Qualitative Analysis[1] of a Differential Equation

Differential Equations

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Consider the following differential equation:

$$\frac{dN}{dt} = 0.15N \,, \tag{1}$$

for non-negative N. The variable N is population[2] of some animal. They could be unicorns. They don't have to be, but they could be. The variable t is time, measured in months.

1. What does Eq. (1) say? Translate the equation into words. What are the units on N and dN/dt?

2. Yes, ok, but what does it *mean*?

3. Very roughly, what do you think the solutions to Eq. (1) look like? Don't write a formula—just make a rough sketch. Think about the concavity of your graph.

4. One the same axes, sketch another solution of the differential equation.

5. Sketch the right-hand side of the equation. How does this help see what the shape of solution of the differential equation is going to be?

6. What is the long-term fate of all starting values for N?

7. Write down the formula for a solution $N(t)$ to Eq. (1).

8. Write down the formula for a different solution $N(t)$ to Eq. (1).

9. Write down the formula for the solution to Eq. (1) for the case where there are 300 unicorns at time $t = 0$.

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[1] And an analytic solution

[2] If it bothers you that N isn't always an integer, then you can think of N as the total biomass of the unicorns instead of the total number of unicorns.