Introducing the SIR Model: Adding Birth and Death

Differential Equations

College of the Atlantic. January 28, 2026

Yesterday we constructed, coded up, and began to analyze the SIR model of epidemics. The model we came up with is:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

$$\frac{dI}{dt} = +\beta SI - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \ . \tag{3}$$

The parameter γ is related to the rate at which infected people recover from the disease, and β is related* to how infectious† the disease is.

Today we are going to add births and deaths to the model. This is sometimes referred to as the SIR model with demography.

Let μ be the birth rate, and for the sake of simplicity, let's assume that the death rate equals the birth rate. When people are born, they are not immune to the disease.

1. Modify the SIR equations to account for the birth and death rate, as described above.

2. Since the birth and death rates are assumed to be equal, the total derivative (i.e. $dS/dt + dI/dt + dR/dt$) should still be zero. Is it? If not, you know you need to adjust your equations.

3. Write down a modified compartment diagram that corresponds to your equations.

4. Modify your python code to incorporate birth and death rates. Experiment with different parameter values. How does the behavior of the model change?

*In some non-direct and fraught ways. We'll dig into this more in the weeks ahead

†Under a particular set of social conditions