Chapter 4.6: Related Rates (and Implicit Differentiation)

Calculus I

College of the Atlantic. November 14, 2024

1. Implicitly take the derivative of the following functions. Solve for y' if possible.

   (a) $x^2 + y^2 = 16$

   (b) $x^2y + y^3 = x$

2. Consider a spherical balloon of radius r (in cm). The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.

   (a) Find $\frac{dV}{dr}$ when $r = 1$ and $r = 2$, and give a practical interpretation of your answers.

   (b) Suppose the balloon is being inflated in such a way that $r(t) = 2t$ centimeters after t seconds. How fast is the volume of the balloon increasing when $r = 1$? How fast when $r = 2$?

   (c) Now suppose that air is being blown into the balloon at a constant rate of 50 cubic centimeters per second. How fast is the radius of the balloon increasing when $r = 1$? How fast when $r = 2$?

3. A three meter ladder stands against a high wall. The foot of the ladder moves outward at a constant speed of 0.1 m/2.

   (a) When the foot is 1 meter from the wall, how fast is the top of the ladder falling?

   (b) When the foot is 2 meters from the wall, how fast is the top of the ladder falling?