Some Foundations in Complex Systems: Tools and Concepts

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The materials online at `http://hornacek.coa.edu/dave/CSSS` were developed to accompany two series of lectures I gave at the Complex Systems Summer School (CSSS), sponsored by the Santa Fe Institute. The lectures were given in July of 2004 in Qingdao, China, and again in July 2005 in Beijing. The CSSS in 2004 was co-sponsored by Qingdao University, and the school in 2005 was co-sponsored by the Institute for Theoretical Physics and the Graduate School at the Chinese Academic of Sciences in Beijing.

The CSSS is attended by roughly sixty students, most of whom are graduate students. A few post-docs and businesspeople attend as well. Students come from a range of disciplinary backgrounds: mathematics, computer science, physics, chemistry, engineering, biology, and economics and other social sciences. Given this diversity of backgrounds, I strove to deliver a set of lectures that would appeal to both beginners and experts.

Goals and Topics Covered

I have several overall goals for the lectures upon which these notes are based. First, I want to introduce CSSS students to three fields that I believe are important to most of the areas of study that fall under the broad heading of complex systems: chaotic dynamics; information theory; and computation theory. In my view, an important strand of the study of complex systems originated with work on chaotic dynamics in the 1960's, 70's, and 80's. A qualitative understanding of the phenomena of chaos thus seems to me to be important background for the study of complex systems. My reasons for including information theory are twofold. First, and most generally, the study of complex systems often entails probabilities; information theory is a language that allows one to meaningfully compare, say, the differing degrees of uncertainty represented by different probability distributions. Second, information theory is needed for many of the statistical complexity measures (including the excess entropy) developed later in these lectures. Computation theory focuses on the architecture and structure of information processing, and thus provides an excellent complement to the more statistically oriented information theory. In addition, Universal Turing Machines form the basis of several complexity measures and more generally are a very useful abstract framework to use to consider possible definitions of randomness and complexity.

I devoted one 1.5-hour lecture to each topic: chaotic dynamics, information theory, and the theory of computation. This material is contained in sections 2–7 of these lecture notes. Needless to say, it is folly to think one can give a thorough introduction to these fields in this time; each deserves at least a semester of serious study. Given this, my goal was to introduce students to the simple ideas of these fields, discuss the sorts of phenomena and questions that are central to each, with an eye on how these fields are applied and/or form a conceptual backbone for complex systems. My aim here was to expose students to the flavor of each field, present some key vocabulary, and provide references for those who wish to pursue these topics further. Accordingly, I have tried to resist the urge to prove any of the classic results of computation theory or information theory; I believe that doing so doesn't serve the needs of an overview set of lectures such as this. I have found that an early emphasis on proofs can obscure some of the elegant and fairly intuitive ideas, especially for an audience that is not used to such formalism.

My second main goal is to present some thoughts on complexity itself: what is it, and can it be measured? I spend considerable time presenting two measures of complexity: the statistical complexity and the excess entropy. I also survey a number of other complexity measures. I suspect that only a minority of students will find themselves doing work in which they explicitly calculate a complexity measure. Nevertheless, I believe that thinking carefully about different possible measures of complexity and structure is a very valuable exercise for all those interested in complex systems. These topics are treated in sections 9-12 of these notes. This material is somewhat more specialized and less standard.

Finally, I also included in the introduction and conclusion (sections 1 and 13) some general thoughts on the "field" of complex systems: what are its essential features and what common themes (if any) unite the different areas of study that fall under the umbrella of complex systems. Is there a complex systems science? And if so, is it distinct in style or character from other sorts of science? Is there a need for a complex systems science?

Outline

2. Introduction to Dynamical Systems And Chaos, Part I: Terminology, definition of chaos.
3. Introduction to Dynamical Systems and Chaos, Part II. Bifurcation diagram, universality, a detour about power laws, Lyapunov exponents, symbolic dynamics.
4. Information Theory, Part I. Basic definitions and interpretations.
5. Information Theory, Part II. Basic definitions and interpretations, Shannon's coding theorem.
6. Computation Theory, Part I: Automata and Computational Hierarchy.
7. Computation Theory, Part II: Universal Turing Machines, Uncomputability, and Computational Complexity.
8. A Very, Very Little Bit about Time Series Analysis
9. Computational Mechanics. Definition and examples.
10. Complexity vs. Entropy. A survey of complexity vs. entropy behaviors for a variety of different model systems.
11. Survey of Complexity Measures. A critical review and overview of around a dozen proposed complexity measures.
12. Extensions of Shannon Entropy. Rényi entropy, thermodynamic formalism, Tsallis entropy.
13. Conclusions

How to Read and Use These Notes

These notes are somewhere between talk slides and full lecture notes. The format is those of slides, but I have tried to include much more detail than I would in a usual presentation. My intention is that these notes can stand alone and be of benefit to readers who did not attend the lectures.

I also hope that these lecture notes might be used by instructors in courses on complex systems or at workshops or summer schools similar to the CSSS. I would imagine that these notes could be useful as assigned reading in such settings, or possibly could even be used as slides for a presentation, as I used them in the CSSS lectures.

Annotated Bibliography

I have prepared an annotated bibliography to complement these lecture notes. The bibliography is online at `http://hornacek.coa.edu/dave/CSSS`. In addition to giving pointers to references concerning the specific topics I covered in my lectures, I also list and briefly comment upon a number of articles and books in complex systems and mathematical modeling.

Acknowledgments

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