Department of Computer Science, University College London

COMP0043 Numerical Methods for Finance

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48 hours take-home paper 2022–23

Monte Carlo of the Heston stochastic volatility model

In the Heston stochastic volatility model (1993) the variance evolves with a Feller square-root process (1951), used also in the Cox-Ingersoll-Ross model for the term structure of interest rates (1985). The Heston model is more realistic than the older stochastic volatility model of Stein and Stein (1991), where the variance evolves with an Ornstein-Uhlenbeck process (1930), because the latter mean-reverting process is Gaussian and thus its support is $\mathbb{R}$, while the Feller square-root process bounds the variance to $\mathbb{R}_+$. For the system of stochastic differential equations that describe the Heston model, see Cui et al. (2017), Eq. (1), and Ballotta and Fusai (2018), Eqs. (5.10) and (5.11).

Questions

a) Write a script for the Monte Carlo simulation of the Heston stochastic volatility model combining the script fsrp.m that simulates the Feller square root process and the script gbp.m that simulates geometric Brownian motion. The generation of correlated normal random numbers is described in "Tools from Stochastic Analysis for Mathematical Finance: A Gentle Introduction, Chapter 10.9.7, in Seydel's "Course Notes on Computational Finance", Chapter 2.4, and in Press et al.'s "Numerical Recipes in C++: The Art of Scientific Computing", 3rd edition, Chapter 7.4. Keep the approach basic or naive without low-bias and other advanced techniques that you may find in the literature.

b) Add code to price call and put European options with the Heston model like in the Monte Carlo section of our Black-Scholes pricing script bs.m. Use the parameters of Cui et al. (2017), Table 1.

You may use MATLAB, C++, Python, Julia or another programming language. If you have questions, please ask on the Moodle forum.

References

1. Yiran Cui, Sebastian del Baño Rollin, Guido Germano, "Full and fast calibration of the Heston stochastic volatility model", *European Journal of Operational Research* 263 (2), 625–638, 2017, DOI 10.1016/j.ejor.2017.05.018, available on the Moodle page of COMP0043, Section 8.

2. Laura Ballotta, Gianluca Fusai, "Tools from Stochastic Analysis for Mathematical Finance: A Gentle Introduction", SSRN 3183712, 2018, DOI 10.2139/ssrn.3183712, available on the Moodle page of COMP0043, Section Literature and Code.