method is presented. The model is in normal direction: linear spring + viscosity force. In tangential direction is also linear spring + viscosity force. The slipping condition is handled as: .

**Walton, O.R., Braun, R.L. Viscosity, Granular-Temperature, and Stress Calculations for Shearing Assemblies of Inelastic, Frictional Disks. Journal of Rheology 30 (1986), 949-980.**

This is a basic paper in which the method of Walton and Braun is presented. Is an hysteretic model in normal direction and in tangential direction. The tangential direction force model is based in a simplified Mindlin-Deresewicz theory. The limit of the tangential force is , so it would be equivalent to .

**Zhou, Y.C., Wright, B.D., Yang, R.Y., Xu, B.H., Yu, A.B. Rolling friction in the dynamic simulation of sandpile formation. Physica A 269 (1999), 536-553.**

They use the Langston model, but in this paper they add two different rolling friction models. They get better agreement with the angular velocity independent model. The angular velocity dependent friction model has worse experimental agreement.

This phrase is very good:

The interaction between particle and fluid and the long-range forces, such as van der Waals and electrostatic forces, can be ignored in the present work which deals with large particles in static, low-viscosity fluid.

**Brillantov, N.V., Pöschel, T. Rolling friction of a viscous sphere on a hard plane. Europhysics Letters 42 (1998), 511-516.**

**Brillantov, N.V., Spahn F., Hertzsch J.M., Pöschel T. Model for collisions in granular gases. Physical Review E 53 (1996), 5382-5392.**

This second reference (1996) is the one in which all the ideas come from.

They give a quasi-static approximation for the visco-elastic sphere. For this approximation to be true the condition is that the time of collission dissipative relaxation times (I do not have an estimation for this time). For the elastic part to be modeled as quasi-static Hertz approximation, the characteristic velocity has to be much more smaller than the sound velocity in the solid (this is very easy).

One idea is that there is a quasi-stationary state sequence, where the elastic part rules the friction limit to slippage. There is a instantaneous dissipation, so the dissipation force is used to decelerate the solid (macroscopic solid), but immediately is dissipated so in the interface there is only the elastic part interaction. In the case of a restoring force, then only the elastic part should be used to accelerate/decelerate the macroscopic solid.

They cite to forester, bridges (nature) and Goldstein (Impact book).

This paper is an example of a viscoelastic model in normal direction. The dependence of the viscous term is , as opposed as the liggghts dependence of

Also from the equations, it can be seen that they take as friction limit the value given by , and always is applied the viscous force

**Langston P.A., Tüzün U., Continuous potential discrete particle simulations of stress and velocity fields in hoppers: transition from fluid to granular flow. Chemical Engineering Science 49 (1994), 1259-1275.**

Here is defined the normal interaction as a continuous potential, and the tangential interaction as the model of Langston . This expression will be optimized in next papers.

The way in which the friction limit is handled is seen better in the next paper.

**Langston P.A., Tüzün U., Heyes D.M., Discrete element simulation of granular flow in 2D and 3D hoppers: dependence of discharge rate and wall stress on particle interactions. Chemical Engineering Science 50 (1995), 967-987.**

Here it’s specified more in deep the normal and tangential models of interactions. For normal interaction they use linear model, continuous potential, and Hertz model. For the tangential force they improve their model, and also it is clearly stated that they use , and , being only the elastic part. So the limit is always , and always is applied the viscous part to both normal and tangential forces.

**Foerster S.F., Louge M.Y., Chang H., Allia K., Measurements of the collision properties of small spheres. Phys. Fluids 4 (1994), 1108-1115.**

Experimental values. It states the dependence of the normal restitution coefficient with velocity as: nearly constant at low velocities, but as fully plastic deformations occur at large velocities, it may decrease like .

It is also worth that in the graphs representing the non dimensional tangential velocity after collision, as function of the initial non dimensional tangential velocity, in the regions of small , the value of seems to be always negative. This is an obscure point of the Maw model, as in one of their papers they show this value also as negative.

**Schäfer J., Dippel S., Wolf D.E., Force Schemes in Simulations of Granular Materials. J. Phys. I France 6 (1996), 5-20**

It points out that there is a plastic deformation velocity. . It also relates that for Hertz normal interaction model, sometimes a viscous dissipation is introduced “Ad-hoc”, but if the coefficient is constant (dashpot), the restitution coefficient grows with impact velocity, and the experiments shows that this is not the case, but it goes as . Then the model of Kubara & Kono, as well as the proposed by Brilliantov gave similar results.

It also defines the total coefficient of restitution as

**Kuwara G., Kono K., Restitution Coeffcient in a Collision between Two Spheres. Japanese Journal of Applied Physics 26 (1987), 1230-1233.**

Here also it’s deduced the dependence . The variation of the restitution coefficient is very low for steel balls. Only for brass like materials has a big dependence.



**Johnson D.L., Norris A.N., Rough elastic spheres in contact: memory effects and the transverse force. J. Mech. Phys. Solids 45 (1997), 1025-1036.**

It analyzes the path dependent tangential force. Also points out that the hertz model should be numerically formulated as a incremental method, and not as a integral model. It also points out that there is a work done by tangential forces that is path dependent.

**Brendel L., Dippel S. Lasting Contacts in Molecular Dynamics Simulations. In: Herrmann, H.J., Hovi, J.-P., Luding, S. (Eds.), Physics of Dry Granular Media. Kluwer Academic Publishers. (1998), p. 313.**

They show that the model for tangential elongation calculation presented in Cundall & Strack, is not correct. It shall be cut up to the value in which .

**Zhang D., Whiten W.J., An efficient calculation method for particle motion in discrete element simulations. Powder Technology 98 (1998), 223-230.**

It’s almost no use. Only cite to Tsuji 1992, and express the dissipation force as

**Tsuji Y., Tanaka T., Ishida T. Lagrangian numerical simulation of plug flow of cohesionless particles in a horizontal pipe. Powder Technology 71 (1992) 239-250.**

The first one in propose a viscoelastic damping model with constant normal coefficient of restitution. It’s derived heuristically from the equations of motion, and looking for a restitution coefficient that only depends on a constant . Also is probably the first in which the stiffness in Hertz model is defined from material constants, and the tangential stiffness is deduced from Mindlin-Deresewicz theory.

**Vu-Quoc L., Zhang X., An accurate and efficient tangential force-displacement model for elastic frictional contact in particle flow simulations. Mechanics of Materials 31 (1999) 235-269.**

A very complex model trying to cover aspects of Mindlin-Deresewicz theory. Its application it’s limited to simple-loading.

**Matchett A.J., Yanagida T., Okudaira Y., Kobayashi S. Vibrating powder beds: a comparison of experimental and Distinct Element method simulated data. Powder Technology 107 (2000), 13-30.**

They measure the energy input to the system, and compare it with the calculated by the DEM code. With the DEM code they calculate the internal Energy as , and the dissipated in the time increment , is . The change in internal Energy is , so the change in total energy is . This is for each step. If summed for all time steps and dividing by t, they have the power loss. They represent the power loss for each acceleration value in stationary conditions. The experimental and numerical results do not agree. Only some qualitative observations can be made by DEM. They also represent as an attractor the energy dissipation rate as wall position. The Energy dissipation rate is given by .

It can be worth to explore how to get this value in our applications.

**Rajamani R.K., Mishra B.K., Venugopal R., Datta A., Discrete element analysis of tumbling mills. Powder Technology 109 (2000), 105-112.**

They calculate the dissipated energy and compare it with the experimental power needed for the tumbling mill to work. In stationary conditions, the kinetic and gravitational terms smear out, and the power needed is due only because of dissipation. Do not consider how the different energy terms are conserved, but only compares de dissipated power with the input power to the mill. Also they introduce the rotation tensor in 3D, and derives one approximation that needs iteration. Also is noticeable that they calculate the moments of the tangential forces only with the spring force, and not with the damping force.

It is seems that they use the condition

**Kharaz A.H., Gorham D.A., Salman A.D. An experimental study of the elastic rebound of spheres. Powder Technology 120 (2001) 281-291.**

Experimental study.

They state that:

*Another potentially important issue is the adhesive effects due to van der Waals forces, which become increasingly important as the particle size decreases and which dominate the impact behavior for particles in the micronsize range. However, the present work deals with particles of millimeter dimensions in which adhesive effects are negligible and so these will not be considered further in this paper.*

It’s a nice problem definition.

Here in this experiment it’s shown clearly the positive region of for , then the negative region of for , and the linear ascending shape of for .

Also they study the energy partition after the collision for each collision angle. It could be interesting to try to postprocess our results to obtain this, but their partition study the linear kinetic energy, the rotational, the associated with the normal coefficient of restitution (the viscous term in our case), and the slipping term.

**C. Thornton, Z. Ning, C.-Y. Wu, M. Nasrullah, L.-Y. Li, Contact mechanics and coefficients of restitution, in: T. Pöschel, S. Luding (Eds.), Granular Gases, Springer, Berlin, 2001, pp. 184–194**

It’s an interesting paper. They compare a viscoelastic model with a elastoplastic model. One important conclusion is that:

*“It is worth noting that the elastoplastic model of impact predicts that the unloading period is shorter than the loading period, in contrast to the predictions of viscoelastic impact models”*

This is probably related to the normal force artifact with the viscoelastic model. It would also to be shorter, so it’s interesting to try to cut the force as soon as it becomes attractive. There is also an expression to approximate the time of collision as:

**Asmar B.N., Langston P.A., Matchett A.J., Walters J.K. Validations tests on a distinct element model of vibrating cohesive particle systems. Computers and chemical Engineering 26 (2002) 785-802.**

It covers a detailed description of the DEM method. It’s centered around the linear model. It limits the force with the elastic part . The limit is written as with as in (Langston et all 1995).

They also introduce energy in the system by the oscillating walls of the container. The only test they made is with one particle and they represent different magnitudes (elastic forces, damping forces, etc…). It is also shown how the normal damping force has negative values so the total force becomes attractive.

**Wu C.Y., Li L.Y., Thorton C. Rebound behavior of spheres for plastic impacts. International Journal of Impact Engineering 28 (2003), 929-946.**

Analysis by FEM of the plastic behavior in collisions. They conclude that for small deformations, the coefficient of restitution is dependent on , and if finite plastic deformation is present, then it depends on .

Also defines a high velocity above which heating effects are present

**Asmar B.N., Langston P.A., Matchett A.J., Walters J.K. Energy monitoring in distinct element models of particle systems. Advanced Powder Technol. 14 (2003) 43-69.**

In the paper is written:

*Despite extensive research into developing DEM and several models to enhance the calculations of contact forces, velocities and displacements, very little has been found that has considered energy modeling. With the exception of a few studies [4, 11, 25–27] that investigated energy, most studies have paid little attention to monitoring energy. The majority assumed that the energy conservation law is applicable by default. Furthermore, those who looked at energy evaluated the stored potential energies and the kinetic energies, and then evaluated the dissipated energies as the remainder term of total energy minus other energies tracked.*

They cite 5 papers that deals with energy monitoring in DEM

They define each of the terms in energy balance, and they do not deduce one from the others. They use a linear model, and for calculating work terms, they calculate the difference between the radio vector for each particle. As I do use the distance between particles, I have to use a factor of .

They do somewhat substep determination when calculating the energy dissipation in gross sliding.

The tangential spring lost energy has to do with our model when the normal force is diminishing and “when slip is from the beginning of the time step because the previous step tangential elastic force has a higher value than for this new step”.

They define the artificially created energy in equation 27 and the previous (25, 26).

In the preliminary test (Figure 9) they have errors of around 5% depending on the number of particles. When running a vibrating case, they show that the artificial energy is very high, it has a value of the order of the injected energy. They do not show the error, because would be around 20% of the total energy.

This is a very important paper, and they can not model accurately this problem.

**Zhu H.P., Yu A.B. Steady-state granular flow in a three-dimensional cylindrical hopper with flat bottom microscopic analysis. J. Phys. D: Appl. Phys. 37 (2004) 1497-1508.**

It uses microdynamics, coordination number, porosity and force networks. From these studies they get that there are four zones in the flow. Not much interesting. Only for citing about the importance of microdynamics.

**Di Renzo, A., Di Maio F.P. Comparison of contact-force models for the simulation of collisions in DEM-based granular flow codes. Chemical Engineering Science 59 (2004) 525-541.**

Good review of previous papers. It has comparisons between the linear, MDns, and full MD codes. Interesting to see how they compare with experiments of Kharaz, and how probably our model looks like the MD code, the one which agrees better with experimental results. It would be good to make a comparison with these results.

It also addresses that the MDns gives worse results than the linear model.

**Raji, A.O., Favier J.F. Model for the deformation in agricultural and food particulate materials under bulk compressive loading using discrete element method. I: Theory, model development and validation. Journal of Food Engineering 64 (2004) 359-371.**

General presentation of method of EDEM. They give the expressions for the non linear damping model proposed by Tsuji. They also give a general expression for the different powers of displacement.

; ;

Tsuji: If ; ;

Kuwabara: If ; ;

It do not give any detail about friction limit.

**Di Renzo, A., Di Maio F.P. An imprived integral non-linear model for the contact of particles in distinct element simulations. Chemical Engineering Science 60 (2005) 1303-1312.**

They only modify the coefficients of the integral MDns. Their solution it’s not correct, but it’s a prove that this model is not correct.

**Stevens A.B., Hrenya C.M. Comparison of soft-sphere models to measurements of collision properties during normal impacts. Powder Technology 154 (2005) 99-109.**

It compares a lot of DEM contact force models. It concludes that the better aggrement of the restitution coefficient and collision duration is given by the Kuwabara & Kono model (also Brilliantov). Probably it has the better physical foundations.

**Karrech A., Duhamel D., Bonnet G., Roux J.N., Chevoir F., Canou J., Dupla J.C., Sab K. A computational procedure for the prediction of settlement in granular materials under cyclic loading. Comput. Methods Appl. Mech. Engrg. 197 (2007) 80-94.**

Only worth to mention that they calculate the friction limit and the gross slipping force as:

if

They uses the integral Hertz Mindlin-Deresewicz tangential model.

**Van der Hoef M.A., Ye M., van Sint Annaland M., Andrews A.T., Sundaresan S., Kuipers J.A.M. Multiscale modeling of gas-fluidized beds. Advances in Chemical Engineering 31 (2006) 65-149.**

**Deen N.G., Van Sing Annaland M., Van der Hoef M.A., Kuipers J.A.M. Review of discrete particle modeling of fluidized beds. Chemical Engineering Science 62 (2007) 28-44.**

Review of fluidized bed simulations. The first is a general review covering also CFD aspects including LBM simulations, etc… The second is centered around the DEM simulation.

They use a linear model. Important is that they say that for energy conservation reasons. This is not the case, and can be cited as example as this option. They also cites the Link 2005 method for averaging particles properties in a CFD cell.

**Kruggel-Emden H., Simsek E., Rickelt S., Wirtz S., Sherer V. Review and extension of normal force models for the Discrete Element Method. Powder Technology 171 (2007) 157-173.**

Review of normal contact models. It classify them in: Continuous Potential, Linear viscoleastic models, Non-linear viscoelastic models (including fully non-linear – viscous term), and Hysteretic models.

Also they propose to adjust the different coefficients from experiments.

**Schwager T., Pöschel T. Coefficient of restitution and linear-dashpot model revisited. Granular Matter 9 (2007) 465-469.**

**Schwager T., Pöschel T. Coefficient of restitution for viscoelastic spheres: The effect of delayed recovery. Physical Review E. 78 (2008)**

They study the “delayed recovery” effect, or the attractive force appearance due to viscous forces in the last moments of the collision.

No conclusion.

**Zhu H.P, Zhou Z.Y., Yang R.Y., Yu A.B. Discrete particle simulation of particulate systems: A review of major applications and findings. Chemical Engineering Science 63 (2008) 5728-5770.**

A very detailed review of applications of DEM.

**Kruggel-Emden H., Wirtz S., Scherer V. A study on tangential force laws applicable to the discrete element method (DEM) for materials with viscoelastic or plastic behavior. Chemical Engineering Science 63 (2008) 1523-1541.**

It is important that here is given for Non-linear models.

They extend the study of DiRenzo and DiMaio. It’s confusing what is considered in each model.

**Luding S. Cohesive, frictional powders: contact models for tension. Granular Matter 10 (2008) 235-246.**

It’s a hysteretic model with linear cohesion.

They derive the calculation of the tangential elongation so, when slipping and considering the viscous force, the tangential force gives the value . If viscous force is big enough, then the tangential elongation can be negative.

**Yang Y., Ji S. Energy Conservatino in a Granular Shear Flow and Its Quasi-Solid-Liquid Transition. Particulate Science and Technology 27 (2009) 126-138.**

They study energy conservation. They say that for this kind of studies, the linear model shall be used (it’s more convenient), and probably this is because with the non-linear, the energy is not conserved up today. They use an instantaneous energy balance, and then the errors do not accumulate. They say that:

*It is shown that energy dissipation rate and external power input are in dynamic equilibrium and have similar average values. So energy is conserved and the granular system exhibits a steady-state flow.*

So, the instantaneous energy balance equation, is not exactly fulfilled (have similar average values). They use , with kinetic energy, potential energy in normal and tangential springs (linear model), is the external power input, and is the dissipated energy.

**Wojtkowski M., Pecen J., Horabik J., Molenda M. Rapeseed impact against a flat surface: Physical testing and DEM simulation with two contact models. Powder Technology 198 (2010) 61-68.**

They compare a hysteretic elastoplastic model with a non linear viscoelastic model. Their conclusions is that there exists some indications where to use one model or the other. One indication is dry (elastoplastic) or wet (viscoelastic) seeds.

**Thornton C., Cummins S.J., Cleary P.W. An investigation of the comparative behavior of alternative contact force models during elastic collisions. Powder Technology 210 (2011) 189-197.**

Only deals with elastic collisions. There is no viscous or plastic dissipation.

With rigid body dynamics, gets the non-dimensional parameter to be represented.

The HMD model is the normal force-displacement due to Hertz, and the theory of Mindlin Deresiewicz for the tangential force-displacement relationship.

This is the model taken as the reference to compare the others.

The LS is the linear model in normal and tangential directions. The relationship between normal and tangential stiffnesses is studied, and they conclude that the value of has to be the correct one from tangential compliance. They give the expression for similar materials. For dissimilar materials is:

The HM model is the hertz in normal direction, and the integral non-slip theory of Mindlin. They use the incremental approach, and study how the integral approach is not correct.

Also they analyze the LHT or the Langston, Tüzun and Heyes. The results with this model are the worst of all.

In the results sections, the following findings are important:

In the LS model, the kappa value is the one given by the expression (10). The kappa = 2/7 is not physically correct.

Comparing the different models, there is an excellent agreement between HMD and LS. The HM is not as good. The HMns from DiRenzo and DiMaio, is an integral method and they analyze after this point. The LTH method also an integral one, differ significantly, and they conclude that it should not be used.

When analyzing the HMns, they see that do create energy, and it should not be used also.

They also comment the tangential displacement correction when slipping occurs.

In their conclusions, they say that LS performs better, but as it has constant collision duration, the HM can be in some cases better. Also, in the appendix A, they analyze the tangential displacement and they consider the deformation part + the sliding part.