Bayesian Compressed Vector Autoregressions

Gary Koop
University of Strathclyde*

Dimitris Korobilis
University of Glasgow†

Davide Pettenuzzo
Brandeis University‡

December 21, 2015

Abstract

Macroeconomists are increasingly working with large Vector Autoregressions (VARs) where the number of parameters vastly exceeds the number of observations. Existing approaches either involve prior shrinkage or the use of factor methods. In this paper, we develop an alternative based on ideas from the compressed regression literature. It involves randomly compressing the explanatory variables prior to analysis. A huge dimensional problem is thus turned into a much smaller, more computationally tractable one. Bayesian model averaging can be done over various compressions, attaching greater weight to compressions which forecast well. In a macroeconomic application involving up to 130 variables, we find compressed VAR methods to forecast better than either factor methods or large VAR methods involving prior shrinkage.

Keywords: multivariate time series, random projection, forecasting

JEL CLassifications: C11, C32, C53

*University of Strathclyde, Department of Economics, Room 4.06, Sir William Duncan Building, 130 Rottenrow, Glasgow G4 0GE, United Kingdom. gary.koop@strath.ac.uk

†University of Glasgow, Adam Smith Business School, Room 204c, Gilbert Scott building, Glasgow, G12 8QQ, United Kingdom. Dimitris.Korobilis@glasgow.ac.uk

‡Brandeis University, Sachar International Center, 415 South St, Waltham, MA. dpettenu@brandeis.edu

1 Introduction

Vector autoregressions (VARs) have been an important tool in macroeconomics since the seminal work of Sims (1980). Recently, many researchers in macroeconomics and finance have been using large VARs involving dozens or hundreds of dependent variables (see, among many others, Banbura, Giannone and Reichlin, 2010, Carriero, Kapetanios and Marcellino, 2009, Koop, 2013, Koop and Korobilis, 2013, Korobilis, 2013, Giannone, Lenza, Momferatou and Onorante, 2014 and Gefang, 2014). Such models often have much more parameters than observations, over-fit the data in-sample, and, as a consequence, forecast poorly out-of-sample. Researchers working in the literature typically use prior shrinkage on the parameters to overcome such over-parameterization concerns. The Minnesota prior is particularly popular, but other approaches such as the LASSO (least absolute shrinkage and selection operator, see Park and Casella, 2008 and Gefang, 2014) and SSVS (stochastic search variable selection, see George, Sun and Ni, 2008) have also been used. Most flexible Bayesian priors that result in shrinkage of high-dimensional parameter spaces rely on computationally intensive Markov Chain Monte Carlo (MCMC) methods and their application to recursive forecasting exercises can, as a consequence, be prohibitive or even infeasible. The only exception is a variant of the Minnesota prior that is based on the natural conjugate prior, an idea that has recently been exploited by Banbura, Giannone and Reichlin (2010) and Giannone, Lenza and Primiceri (2015), among others. While this variant allows for analytical formulas for the regularized parameter posterior, there is a cost in terms of flexibility in that a priori all VAR equations are assumed to be symmetric; see Koop and Korobilis (2010) for a further discussion of this issue with the natural conjugate prior.

An alternative approach to shrinkage is achieved by compressing the data instead of the parameters. A considerably smaller set of unobserved variables (factors) is estimated at a first step, and then a lower dimensional VAR model is specified on these factors. In economics, principal components (PC) methods are commonly used (see, for instance, the factor augmented VAR, FAVAR, of Bernanke, Boivin and Eliasz, 2005 or the dynamic factor model, DFM, of, e.g., Stock and Watson, 2002). The gains in computation from such an approach are quite significant, since principal components are relatively easy to compute and under mild conditions provide consistent estimates of unobserved factors for a wide variety of factor models, including the case of structural instabilities in coefficients (Bates, Plagborg-Møller, Stock and Watson, 2013). However, data compression using principal components is done without reference to the dependent variable in the regression. As a result, estimated factors might lack economic interpretation, and PC is, thus, referred to as an "*unsupervised* data compression method".

In this paper we propose a new quasi-Bayesian procedure for estimating large VARs, inspired by the compressed regression literature (see Guhaniyogi and Dunson, 2014). The desire to work with Big Data[1] using computationally efficient methods has resulted in new data compression methods, now commonly used in various fields (e.g. neuroimaging, machine learning, astronomy). In particular, we use methods that randomly compress the data. Similar to PC these compression methods project the high-dimensional data into a lower dimensional matrix. In contrast to PC methods, our projections are generated randomly from simple schemes, and are, thus, computationally trivial and do not depend on the information in the original data ("data-oblivious"). Additionally, the data compression of the explanatory variables is done taking into consideration the explanatory power the compressed variables have for the dependent variable using Bayesian model averaging (BMA), making our approach a "*supervised* data compression method". To our knowledge, supervised compressed regression methods of this sort have not been used before in the VAR literature.[2]

We first establish the theory behind random compression, and subsequently we specify our proposed Bayesian Compressed VAR (BCVAR) model for forecasting using Big Data. This model is a natural mutlivariate extension of the compressed regression model of Guhaniyogi and Dunson (2014). We carry out a substantial macroeconomic forecasting exercise involving VARs with up to 129 dependent variables and 13 lags. We compare the forecasting performance of seven key macroeconomic variables using the BCVAR, to various popular alternatives: univariate AR models, the FAVAR, and the Minnesota prior VAR (implemented as in Banbura, Giannone and Reichlin, 2010). Our results are encouraging for the BCVAR, showing substantial forecast improvements in many cases and comparable forecast performance in the remainder.

[1]Big data comes in two forms that are often called Tall and Fat. Tall data involves a huge number of observations, whereas Fat data involves a huge number of variables. In this paper, we fall in the Fat data part of the literature.

[2]Carriero, Kapetanios and Marcellino (2015) use a reduced rank VAR framework they refer to as a multivariate autoregressive index model that shares similarities with the compressed regressions used in this paper. However, they use computationally-burdensome MCMC methods which would likely preclude their use in very high dimensional models.

2 The Theory and Practice of Random Compression

Random compression methods have been used in fields such as machine learning and image recognition as a way of projecting a huge number of variables into a lower dimensional space, with minimal distortion of the properties of the original space. In this way, they are similar to PC methods which take as inputs many variables and produce orthogonal factors. Random compression is similar in that respect to PC, but is computationally simpler and capable of dealing with massively huge numbers of variables. For instance, in a regression context, Guhaniyogi and Dunson (2014) have an application involving 84,363 explanatory variables.

To fix the basic ideas of random compression, let X be a $T \times k$ data matrix involving T observations on k variables where $k >> T$. X_t is the t^{th} row of X. Define the projection matrix, Φ, which is $m \times k$ with $m << k$ and $\widetilde{X}_t' = \Phi X_t'$. Then $\widetilde{X}_t$ is the t^{th} row of the compressed data matrix, $\widetilde{X}$. Since $\widetilde{X}$ has m columns and X has k, the former is much smaller and is much easier to work with in the context of a statistical model such as a regression or a VAR. The question is: what information is lost by compressing the data in this fashion? The answer is that, under certain conditions, the loss of information may be small. The underlying motivation for compression arises from the Johnson-Lindenstrauss lemma (see Johnson and Lindenstrauss, 1984). This states that any k point subset of Euclidean space can be embedded in $m = O\left(\log(k)/\epsilon^2\right)$ dimensions without distorting the distances between any pair of points by more than a factor of $1 \pm \epsilon$ for any $0 < \epsilon < 1$.

The random compression literature recommends treating Φ as a random matrix and drawing $\Phi^{(r)}$ for $r = 1,..,R$ in some fashion. A key early paper is Achlioptas (2003) which provides theoretical justification for various ways of drawing Φ in a computationally-trivial manner. One scheme, which we use in our empirical work, is to draw Φ_{ij}, which is the ij^{th} element of Φ, $i = 1,..,m$, $j = 1,..,k$ from the following distribution:

$$\begin{array}{l} \Pr\left(\Phi_{ij} = \frac{1}{\sqrt{\varphi}}\right) = \varphi^2 \\ \Pr\left(\Phi_{ij} = 0\right) = 2\left(1-\varphi\right)\varphi \\ \Pr\left(\Phi_{ij} = -\frac{1}{\sqrt{\varphi}}\right) = \left(1-\varphi\right)^2 \end{array}, \tag{1}$$

where φ and m are unknown parameters. Theory suggests that Φ should be a random matrix whose columns have unit lengths and, hence, Gram-Schmidt orthonormalization is done on the rows of the matrix Φ.

Such methods are referred to as data oblivious since Φ is drawn without reference to the data. But the statistical theory proves that even data oblivious random compression can lead to good properties. For instance, in the compressed regression model, Guhaniyogi and Dunson (2014) provide proofs of its theoretical properties asymptotically in T and k. Under some weak assumptions, the most significant relating to sparsity (e.g. on how fast m can grow relative to k as sample size increases), they show that their Bayesian compression regression algorithm produces a predictive density which converges to the true predictive density. The convergence rate depends on how fast m and k grow with T. With some loose restrictions on this, they obtain near parametric rates of convergence to the true predictive density. In a simulation study and empirical work, they document excellent coverage properties of predictive intervals and large computational savings relative to popular alternatives. In the large VAR, there is likely to be a high degree of sparsity since most VAR coefficients are likely to be zero, especially in very large dimensions or for more distant lag lengths. In such a case, the theoretical results of Guhaniyogi and Dunson (2014) suggest fast convergence should occur, and computational benefits will be great.

These desirable properties of random compression hold even for a single, data-oblivious, random draw of Φ. However, random compression methods result in obtaining a lower dimensional representation of our data which might not be optimal in a mean-square error sense. In practice, when working with random compression several random draws have to be taken and then averaged. In compressed regression, Guhaniyogi and Dunson (2014) do BMA. They treat each $\Phi^{(r)}$ for $r = 1, .., R$ as defining a new model, calculate the marginal likelihood for each model, and average across models using weights proportional to the marginal likelihoods. m and φ can be simulated in the BMA exercise. Guhaniyogi and Dunson (2014) recommend simulating φ from the $U[a, b]$ distribution, where a (b) is set to a number slightly above zero (below one) to ensure numerical stability. They simulate their m from the $U[2\log(k), \min(T, k)]$ distribution.

To see precisely how this works in a regression context, let y_t be the dependent variable and consider the regression:

$$y_t = X_t \alpha + \varepsilon_t. \tag{2}$$

If $k >> T$, then working directly with (2) is impossible with some statistical methods (e.g. maximum likelihood estimation) and computationally demanding with others (e.g. Bayesian approaches which require the use of MCMC methods). Some of the computational burden can arise simply due to the need to store in memory huge data matrices. Manipulating such data matrices even a single time can be very demanding. For instance, calculation of the Bayesian posterior mean under a natural conjugate prior requires, among other manipulations, inversion of a $k \times k$ matrix involving the data. This can be difficult if k is huge.

However, the compressed regression variant of (2) is

$$y_t = (\Phi X_t)\,\beta + \varepsilon_t. \tag{3}$$

Thus, the k explanatory variables are squeezed into the lower dimensional ΦX_t. For a given Φ, the problem is reduced to the very simple one of estimating a regression with a small number of explanatory variables given by $\widetilde{X}_t = \Phi X_t$. Once the explanatory variables have been compressed, standard Bayesian regression methods can be used for the regression of y_t on $\widetilde{X}_t$. If a natural conjugate prior is used, then analytical formulae exist for the posterior, marginal likelihood and predictive density and computation is trivial.

It is clear that there are huge computational gains by adopting specification (3) instead of (2). In addition, the use of BMA will ensure that bad compressions (i.e. those that lead to the loss of information important for explaining y_t) are avoided or down-weighted. To provide some more intuition, note that if we were to interpret m and φ and, thus, Φ, as random parameters (instead of specification choices defining a particular compressed regression), then BMA can be interpreted as importance sampling. That is, the $U[a,b]$ and $U[2\log(k), \min(T,k)]$ distributions that Guhaniyogi and Dunson (2014) use for drawing φ and m, respectively, can be interpreted as importance functions. Importance sampling weights are proportional to the posterior for m and φ. But this is equivalent to the marginal likelihood which arises if Φ is interpreted as defining a model. Thus importance sampling is equivalent to BMA. In the same manner that importance sampling attaches more weight to draws from high regions of posterior probability, doing BMA with randomly compressed regressions attaches more weight to good draws of Φ which have high marginal likelihoods.

In a VAR context, doing BMA across models should only improve empirical performance since this will lead to more weight being attached to choices of Φ which are effective in explaining the dependent variables. Methods with this property are referred to as supervised dimension reduction techniques, in contrast with unsupervised techniques such as principal components. It is likely that supervised methods such as this should forecast better than unsupervised methods, a point we investigate in our empirical work.

In summary, for a given compression matrix, Φ, the huge dimensional data matrix is compressed into a much lower dimension. This compressed data matrix can then be used in a statistical model such as a VAR. The theoretical statistical literature on random compression has developed methods such as (1) for randomly drawing the compression matrix and showed them to have desirable properties under weak conditions which are likely to hold in large VARs. By averaging over different draws for Φ (which can differ both in terms of m and φ) BMA can be done. All this can be done in a computationally simple manner, working only with models of low dimension.

3 Random Compression of VARs

We start with the standard reduced form VAR model,[3]

$$Y_t = BY_{t-1} + \epsilon_t$$

where Y_t for $t = 1, ..., T$ is an $n \times 1$ vector containing observations on n time series variables, ϵ_t is i.i.d. $\mathcal{N}(0, \Omega)$ and B is an $n \times n$ matrix of coefficients. Note that, with $n = 100$, the uncompressed VAR will have $10,000$ coefficients in B and $5,050$ in Ω. In a VAR(13), such as the one used in this paper, the former number becomes $130,000$. It is easy to see why computation can become daunting in large VARs and why there is a need for shrinkage.

To compress the explanatory variables in the VAR, we can use the matrix Φ given in (1) but now it will be an $m \times n$ matrix where $m << n$. In a similar fashion to (3), we can define the compressed VAR:

$$Y_t = B^c (\Phi Y_{t-1}) + \epsilon_t, \tag{4}$$

where B^c is $m \times n$. Thus, we can draw upon the motivations and theorems of, e.g., Guhaniyogi and Dunson (2014) to offer theoretical backing for the compressed VAR. If a natural conjugate prior is used, the posterior, marginal likelihood and predictive density of the compressed VAR in (4) have familiar analytical forms (see, e.g., Koop and Korobilis, 2009) for a given draw of Φ. These, along with a method for drawing Φ, are all that are required to forecast with the

[3]For notational simplicity, we explain our methods using a VAR(1) with no deterministic terms. These can be added in a straightforward fashion. In our empirical work, we have monthly data and use 13 lags and an intercept.

BCVAR. And, if m is small, the necessary computations of the natural conjugate BCVAR are straightforward.

However, the natural conjugate prior has some well-known restrictive properties in VARs.[4] In the context of the compressed VAR, working with a Φ of dimension $m \times n$ as defined in (4), with only n columns instead of n^2 would likely be much too restrictive in many empirical contexts. For instance, it would imply if you want to delete a variable in one equation, then that same variable must be deleted in all equations. In macroeconomic VARs, where the first own lag in each equation is often found to have important explanatory power, such a property is problematic. It would imply, say, that lagged money could either be included in every equation or none when what we really want is for lagged money to be included in the inflation equation but not most of the other equations in the VAR.

An additional problem with the natural conjugate BCVAR is that it does not compress the error covariance matrix. In large VARs, Ω contains a large number of parameters and we want a method which allows for their compression. This issue does not arise in the regression model but is potentially very important in large VARs. These considerations motivate working with a re-parameterized version of the VAR. In particular, following common practice (see, e.g., Primiceri, 2005, Eisenstat, Chan and Strachan, 2015 and Carriero, Clark and Marcellino, 2015) we use a triangular decomposition of Ω:

$$A\Omega A' = \Sigma\Sigma,$$

where Σ is a diagonal matrix with diagonal elements σ_i and A is a lower triangular matrix with ones on the diagonal. We can write $A = I + \widetilde{A}$, where $\widetilde{A}$ is lower triangular but with zeros on the diagonal.

Using these definitions, we can write the VAR in structural form as

$$\begin{aligned} Y_t &= \Gamma Y_{t-1} + \widetilde{A}\left(-Y_t\right) + \Sigma E_t \\ &= \Theta Z_t + \Sigma E_t \end{aligned} \tag{5}$$

where $E_t \sim N\left(0, I_n\right)$, $Z_t = \left[Y_{t-1}, -\widetilde{Y}_t\right]$ and $\Theta = \left[\Gamma, \widetilde{A}\right]$. $\widetilde{Y}_t$ is defined taking into account the lower triangular structure of $\widetilde{A}$ so that the first equation of the VAR includes only Y_{t-1} as explanatory variables, the second includes $(Y_{t-1}, -Y_{1,t})$, the third includes $(Y_{t-1}, -Y_{1,t}, -Y_{2,t})$,

[4] These are summarized on pages 279-280 of Koop and Koroblis (2009).

etc. Note that this lower triangular structure along with the diagonality of Σ means that equation-by-equation estimation of the VAR can be done, a fact we exploit in our algorithm. Furthermore, since the elements of $\widetilde{A}$ control the error covariances, by doing compression in the model in (5) we can compress the error covariances as well as the reduced form VAR coefficients.

The simplest way of doing compression in (5) is through the following specification:

$$Y_t = \Theta^c (\Phi Z_t) + \Sigma E_t \tag{6}$$

where Φ is now an $m \times (2n-1)$ random compression matrix (and $m \times (n + pn - 1)$ in the VAR with p lags). However, it shares the disadvantage of the natural conjugate approach that it compresses an explanatory variable in the same way in every equation. Accordingly, we use another BCVAR where each equation has its own random compression matrix:

$$Y_{t,i} = \Theta_i^c (\Phi_i Z_{i,t}) + \sigma_i E_{i,t}. \tag{7}$$

Having n compression matrices (each of potentially different dimension and with different randomly drawn elements) does raise the computational burden somewhat, but has the advantage that explanatory variables in different equations can be compressed in different ways.

For given draws of Θ^c or Θ_i^c (for $i = 1, .., n$), estimation and prediction can be done in a computationally-fast fashion using a variety of methods since each model will be of low dimension and, for the reasons discussed previously, all these can be done one equation at a time. In the empirical work in this paper, we use standard Bayesian methods suggested in Zellner (1971) for the seemingly unrelated regressions model. In particular, for each equation we use the prior:

$$\begin{aligned} \Theta_i^c | \sigma_i^2 &\sim N\left(\underline{\Theta}_i^c, \sigma_i^2 \underline{V}_i\right) \\ \sigma_i^{-2} &\sim G\left(\underline{s}_i^{-2}, \underline{\nu}_i\right), \end{aligned} \tag{8}$$

where $G\left(\underline{s}_i^{-2}, \underline{\nu}_i\right)$ denotes the Gamma distribution with mean $\underline{s}_i^{-2}$ and degrees of freedom $\underline{\nu}_i$. In our empirical work, we set set $\underline{\Theta}_i^c = 0$, $\underline{V}_i = 0.5 \times I$ and, for σ_i^{-2} use the non-informative version of the prior (i.e. $\underline{\nu}_i = 0$). We then use familiar Bayesian results for the Normal linear regression model (e.g. Koop, 2003, page 37) to obtain an analytical posterior. The one-step ahead predictive density is also available analytically. However, h-step ahead predictive densities for $h > 1$ are not available analytically. Point forecasts can be iterated forward in the usual fashion, but predictive simulation is required to produce h-step ahead predictive densities. We also consider a version of the model where the error covariances are not compressed. In this case, we use Normal priors with large variances for the inverse of the error covariances.

We do BMA by taking a weighted average over predictions using $\Phi_i^{(r)}$ for $r = 1, .., R$ in the same manner as Guhaniyogi and Dunson (2014) as described in the preceding section. The only difference is that we use the Bayesian information criterion (BIC) to construct model probabilities used in the weighted average instead of the marginal likelihood.[5] To be precise, conditional on $\Phi_i^{(r)}$, we can approximate the marginal likelihood of model r using the exponential of the BIC. BMA weights are these approximate marginal likelihoods normalized to sum to one.

Several different ways of drawing Φ_i have been proposed in the literature (see, e.g., Achlioptas, 2003). We have described the implementation of Guhaniyogi and Dunson (2014) in the preceding section. Note that this requires draws of φ and m. In the BCVAR, for the former we draw from the $U[0.1, 0.8]$ and for the latter from the discrete $U[1, 5\ln(k_i)]$ where k_i is the number of explanatory variables $Z_{i,t}$. We note in passing that Achlioptas (2003) also considers simpler random projection schemes such as

$$\Phi_{ij} \sim N(0,1),$$

as well as the sparse random projection scheme

$$\Phi_{ij} = \begin{cases} -\sqrt{s} & \text{, with probability} \quad \frac{1}{2s} \\ 0 & \text{, with probability} \quad 1 - \frac{1}{s} \\ \sqrt{s} & \text{, with probability} \quad \frac{1}{2s} \end{cases}$$

for a constant s (e.g. $s = 3$). In our macroeconomic application, we found that these different random projection schemes produced very similar forecasts and present results for the Guhaniyogi and Dunson (2014) approach.

4 Alternative methods

The preceding sections described our implementation of the BCVAR. However, we wish to compare its performance to popular alternatives. Reasoning that previous work with large numbers of dependent variables have typically used factor methods or large VAR methods using the Minnesota prior, we focus on these. We compare forecasts using all of these methods to a benchmark approach which uses OLS forecasts from individual AR(1) models.

[5] We use the BIC instead of the marginal likelihood to allay worries about prior sensitivity and since, in extensions of the present approach, the marginal likelihood will not be available in closed form.

4.1 The FAVAR

We use the FAVAR of Bernanke, Boivin, Eliasz (2005) dividing Y_t into a set of primary variables of interest, Y_t^*, and the remainder $\widetilde{Y}_t$ and working with the model:

$$
\begin{aligned}
\widetilde{Y}_t &= \Lambda F_t + \epsilon_t^Y \\
\begin{bmatrix} F_t \\ Y_t^* \end{bmatrix} &= B \begin{bmatrix} F_{t-1} \\ Y_{t-1}^* \end{bmatrix} + \epsilon_t^*.
\end{aligned}
$$

We use principal components methods to estimate the factors and noninformative prior Bayesian methods. We use BIC to select the optimal lag length and factor dimension at each point in time.

4.2 Bayesian VAR using the Minnesota Prior

We follow closely Banbura et al (2010)'s implementation of the Minnesota prior VAR which involves a single prior shrinkage parameter, ω. However, we select ω in a different manner than Banbura et al (2010). In particular, we estimate ω relying on marginal likelihoods. Giannone, Lenza and Primiceri (2015) propose a formal procedure in order to choose the value of ω that maximizes the data marginal likelihood. In our case, in order to maintain comparability with the way we implement model averaging on BCVAR, we use a computationally simpler alternative. We choose a grid of values for the inverse of the shrinkage factor which goes from $0.5 \times \sqrt{np}$ to $10 \times \sqrt{np}$ with increments of $0.1 \times \sqrt{np}$. At each forecast period, we use BIC to choose an optimal degree of shrinkage. All remaining specification and forecasting choices are exactly the same as in Banbura et al (2010) and, hence, are not given here. In our empirical results, we use the acronym BVAR to refer to this approach.

5 Forecasting with Large Macroeconomic Data Sets

5.1 Data

We use the FRED-MD data base of monthly US variables from January 1960 through December 2014. The reader is referred to McCracken and Ng (2015) for a description of this macroeconomic data set which includes a range of variables from a broad range of categories (e.g. output, capacity, employment and unemployment, prices, wages, housing, inventories and orders, stock prices, interest rates, exchange rates and monetary aggregates). We use the 129 variables for which complete data was available, after transforming all variables using the transformation codes provided in the appendix. We present forecasting results for seven variables of interest: the growth of industrial production (INDPRO), the unemployment rate (UNRATE), employment growth (PAYEMS), the Fed funds rate (FEDFUNDS), the 10 year T-bill rate (GS10), producer price inflation (PPIFGS) and consumer price inflation (CPIAUCSL). We present results for VARs of different dimensions and these seven variables are included in all of our VARs. These are also the variables which we label $\widetilde{Y}_t$ in the FAVAR. We have a Medium VAR with 19 variables, a Large VAR with 46 variables and a Huge VAR will all 129 variables. A listing of all variables (including which appear in which VAR) is given in the appendix. Note that most of our variables have substantial persistence in them and, accordingly, the first own lag in each equation almost always has important explanatory power. Accordingly, we do not compress the first own lag. This is included in every equation, with compression being done on the remaining variables. We also standardize our variables at each point time by subtracting off the mean and dividing by the standard deviation in a recursive manner.

5.2 Forecasting Results

We start our recursive forecasting exercise in July 1987 (i.e. after the first half of the sample) and use the remaining half of the sample to evaluate the forecasts. We evaluate the quality of point forecasts using Mean Squared Forecast Errors (MSFEs) and the quality of the predictive densities using log scores. We choose a relatively large value for lag length ($p = 13$), trusting in the compression or shrinkage of the various methods to remove unnecessary lags. All forecasts are iterated when $h > 1$ and predictive simulation is used to produce the predictive density.

We present results using our compressed VAR methods in two ways: the first of these, which compresses both the VAR coefficients and the error covariances as in (7) is labelled BCVAR$_C$ in the tables. The second, which is the same but does not compress the error covariances, is labelled BCVAR. We also present results for the BVAR and FAVAR as described in Section 4.

Tables 1 to 3 present evidence on the quality of our point forecasts. In general, we are finding that BCVARs tend to forecast best. This holds, with several exceptions, for every VAR dimension, variable and forecast horizon. Random compression of the VAR coefficients is leading to improvements in forecast performance. Evidence relating to compression of the error covariance is more mixed. That is, sometimes it is the BCVAR which forecasts best and other times it is BCVAR$_C$. But for the Huge VAR, where the dimensionality of the error covariance matrix is huge, there are clear benefits in compressing error covariances as well as VAR coefficients.

One notable pattern in these tables is that BCVAR and BCVAR_C are (with some exceptions) forecasting particularly well for prices and short term interest rates (i.e. CPIAUCSL, FEDFUNDS and PPIFIGS). It is also interesting to note that for UNEMP all of our approaches are doing much better than the AR(1) benchmark. In contrast, for the long-term interest rate (GS10), our Huge or Large VAR methods are almost never beating the benchmark. But at least in this case, where small models are forecasting well, it is reassuring to see that MSFEs obtained using random compression methods are only slightly worse than the benchmark ones. This indicates that random compression methods are finding that the GS10 equation in the Huge VAR is hugely over-parameterized, but is successfully compressing the explanatory variables so as obtain results that are nearly the same as those from parsimonious univariate models.

Figures 1 to 6 present evidence on when the forecasting gains of BCVARs relative to the other approaches are achieved. These plots the cumulative sum of squared forecasting errors for the benchmark minus those for a competing approach (i.e. positive values for this metric imply that an approach is beating the benchmark) for different sized VARs, different variables and different forecasting horizons. For the sake of brevity, we only present results for $h = 1$ and 12, with intermediate forecast horizons tending to display similar patterns. With some exceptions, a few main stories come out of these figures. First, prior to 2000, it is often the case that all methods are forecasting roughly the same. It is only after 2000 that the benefits of random compression come to be seen. Second, random compression methods usually handle the financial crisis better than other approaches. Note, in particular, that the forecast performance of the BVAR deteriorates substantially for GS10 and FEDFUNDS around the time of the financial crisis. In contrast, the poor forecast performance of the FAVAR for CPIAUCSL is not solely associated with the financial crisis, but tends to occur throughout our forecast evaluation period.

Tables 4 to 6 present the average log predictive likelihood differentials for the VARs of different dimensions, while Figures 7 to 12 plot the cumulative sums of log predictive likelihood differentials for the different VARs. These plots show the cumulative sum of log predictive likelihoods generated by a competing approach minus the sum of log predictive likelihoods generated by the AR(1) model (i.e. positive values for this metric imply that an approach is beating the benchmark) for different sized VARs, different variables and different forecasting horizons. Results for the average log predictive likelihood differentials in Tables 4 to 6 are similar to those from the MSFE ratios reported in Tables 1 to 3, but evidence in favor of random compression is slightly weaker than what we found with MSFEs. However, we find a similar pattern with the financial crisis playing a particularly important role in the good forecast performance of BCVAR and BCVAR$_C$. Note, in particular, how for CPIAUCSL, INDPRO, and PPIFGS the forecast performance of the Minnesota prior BVAR deteriorates dramatically in 2008–2009 while our BCVAR or BCVAR$_C$ predictive likelihoods do not do so. As Figures 7 to 12 show, this pattern is present for all VARs dimensions, and holds both at very short horizons as well as at longer ones. In general, our compressed VAR approaches may not be best in every case, but even when they are not they are close to the best.

Finally, it is worth stressing that this section is simply comparing the forecast performance of different plausible methods for a particular data set. However, the decision whether to use compression methods should not be based solely on this forecasting comparison. In other, larger applications, plausible alternatives to random compression such as the Minnesota prior BVAR or any VAR approach which requires the use of MCMC methods, may simply be computationally infeasible. Random compression works well using the present data set, it may be the only thing which is computationally feasible in larger data sets.

6 Conclusions

In this paper, we have drawn on ideas from the random projection literature to develop methods suitable for use with large VARs. For such methods to be suitable, they must be computationally simple, theoretically justifiable and empirically successful. We argue that the BCVAR methods developed in this paper meet all these goals. In a substantial macroeconomic application, involving VARs with up to 129 variables, we find BCVAR methods to be fast and yield results which are at least as good as or better than competing approaches. And, in contrast to the Minnesota prior BVAR, BCVAR methods can easily be scaled up to much higher dimensional models.

References

Achlioptas, D. (2003). Database-friendly random projections: Johnson-Lindenstrauss with binary coins. Journal of Computer and System Sciences, 66, 671–687.

Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. Econometrica, 70, 191-221.

Banbura, M., Giannone, D. and Reichlin, L. (2010). Large Bayesian vector autoregressions. Journal of Applied Econometrics, 25, 71-92.

Bates, B. J., Plagborg-Møller, M., Stock, J. H. and Watson, M. W. (2013). Consistent factor estimation in dynamic factor models with structural instability. Journal of Econometrics, Available online 17 April 2013, doi: 10.1016/j.jeconom.2013.04.014

Bernanke, B., Boivin, J., Eliasz, P. (2005). Measuring monetary policy: A factor augmented vector autoregressive (FAVAR) approach. Quarterly Journal of Economics 120, 387-422.

Carriero, A., Clark, T. and Marcellino, M. (2015). Large Vector Autoregressions with asymmetric priors and time varying volatilities, manuscript.

Carriero, A., Kapetanios, G. and Marcellino, M. (2009). Forecasting exchange rates with a large Bayesian VAR. International Journal of Forecasting, 25, 400-417.

Carriero, A., Kapetanios, G. and Marcellino, M. (2015). Structural analysis with multivariate autoregressive index models. Centre for Economic Policy Research Discussion Paper DP10801.

Eisenstat, E., Chan, J. and Strachan, R. (2015). Stochastic model specification search for time-varying parameter VARs, Econometric Reviews, forthcoming.

Gefang, D. (2014). Bayesian Doubly Adaptive elastic-net Lasso for VAR Shrinkage. International Journal of Forecasting, 30, 1-11.

George, E., Sun, D. and Ni, S. (2008). Bayesian stochastic search for VAR model restrictions. Journal of Econometrics, 142, 553-580.

Giannone, D., Lenza, M., Momferatou, D. and Onorante, L. (2014). Short-term inflation projections: a Bayesian vector autoregressive approach. International Journal of Forecasting, 30, 635-644.

Giannone, D., Lenza, M. and Primiceri, G. (2015). Prior selection for Vector Autoregressions. Review of Economics and Statistics, 97, 436-451.

Guhaniyogi, R. and Dunson, D. (2014). Bayesian compressed regression. Journal of the American Statistical Association, DOI: 10.1080/01621459.2014.969425.

Johnson, W. and Lindenstrauss, J. (1984). Extensions of Lipshitz mapping into Hilbert space. Contemporary Mathematics, 26, 189-206.

Kadiyala, K. and Karlsson, S. (1997). Numerical methods for estimation and inference in

Bayesian VAR models. Journal of Applied Econometrics, 12, 99-132.

Koop, G. (2008). Bayesian Econometrics. Chichester: John Wiley and Sons.

Koop, G. (2013). Forecasting with medium and large Bayesian VARs. Journal of Applied Econometrics, 28, 177-203.

Koop, G. and Korobilis, D. (2009). Bayesian multivariate time series methods for empirical macroeconomics. Foundations and Trends in Econometrics, 3, 267-358.

Koop, G. and Korobilis, D. (2013). Large time-varying parameter VARs. Journal of Econometrics, 177, 185-198.

Koop, G. and Potter, S. (2004). Forecasting in dynamic factor models using Bayesian model averaging. The Econometrics Journal, 7, 550-565.

Korobilis, D. (2013). VAR forecasting using Bayesian variable selection. Journal of Applied Econometrics, 28, 204-230.

McCracken, M. and Ng, S. (2015). FRED-MD: A Monthly Database for Macroeconomic Research. Federal Reserve Bank of St. Louis, working paper 2015-012A.

Park, T. and Casella, G. (2008). The Bayesian Lasso. Journal of the American Statistical Association, 103, 681-686.

Primiceri. G., (2005). Time varying structural vector autoregressions and monetary policy. Review of Economic Studies, 72, 821-852.

Sims, C. (1980). Macroeconomics and reality. Econometrica, 48, 1-48.

Stock, J. and Watson, M. (2002). Macroeconomic forecasting using diffusion indexes. Journal of Business & Economic Statistics, 20,147-162.

Zellner, A. (1971). An Introduction to Bayesian Inference in Econometrics. New York: John Wiley and Sons.

Table 1. Out-of-sample forecast performance, Medium VAR

Variable	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*
	$h=1$				$h=2$			
PAYEMS	1.270	0.892	**0.799**	0.813	0.923	**0.571**	0.688	0.711
CPIAUCSL	1.078	**0.925**	0.951	0.935	1.175	0.964	0.940	**0.925**
FEDFUNDS	1.413	2.727	1.027	**0.952**	1.055	2.437	**0.999**	1.002
INDPRO	0.869	**0.821**	0.830	0.894	0.914	**0.835**	0.914	0.934
UNRATE	0.767	0.764	**0.762**	0.798	0.643	**0.582**	0.626	0.662
PPIFGS	0.990	0.968	**0.966**	0.982	1.041	1.060	1.030	**1.000**
GS10	1.015	1.095	1.007	**0.997**	1.028	1.082	**0.995**	1.002
	$h=3$				$h=6$			
PAYEMS	0.790	**0.530**	0.630	0.649	0.763	**0.687**	0.694	0.703
CPIAUCSL	1.240	0.993	0.969	**0.956**	1.291	0.983	**0.962**	0.967
FEDFUNDS	**0.934**	1.862	1.063	1.030	**0.860**	1.230	0.976	0.987
INDPRO	0.932	0.924	**0.917**	0.939	0.966	1.022	0.965	**0.963**
UNRATE	0.625	**0.526**	0.563	0.603	0.639	**0.512**	0.535	0.575
PPIFGS	1.042	1.070	1.051	**1.026**	1.061	1.092	1.046	**1.042**
GS10	1.048	1.144	1.058	**1.031**	**1.032**	1.123	1.054	1.038
	$h=9$				$h=12$			
PAYEMS	**0.778**	0.798	0.779	0.798	**0.815**	0.901	0.887	0.893
CPIAUCSL	1.292	0.968	0.956	**0.943**	1.273	0.985	**0.966**	0.970
FEDFUNDS	0.907	1.065	**0.904**	0.925	**0.957**	1.135	0.960	0.990
INDPRO	0.991	1.022	**0.953**	0.969	0.997	0.992	**0.965**	0.975
UNRATE	0.675	0.569	**0.562**	0.601	0.704	0.640	**0.601**	0.636
PPIFGS	1.053	1.075	1.051	**1.031**	1.054	1.105	1.062	**1.046**
GS10	1.030	1.047	1.023	**1.016**	**1.017**	1.054	1.031	1.019

This table reports the ratio between the MSFE of model i for the Medium size VAR and the MSFE of the benchmark AR(1), computed as

$$MSFE_{ijh} = \frac{\sum_{\tau=\underline{t}}^{\bar{t}-h} e^2_{i,j,\tau+h}}{\sum_{\tau=\underline{t}}^{\bar{t}-h} e^2_{bcmk,j,\tau+h}},$$

where $e^2_{i,j,\tau+h}$ and $e^2_{bcmk,j,\tau+h}$ are the squared forecast errors of variable j at time τ and forecast horizon h generated by model i and the AR(1) model, respectively. $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period, $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, and $h \in \{1, 2, 3, 6, 9, 12\}$. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Bold numbers indicate the lowest MSFE across all models for a given variable-forecast horizon pair.

Table 2. Out-of-sample point forecast performance, Large VAR

Variable	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*
	$h=1$				$h=2$			
PAYEMS	1.106	**0.788**	0.888	0.905	0.840	**0.521**	0.805	0.837
CPIAUCSL	1.188	1.009	0.998	**0.953**	1.186	1.110	0.943	**0.909**
FEDFUNDS	1.728	2.461	**1.093**	1.103	1.438	2.594	**0.991**	1.150
INDPRO	0.885	**0.783**	0.838	0.914	0.967	**0.772**	0.934	0.929
UNRATE	**0.750**	0.824	0.798	0.855	**0.615**	0.666	0.722	0.758
PPIFGS	1.032	1.045	0.987	**0.986**	1.052	1.162	1.012	**1.004**
GS10	1.011	1.106	1.006	**0.992**	1.050	1.149	1.052	**1.044**
	$h=3$				$h=6$			
PAYEMS	0.871	**0.489**	0.757	0.748	0.911	**0.647**	0.773	0.762
CPIAUCSL	1.159	1.154	0.948	**0.913**	1.101	0.999	0.910	**0.902**
FEDFUNDS	1.098	2.241	**1.034**	1.108	**0.947**	1.224	1.098	1.088
INDPRO	0.967	**0.862**	0.957	0.952	0.974	0.980	**0.959**	0.984
UNRATE	**0.600**	0.615	0.677	0.731	0.634	**0.617**	0.671	0.712
PPIFGS	1.031	1.177	1.021	**1.013**	1.029	1.095	1.012	**0.998**
GS10	**1.036**	1.222	1.057	1.059	**1.020**	1.115	1.043	1.029
	$h=9$				$h=12$			
PAYEMS	0.959	**0.840**	0.865	0.844	0.993	0.999	0.972	**0.940**
CPIAUCSL	1.092	0.932	0.887	**0.867**	1.085	0.904	0.902	**0.879**
FEDFUNDS	**1.006**	1.139	1.060	1.028	**1.014**	1.259	1.092	1.041
INDPRO	**0.977**	1.018	1.004	0.999	**0.979**	1.056	1.002	1.020
UNRATE	**0.657**	0.715	0.698	0.739	**0.688**	0.831	0.728	0.756
PPIFGS	1.009	1.051	**0.985**	0.992	1.013	1.039	1.004	**0.972**
GS10	**1.009**	1.050	1.009	1.019	1.018	1.054	1.023	**1.014**

This table reports the ratio between the MSFE of model i for the Large size VAR and the MSFE of the benchmark AR(1), computed as

$$MSFE_{ijh} = \frac{\sum_{\tau=\underline{t}}^{\bar{t}-h} e^2_{i,j,\tau+h}}{\sum_{\tau=\underline{t}}^{\bar{t}-h} e^2_{bcmk,j,\tau+h}},$$

where $e^2_{i,j,\tau+h}$ and $e^2_{bcmk,j,\tau+h}$ are the squared forecast errors of variable j at time τ and forecast horizon h generated by model i and the AR(1) model, respectively. $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period, $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, and $h \in \{1,2,3,6,9,12\}$. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Bold numbers indicate the lowest MSFE across all models for a given variable-forecast horizon pair.

Table 3. Out-of-sample point forecast performance, Huge VAR

Variable	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*
	$h=1$				$h=2$			
PAYEMS	1.125	**0.702**	0.761	0.779	0.821	**0.447**	0.641	0.661
CPIAUCSL	1.157	**0.872**	0.947	0.943	1.145	0.939	0.890	**0.879**
FEDFUNDS	1.501	1.984	**0.891**	0.927	1.287	2.187	**0.929**	0.961
INDPRO	0.926	**0.785**	0.856	0.913	0.980	**0.786**	0.924	0.939
UNRATE	0.785	0.883	**0.781**	0.813	0.646	0.646	**0.644**	0.703
PPIFGS	1.021	**0.947**	0.985	1.004	1.054	1.071	1.029	**1.016**
GS10	1.012	1.100	**0.998**	1.019	1.029	1.150	**1.024**	1.032
	$h=3$				$h=6$			
PAYEMS	0.800	**0.412**	0.591	0.591	0.831	**0.516**	0.645	0.662
CPIAUCSL	1.124	0.978	**0.903**	0.923	1.044	1.019	0.916	**0.893**
FEDFUNDS	0.970	1.848	**0.961**	0.985	**0.899**	1.275	0.995	0.986
INDPRO	0.973	**0.865**	0.951	0.947	0.979	0.972	**0.961**	0.970
UNRATE	0.633	**0.540**	0.590	0.643	0.640	**0.435**	0.562	0.601
PPIFGS	**1.034**	1.089	1.040	1.043	**1.010**	1.130	1.058	1.055
GS10	**1.040**	1.212	1.058	1.057	**1.018**	1.176	1.053	1.033
	$h=9$				$h=12$			
PAYEMS	0.881	**0.649**	0.744	0.748	0.914	**0.788**	0.854	0.842
CPIAUCSL	1.034	0.994	0.876	**0.871**	1.033	0.992	0.880	**0.862**
FEDFUNDS	**0.934**	1.066	0.960	0.973	**0.996**	1.141	1.037	1.007
INDPRO	0.986	1.042	**0.973**	0.990	**0.977**	1.056	0.990	0.985
UNRATE	0.662	**0.448**	0.577	0.610	0.687	**0.497**	0.603	0.640
PPIFGS	**1.000**	1.127	1.062	1.019	**1.006**	1.159	1.069	1.037
GS10	1.019	1.070	1.025	**1.003**	1.018	1.084	1.035	**1.016**

This table reports the ratio between the MSFE of model i for the Huge size VAR and the MSFE of the benchmark AR(1), computed as

$$MSFE_{ijh} = \frac{\sum_{\tau=\underline{t}}^{\overline{t}-h} e^2_{i,j,\tau+h}}{\sum_{\tau=\underline{t}}^{\overline{t}-h} e^2_{bcmk,j,\tau+h}},$$

where $e^2_{i,j,\tau+h}$ and $e^2_{bcmk,j,\tau+h}$ are the squared forecast errors of variable j at time τ and forecast horizon h generated by model i and the AR(1) model, respectively. $\underline{t}$ and $\overline{t}$ denote the start and end of the out-of-sample period, $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, and $h \in \{1, 2, 3, 6, 9, 12\}$. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Bold numbers indicate the lowest MSFE across all models for a given variable-forecast horizon pair.

Table 4. Out-of-sample density forecast performance, Medium VAR

Variable	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*
	$h=1$				$h=2$			
PAYEMS	0.016	**0.205**	0.084	0.081	0.116	**0.360**	0.165	0.162
CPIAUCSL	-0.053	-0.612	0.031	**0.091**	-0.254	-1.516	-0.146	**-0.008**
FEDFUNDS	0.047	**0.131**	-0.015	-0.014	0.044	**0.116**	0.001	-0.004
INDPRO	-0.077	-0.084	**0.151**	0.010	**0.136**	-0.027	0.084	0.043
UNRATE	0.147	**0.167**	0.115	0.093	0.211	**0.325**	0.217	0.213
PPIFGS	0.023	-0.353	**0.205**	0.055	**-0.082**	-0.780	-0.127	-0.149
GS10	-0.002	**0.010**	-0.013	-0.009	**-0.005**	-0.011	-0.011	-0.014
	$h=3$				$h=6$			
PAYEMS	0.150	**0.362**	0.198	0.189	0.120	**0.246**	0.172	0.174
CPIAUCSL	-0.113	-1.090	-0.161	**-0.027**	-0.131	-0.760	-0.102	**-0.081**
FEDFUNDS	0.027	**0.112**	-0.008	-0.014	0.025	**0.117**	-0.001	-0.005
INDPRO	0.042	-0.013	**0.066**	0.066	0.181	-0.070	0.030	**0.240**
UNRATE	0.369	**0.459**	0.410	0.403	0.775	0.637	**0.951**	0.913
PPIFGS	-0.053	-0.482	-0.105	**0.101**	**-0.012**	-0.644	-0.055	-0.015
GS10	**-0.005**	-0.010	-0.020	-0.016	-0.006	**0.006**	-0.010	-0.008
	$h=9$				$h=12$			
PAYEMS	0.114	**0.131**	0.123	0.122	**0.077**	0.042	0.064	0.074
CPIAUCSL	-0.283	-0.794	**-0.259**	-0.265	-0.159	-0.783	-0.168	**-0.039**
FEDFUNDS	0.013	**0.115**	-0.003	-0.010	0.010	**0.116**	-0.010	-0.015
INDPRO	-0.012	-0.103	**0.057**	-0.017	0.027	-0.053	0.007	**0.043**
UNRATE	**1.310**	0.688	1.297	1.102	**2.393**	1.330	2.108	1.715
PPIFGS	-0.092	-0.390	0.021	**0.050**	**0.011**	-0.392	-0.007	0.002
GS10	0.002	**0.032**	-0.009	-0.028	-0.017	**0.003**	-0.019	-0.028

This table reports the average log predictive likelihood (ALPL) differential between the model i for the Medium size VAR and the benchmark AR(1), computed as

$$ALPL_{ijh} = \frac{1}{\bar{t} - \underline{t} - h + 1} \sum_{\tau=\underline{t}}^{\bar{t}-h} \left(LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h} \right),$$

where $LPL_{i,j,\tau+h}$ and $LPL_{bcmk,j,\tau+h}$ are the log predictive likelihoods of variable j at time τ and forecast horizon h generated by model i and the AR(1) model, respectively. $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period, $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, and $h \in \{1, 2, 3, 6, 9, 12\}$. All density forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Bold numbers indicate the highest ALPL across all models for a given variable-forecast horizon pair.

Table 5. Out-of-sample density forecast performance, Large VAR

Variable	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR$_c$*
	$h=1$				$h=2$			
PAYEMS	0.060	**0.259**	0.065	0.063	0.134	**0.399**	0.120	0.113
CPIAUCSL	-0.480	-0.775	-0.078	**0.104**	-0.631	-2.312	-0.222	**-0.220**
FEDFUNDS	0.056	**0.149**	-0.017	-0.018	**0.025**	-0.023	-0.004	-0.012
INDPRO	-0.104	-0.059	-0.011	**0.045**	0.020	**0.240**	0.102	0.146
UNRATE	**0.158**	0.122	0.096	0.061	0.189	**0.235**	0.169	0.144
PPIFGS	-0.211	-0.711	-0.023	**0.028**	-0.129	-1.207	**0.023**	-0.144
GS10	**0.028**	-0.012	0.001	0.011	-0.021	**-0.010**	-0.021	-0.027
	$h=3$				$h=6$			
PAYEMS	0.093	**0.407**	0.148	0.149	0.075	**0.282**	0.114	0.157
CPIAUCSL	-0.270	-1.949	-0.446	**-0.269**	-0.151	-0.889	-0.189	**-0.002**
FEDFUNDS	0.030	**0.032**	0.001	-0.001	0.009	**0.158**	-0.013	-0.014
INDPRO	0.023	0.044	**0.223**	0.074	-0.073	-0.294	**-0.045**	-0.081
UNRATE	0.098	0.356	**0.357**	0.340	0.599	0.502	**0.958**	0.869
PPIFGS	**0.079**	-1.109	-0.020	-0.009	-0.186	-0.857	-0.116	**-0.060**
GS10	**0.005**	-0.025	-0.002	-0.029	-0.014	**-0.009**	-0.021	-0.024
	$h=9$				$h=12$			
PAYEMS	0.036	**0.105**	0.065	0.099	0.020	0.016	**0.041**	0.024
CPIAUCSL	-0.095	-0.943	**-0.081**	-0.228	-0.062	-0.784	-0.106	**-0.053**
FEDFUNDS	0.003	**0.148**	-0.024	-0.022	0.002	**0.136**	-0.028	-0.016
INDPRO	**0.138**	-0.168	0.092	-0.002	**0.071**	-0.231	-0.109	0.058
UNRATE	**1.539**	0.180	1.326	1.136	**1.901**	-0.016	1.367	1.040
PPIFGS	0.004	-0.629	0.029	**0.061**	**-0.025**	-0.711	-0.150	-0.145
GS10	-0.018	**0.024**	-0.022	-0.024	-0.013	**0.010**	-0.018	-0.040

This table reports the average log predictive likelihood (ALPL) differential between the model i for the Large size VAR and the benchmark AR(1), computed as

$$ALPL_{ijh} = \frac{1}{\bar{t} - \underline{t} - h + 1} \sum_{\tau=\underline{t}}^{\bar{t}-h} \left(LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h} \right),$$

where $LPL_{i,j,\tau+h}$ and $LPL_{bcmk,j,\tau+h}$ are the log predictive likelihoods of variable j at time τ and forecast horizon h generated by model i and the AR(1) model, respectively. $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period, $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, and $h \in \{1, 2, 3, 6, 9, 12\}$. All density forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Bold numbers indicate the highest ALPL across all models for a given variable-forecast horizon pair.

Table 6. Out-of-sample density forecast performance, Huge VAR

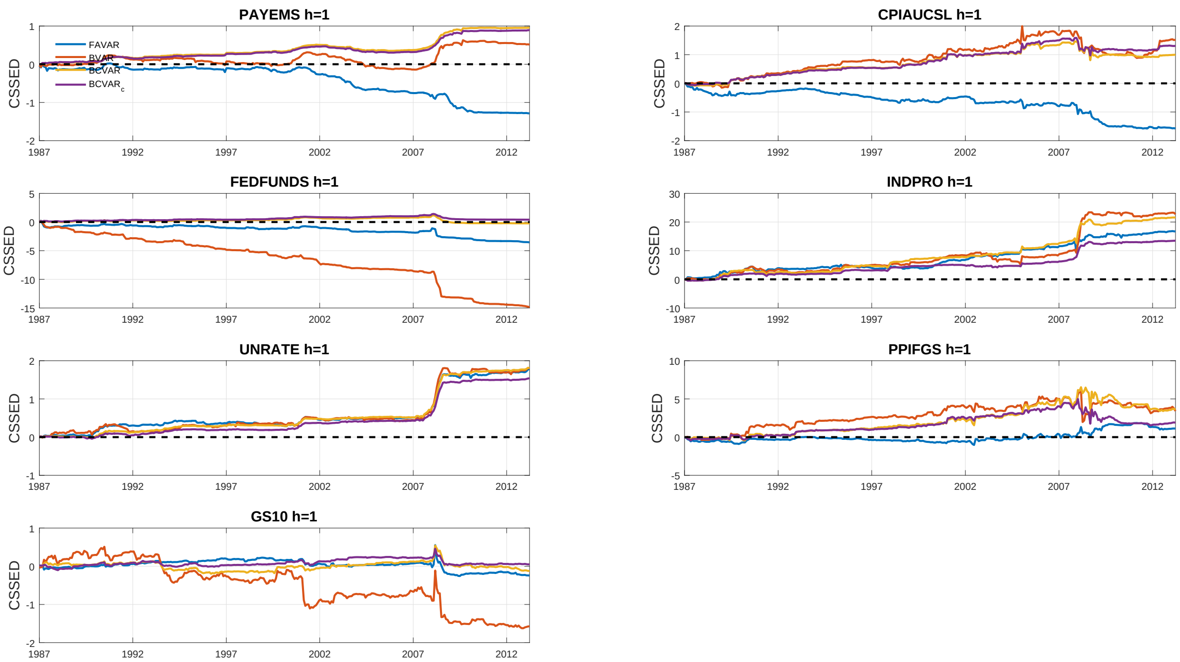
Variable	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR*$_c$	*FAVAR*	*BVAR*	*BCVAR*	*BCVAR*$_c$
	$h=1$				$h=2$			
PAYEMS	0.056	**0.328**	0.094	0.094	0.151	**0.498**	0.189	0.193
CPIAUCSL	-0.426	-0.413	-0.018	**0.021**	-0.734	-2.113	**0.107**	0.028
FEDFUNDS	0.053	**0.283**	-0.013	-0.010	0.028	**0.248**	-0.006	-0.005
INDPRO	-0.251	-0.389	-0.007	**0.002**	0.067	-0.100	**0.168**	0.126
UNRATE	**0.136**	0.044	0.106	0.101	0.107	**0.223**	0.202	0.190
PPIFGS	-0.053	-1.023	**0.083**	0.053	0.015	-1.737	**0.097**	0.092
GS10	**0.010**	0.010	-0.004	-0.002	**-0.003**	-0.010	-0.005	-0.010
	$h=3$				$h=6$			
PAYEMS	0.123	**0.487**	0.207	0.210	0.092	**0.374**	0.192	0.200
CPIAUCSL	-0.295	-2.392	-0.209	**-0.122**	-0.179	-2.254	**0.113**	-0.223
FEDFUNDS	0.030	**0.229**	-0.004	-0.003	0.014	**0.190**	-0.010	-0.008
INDPRO	0.124	0.122	**0.147**	0.087	-0.048	-0.160	**0.031**	-0.046
UNRATE	0.145	0.244	0.367	**0.373**	0.563	-0.666	0.940	**0.987**
PPIFGS	-0.112	-1.296	0.021	**0.073**	-0.131	-1.533	**-0.002**	-0.115
GS10	-0.004	-0.041	**0.002**	-0.008	-0.010	-0.024	-0.011	**-0.009**
	$h=9$				$h=12$			
PAYEMS	0.071	**0.249**	0.161	0.166	0.043	**0.146**	0.091	0.098
CPIAUCSL	-0.087	-1.029	-0.076	**0.059**	0.034	-1.486	0.055	**0.162**
FEDFUNDS	0.015	**0.278**	-0.013	-0.007	0.002	**0.218**	-0.021	-0.020
INDPRO	-0.006	-0.208	**0.052**	0.035	0.000	-0.214	-0.024	**0.003**
UNRATE	1.261	-0.519	**1.447**	1.142	**2.109**	-1.021	2.095	1.296
PPIFGS	-0.245	-1.370	-0.264	**-0.189**	0.004	-0.533	**0.180**	0.001
GS10	-0.004	**0.032**	-0.012	-0.015	0.011	**0.039**	-0.002	-0.016

This table reports the average log predictive likelihood (ALPL) differential between the model i for the Huge size VAR and the benchmark AR(1), computed as

$$ALPL_{ijh} = \frac{1}{\bar{t} - \underline{t} - h + 1} \sum_{\tau=\underline{t}}^{\bar{t}-h} \left(LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h} \right),$$

where $LPL_{i,j,\tau+h}$ and $LPL_{bcmk,j,\tau+h}$ are the log predictive likelihoods of variable j at time τ and forecast horizon h generated by model i and the AR(1) model, respectively. $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period, $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, and $h \in \{1, 2, 3, 6, 9, 12\}$. All density forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Bold numbers indicate the highest ALPL across all models for a given variable-forecast horizon pair.

Figure 1. Cumulative SSE diffs, Medium VAR, $h = 1$

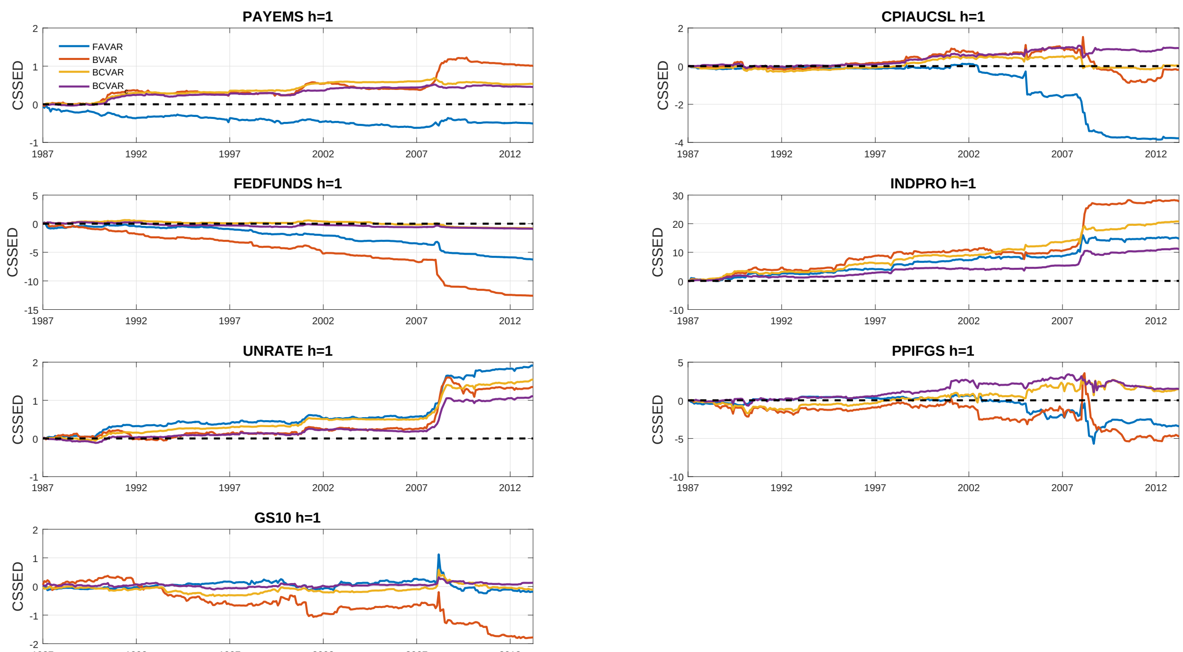


This figure plots the sum of squared forecast errors generated by the AR(1) model minus the sum of the squared forecast errors generated by model i for a Medium size VAR and forecast horizon $h = 1$,

$$CSSED_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} \left(e_{bcmk,j,\tau+h}^{2} - e_{i,j,\tau+h}^{2}\right)$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 2. Cumulative SSE diffs, Large VAR, $h = 1$

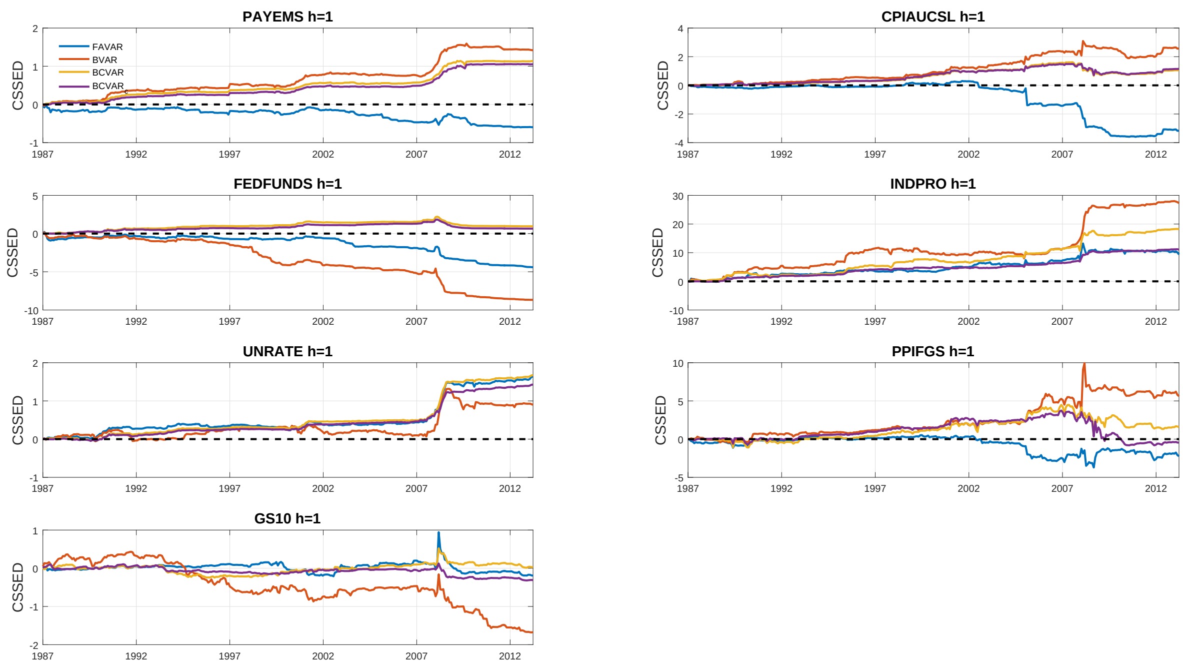


This figure plots the sum of squared forecast errors generated by the AR(1) model minus the sum of the squared forecast errors generated by model i for a Large size VAR and forecast horizon $h = 1$,

$$CSSED_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} \left(e_{bcmk,j,\tau+h}^{2} - e_{i,j,\tau+h}^{2} \right)$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 3. Cumulative SSE diffs, Huge VAR, $h = 1$

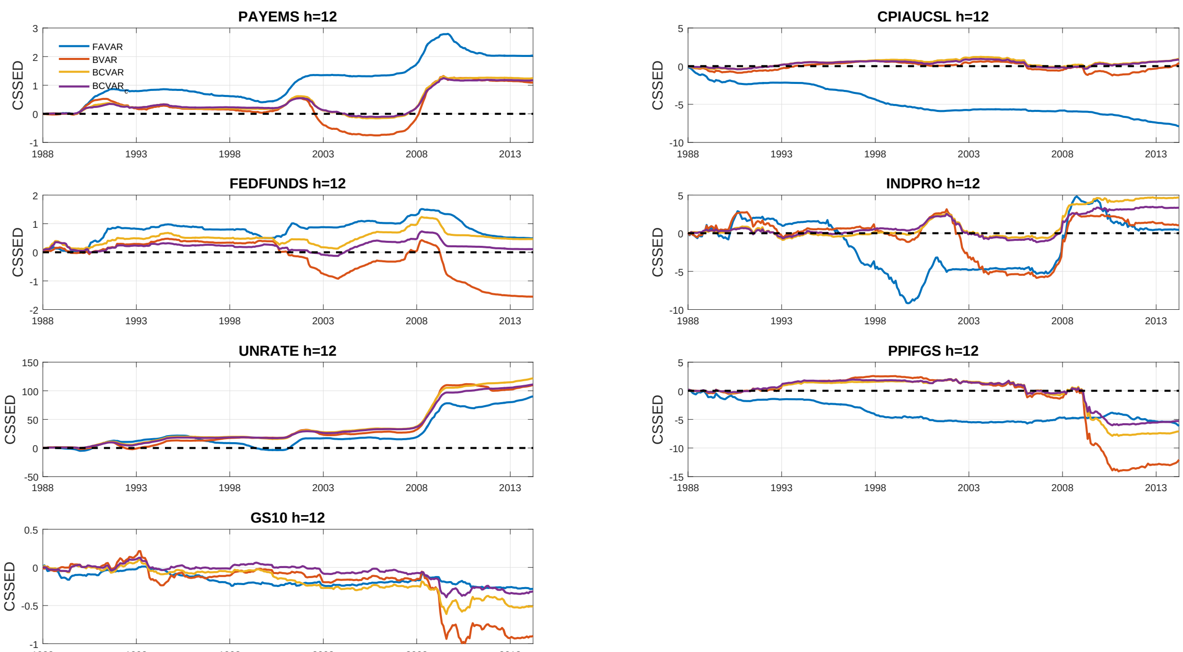


This figure plots the sum of squared forecast errors generated by the AR(1) model minus the sum of the squared forecast errors generated by model i for a Huge size VAR and forecast horizon $h = 1$,

$$CSSED_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} \left(e_{bcmk,j,\tau+h}^{2} - e_{i,j,\tau+h}^{2}\right)$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 4. Cumulative SSE diffs, Medium VAR, $h = 12$

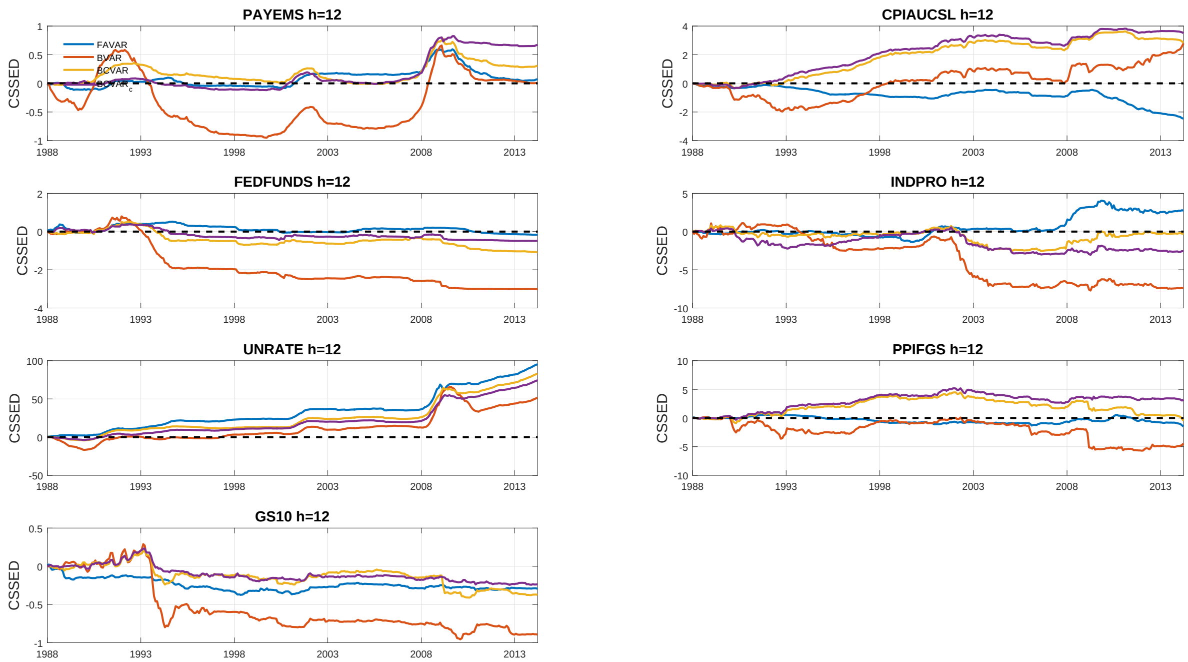


This figure plots the sum of squared forecast errors generated by the AR(1) model minus the sum of the squared forecast errors generated by model i for a Medium size VAR and forecast horizon $h = 12$,

$$CSSED_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} \left(e_{bcmk,j,\tau+h}^{2} - e_{i,j,\tau+h}^{2} \right)$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 5. Cumulative SSE diffs, Large VAR, $h = 12$

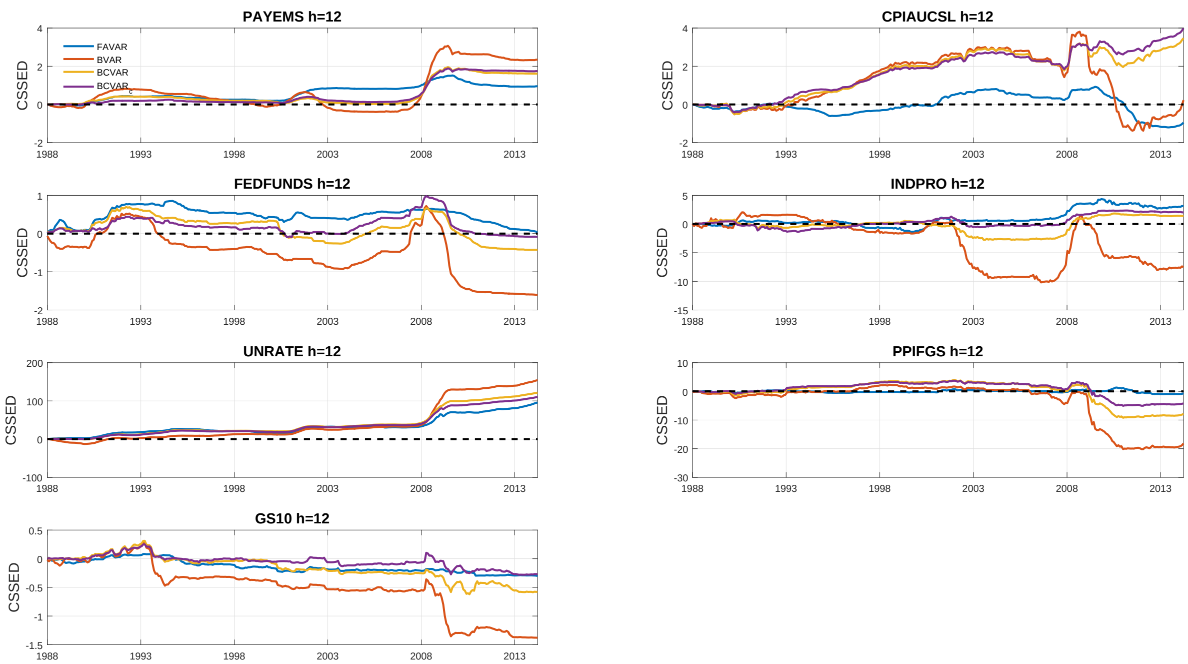


This figure plots the sum of squared forecast errors generated by the AR(1) model minus the sum of the squared forecast errors generated by model i for a Large size VAR and forecast horizon $h = 12$,

$$CSSED_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} \left(e_{bcmk,j,\tau+h}^{2} - e_{i,j,\tau+h}^{2} \right)$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 6. Cumulative SSE diffs, Huge VAR, $h = 12$

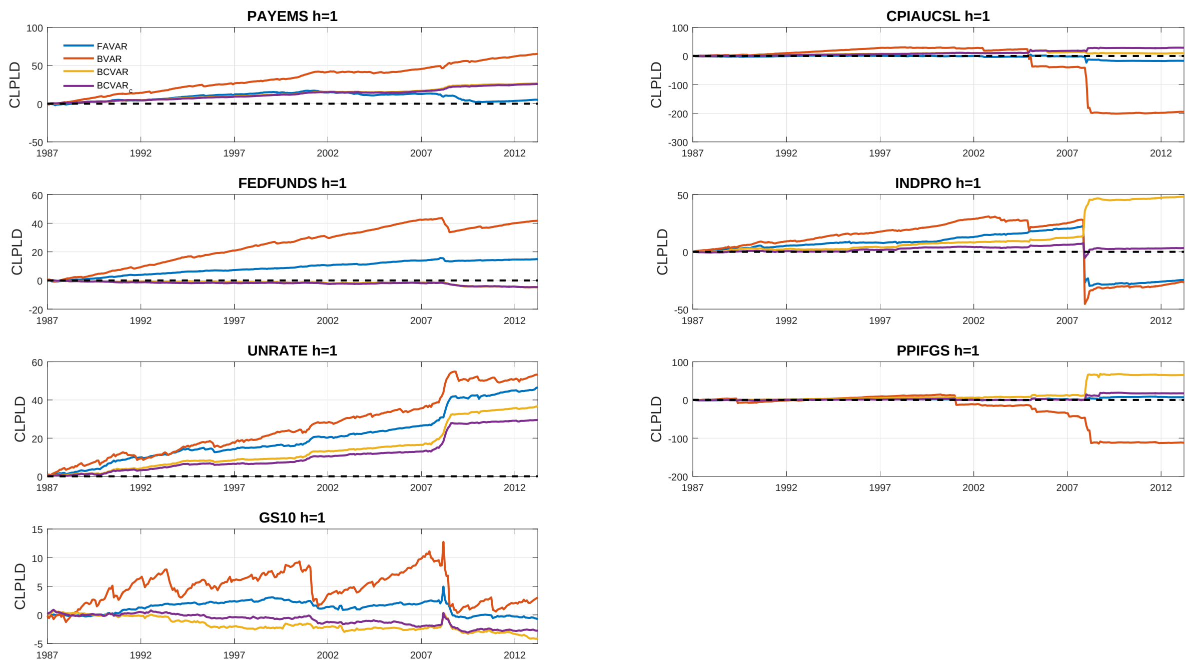


This figure plots the sum of squared forecast errors generated by the AR(1) model minus the sum of the squared forecast errors generated by model i for a Huge size VAR and forecast horizon $h = 12$,

$$CSSED_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} \left(e_{bcmk,j,\tau+h}^{2} - e_{i,j,\tau+h}^{2}\right)$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 7. Cumulative log predictive likelihood diffs, Medium VAR, $h = 1$

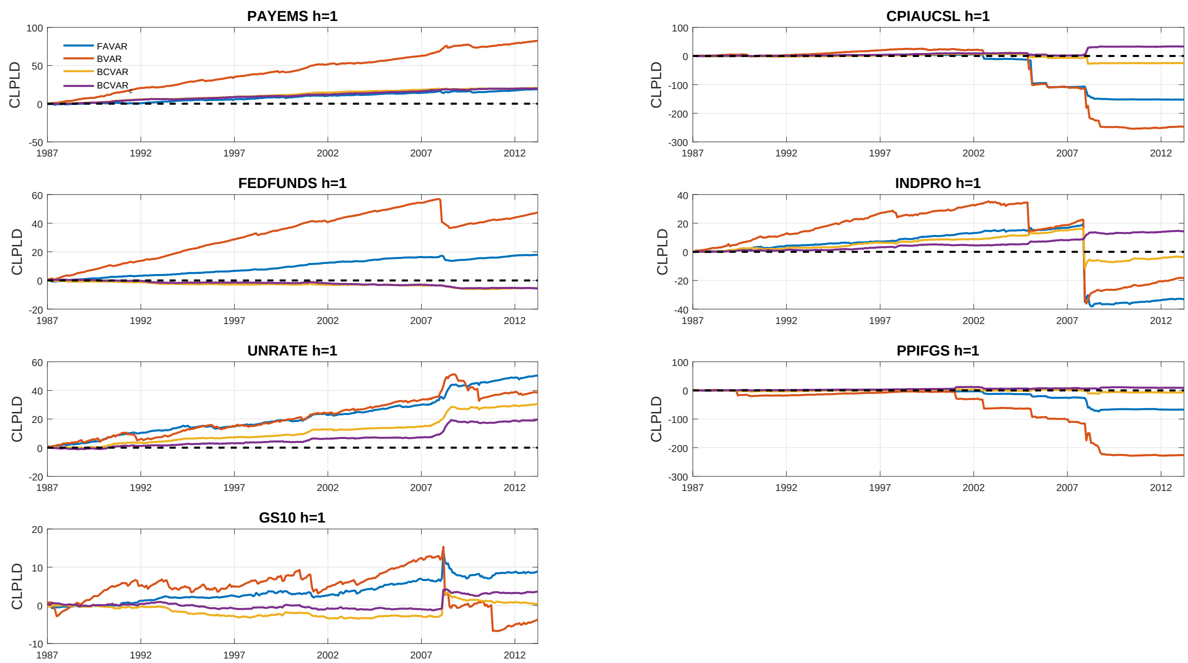


This figure plots the sum of log predictive likelihoods generated by model i minus the sum of log predictive likelihoods generated by the AR(1) model for a Medium size VAR and forecast horizon $h = 1$,

$$CLPLD_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} (LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h})$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 8. Cumulative log predictive likelihood diffs, Large VAR, $h = 1$

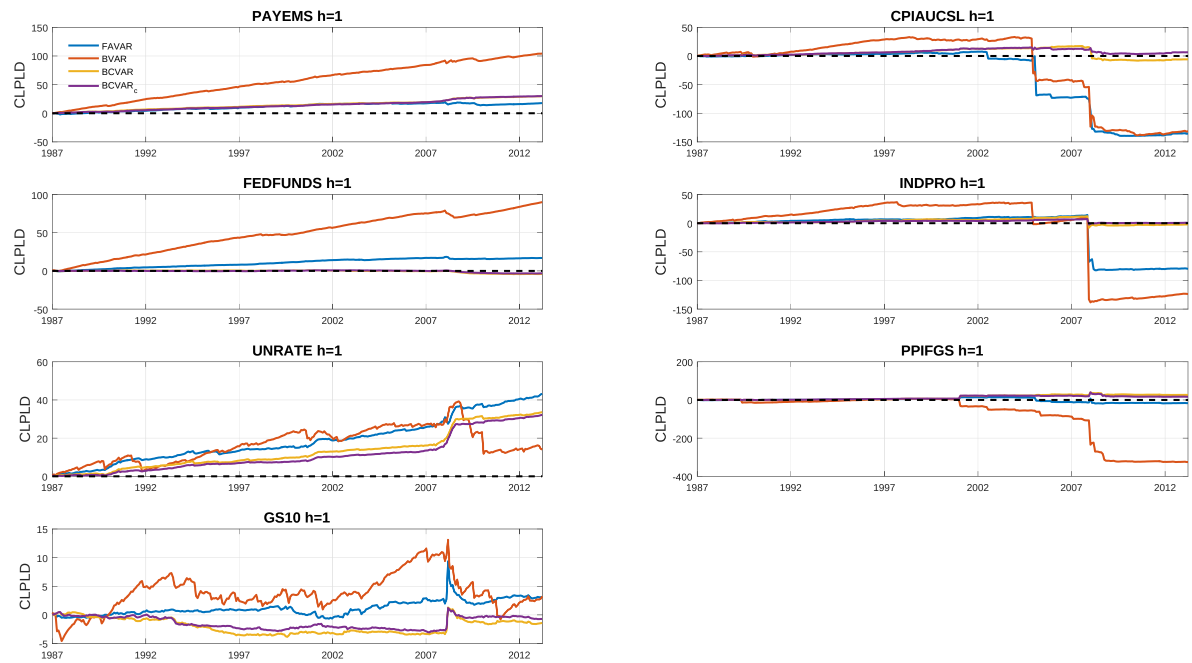


This figure plots the sum of log predictive likelihoods generated by model i minus the sum of log predictive likelihoods generated by the AR(1) model for a Large size VAR and forecast horizon $h = 1$,

$$CLPLD^h_{ijh} = \sum_{\tau=\underline{t}}^{t} (LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h})$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 9. Cumulative log predictive likelihood diffs, Huge VAR, $h = 1$

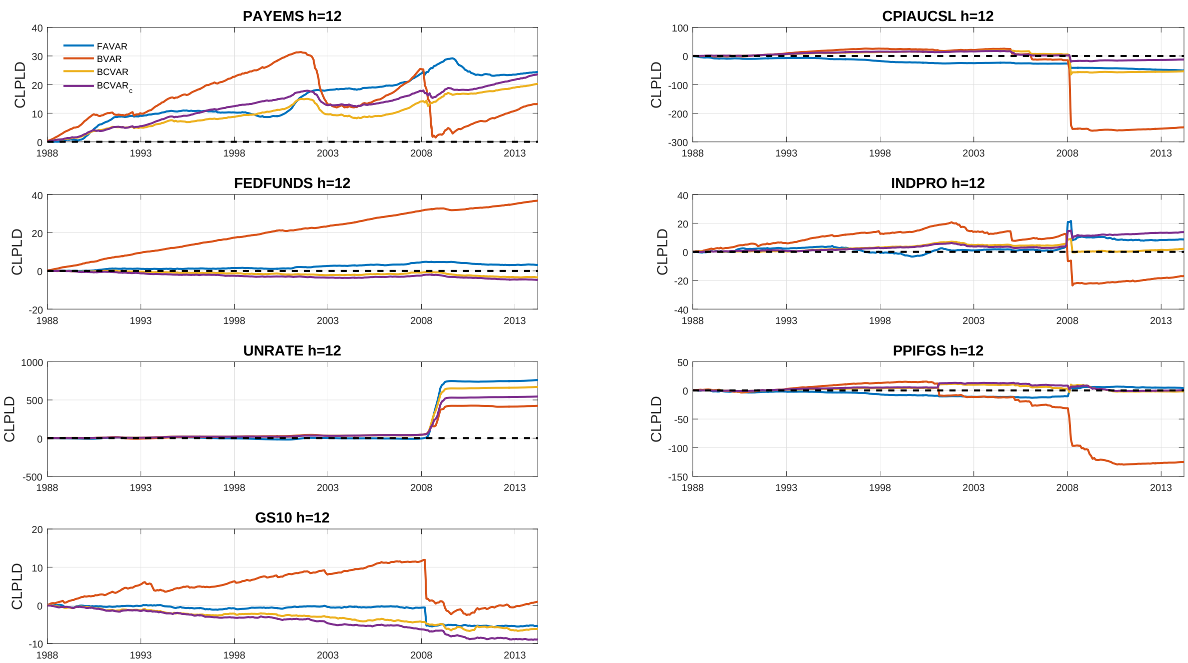


This figure plots the sum of log predictive likelihoods generated by model i minus the sum of log predictive likelihoods generated by the AR(1) model for a Huge size VAR and forecast horizon $h = 1$,

$$CLPLD_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} (LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h})$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 10. Cumulative log predictive likelihood diffs, Medium VAR, $h = 12$

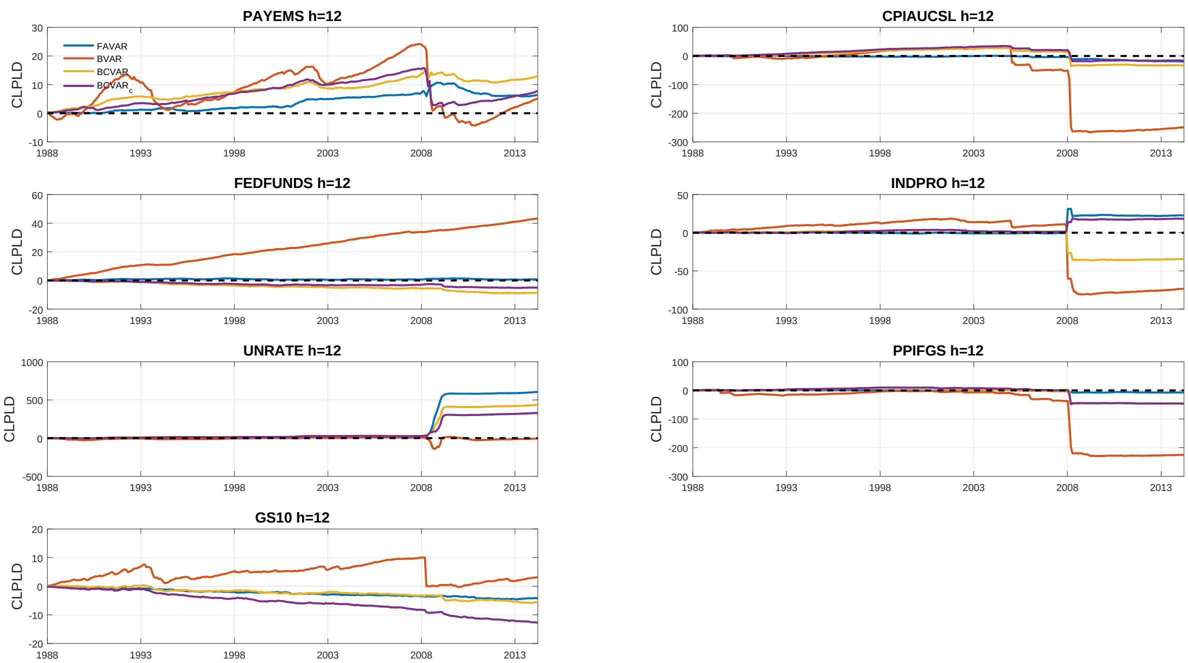


This figure plots the sum of log predictive likelihoods generated by model i minus the sum of log predictive likelihoods generated by the AR(1) model for a Medium size VAR and forecast horizon $h = 12$,

$$CLPLD^h_{ijh} = \sum_{\tau=\underline{t}}^{t} (LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h})$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 11. Cumulative log predictive likelihood diffs, Large VAR, $h = 12$

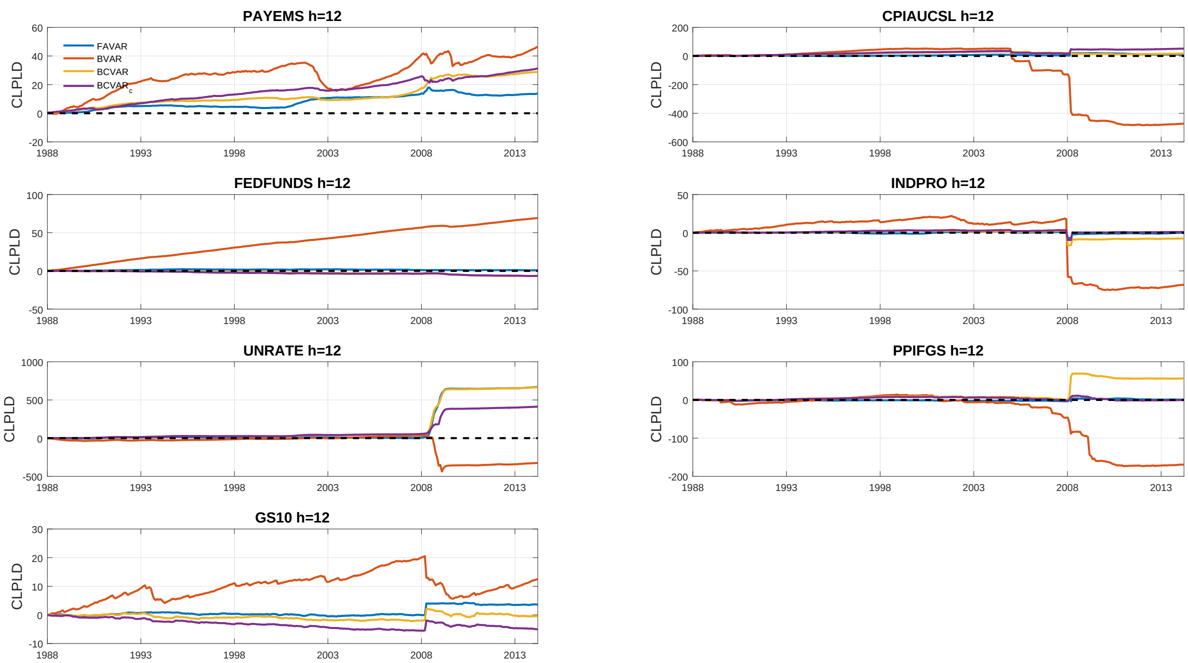


This figure plots the sum of log predictive likelihoods generated by model i minus the sum of log predictive likelihoods generated by the AR(1) model for a Large size VAR and forecast horizon $h = 12$,

$$CLPLD_{ijh}^{h} = \sum_{\tau=\underline{t}}^{t} (LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h})$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Figure 12. Cumulative log predictive likelihood diffs, Huge VAR, $h = 12$



This figure plots the sum of log predictive likelihoods generated by model i minus the sum of log predictive likelihoods generated by the AR(1) model for a Huge size VAR and forecast horizon $h = 12$,

$$CLPLD^h_{ijh} = \sum_{\tau=\underline{t}}^{t} (LPL_{i,j,\tau+h} - LPL_{bcmk,j,\tau+h})$$

where $t = \underline{t}, ..., \bar{t} - h$. Values above zero indicate that model i generates better performance than the benchmark, while negative values suggest the opposite. $i \in \{FAVAR, BVAR, BCVAR, BCVAR_c\}$, $j \in \{PAYEMS, CPIAUCSL, FEDFUNDS, INDPRO, UNRATE, PPIFGS, GS10\}$, $\underline{t}$ and $\bar{t}$ denote the start and end of the out-of-sample period. All forecasts are generated out-of-sample using recursive estimates of the models, with the out of sample period starting in 1987:01 and ending in 2014:12. Each panel displays results for a different series.

Appendix A: Data and transformations

The column Tcode denotes the following data transformation for a series x: (1) no transformation; (2) Δx_t; (3) $\Delta^2 x_t$; (4) $\log(x_t)$; (5) $\Delta \log(x_t)$; (6) $\Delta^2 \log(x_t)$. The FRED column gives mnemonics in FRED followed by a short description. The comparable series description in Global Insight is given in the column GSI:Description.

Table A.1. **Output and Income**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
1	5	X	X	RPI	Real Personal Income	PI
2	5		X	W875RX1	RPI ex. Transfers	PI less transfers
6	5	X	X	INDPRO	IP Index	IP: total
7	5			IPFPNSS	IP: Final Products and Supplies	IP: products
8	5			IPFINAL	IP: Final Products	IP: final prod
9	5			IPCONGD	IP: Consumer Goods	IP: cons gds
10	5			IPDCONGD	IP: Durable Consumer Goods	IP: cons dble
11	5			IPNCONGD	IP: Nondurable Consumer Goods	IP: cons nondble
12	5			IPBUSEQ	IP: Business Equipment	IP: bus eqpt
13	5			IPMAT	IP: Materials	IP: matls
14	5			IPDMAT	IP: Durable Materials	IP: dble matls
15	5			IPNMAT	IP: Nondurable Materials	IP: nondble matls
16	5			IPMANSICS	IP: Manufacturing	IP: mfg
17	5			IPB51222S	IP: Residential Utilities	IP: res util
18	5			IPFUELS	IP: Fuels	IP: fuels
19	1		X	NAPMPI	ISM Manufacturing: Production	NAPM prodn
20	1			CAPUTLB00004S	Capacity Utilization: Manufacturing	Cap util

Table A.2. **Labor Market**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
21	1	X	X	HWI	Help-Wanted Index for US	Help wanted indx
22	1		X	HWIURATIO	Help Wanted to Unemployed ratio	Help wanted/unemp
23	5		X	CLF16OV	Civilian Labor Force	Emp CPS total
24	5			CE16OV	Civilian Employment	Emp CPS nonag
25	1	X	X	UNRATE	Civilian Unemployment Rate	U: all
26	1			UEMPMEAN	Average Duration of Unemployment	U: mean duration
27	5			UEMPLT5	Civilians Unemployed $\leq$ 5 Weeks	U $\leq$ 5 wks
28	5			UEMP5TO14	Civilians Unemployed 5-14 Weeks	U 5-14 wks
29	5			UEMP15OV	Civilians Unemployed $>$ 15 Weeks	U $>$ 15 wks
30	5			UEMP15T26	Civilians Unemployed 15-26 Weeks	U 15-26 wks
31	5			UEMP27OV	Civilians Unemployed $>$ 27 Weeks	U $>$ 27 wks
32	5			CLAIMSx	Initial Claims	UI claims
33	5	X	X	PAYEMS	All Employees: Total nonfarm	Emp: total
34	5			USGOOD	All Employees: Goods-Producing	Emp: gds prod
35	5			CES1021000001	All Employees: Mining and Logging	Emp: mining
36	5			USCONS	All Employees: Construction	Emp: const
37	5			MANEMP	All Employees: Manufacturing	Emp: mfg
38	5			DMANEMP	All Employees: Durable goods	Emp: dble gds
39	5			NDMANEMP	All Employees: Nondurable goods	Emp: nondbles
40	5			SRVPRD	All Employees: Service Industries	Emp: services
41	5			USTPU	All Employees: TT&U	Emp: TTU
42	5			USWTRADE	All Employees: Wholesale Trade	Emp: wholesale
43	5			USTRADE	All Employees: Retail Trade	Emp: retail
44	5			USFIRE	All Employees: Financial Activities	Emp: FIRE
45	5			USGOVT	All Employees: Government	Emp: Govt
46	1		X	CES0600000007	Hours: Goods-Producing	Avg hrs
47	1			AWOTMAN	Overtime Hours: Manufacturing	Overtime: mfg
48	1			AWHMAN	Hours: Manufacturing	Avg hrs: mfg
49	1			NAPMEI	ISM Manufacturing: Employment	NAPM empl
128	5			CES0600000008	Ave. Hourly Earnings: Goods	AHE: goods
129	5			CES2000000008	Ave. Hourly Earnings: Construction	AHE: const
130	5			CES3000000008	Ave. Hourly Earnings: Manufacturing	AHE: mfg

Table A.3. **Housing**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
50	4	X		HOUST	Starts: Total	Starts: nonfarm
51	4			HOUSTNE	Starts: Northeast	Starts: NE
52	4			HOUSTMW	Starts: Midwest	Starts: MW
53	4			HOUSTS	Starts: South	Starts: South
54	4			HOUSTW	Starts: West	Starts: West
55	4	X		PERMIT	Permits	BP: total
56	4			PERMITNE	Permits: Northeast	BP: NE
57	4			PERMITMW	Permits: Midwest	BP: MW
58	4			PERMITS	Permits: South	BP: South
59	4			PERMITW	Permits: West	BP: West

Table A.4. **Consumption, Orders and Inventories**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
3	5		X	DPCERA3M086SBEA	Real PCE	Real Consumption
4	5		X	CMRMTSPLx	Real M&T Sales	M&T sales
5	5		X	RETAILx	Retail and Food Services Sales	Retail sales
60	1	X		NAPM	ISM: PMI Composite Index	PMI
61	1		X	NAPMNOI	ISM: New Orders Index	NAPM new ordrs
62	1		X	NAPMSDI	ISM: Supplier Deliveries Index	NAPM vendor del
63	1		X	NAPMII	ISM: Inventories Index	NAPM Invent
65	5			AMDMNOx	Orders: Durable Goods	Orders: dble gds
67	5			AMDMUOx	Unfilled Orders: Durable Goods	Unf orders: dble
68	5			BUSINVx	Total Business Inventories	M&T invent
69	1			ISRATIOx	Inventories to Sales Ratio	M&T invent/sales

Table A.5. **Money and Credit**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
70	5	X	X	M1SL	M1 Money Stock	M1
71	5		X	M2SL	M2 Money Stock	M2
73	5		X	M2REAL	Real M2 Money Stock	M2 (real)
74	5		X	AMBSL	St. Louis Adjusted Monetary Base	MB
75	5		X	TOTRESNS	Total Reserves	Reserves tot
77	5	X	X	BUSLOANS	Commercial and Industrial Loans	C&I loan plus
78	5			REALLN	Real Estate Loans	DC&I loans
79	5		X	NONREVSL	Total Nonrevolving Credit	Cons credit
80	1		X	CONSPI	Credit to PI ratio	Inst cred/PI
132	5			MZMSL	MZM Money Stock	N.A.
133	5			DTCOLNVHFNM	Consumer Motor Vehicle Loans	N.A.
134	5			DTCTHFNM	Total Consumer Loans and Leases	N.A.
135	5	X		INVEST	Securities in Bank Credit	N.A.

Table A.6. **Interest rates and Exchange rates**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
85	2	X	X	FEDFUNDS	Effective Federal Funds Rate	Fed Funds
86	2		X	CP3M	3-Month AA Comm. Paper Rate	Comm paper
87	2		X	TB3MS	3-Month T-bill	3 mo T-bill
88	2		X	TB6MS	6-Month T-bill	6 mo T-bill
89	2		X	GS1	1-Year T-bond	1 yr T-bond
90	2		X	GS5	5-Year T-bond	5 yr T-bond
91	2	X	X	GS10	10-Year T-bond	10 yr T-bond
92	2		X	AAA	Aaa Corporate Bond Yield	Aaa bond
93	2		X	BAA	Baa Corporate Bond Yield	Baa bond
94	1			COMPAPFF	CP - FFR spread	CP-FF spread
95	1			TB3SMFFM	3 Mo. - FFR spread	3 mo-FF spread
96	1			TB6SMFFM	6 Mo. - FFR spread	6 mo-FF spread
97	1			T1YFFM	1 yr. - FFR spread	1 yr-FF spread
98	1			T5YFFM	5 yr. - FFR spread	5 yr-FF spread
99	1	X		T10YFFM	10 yr. - FFR spread	10 yr-FF spread
100	1			AAAFFM	Aaa - FFR spread	Aaa-FF spread
101	1			BAAFFM	Baa - FFR spread	Baa-FF spread
103	5		X	EXSZUS	Switzerland / U.S. FX Rate	Ex rate: Switz
104	5		X	EXJPUS	Japan / U.S. FX Rate	Ex rate: Japan
105	5	X	X	EXUSUK	U.S. / U.K. FX Rate	Ex rate: UK
106	5		X	EXCAUS	Canada / U.S. FX Rate	EX rate: Canada

Table A.7. **Prices**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
107	5	X	X	PPIFGS	PPI: Finished Goods	PPI: fin gds
108	5		X	PPIFCG	PPI: Finished Consumer Goods	PPI: cons gds
109	5		X	PPIITM	PPI: Intermediate Materials	PPI: int materials
110	5		X	PPICRM	PPI: Crude Materials	PPI: crude materials
111	5	X		oilprice	Crude Oil Prices: WTI	Spot market price
112	5			PPICMM	PPI: Commodities	PPI: nonferrous
113	1			NAPMPRI	ISM Manufacturing: Prices	NAPM com price
114	5	X	X	CPIAUCSL	CPI: All Items	CPI-U: all
115	5			CPIAPPSL	CPI: Apparel	CPI-U: apparel
116	5			CPITRNSL	CPI: Transportation	CPI-U: transp
117	5			CPIMEDSL	CPI: Medical Care	CPI-U: medical
118	5			CUSR0000SAC	CPI: Commodities	CPI-U: comm.
119	5			CUUR0000SAD	CPI: Durables	CPI-U: dbles
120	5			CUSR0000SAS	CPI: Services	CPI-U: services
121	5			CPIULFSL	CPI: All Items Less Food	CPI-U: ex food
122	5			CUUR0000SA0L2	CPI: All items less shelter	CPI-U: ex shelter
123	5			CUSR0000SA0L5	CPI: All items less medical care	CPI-U: ex med
124	5			PCEPI	PCE: Chain-type Price Index	PCE defl
125	5			DDURRG3M086SBEA	PCE: Durable goods	PCE defl: dlbes
126	5			DNDGRG3M086SBEA	PCE: Nondurable goods	PCE defl: nondble
127	5			DSERRG3M086SBEA	PCE: Services	PCE defl: service

Table A.8. **Stock Market**

Series id	*Tcode*	*Medium*	*Large*	*FRED*	*Description*	*GSI:Description*
81	5	X	X	S&P 500	S&P: Composite	S&P 500
82	5		X	S&P: indust	S&P: Industrials	S&P: indust
83	1		X	S&P div yield	S&P: Dividend Yield	S&P div yield
84	5		X	S&P PE ratio	S&P: Price-Earnings Ratio	S&P PE ratio