Qualitative Analysis of another Differential Equation

Differential Equations

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Yesterday we worked through this differential equation:

$$\frac{dN}{dt} = 0.15N \; , \tag{1}$$

for non-negative N, where N is the population* of some animal. Using qualitative analysis, we could see that solutions $N(t)$ were increasing and concave up. Using some calculus facts, we saw that the general analytic† solution is:

$$N(t) = N_0 e^{0.15t} \; , \tag{2}$$

where N_0 is the population at time $t = 0$.

Note that Eq. (1) is of the form:

$$\frac{dN}{dt} = f(N) \; . \tag{3}$$

The idea is that the growth rate dN/dt of the population N depends on (i.e. is a function of), the current population N. Equations of the form of Eq. (3) can describe a very wide range of situations. We will consider one such situation today.

1. Exponential growth can't continue forever. As the population grows, the growth rate stops increasing, starts decreasing, and then becomes zero. Make a sketch a possible $f(N)$ that describes this situation.

2. Based on the $f(N)$ you drew, sketch a solution to Eq. (3). Think about the concavities of your graph.

3. Sketch a few other solutions.

4. What is the the long-term fate(s) of all starting values for N?

5. A possible formula the $f(N)$ that you sketched previously is:

   $$f(N) = kN(1 - \frac{N}{K}) \; , \tag{4}$$

   where K and k are positive constants.

   (a) Convince yourself that the graph of Eq. (4) looks like the figure.
   (b) What is the practical meaning of K? If K is doubled, how do the solutions $N(t)$ change?
   (c) What is the practical meaning of k? If k is doubled, how do the solutions $N(t)$ change?

6. To what real-world-ish scenarios might one see growth behavior like the one we've analyzed here?

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*Is that better?

†I.e., an equation