

Class 20: Probability Density

Calculus II

College of the Atlantic. Feb 19, 2025



Figure 1: Left: A unicorn with wings. Image by brgfx on Freepik. Right: Mini marshmallows. Image by wikipedia contributor Dvortygirl. Image source: wikipedia.licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license.

Unicorns are created when a cosmic ray with an energy greater than 50 TeV (Tera Electron Volts) interacts with a marshmallow. See figures above. An international team of scientists has created a unicorn generator device by densely packing a large room with marshmallows.

Cosmic rays hit the earth at a constant probabilistic rate. In any hour, the probability that cosmic ray with a sufficient energy to create a unicorn hits the marshmallow unicorn generator, is 0.03. The waiting time between unicorn creation events is a random variable t that is distributed according to:

$$\rho(t) = \lambda e^{-\lambda t}, \quad (1)$$

Where $\lambda = 0.03$, in units of probability per hour, and t is measured in hours.

At exactly 8:00am you observe a unicorn creation event. After seeing this event:

1. What is the probability that you need to wait two hours or less for the next unicorn creation event?
2. What is the probability that you need to wait less than forever for the next unicorn creation event?
3. What is the probability that you have to wait exactly 4 hours for the next unicorn to be created?
4. How long would you have to wait so there is a 50% chance that you see another unicorn created?