Unit Five Homework

Physics II

Due Monday, May 4, 2020

College of the Atlantic. Spring 2020

Instructions:

- Do problem two in pairs.
- If you want, you can do the other problems in this assignment in pairs, too. If so, it's fine to hand in only one write-up.
- "Hand in" the problem on google classroom. Please don't email me the homework; it's a lot easier for me if you submit it on classroom.
- In addition to these problems, there are some problems that you should do on Edfinity. There is one Edfinity assignment: Homework 05. You should "hand in" these assignments individually, but of course it's totally fine—and in fact encouraged—to work on the Edfinity problems with others.

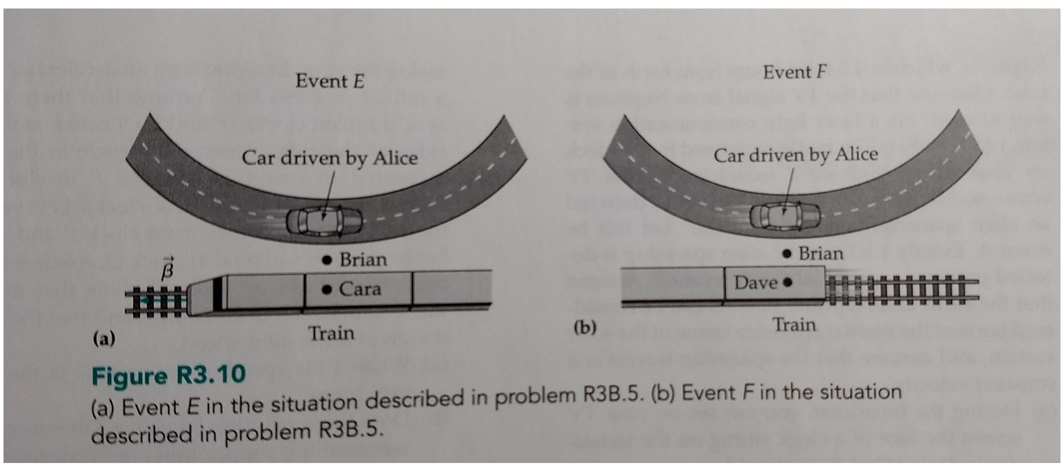

Figure 1: Figure from Moore, *Six Ideas that Shaped Physics: Unit R (2nd edition)*, 2003, p.92.

1. (Based very closely on a problem from Tom Moore *Six Ideas that Shaped Physics: Unit R (second edition)*, (2003).) Alice is driving a race car around an essentially circular track at a constant speed of 60 m/s. Brian, who is sitting at a fixed position at the edge of the track, measures the time that Alice takes to complete a lap by starting his watch when Alice passes by his position (call this event E) and stopping it when Alice passes by his position again (call this event F). This situation is also observed by Cara and Dave, who are passengers in a train that passes very close to Brian. Cara happens to be passing Brian just as Alice passes Brian the first time, and Dave happens to pass Brian just as Alice passes Brian the second time. This is shown in Fig. 1. Assume that the clocks used by Alice, Brian, and Care are close enough that we can consider them all to be present at event E. Similarly, assume that the closed used by Alice, Brian, and Dave are present at event F. Assume that the ground frame is an inertial reference frame.

   (a) Who measures the shortest time between these events. Who measures the longest?

(b) If Brian measures 100 s between the events, how much less time does Alice measure between the events?

(c) If the train carrying Cara and Dave moves at a speed of 30 m/s, how much larger or smaller is the time that they measure compared to Brian's time?

2. This problem describes a 1977 experiment that provided further support for special relativity. The experiment is described in: Bailey, J., et al. "Measurements of relativistic time dilatation for positive and negative muons in a circular orbit." *Nature* 268.5618 (1977): 301-305. The experiment concerns muons. The experimenters measured the half-life of muons at rest to have a value of 1.52 μs. They then produced some high-energy muons in a particle accelerator. These muons were immediately injected into a storage ring upon creation. In the storage ring the muons travel at a constant speed of $v = 0.99942$ as measured in the laboratory frame.

   (a) What would you expect to observe for the half-life of the muons in the circular storage ring? (The measured value is within 0.02% of the predicted value.)

   (b) If you use the binomial approximation for this problem, you'll get a very wrong answer. Why?

3. Consider two identical twin emperor penguins, each of which is going to live to be exactly 20 years old, as measured by their own biological clocks. Since infancy, one twin lives at the equator and the other lives at the south pole. How much longer will the equatorial twin live than the south-pole twin? In thinking about this problem, it will probably be easiest to use a reference frame based in the center of the earth.

Figure 2: Emperor Penguins. Image in the public domain. Image ID: corp2417, NOAA Corps Collection Photographer: Giuseppe Zibordi Credit: Michael Van Woert, NOAA NESDIS, ORA. `https://commons.wikimedia.org/wiki/File:Kaiserpinguine_mit_Jungen.jpg`.

4. **Optional.** Recommended. Uses a bit of algebra to motivate the binomial approximation.

   (a) If $x \ll 1$, then $x^2 \ll x$. Convince yourself that this is true with an example. If $x = 0.001$, what is x^2?

   (b) What is $(1 + x)^2$?

   (c) What is $(1+x)^2$ if we ignore x^2 because it is so small? You should have an expression that is equal to the binomial approximation for $a = 2$.

(d) Repeat the above analysis for $(1+x)^3$. If you ignore x^2 and x^3 terms, you'll should get binomial approximation for $a = 3$.

5. **Optional.** Uses a tiny bit of calculus to derive the binomial approximation. Consider the following function:

$$f(x) = (1+x)^a . \tag{1}$$

We would like to approximate this function with a line. I.e., we would like to find the slope m and intercept b such that the line is the best approximation of $f(x)$:

$$f(x) \approx b + mx . \tag{2}$$

(a) Find b by plugging in $x = 0$ to each side of the equation.

(b) Take the derivatives of both sides of Eq. (2). Then plug in $x = 0$ and solve for m.

(c) Plug your b and m values in to Eq. (2). Congratulations. You've derived the binomial approximation. What we've done, as you have likely suspected, is determine the tangent line approximation for $f(x)$ at $x = 0$.