Introduction to Coupled ODEs

Differential Equations

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Here is a system of coupled ODES:

$$\frac{dP_1}{dt} = \alpha P_1 - \beta P_1 P_2 \,, \tag{1}$$

$$\frac{dP_2}{dt} = -\gamma P_2 + \delta P_1 P_2 \,. \tag{2}$$

Here's the deal. The idea is that P_1 and P_2 are the populations of two different species. The above equations are a rule for how P_1 and P_2 change. Their rates of change depend on both populations. For example, the rate of change of P_1 depends on the current values of P_1 *and* P_2.

A solution to Eqs. (1) and (2) consists of $P_1(t)$ and $P_2(t)$: the values of both populations as a function of time t.

The Greek letters are model parameters, similar to r and K for logistic* growth. Part of this exercise is to puzzle through what each of the Greek letters represent. Each of the Greek letters is a positive number.

1. What does α represent? To think about this, what happens if there are no P_2s? That is, if $P_2 = 0$, what is Eq. (1)?

2. What does γ represent? To think about this, what happens if there are no P_1s? That is, if $P_1 = 0$, what is Eq. (2)?

3. Now let's think about β. What does the presence of P_2s mean from the point of view of the P_1s? What might the parameter β represent?

4. Similarly, let's think about δ. What does the presence of P_1s mean from the point of view of the P_2s? What might the parameter δ represent?

5. What's going on with P_1 and P_2? What animals† could P_1 and P_2 be?

6. What do you think solutions P_1 and P_2 might look like?

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*Recall that the logistic equation is:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \,. \tag{3}$$

†Real or fictional