

Homework Ten

Calculus I

College of the Atlantic

Due Friday, November 22, 2024

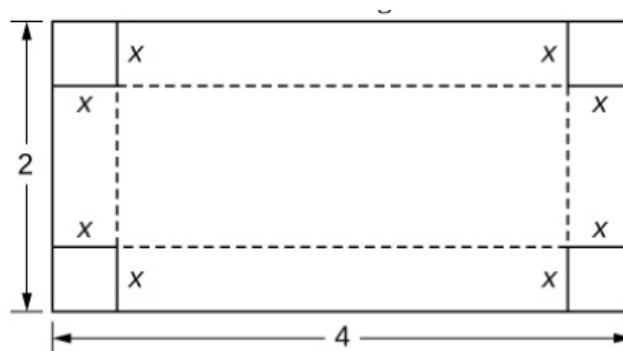
Part 1: WeBWorK. Do Homework 10A¹ on WeBWorK. The WeBWorK page is here: <https://webwork-hosting.runestone.academy/webwork2/coa-feldman-es1024-fall12024/> I recommend doing the WeBWorK part of the homework first. This will enable you to benefit WeBWorK's instant feedback before you do part two.

Part 2: Non-WeBWorK problems. Here are some instructions for how to submit this part of the assignment.

- Do the problems by hand using pencil (or pen) and paper. There is no need to type this assignment.
- Make a pdf scan of your work using genius scan or some similar scanning app. Please make the homework into a single pdf, not multiple pdfs.
- Submit the assignment on google classroom. Please don't email it to me.

Here are some non-WeBWorK problems.

1. You have a piece of cardboard that is 4 inches by two inches. You wish to use the cardboard to make a rectangular box. To do so you will cut an x by x square out of each corner and the fold up along the dotted lines, as shown in the figure. What value of x leads to the box with the greatest volume?



2. You and your friend both traveled to Des Moines, Iowa, for a fun-filled weekend get together. Monday morning comes and you head your separate ways. You drive due north toward Minneapolis at 100 km/hr, and your friend drives due west toward Omaha at 50 km/hr. As you head northward, you think about how much you miss your friend, and you picture them heading west, getting ever farther away from you. You begin to wonder: how fast is the distance between me and my friend growing?

¹Note: There is no 10B. This is the only WeBWorK part for this week.

- (a) How fast is the distance between you and your friend growing after you have each been driving for one hour?
 - (b) How fast is the distance between you and your friend growing after you have each been driving for two hours?
3. It is winter break and you are bored. You decide to pass the time by watching cars drive by on a lonely road near your home. The road happens to run directly east-west. You position yourself 5 meters south of the road.

A car appears in the distance to your right. You watch it as it gets closer and closer to you. It passes directly in front of you and then recedes to the left. Eventually it is so far away that you can no longer see it.

This scene repeats itself several times over the next hour. You begin to wonder, how is the speed at which I have to move my head related to how fast the car is going and the position of the car?

Let θ refer to the angle between your line of sight and due east. So if you are looking due east, $\theta = 0$, and if you are looking due north, $\theta = \pi/2$. On this road cars always travel at a constant speed of 60 km/hr.

- (a) You are watching a car. At the moment in time at which $\theta = \pi/3$, at what rate (in radians/sec) must you turn your head so that the car remains in the center of your line of sight?

By the way, it is possible that you'll need to take the derivative of a tangent function. Since

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} , \quad (1)$$

once can find the derivative by using the chain and product² rules. Doing so, and using the identity $\cos^2(\theta) + \sin^2(\theta) = 1$, one arrives at:

$$\frac{d}{d\theta} \tan(\theta) = \frac{1}{\cos^2(\theta)} . \quad (2)$$

Heads-up: you'll need to pay attention to units.

4. **Optional Challenge Problem.** A spherical rock on a beach on the coast of Maine erodes at a rate proportional to its surface area. As it erodes it maintains its spherical shape. The rock erodes in such a way that it loses half of its volume in 40 years. How long will it take for the rock to erode completely?

²Or the quotient rule, if you're one of *those* people