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SUSTAINABLE ENERGY: FOUNDATIONS AND SKILLS

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Contents

Acknowledgments

This book began as a class that was developed at College of the Atlantic with my former colleague Anna E. Demeo. I am grateful for her friendship, insight, humor, and enthusiasm. Anna is currently Associate Professor of Sustainable Design Engineering at the University of Prince Edward Island.

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0 Preface

The goal of this textbook is to give readers the knowledge and skills needed to be effective advocates for sustainable energy and to make wise choices among different options for renewable power generation and energy conservation. These skills and knowledge include basic understanding of energy and power as well as a facility with simple financial tools used to evaluate investments. But most important is a willingness to do basic calculations, make estimates, and use these numbers to think critically. In my experience these skills and habits of mind are accessible to almost all students, including those with only a modest preparation in mathematics.

This book is based on a class I have taught almost yearly for since 2010 at College of the Atlantic. The class was developed together by Anna Demeo and me, and we team-taught the class for a number of years. We originally used parts of David MacKay's *Sustainable Energy—Without Hot Air* (UIT Cambridge, 2008) for the course. I love this book: it is engaging, accessible, grounded, and MacKay does a masterful job of making potentially abstract quantities tangible and meaningful. MacKay's book is an inspiration for this text.

0.1 Central Premises

There are several fundamental beliefs that inform my thinking about this book.

- **The climate problem is an energy problem.** Burning fossil fuels for the production of energy is responsible for the vast majority of the carbon dioxide. This carbon dioxide is the primary driver of climate change. Agriculture and deforestation are important, as well, but energy is by far the largest contributor. There is widespread agreement that avoiding catastrophic climate change must involve a transition almost entirely away from fossil fuels.

- **Solutions that scale.** Climate/energy is a big problem that requires big solutions. We produce and use energy on a large scale, and the vast majority of this energy derives from fossil fuels. Thus, to successfully prevent catastrophic climate change we must employ large solutions that move us away from fossil fuels. This requires solutions that scale up to address the magnitude of the issues we face. Learning how a residential wind turbine works is interesting, and learning how to build your own is a fun hobby. But understanding the land area and cost associated with large wind farms is essential for developing serious solutions to the climate/energy problem.

- **Energy numeracy[1].** I believe that energy and power units—the kilowatt-hour and the kilowatt—need to be as commonly understood as inches and miles, pounds and tons. Is forty kilometers too far to walk in a day? The answer to this question is clearly yes for most people, including me. Is forty kilowatt-hours a lot of energy to use in one day? I want readers to be able to answer questions about kilowatt hours as confidently as they can answer questions about kilometers.

[1] Find a better word than numeracy?

- **Estimation.** A key part of working numerically is the ability to make quick numerical estimations, a skill that I find many students are surprisingly uncomfortable with. The text gives explicit instruction in estimation techniques and include multiple examples of back-of-the-envelope calculations.

- **Understanding large numbers.** Energy numbers tend to be huge and difficult to understand. Large numbers are often used, perhaps unintentionally, to obscure or intimidate. Megawatts, gigawatt-hours, and quadrillions of BTUs are all common units in energy and climate discussions. Large numbers also arise when considering the land mass needed for solar or wind or biomass: tens of thousands of square miles or millions of acres. This book will teach simple but important techniques for understanding large numbers and making them less abstract and more personal. I think this is absolutely essential for rational discussions about our energy and climate future.

- **Numbers not adjectives.** This is a mantra of MacKay in his book. Energy is an inherently quantitative issue. It is not enough to say that there is huge potential in solar power. How huge? And how much area would the solar cells take? It's not enough to say that insulating your attic will save a lot of energy. But *how much* energy? And how does that compare to the energy you could generate if you spent a certain amount of money to buy solar panels? How much less greenhouse gas emissions would you be responsible for if you got a more fuel efficient car, and how would this deduction compare to shifting to a vegetarian diet?

- **Just a bit of physics.** There are a relatively small handful of basic notions from physics that I think greatly deepen ones understanding of sustainable energy. These topics include the law of conservation of energy, the difference between power and energy, and some basic formulas for kinetic energy, thermal energy, and so on. I construct a foundation in these areas, assuming essentially no prior knowledge of physics. I then use this foundation to build an understanding of some key results, such as the fact that the power in wind of speed v is proportional to v^3, and that the amount of energy needed to heat a home is directly proportional to the difference in temperature between the inside and the outside. Throughout this book the central goal is to prepare readers to understand and do effective work in sustainable energy. The physics we introduce is in service of this goal.

- **Finance.** It is fairly easy to come up with good ideas for generating energy or reducing energy use. Usually the hard part is securing funding for a project. In this area some basic knowledge of financial mathematics goes a very long way. Understanding the vocabulary of finance is essential for conversation with banks and other funders, something that is necessary for all but the smallest projects. Additionally, financial metrics give one a tool to use when choosing among different potential investments.

- **Choosing among options.** There are many ways to use less energy or to generate energy without fossil fuels. However, given the reality of limited time and resources, it can be a challenge to choose among these many options. The initial cost, as well as the payback period, factor into these choices as do numerous other considerations, including community support (or lack thereof), local ordinances, state regulations, and utility rules. How does insulating my house compare to putting up solar panels on my roof? Should a state government spend funds to help homeowners insulate their homes, or should it invest in public transportation?

- **You can't stop reality from being real.** A plan to get the world off fossil fuels has to be grounded in reality, not sentiment. The approach in this book is data driven and considers bounds on energy use and production arising from the law of conservation of energy and other constraints.

- **The truth.** The situation around climate change looks bleak and there are enormous challenges before us. I am confident that any solution has to begin with an honest accounting of our current situation. Downplaying the scale of the problem or ignoring it entirely will not fix anything. However, I have found both a sense of empowerment and a measure of comfort come out of facing reality head on. Instead of getting discouraged, let's get to work thinking about solutions that add up and that are based on actual numbers and not wishful thinking.

- **Intellectural Curiosity and Fun.** There's a lot of hard work to do, and the negative impacts of a changing climate are already here, and will very likely worsen in the years ahead. While I don't mean to make light of the seriousness of the situation, I nevertheless find these topics interesting, and even fun, to learn about and think through I really like figuring this stuff out and sharing fun and creative ways of thinking of things. And I also like good (and bad) jokes.

0.2 *What this book isn't*

To get a better sense of the scope of our book, it might be helpful to say a bit about what this book is not about:

- Energy policy
- Social and environmental impacts of energy technologies
- How to weather-proof your house
- How to build your own wind turbine or install solar cells
- The science of climate change
- A textbook designed to systematically cover a set of topics typically found in an introductory physics course.

While I do not cover these topics at length, this book should help readers think about about the items above list in a deeper and more grounded way.

0.3 *Audience*

This text covers the basic knowledge and analytical skills needed by almost anyone working in sustainable energy. The topics in this book will be of interest to students focusing in sustainability and environmental studies, as well as those whose primary interests are business, energy policy, or physics. Additionally, the material should prove valuable to solar PV installers, home insulators, business owners interested in saving money and greening their operations, climate change activists, planning board members, state representatives, and anyone interested in getting involved in a community renewable energy project.

This book should be well suited for readers with a range of backgrounds. At a minimum, readers should be comfortable with basic math and very simple algebra. Some facility with units and unit conversions, at the level one encounters in a high school chemistry or physical science class, is certainly helpful, but not strictly necessary. We review unit conversions in the book.

This text is intended to be used in a classroom setting. The format of the book encourages flexibility and should enable it to be adopted for courses with different emphases. I see this book as a good match for an interdisciplinary, foundational sustainability course in an environmental studies program, a business program, and perhaps also in masters programs emphasizing sustainability. While not a comprehensive physics of energy textbook, there is nevertheless enough physics content that it might also be used as a text for a physics of energy course, although likely the instructor would need to supplement the course with a bit more advanced material.

0.4 *Notes on the Book's Organization*

The book is divided into five[2] parts.

There is far more material in this book than can be taught in a typical one-term college course. The structure of the course is sufficiently flexible that instructors should be able to choose a set of topics that works for their goals and their students' backgrounds.

Mention something about exercises.

[2] **TODO!** This will need significant updating.

Part I

Introducing Climate & Energy

1
Greenhouse Gas Emissions

As the title suggests, this chapter is about greenhouse gas (GHG) emissions. What are the different ways GHG emissions are tallied up, and what units are used to measure GHG emissions? This chapter is our first encounter with very large numbers—gigatons of emissions—and I discuss some ways to wrap our heads around such large numbers so they are meaningful and relatable.

I suspect much of this material will be familiar to many readers, but I'd encourage you to at least skim this chapter, since it helps to set the stage for much of what follows. But If you're eager to get started with energy, you can certainly skip ahead to Chapter 3.

1.1 World Emissions

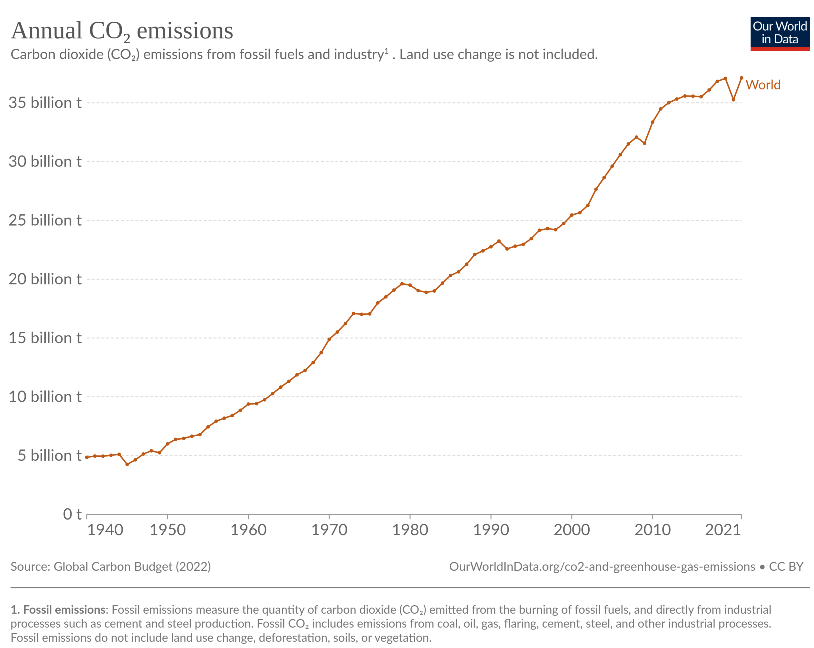


Figure 1.1: Worldwide annual CO_2 emissions, 1940–2021, not including land use change and forestry. Figure from Ritchie (2020), available at `https://ourworldindata.org/co2-and-greenhouse-gas-emissions`.

We'll start by looking at the annual greenhouse gas (GHG) emissions for the entire world. Figure 1.1[1] shows the worldwide GHG emissions from 1940 to 2021. Before we dissect this graph, note the big picture: global emissions have been rising more or less steadily. The small decreases in emissions are associated with large world events. For example, we can see that emissions went down in 2009, during a worldwide economic recession, and during 2020, as a result of the shutdowns that occured as a result of the COVID-19 pandemic.

[1] All figures in this chapter were made on the ourworldindata.org website. See Sec.1.6 for more about data and graphs.

Let's now look at the graph a little more closely. First, note the fine print near the top of Fig. 1.1: "Carbon dioxide (CO_2) emissions from fossil fuels and industry." This means that the graph shows only the CO_2 from burning fossil fuels[2] and from industrial processes, like the production of cement. The fine print in Fig. 1.1 continues, "Land use change is not included." Emissions associated with land-use change are those associated with forestry (growing forests or burning them) and with agricultural practices. This is often abbreviated LUCF, for land-use change and foresty.

[2] We'll see in Section 2.4 that emissions from industrial processes are quite small compared to the emissions that result from directly burning fossil fuels.

1.2 *Carbon (CO_2) vs. Carbon Dioxide Equivalent (CO_2e)*

The principal GHG is is carbon dioxide: CO_2. This gas, which is produced when fuel is burned, contributes to the greenhouse[3] effect. But carbon dioxide is not the only greenhouse gas. Other gases, mainly methane (CH_4) and nitrous oxide (N_2O) also contribute to the greenhouse effect; like CO_2, they act as planetary blankets. Rather than reporting the amounts of all the different greenhouse gases that are getting emitted, one usually converts other greenhouse gases to equivalent amounts of CO_2. Methane has a 100-year global warming potential[4] of 28. This means that one kilogram of methane will warm the planet as much as 28 kilograms of carbon dioxide, over a 100-year period. One would say that the CO_2 equivalent of one kg of methane is 28 kg. Carbon dioxide equivalent is abbreviated CO_2e, or sometimes CO_2eq.

[3] I briefly discuss the greenhouse effect in Section 2.1.

[4] The global warming potential of methane and many, many other gases are compiled in the Intergovernmental Panel on Climate Change (IPCC) Fifth Assessment Report by Working Group One: *Climate Change 2013: The Physical Science Basis* (Stocker et al., 2014, pp. 731-738).

Returning to Fig. 1.1, the data shown is the total world emissions of CO_2 ; a tallying up of all the CO_2 that resulted from burning fossil fuels each year. What would this graph look like if we tallied up *all* the greenhouse gases, not just CO_2? Such a graph—a plot of CO_2e vs. year—is shown in Fig. 1.2. Noting the different scales on the plots in Fig. 1.1 and Fig. 1.2. One can see that CO_2e emissions are larger than CO_2 emissions. This makes sense, since CO_2e includes all greenhouse gases (converted to their CO_2 warming equivalent), not just CO_2.

Let's now look at the vertical scale on Fig. 1.2. The units are "billion t", which stands for billions of tons. In 2019, the last year shown in Figs. 1.1 and 1.2 the world emissions were around 37 Gt of CO_2 and 48 Gt of CO_2e.

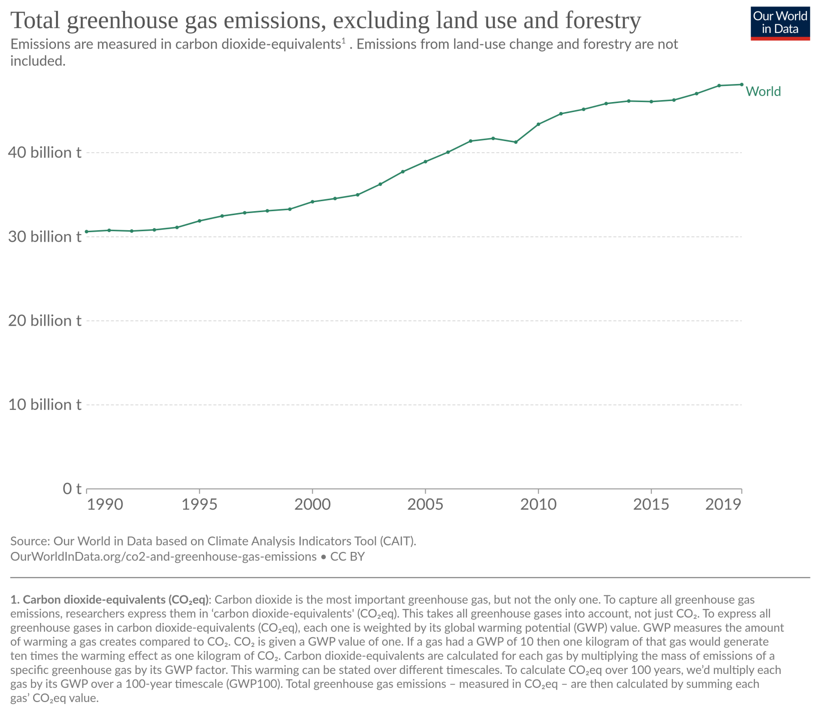


Figure 1.2: Worldwide annual CO_2 emissions, 1940–2021, not including land use change and forestry. Figure from Ritchie (2020), available at https://ourworldindata.org/co2-and-greenhouse-gas-emissions.

1.3 *Making Sense of GigaTonnes*

So the 2019 world emissions were 48 billion tons. How should we think about this? We'll need to talk about tons, and billions. Tons first.

A ton is a unit of mass–which roughly speaking can be thought of as how heavy something is or how much it weighs. In this context, "ton" refers to a metric ton, which is is 1000 kilograms. Metric tons are sometimes denoted "tonnes". A metric ton is different from tons based on pounds.In the United Kingdom Imperial system of units, a ton is 2,240 pounds. In the US and Canada, a ton is 2000 pounds. UK tons are sometimes called long tons, and US/Canada tons are sometimes called short tons. So far as I know, just about everyone who studies energy and climate uses the much simpler metric ton, or tonne. In this book we will only use metric tons, I will refer to them simply as tons and not tonnes.

By the way, one kilogram is equivalent to 2.2 pounds, so we can convert metric tons to pounds:

$$1 \text{ ton} = 1000\,\text{kg} \left(\frac{2.2\,\text{lbs}}{1\,\text{kg}}\right) = 2200\,\text{lbs}\ . \tag{1.1}$$

So one metric ton is 2200 pounds.

Gt, where Gt stands for gigatonnes.

Now let's think about billions. In the US and many other places,

$$\text{One billion} = 1,000,000,000 = 10^9\ . \tag{1.2}$$

One billion, or 10^9. So:

$$1\,\text{Gt} = \text{one billion tons} = 10^9\ \text{tons}\,. \tag{1.3}$$

That's a lot of tons. A billion is a number that is hard for almost everyone to picture and think about. A convenient way to get a handle on the worldwide total emissions of 46 Gt is to instead think about emission *per capita*—the average emissions per person.

In 2016 the world population was around 7.4 billion. Another way to say this is that the population was 7.4 gigapeople. The unit gigapeople[5] is 10^9 people. We can divide the total emissions by the world population to get the emissions per person:

[5] This is not a standard unit. But it's very convenient for describing the world population.

$$\text{Emissions per capita} = \frac{46\,\text{Gt}}{7.4\text{Gp}} \approx 6.2\text{t/p}. \tag{1.4}$$

That is, the worldwide average emissions is 6.2 tons of CO_2e per person. I usually round this down to 6:

$$\textbf{Average world } \mathbf{CO_2}\textbf{e emissions} \approx \textbf{6 tons per person}\,. \tag{1.5}$$

This number is easy to remember, and we will use it often as a benchmark throughout the book.

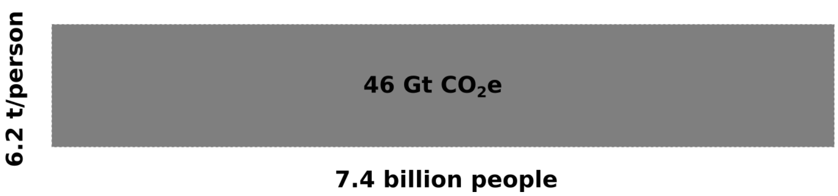


Figure 1.3: Visualizing world CO_2e emissions. The area of the rectangle represents the total world emissions: 46 Gt.

Figure 1.3 gives us another way to think about and visualize worldwide emissions. Here, the *area* of the rectangle represents the total emissions: 46 Gt. The base on the rectangle represents the number of people in the world: 7.4 billion (or 7.4 Gp). And the height of the rectangle is the *per capita* emissions of 6.2 t/p. To get the area of the rectangle, we multiply the base and the height:

$$\text{Area of rectangle} = \text{Base} \times \text{Height} = 7.4\,\text{Gp} \times 6.2\text{t/p} = 46\,\text{Gt}\,. \tag{1.6}$$

1.4 *Emissions by Country*

Figure 1.2 shows the total world CO_2e emissions. How does this break down by country? Figure 1.4 shows the emissions associated with six different countries: China, the United States, India, France, Argentina, and Nigeria. China is the country with the largest total emissions, passing the US into first place around 2004. We see on the graph that China's emissions have risen fairly sharply since 2002.

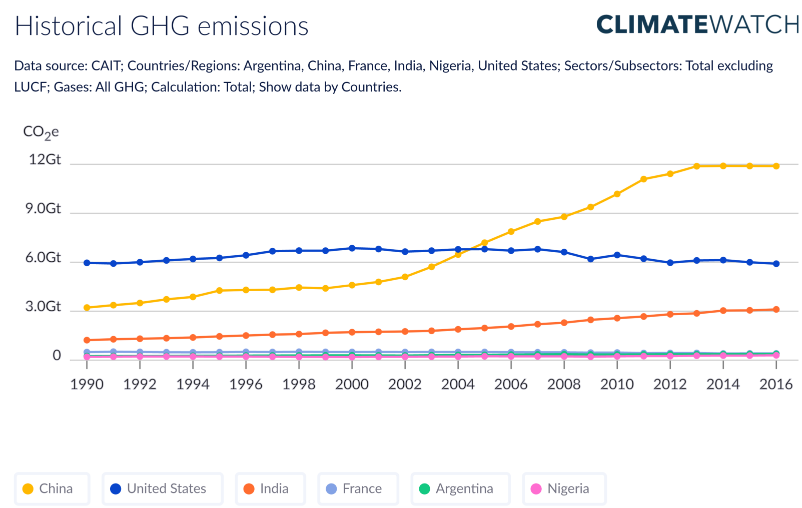


Figure 1.4: CO_2e emissions (not including LUCF) for several countries. Figure from `www.climatewatch.org/ghg-emissions`.

Let's focus our attention on the US. In 2016 the US emissions were around 6 Gt of CO_2e. The US population in 2016 was around 330 million people. This is 0.33 gigapeople, which is almost 1/3 of a gigaperson. We can then calculate the US emissions *per capita*:

$$\text{US Emissions per capita} = \frac{6\,\text{Gt}}{\frac{1}{3}\text{Gp}} \approx 18\,\frac{\text{t}}{\text{p}}\,. \tag{1.7}$$

This is another very useful number to remember:

$$\textbf{Average US CO}_2\textbf{e emissions} \approx \textbf{18 tons per person}\,. \tag{1.8}$$

Note that the US *per capita* emissions are roughly three times the world *per capita* emissions.

The *per capita* emissions of the US and five other countries are shown in Fig. 1.5. Note that, of the countries shown, the US *per capita* emissions are by far the largest. China's *per capita* emissions are roughly half that of the US. The *per capita* emissions of India and Nigeria are relatively small—just 2.6 and 1.6 tons of CO_2e per person, respectively.

We can also make a rectangle for the US. This is shown in Fig. 1.6

1.5 *Details about Measuring Emissions*

When looking at GHG emissions data, it's important to know that what is generically called emissions can actually refer to different things. So when working with emissions data it can be important to read the "fine print." There are different ways that emissions are tallied up. Some of these different tallying methods are discussed below. Emissions data

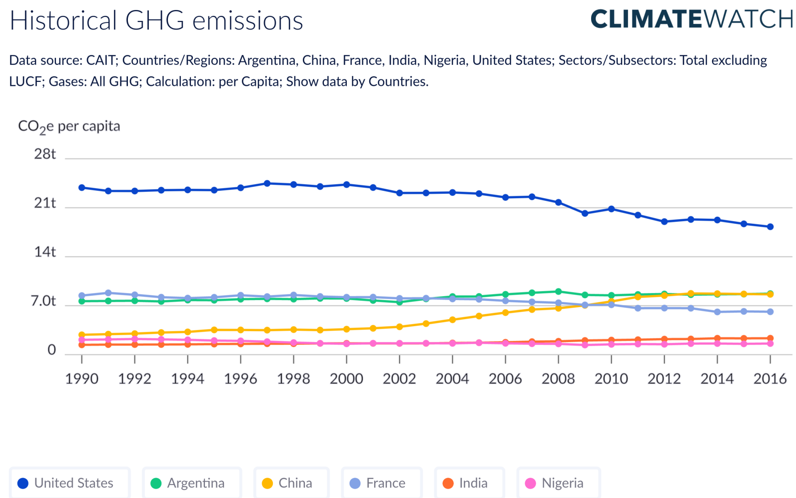


Figure 1.5: CO_2e emissions *per capita* (not including LUCF) for several countries. Figure from www.climatewatch.org/ghg-emissions.

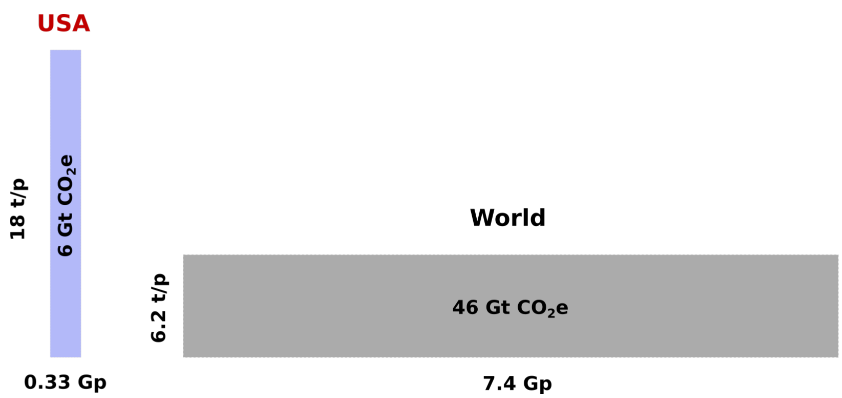


Figure 1.6: Visualizing CO_2e emissions for the world and the US. The area of the rectangles represents the total emissions.

tallied different ways for the world and several countries are shown in Table 1.1.

1.5.1 *hello world*

- **CO_2 or CO_2e?** As we've discussed, there are other greenhouse gases besides carbon dioxide. Greenhouse gases other than CO_2 are converted to their CO_2 equivalent.
- **Is land-use change and forestry (LUCF) included?**. LUCF includes the CO_2 emitted when forests are burned, the CO_2 absorbed by trees as they grow, and changes in the carbon stored in soil. LUCF can be positive or negative, depending on the nature of the land-use changes. Negative LUCF emissions usually arise in countries which cut down a lot of their forests many years ago and whose forests are now growing back. A good, concise overview of some of the issues associated with quantifying LUCF emissions is (Baumert et al., 2005,

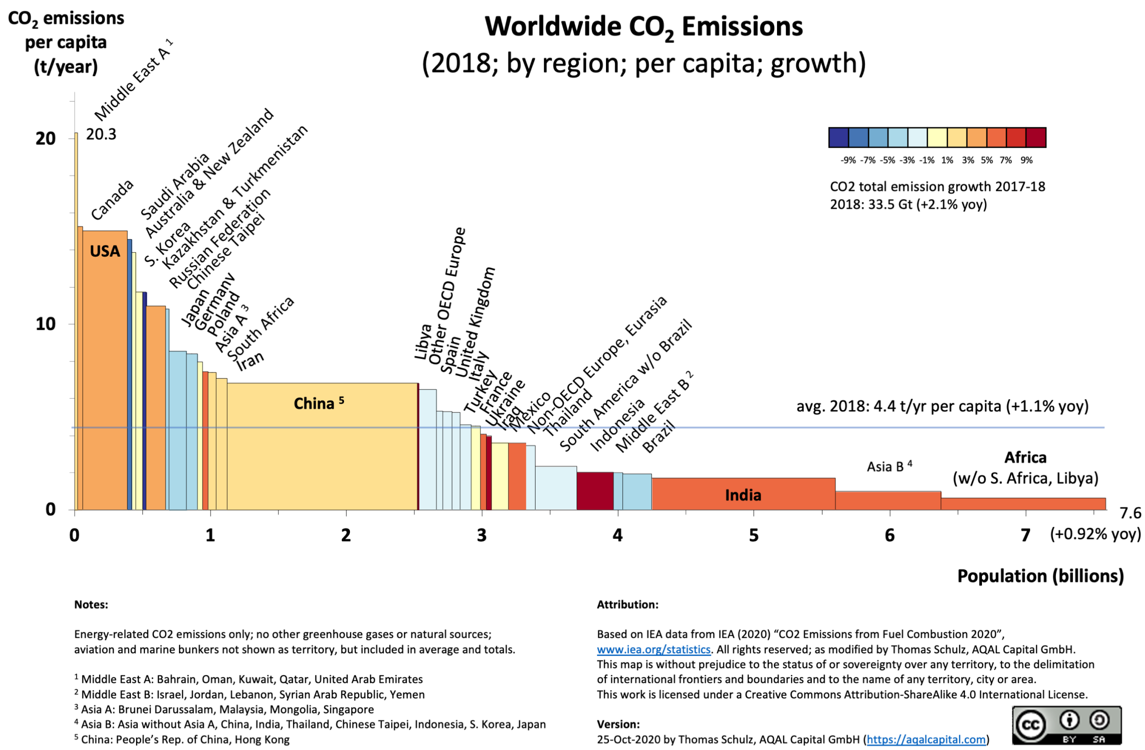


Figure 1.7: A rectangle view of worldwide emissions. These data include only CO_2e (and not CO_2e) associated directly with burning fossil fuels. Figure source: Tom Schulz, AQAL Capital, available at `https://aqalgroup.com/2018-worldwide-co2-emissions/` licensed under CC BY-SA 4.0: `https://creativecommons.org/licenses/by-sa/4.0`.

Chapter 17).

- **Carbon Dioxide or Carbon?** Ecologists measure emissions in terms of tons of carbon as opposed to tons of CO_2. This makes sense, as ecologists are interested in tracking the carbon cycle: atmospheric carbon dioxide is pulled from the atmosphere by plants who turn carbon dioxide into carbohydrates. These carbohydrates are then eaten (bugs, animals, or microorganisms in the soil), producing carbon dioxide. Since ecologists track carbon in different forms (carbon dioxide and various carbohydrates), they usually measure emissions in terms of carbon instead of carbon dioxide. To convert from C to CO_2, or vice versa:

$$1\text{kg of Carbon} = 3.67 \text{ kg of } CO_2 \,. \tag{1.9}$$

(The origins of this conversation factor are discussed in the narrative before Exercise 1.9.) The distinction between C and CO_2 is an important one, as their numerical values differ by almost a factor of four. exercises about this.

- Include only emissions from burning fuels? Or all CO_2e emissions? Examples of non-burning emissions might be natural gas that escapes from pipelines. This is referred to as *fugitive emissions*[6] Fugitive emissions are significant. 1.5% of world emissions of CO_2e is due to fugitive emissions in Russia. Another source of CO_2e that is not due to burning fuels is cement production. Carbon dioxide is a byproduct of the chemical process produces calcium oxide, a key ingredient in cement. (See Section 14.5 for details.)

[6] I think this would make a good name for a band.

- How account for international shipping? If I take a flight from New York to Amsterdam, or if my smart phone takes a boat from Taipei to Los Angeles, to which countries should the emissions associated with this transport be tallied?

- Production-based vs. consumption-based estimates. Need reference.

	Region			
Measurement	US	India	Brazil	World
CO_2e including LUCF	5.8	3.2	1.38	49.4
CO_2e excluding LUCF	5.9	3.1	1.05	46.1
CO_2 including LUCF	4.8	2.3	0.74	36.7
CO_2 excluding LUCF	4.9	2.2	0.44	34.0
CO_2e energy only	5.2	2.2	0.45	36.0
CO_2 energy only	4.9	2.1	0.42	32.6

Table 1.1: GHG Emissions in 2016 calculuated six different ways. Units are Gt. All data from Climate-Watch `https://www.climatewatchdata.org/ghg-emissions`.

1.6 *Where to get Data and Make Graphs*

There are some excellent websites that have compiled data and let one make all sorts of plots and visualizations. I used these sites to make most of the graphs in this chapter. I definitely recommend exploring some of these sites. Doing so can be a great way get a feel for past and present emissions.

There are two main sites I use to look at emissions. The first is **Climate Watch** (`www.climatewatchdata.org`), which is managed the the World Resources Institutes. The second is **Our World in Data** (`www.ourworldindata.org`), which is a collaboration between researchers at the University of Oxford and the Global Change Data Lab. I used their articles on energy and CO_2 and other emissions when assembling this chapter.

Another great resource is **The Carbon Map** (`www.carbonmap.org`). This site lets users explore maps of the world where the sizes of countries are scaled to, among other things, current and historical emissions. I find this to be a powerful way to visualize just how different the emissions are among different regions and countries.

1.7 *Exercises*

Exercise *1.1:* The world population in 2016 was around 7.4 billion.

1. Estimate the mass of all the people in the world in 2016. Express your answer in gigatonnes.
2. How does your answer above compare the 2016 worldwide emissions of CO_2e ?

Exercise *1.2:* The certain country emits 20 Mt of CO_2e in one year. The country's population is 28.3 million.

1. What are the CO_2e emissions per capita for this country?
2. How does this compare with the worldwide average CO_2e per person per year?
3. How does this compare to the US's CO_2e per person per year?

Exercise *1.3:* Go to the climate watch ghg emissions page at: `https://www.climatewatchdata.org/ghg-emissions`

1. Reproduce Fig. 1.4.
2. Reproduce Fig. 1.5.

For the next four problems you will need to use either the Climatewatch or Our World in Data websites (see Section 1.6) look up the CO_2e emissions for certain countries. Use the the most recent data available.

Exercise *1.4:* Mexico

1. What is the yearly CO_2e (without LUCF) emissions for Mexico?
2. Express these emission *per capita*.
3. How do these emissions compare to the US and the worldwide average?

Exercise *1.5:* Repeat Problem 1.4 using Uganda instead of Mexico.

Exercise *1.6:* Repeat Problem 1.4 using Spain instead of Mexico.

Exercise *1.7:* Repeat Problem 1.4 using Australia instead of Mexico.

Exercise *1.8:* Collectively, in 2014 the countries referred to as "Least Developed Countries" or LDCs, emitted 1.4 Gt of CO_2e . In 2014 the total population of LDCs was around 950 million.

1. What is the CO_2e emissions per year per capita for LDCs?
2. How does this compare to the CO_2e emissions per year per person for the US?

The next several problems are about measuring emissions in terms of the amount of carbon (C) instead of carbon dioxide (CO_2). Ecologists and others interested in the global carbon cycle often think in terms of C and not CO_2, since they are interested not only in the carbon in the atmosphere. Carbon dioxide in the atmosphere is "eaten" by plants as they grow. So CO_2 from the atmosphere might end up as an oak tree or a tomato or a calendula flower. The plant incorporates the carbon from CO_2 into its structure. There isn't CO_2 in the plant, but rather the C gets turned into a carbohydrate. So, since ecologists are tracking the flow of carbon atoms, not carbon dioxide molecules, they measure C and not CO_2.

Converting from C to CO_2 or vice-versa follows from knowledge of the atomic masses of carbon (C) and oxygen (O) molecules. The atomic mass of C is 12, and the atomic mass of O is 16. This means that

$$\frac{\text{Mass of } CO_2}{\text{Mass of C}} = \frac{12 + 2(16)}{12} = \frac{44}{12} \approx 3.67 \,. \tag{1.10}$$

This means that:

$$\mathbf{1}\,\text{kg of C} = \mathbf{3.67}\,\text{kg of } CO_{\mathbf{2}} \,. \tag{1.11}$$

This lets one convert from C to CO_2 or vice-versa.

It makes sense to me that ecologists work with C and not CO_2. Less understandable to me is units that ecologists prefer to use for measuring carbon: petagrams, abbreviated Pg. Peta is a prefix indicating 10^{15}. (A list of prefixes can be found in Section A.2.1.) Let's see how many petagrams there are in a gigaton:

$$1\,\text{Gt}\left(\frac{10^9\,\text{tons}}{\text{Gt}}\right)\left(\frac{10^3\,\text{kg}}{1\text{tonne}}\right)\left(\frac{10^3\,\text{g}}{\text{kg}}\right) = 10^{15}\,\text{g} \,. \tag{1.12}$$

So one petagram is the same thing as one gigaton. So why don't ecologists just use gigatons like everybody else? I guess this isn't that big a deal, since they're the exact same thing. But still. Anyway...

Exercise *1.9:* 650 Gt of carbon is how many Gt of CO_2?

Exercise *1.10:* 500 Gt of CO_2 is how many Gt of carbon?

Exercise *1.11:* In a paper in the journal *Science Advances*, Chazdon et al. (2016) estimate that over the next 40 years, second-growth[7] forests in Latin America could remove 8.48 petagrams of carbon from the atmosphere.

[7] These are forests which are 1–60 years old at the time this study was conducted.

1. Convert 8.48 Pg of C to Gt of CO_2.
2. Divide your answer to the question above by 40 years to get the yearly rate of CO_2 removal by Latin American second-growth forests
3. This rate of CO_2 removal would balance the yearly CO_2e emissions associated with how many people, assuming the people emit at a rate equal to the worldwide average?
4. This rate of CO_2 removal would balance the yearly CO_2e emissions associated with how many people from the US?
5. How does your answer to the above question compare to the population of the US?

Exercise *1.12:* In the journal *Earth Systems Science Data* van der Werf et al. (2017) estimate that during 1997–2016, the average yearly emissions of carbon due to forest fires is 2.2 Pg.

1. How much CO_2 is this? Express your answer in Gt.
2. Put this number in perspective. These emissions are equal to the emissions of how many average people? How many people from the US?
3. Bob Berwin, writing in *Inside Climate News* (2018) reports on the amount of CO_2 emitted by fires as estimated by van der Werf, et al. What number does Berwin use? Does he get it right? This number is mentioned in the fourth paragraph of his article.

2
Climate Change and Energy

In the previous chapter we discussed ways to quantify and think about greenhouse gas emissions and looked at emissions data for various countries. In this chapter I'll say a little bit more about emissions and then we'll look at how energy use is related to greenhouse gas emissions.

2.1 The Greenhouse Effect

Here is a very quick sketch of the greenhouse effect. The sun delivers energy to the earth. This energy arrives as electromagnetic radiation all across the spectrum: not only visible light but also lower-energy (and longer wavelength) infrared radiation, higher energy (shorter wavelength) ultraviolet radiation. Most of this radiation passes through the clear atmosphere and reaches the earth's surface. In particular, most of the radiation in the visible and ultraviolet parts of the spectrum make it to the surface of the earth, where they warm up whatever surface they land on. The earth, like all objects (except for those at a temperature of absolute zero) emit radiation. This radiation is of a different character than the radiation emitted by the sun. The earth is much cooler than the sun, so more of the earth's radiation falls in the infrared section of the electromagnetic spectrum.

This radiation from the earth heads upward, where some of it is absorbed by CO_2 molecules in the atmosphere. The molecule then re-emits this energy in a random direction, so some of it heads back to earth. This is how CO_2 warms the planet—it prevents some of the infrared radiation from the earth from escaping into outer space. As we add more and more carbon dioxide to the atmosphere, less infrared radiation can leave the earth. So CO_2 thus acts as a blanket. There has been significant amounts of CO_2 in the earth's atmosphere for hundreds of thousands of years, but this amount hasn't changed much. The result is that the earth has been at a more or less constant temperature. Similarly, if you keep a blanket on you for a while, you warm up a little and then reach a constant temperature.

2.2 *Atmospheric* CO_2 *Concentrations*

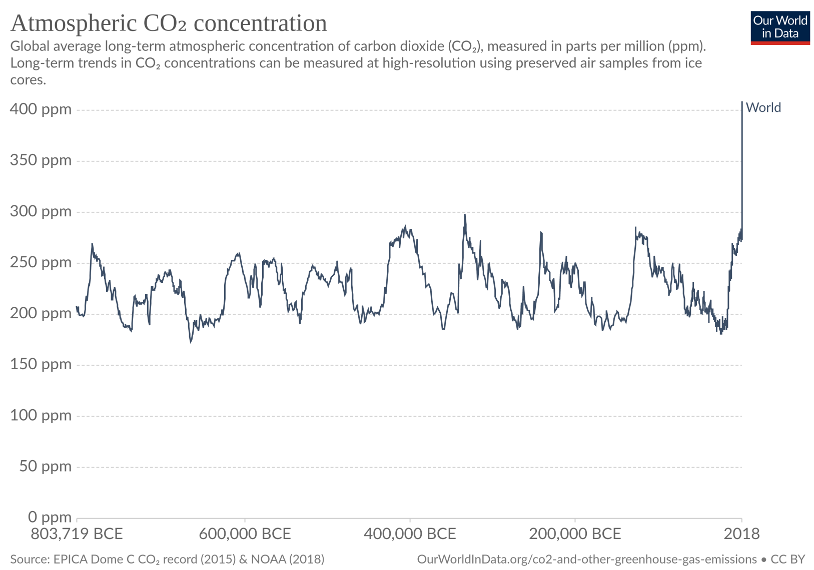


Figure 2.1: Atmospheric CO_2 levels, measured in units of ppm, parts per million. Figure from `https://ourworldindata.org/co2-and-other-greenhouse-gas-emissions`. **DO I NEED THIS FIGURE????**

Atmospheric CO_2 concentrations are plotted in Fig. 2.1. We can see for approximately the last 800,000 years, the concentration level of CO_2 has stayed between 175 and 300 ppm[1]. In the last several hundred years the atmosphere's CO_2 concentration has increased to over 400 ppm. And it is still rising. Figure 2.1 makes clear that we are in uncharted territory. Atmospheric concentrations of CO_2 unlike anything that has been seen for almost a million years.

[1] Ppm stands for parts per million. So if, say, the CO_2 concentration was 250ppm, this means that if you grabbed a million molecules of air at random, around 250 of them would be CO_2. The rest of the would be mostly gaseous nitrogen and oxygen (N_2 and O_2) with very small amounts of Argon and other gases.

2.3 *The Relationship Between Emissions and Warming: Emissions are Cumulative*

Before turning our attention to energy, there is another key fact about greenhouse gas emissions to consider: the effects of greenhouse gases are cumulative. Carbon dioxide and other greenhouse gases stay in the atomosphere for a very long time.

- Atmospheric space
- Carbon Budget
- Bathtub

How much atmospheric space is left? See Anderson et al. (2020). For a popular account, see Anderson and Stoddard (2020).

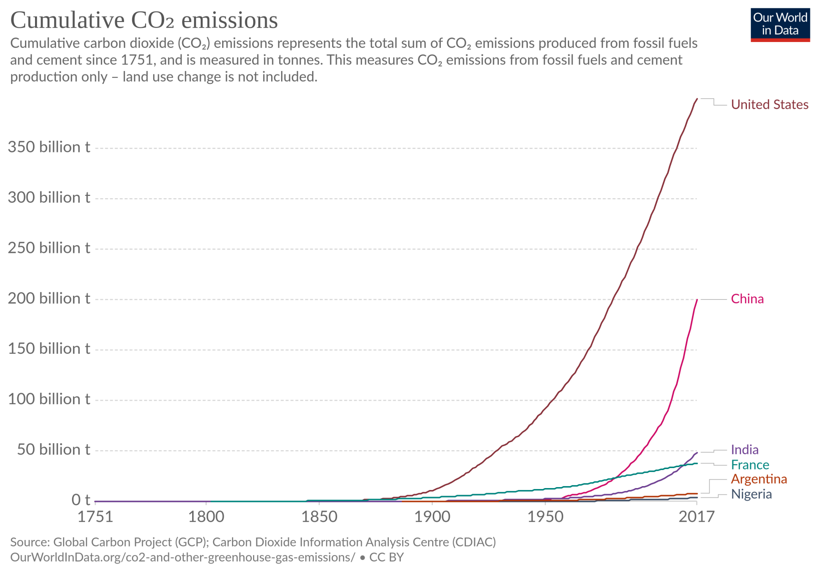


Figure 2.2: Visualizing CO_2e emissions for the world and the US. The area of the rectangles represents the total emissions. Figure from https://ourworldindata.org/grapher/cumulative-co-emissions.

2.4 *The Climate Problem is (Mostly) an Energy Problem*

By the way, there are other reasons to care about energy beside climate change.

1. Saving money

2. Geopolitics

3. Not peak oil. We will be doomed long before all the oil runs out. Need citations. There is not quite 100% agreement on just how much oil is left and how hard/easy it would be to get it.

4. Environment. Fossil fuels, especially coal, produce ozone and particulate matter that is fantastically unhealthy. Look up some estimates for deaths due to asthma. Both in US and developing world, if possible.

2.5 *Energy Use is Associated with Wellness*

See Day et al. (2016)

2.6 *Energy and Climate Justice*

Carbon Inequality Kartha et al. (2020)

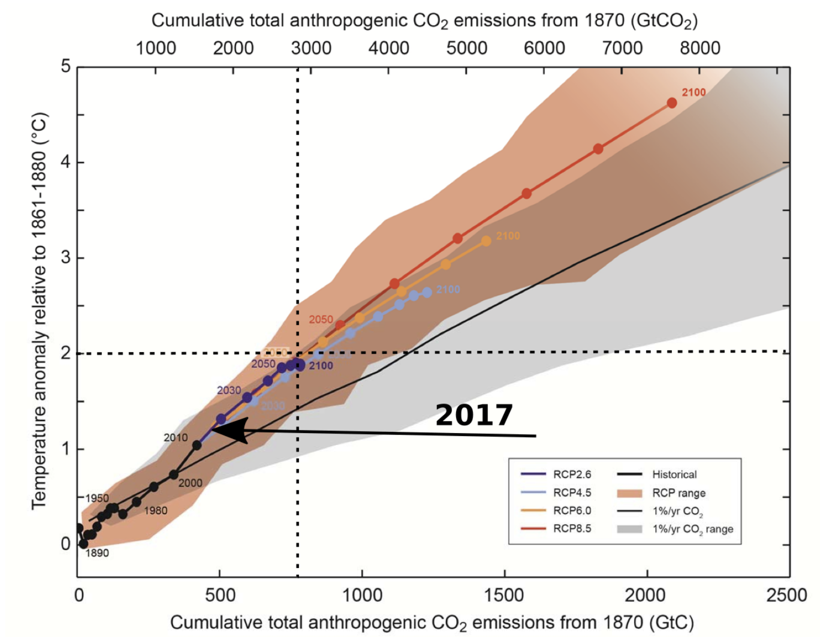


Figure 2.3: Visualizing CO_2e emissions for the world and the US. The area of the rectangles represents the total emissions.

2.7 *The US Energy System: A Look Ahead*

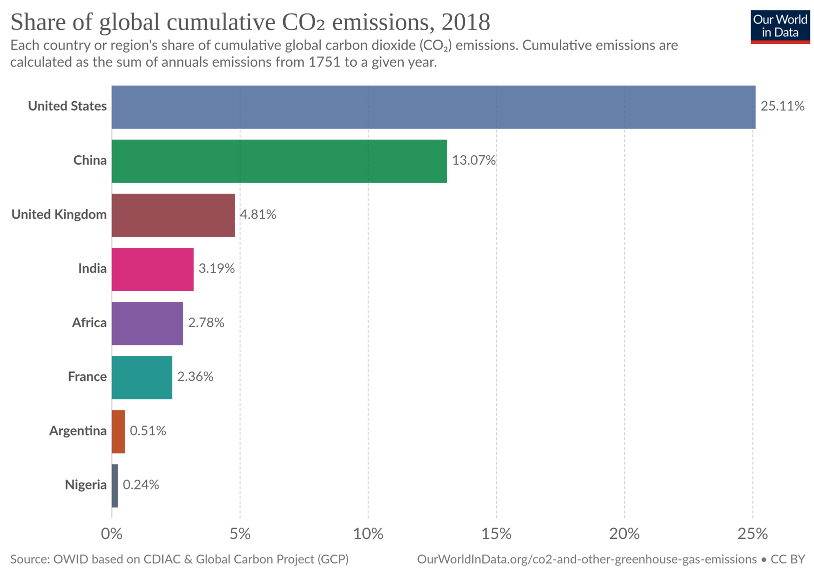


Figure 2.4: Visualizing CO_2e emissions for the world and the US. The area of the rectangles represents the total emissions.

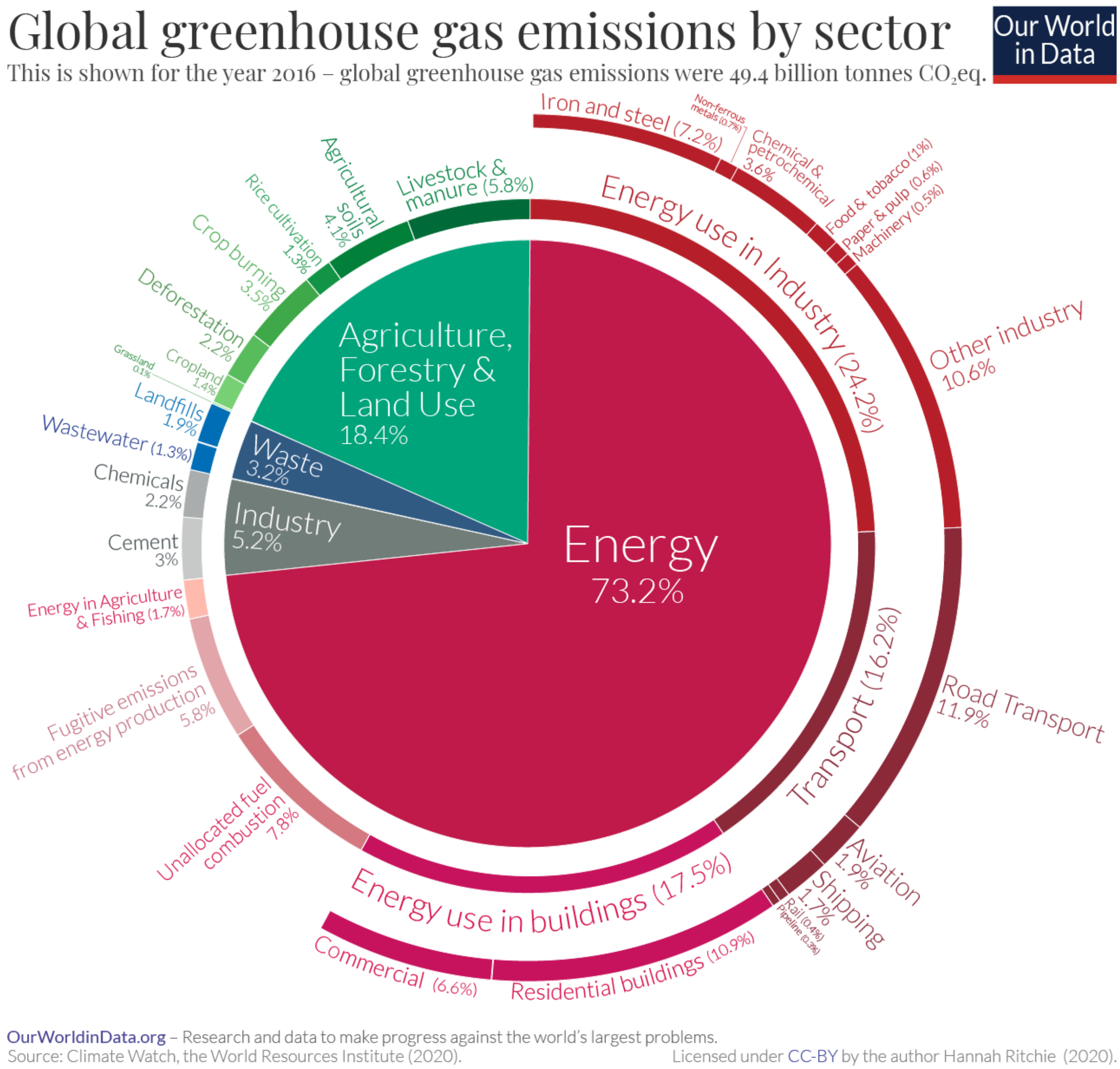


Figure 2.5: Greenhouse gas emissions by sector. Figure by Hannah Ritchie (2020), based on data and methods from IPCC (2014) and Baumert et al. (2005). Figure from `https://ourworldindata.org/ghg-emissions-by-sector`. A flowchart representation of these data can be found at `https://www.wri.org/resources/data-visualizations/world-greenhouse-gas-emissions-2016`.

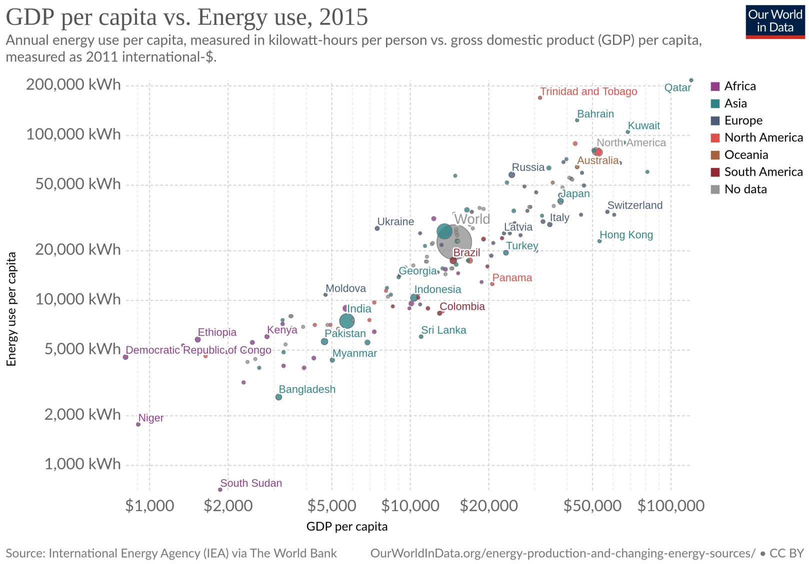


Figure 2.6: Visualizing CO_2e emissions for the world and the US. The area of the rectangles represents the total emissions. **Double-check the scales on the rectangles.**

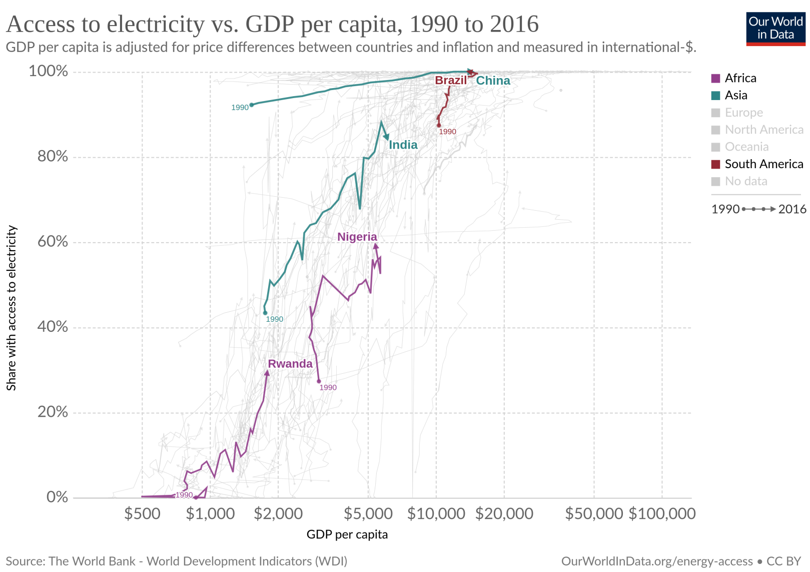


Figure 2.7: Visualizing CO_2e emissions for the world and the US. The area of the rectangles represents the total emissions.

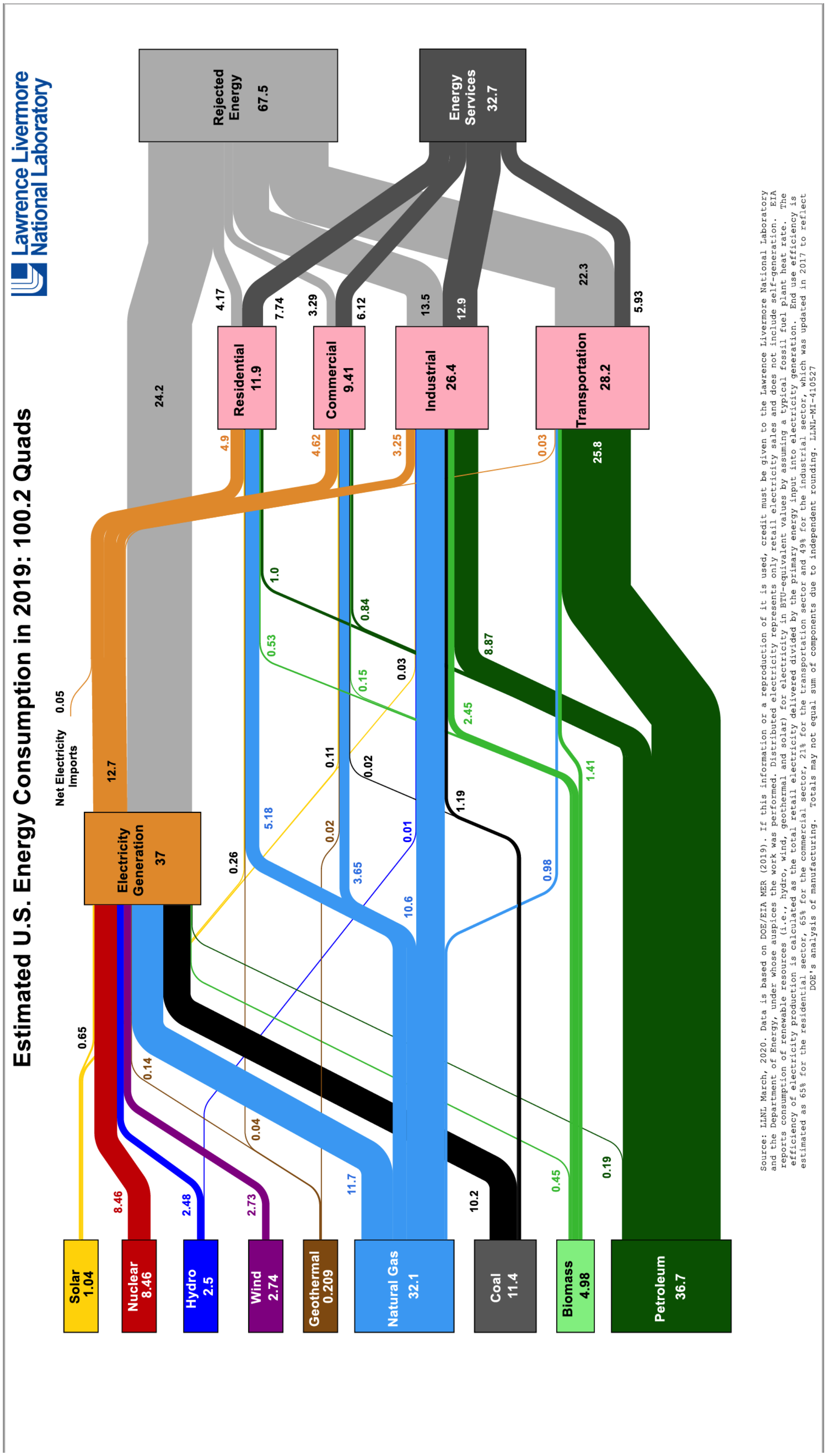


Figure 2.8: A flow-chart representing the 2019 US Energy System. Figure from https://flowcharts.llnl.gov/commodities/energy.

2.8 *Exercises*

Exercise *2.1:* The world population in 2016 was around 7.4 billion.

1. Estimate the mass of all the people in the world in 2016. Express your answer in gigatonnes.
2. How does your answer above compare the 2016 worldwide emissions of CO_2e ?

Part II

Energy Basics

3 Energy

3.1 Jane's Blocks

This is a book about energy, so we should begin by talking about what what energy is. To do so, I'll using an example that is due to Richard Feynman[1], and was published in his introductory lectures on physics (1977). I'll tell a slightly modified version Feynman's example below.

In this story there are two characters, Jane, a smallish child who likes to play with a set of blocks, and Anna, Jane's mom. Anna happens to be an engineer. Jane spends hours alone in her room playing with her blocks, and when Anna pops in to check on her daughter, she notices that the blocks are in different configurations. Sometimes they are piled on the bed, sometimes they are neatly stacked, other times Jane has used them to build elaborate structures. Although the blocks are in different locations in the room every day, Anna notices that there is one thing that stays constant: the total number of blocks is always the same: 28.

Anna expresses this fact via an equation:

$$\text{No. of blocks seen} \;=\; 28\,. \tag{3.1}$$

For many days this equation holds true. Anna always counts 28 blocks in Jane's room.

One afternoon, however, Anna visits Jane's room and discovers that there are only 26 blocks. An exhaustive search of the room does not yield the two missing blocks. Anna then thinks to look out the window and notices two of Jane's toy blocks outside on the yard. Apparently Jane tossed two blocks out the window. Anna retrieves the blocks and installs a secure screen in Jane's window. This ensures that no blocks can escape from the room. Order is restored. Anna always tallies up 28 blocks when she checks in on Jane in her room.

One day, however, the blocks don't add up to 28. After some investigation, Anna suspects that Jane has put some blocks in a box that she has. Anna starts to open the box, but Jane objects strenuously,

[1] Feynman is a nobel-prize winning US physicist. He is well known and much admired, by physicists and others. However, his elevated status is fraught, to say the least. My feelings on Feynman are well summarized by the astronomer Aida Behmard, who writes, "But we must consider Feynman in all his manifestations—a brilliant scientist, but also a narcissist whose sexist behavior did undeniable harm. And while his contributions to physics are immense, it is worth considering that science is not, nor has it ever been, a level playing field." Behmard's entire essay, "Feynman, Harassment, and the Culture of Science" is well worth reading 2019. See also "Surely You're a Creep, Mr. Feynman, McNeill (2019).

demanding that her mom not open the box. Anna is curious about the contents of the box, but she relents, realizing that this is a battle she doesn't need to fight.

The next time Anna comes in Jane's room and she sees 28 blocks, Anna goes and gets her kitchen scale. She puts a block on the scale and finds that it weighs 4 oz. She also places the empty box on the scale, and sees that it weighs 16 oz. Anna can use this information to figure out if there are any blocks in the box without having to upset Jane by looking inside: she just weighs the box. For example, if she puts the box on the scale and finds that it weighs 24 oz, she can infer that there are two blocks in the box. The box weighs 16 oz and there are two blocks, each of which weighs 4 oz: 16 oz + 4 oz + 4oz = 24 oz.

Anna can modify Eq. (3.1) to account for the blocks in the box as follows:

$$\text{No. of blocks seen} + \frac{\text{Weight of box} - 16\text{oz}}{4\text{oz}} = 28\,. \qquad (3.2)$$

To see how this works, you could try plugging in on the left side 26 for the number of blocks seen and 24 oz for the weight of the box. You should find that the left-hand-side of Eq. (3.2) evaluates to 28. (We're not going to do anything with these box formulas, so don't worry about getting too bogged down in the details. The point is that such a formula exists.)

Equation (3.2) says that the number of blocks remains the same; it's always 28. Not all the blocks are directly seen. The number of blocks inside the box needs to be calculated; the number is inferred from the box weight.

Everything proceeds smoothly for a little while, but one day Anna applies Eq. (3.2) and doesn't get 28. Something has gone wrong—where are the blocks hiding now? Jane has an aquarium in her room that is filled with murky, algae-filled water. Anna suspects that the block[2] is in the aquarium, but the water is so cloudy that she can't see if there are blocks there or not. She could reach in and feel around to see if there is a block in the tank, but she's rather not, since the water is pretty gross. What can Anna do?

[2] The block are heavy enough that they don't float.

The next time Anna uses Eq. (3.2) and verifies that there are 28 blocks, she can be sure that there no blocks in the aquarium. She then measures the height of the water, and finds that it is 14 inches. She drops a block in the aquarium and finds that the water level rises a quarter of an inch. She now has a way to figure out how many blocks are in the aquarium without having to put her hands in the murky water. There is one block in the aquarium for every quarter of an inch the water level is above 14 inches. She uses this information to modify

Eq. (3.2) as follows:

$$\text{No. of blocks seen} + \frac{\text{Weight of box} - 16\,\text{oz}}{4\,\text{oz}} + \frac{\text{Height of water} - 14\,\text{in}}{\frac{1}{4}\,\text{in}} = 28\,. \qquad (3.3)$$

The game of "hide the blocks" continues. Jane finds more and more places to stash her blocks, and each time Anna is able to come up with a clever way to infer the presence of unseen blocks. And so the number of blocks—seen and unseen—remains the same.

3.2 *Conservation of Energy*

The main point of the above story is that energy an abstract quantity that stays the same no matter what. It is like Jane's blocks, except there are no blocks. Energy is always hidden. It is something that is measured indirectly. There are a number of different forms of energy, many of which you are no doubt familiar with—if not from physics or chemistry c lasses, then definitely from everyday life:

- Kinetic. This is the energy associated with motion.
- Gravitational. This is the energy associated with an object's change in altitude. For example, an object on top of a table has more gravitational potential energy than when that object is on the floor.
- Thermal. This is the "hidden" kinetic energy associated with the motion and vibration of molecules. Thermal energy is more colloquially known as heat.
- Radiant. This is the energy associated with electromagnetic radiation. This includes visible light, X-rays, UV radiation, and so on. Energy from the sun arrives on Earth as radiant energy.
- Chemical. This is the energy stored in chemical bonds. When we eat food, we take the chemical energy in the food and convert it into heat and kinetic energy. Similarly, burning oil or wood converts chemical energy into heat.
- Nuclear. This is the energy stored in nuclear bonds—the forces that hold the nucleus of an atom together.

Just as was the case for Jane's blocks, each of these types of energy has a different formula associated with it. The above list is not exhaustive, but does enumerate the main types of energy we'll be working with.

As objects in the universe interact, they exchange energy. But energy is never created or destroyed; it just changes form. For example, Anna might drop her cell phone. When she does, the phone falls downward and the gravitational energy of the phone is converted to kinetic energy as it falls. The phone hits the floor, bounces a few times, and comes to rest. The kinetic energy has now been converted to thermal energy in the floor and the phone; their molecules are now vibrating a tiny bit more quickly than they were before the collision. Anna picks up her phone and puts it back in her pocket. This increases the gravitational energy of the phone. To do so, Anna must use some of the chemical energy stored in her body. She can replace this energy later when she eats lunch. And so on.

In this sense conservation of energy may be thought of as a sort of cosmic accounting. It is a form of bookkeeping. Accounting is a way of keeping track of financial transactions, but it does not speak to why those transactions have occurred. We will use the conservation of energy similarly: we will use it to keep track of energy as it changes from one form to another. This view of physics is not quite mechanistic, in that it focus neither on the forces that make objects move nor the details of the interactions between objects. Nevertheless, conservation of energy is one of the most powerful ideas in physics, and is the key to understanding the physics behind sustainable (and not sustainable) energy.

3.3 *Kinetic Energy and the Joule*

There are many different types of energy, just as there are many different places Jane can hide her blocks. Each type of energy has a different formula associated with it. The first type of energy we will consider is the energy associated with a moving object. This type of energy is called *kinetic energy*. The kinetic energy of an object with a mass m and a speed v is given by:[3]

$$E_k = \frac{1}{2} m v^2 \,. \tag{3.4}$$

For example, a 2kg rock moving at 3 m/s has a kinetic energy of

$$E_k = \frac{1}{2}(2\text{kg})(3\text{m/s})^2 = 9\,\text{kg}\,\text{m}^2/\text{s}^2 \,. \tag{3.5}$$

So we see that the units of energy are kilograms-meters-squared-per-second-squared. This is a mouthful. Happily, this awkward unit has a name, so we don't have to use this long phrase. The unit is known as the *Joule* and abbreviated simply as J. I.e., one Joule is defined as:

$$1\,\text{Joule} = \text{kg}\,\text{m}^2/\text{s}^2 \,. \tag{3.6}$$

How much is a Joule of energy?

On a human scale, not much. To paraphrase an example from Moore (2002), one Joule is the kinetic energy of a two-liter bottle soda walking

[3] For the purposes of this book, we will view Eq. (3.4) as fundamental. I.e., a fact of nature that need not be derived—it is just the way the world is. (Do I want to say anything more along these lines?)

down the street at one meter per second.[4] If this two-liter soda bottle walked into you at the speed of one meter per second, it would not do much damage.[5] You would notice this, but it wouldn't hurt. The point is that a Joule is a small unit of energy. Everyday events in our lives are associated with energies of between several thousand and several million Joules. For example, if you left a typical toaster on for two minutes it would use $120,000$ Joules. Raising the temperature of two kilograms of liquid water by 10 degrees Celsius takes $83,600$ Joules. The chemical energy in a typical avocado is around 1.2 million Joules.

[4] One liter of water has a mass of two kilograms. A speed of 1 meter per second is roughly the walking speed of a typical human.

[5] It would be really weird, because bottles of soda don't have legs and can't walk. But the collision wouldn't be physically harmful to you.

Figure 3.1: An avocado. The chemical energy in a typical avocado is around 320 dietary calories, which is equivalent to around 1.2 million Joules. (Image by `Kjokkenutstyr.net`, licensed under the Creative Commons Attribution-Share Alike 4.0 International license. Image source: `https://commons.wikimedia.org/wiki/File:Avocado-board.jpg`.)

The Joule is the standard unit for energy in the metric systems. However, it is awkardly small for most human-scale interactions. So we seek a larger unit—one that will be easier to work with. We will, in fact, be led to several such units, since different types of energy are traditionally measured using different units. This is unfortunate, since it obscures the fact that energy is energy and that all types of energy are interchangeable. Moreover, some of these units are quite awkward. But this is the world we live in, so we're going to have to become comfortable with multiple units. We will encounter our first commonly used, human-scale energy unit, the kilowatt-hour, in Chapter 4. For now, let's ponder further the nature of energy.

3.4 *But What* **is** *Energy??*

So far we've said that energy is this thing that is conserved—that stays the same no matter what.[6] In so doing, we've sidestepped the question of what energy is. It turns out that this is actually a deep question. Energy is an abstract quantity that does not have a single, direct definition. That said, one way to make energy more concrete is to think of energy as the ability to do work.

[6] I'm not sure where this section goes. It also shouldn't be too long. **TODO!** Work on this section.

Work in physics is force exerted through distance. If you push a couch along the floor for two meters, the work you have done is equal to the force you exerted on the couch times the distance you pushed it.[7] The units of work are the same as that of energy: Joules in SI.

[7] Specifically, the work W is given by $W = \vec{F} \cdot \vec{dr}$, where $\vec{F}$ is the force vector and $\vec{dr}$ is the distance though which the object was pushed. The force and displacement are vectors, meaning we need to account for their direction as well as their magnitude. And "$\cdot$" is the dot product, a way of multiplying two vectors that, roughly speaking, multiplies only the parts of the two vectors that are pulling in the same direction. We won't use this formula, but I include it since some of you likely have encountered this in past physics classes.

Energy, then, is the ability to do work. So an object that has a energy of 100 J has the ability to do 100 J of work. Conversely, if one does 100 J of work something, it now has 100 J of energy.

Energy is an abstract and powerful concept that provides a framework for thinking about a vast array of physical phenomena. This realization formed gradually over the 1900s. **TODO!** Finish this paragraph. See Sen (2022).

3.5 *Energy Quality*

TODO! Move all of this to the Thermodynamics and Energy Quality Chapter.

When physicists (and engineers?) say that energy is conserved, we mean that energy is neither created nor destroyed but just changes forms. Given this, what does it mean to conserve energy, as as environmentalists and others admonish us to do? What is there to conserve? After all, the total amount of energy always stays the same.

However, while the total amount of energy stays the same when converted from one form to another, it is not always easy to or feasible to turn the energy back into its original form. For example, one might burn some oil, converting chemical energy into thermal energy. This is easy to do. One just needs some oil and a match. Light the oil on fire. It burns. Things heat up. But it is much, much, much harder to take heat unburn it and end up with oil again.

So at issue is the *quality* of the energy. Talk about high vs. low quality energy. High quality is orderly energy and low-quality is disorderly. Etc.

TODO! I think this should go in a later section and I should expand on it some. Not exactly sure where it goes. But probably not here.

TODO! Mention how "conservation" has two meanings. Physics means stays the same, and in everyday use "conservation" means to preserve or minimize the use of.

3.6 *Exercises*

Exercise *3.1:* A 0.8 kg bird flies at 2 m/s. What is its kinetic energy?

Exercise *3.2:* A 120 pound hockey player skates at 8 m/s. What is her kinetic energy?

Exercise *3.3:* A 80 kg football player runs at 5 m/s. What is his kinetic energy?

Exercise *3.4:* A hockey puck has a mass of 160 grams. How fast would the hockey puck have to be moving so that its kinetic energy was equal to the chemical energy of an avocado, 1.2 MJ?

Exercise *3.5:* A 200g apple has around 440,000 J of chemical energy. At what speed would you have to throw the apple so that its kinetic energy was equal to its chemical energy?

Exercise *3.6:* A 2500 pound car drives on Interstate 95 at 60 miles per hour. What is the car's kinetic energy? Express your answer in both J and MJ.

Exercise *3.7:* Choose a sport that you enjoy playing or watching that involves a ball or other object that is thrown or kicked through the air or along the ground: baseball, cricket, American football, football football, golf[8], tennis, badminton, croquet[9], etc. Look up the mass of the object and estimate the fastest speed that that object travels during play.

[8] I'm not sure this is really a sport, but whatever.

[9] Also not really a sport.

1. Estimate the amount of kinetic energy of the moving sport object. State your answer in Joules and MegaJoules.
2. How does your answer compare to the chemical energy in a large banana, which is roughly 0.5 MJ?

Exercise *3.8:* What happens to the kinetic energy of an object if:

1. its speed is doubled?
2. its speed is tripled?
3. its speed increases by 10%?

4
Power

4.1 Power

Energy is transformed from one type to another. The *rate* at which this transformation occurs in known as *power*. Just like current is the amount of charge that flows per unit time, power is the amount of energy that flows per unit time:

$$\text{Power} = \frac{\text{Energy}}{\text{time}} \,. \tag{4.1}$$

The SI units for power are *watts*, abbreviated W, and defined as:

$$1\,\text{W} = 1\,\frac{\text{J}}{\text{s}} \,. \tag{4.2}$$

A Watt is a reasonable unit of power to describe many everyday phenomena. A typical light bulb might draw 40 watts and a toaster or a hairdryer around 1000 watts. In some contexts—thinking about how much power a town uses or a power station generates—larger units are more convenient, so we will often us kW (kilowatts) and MW (megawatts).[1]

To get a feel for power and Watts, let's work through a few examples:

[1] As with all kilo- and mega- units, 1 kW = 1000 W, and 1 MW = 1000 kW = $1,000,000$ W.

Example 4.1. *In order to bring a pint of water to a boil to make coffee, a heating element transfers* $168,000$ *J to the water in* 4 *minutes. What is the power associated with this energy flow?*

We start with the definition of power $P = E/t$:

$$P = \frac{168,000\,\text{J}}{4\,\text{min}}\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 700\,\frac{\text{J}}{\text{s}} = 700\,\text{W} \,. \tag{4.3}$$

Example 4.2. *A power of* 1 *kW flows for* 1 *hour. How many joules of energy is this?*

Since $P = E/t$, it follows that $E = Pt$. Since 1 kW is 1000 W, and since one watt is one joule per second:

$$E = 1000\frac{\mathrm{J}}{\mathrm{s}}(1\,\mathrm{h})\left(\frac{60\,\mathrm{min}}{1\,\mathrm{h}}\right)\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) = 3600000\,\mathrm{J}\,. \tag{4.4}$$

Or, converting to MJ:

$$E = 3600000\,\mathrm{J}\left(\frac{1\,\mathrm{MJ}}{1,000,000\,\mathrm{J}}\right) = 3.6\,\mathrm{MJ}\,. \tag{4.5}$$

Example 4.3. *A hard-throwing baseball pitcher might throw over the course of a game* 100 *pitches at an average speed of* 90 *miles an hour. The mass of a baseball is* 145 *grams. If the pitcher throws these* 100 *pitches in one hour, what power has he delivered?*

Let's start by calculating the kinetic energy the pitcher gives to a baseball each pitch. To do so, we first convert miles per hour to meters per second:

$$90\mathrm{mi/hr} = 90\mathrm{mi/hr}\left(\frac{1\mathrm{m/s}}{2.24\mathrm{mi/h}}\right) = 40.2\mathrm{m/s}\,. \tag{4.6}$$

(The conversion factor 1 m/s = 2.24 mi/h is a useful one. It is listed in Appendix A for easy reference.) A 145 g ball is 0.145 kg. We can then determine the kinetic energy of the thrown ball:

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(0.145\,\mathrm{kg})(40.2\mathrm{m/s})^2 = 117\,\mathrm{J}\,. \tag{4.7}$$

Power is energy/time. There are 100 such pitches in one hour, so:

$$P = \frac{100 \times 117\,\mathrm{J}}{1\,\mathrm{h}}\left(\frac{1\,\mathrm{h}}{60\,\mathrm{min}}\right)\left(\frac{1\,\mathrm{min}}{60\,\mathrm{s}}\right) = 3.25\,\mathrm{J/\,s} = 3.25\,\mathrm{W}\,. \tag{4.8}$$

4.2 *Kilowatt Hours*

While Joules are the energy unit of choice for physicists and chemists, they are not used much in the energy field. Instead, one of the most common units are *kilowatt hours*, abbreviated kWh. A kilowatt hour is the amount of energy that has flowed if one kilowatt flows for one hour. You have already seen kWh—that's what Example 4.2 was about. There you showed that one kWh is 3.6 million joules of energy.

A kilowatt hour is a unit of energy, just like an amp hour is a unit of charge. A kilowatt hour is the amount of energy that has flowed if one kilowatt (1000 J/s) flows for one hour. An amp hour is the amount of current if one amp (1 C/s) flows for one hour. In both cases we are multiplying a flow rate by a time.

Some information that I might turn into paragraphs at a later date:

- In Maine our local utility Emera sells electric energy to homes. Emera charges approximately 16 cents for one kWh or electricity.[2]
- In the U.S., on average, one kWh of electricity production results in 613 grams of CO_2 being released into the atmosphere. The values for other countries can be found on page 335 of MacKay (2009).
- Worldwide averages for the carbon intensity of different modes of electricity generation can be found in Section A.3.
- The average Maine home uses 520 kWh of electricity per month.
- The average US home uses 900 kWh per month. (http://www.eia.gov/tools/faqs/faq.cfm?id=97&t=3.)
- The average home in Spain uses 344 kWh/month. The average home in Mexico uses 150 kWh/month. (http://shrinkthatfootprint.com/average-household-electricity-consumption)

[2] Emera charges different rates for commercial customers. This is the topic of Section 5.7.

Example 4.4. *A 3 kW electric pump runs for 4 hours a day. How much energy, in kWh, does the pump use?*

Energy is power times time. So

$$E = (3\,\text{kW})(4\,\text{h}) = 12\,\text{kWh}\,. \tag{4.9}$$

Note that kW are units of power and kWh are units of energy.

Example 4.5. *A 800 W electric heater is on for 40 min. How much energy has it used? Answer in both kWh and J. How much would this cost in Maine?*

To end up in kWh, we'd like power in kW and time in hours. Let's convert:

$$800\,\text{W} = 800\,\text{W}\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right) = 0.8\,\text{kW}\,. \tag{4.10}$$

$$40\,\text{min} = 40\,\text{min}\left(\frac{1\,\text{h}}{60\,\text{min}}\right) \approx 0.67\,\text{h}\,. \tag{4.11}$$

Energy is power times time, so:

$$E = (0.8\,\text{kW})(0.67\,\text{h}) = 0.536\,\text{kWh}\,. \tag{4.12}$$

So this heater uses a bit more than half a kWh or energy. How many joules is this? Doing the conversion, we obtain:

$$0.536\,\text{kWh} = \frac{3.6\,\text{MJ}}{1\,\text{kWh}} = 1.93\,\text{MJ}\,. \tag{4.13}$$

So the heater will have used 1.93 million joules.

In Maine, electricity costs 17.2 cents for one kWh. So,

$$\text{Cost} = (0.536\,\text{kWh})\left(\frac{0.172\$}{1\,\text{kWh}}\right) = 0.092\$\,. \tag{4.14}$$

or a bit more than nine cents.

4.3 *An Analogy: Car Hours*

Where I live, if you ask someone how far away Boston is, most people will give an answer of "five hours." Heads will nod. We all know that this is about how far Boston is. But if one thinks about it, this is sort of a strange answer. When one asks how far away something is, one is looking for a distance, and so one would expect the answer to be stated in miles or perhaps kilometers, not time. How is "five hours" a distance?

Well, what is implied is that the distance is five *car hours*. I.e., the distance is how far you would travel if you drove in a car for five hours. Ok, but what does *this* mean? What's understood is that "car" implies a rate—how fast one can typically drive in a car. Of course the speed of a car varies. It's certainly not constant. But for the sake of this example, let's say that car speed is 60 mi/hr. Then we can see how five hours can be a distance:

$$\text{Distance} = \text{rate} \times \text{time}\,, \tag{4.15}$$
$$\text{Distance} = \text{car} \times 5\,\text{hours}\,, \tag{4.16}$$
$$\text{Distance} = 60\,\text{mi/h} \times 5\,\text{hours} = 300\,\text{mi}\,. \tag{4.17}$$

So Boston is 300 miles away.

Ok, let's unpack this a little. What's is going on in Eq. (4.16)? "Car" in this equation is being used as a speed—a rate. Specifically, car $= 60\,\text{mi/h}$. So car hour is a unit of distance:

$$\text{Unit of distance} = \text{car hour}\,. \tag{4.18}$$

A rate (car) time a time (one hour) gives an amount: in this case, distance (a certain number of miles). Similarly, the kilowatt hour is a unit of energy:

$$\text{Unit of energy} = \text{kW hour}\,. \tag{4.19}$$

The kilowatt is a rate: 1 kW = 1 J/s. And a rate (kW) times a time (one hour) gives an amount: in this case energy (a certain number of Joules). So kWh is no more or less weird a unit of energy than "car hours" is a weird unit of distance.

4.4 *Energy Quality*

When physicists (and engineers?) say that energy is conserved, we mean that energy is neither created nor destroyed but just changes forms. Given this, what does it mean to conserve energy, as as environmentalists and others admonish us to do? What is there to conserve? After all, the total amount of energy always stays the same.

However, while the total amount of energy stays the same when converted from one form to another, it is not always easy to or feasible to turn the energy back into its original form. For example, one might burn some oil, converting chemical energy into thermal energy. This is easy to do. One just needs some oil and a match. Light the oil on fire. It burns. Things heat up. But it is much, much, much harder to take heat unburn it and end up with oil again.

So at issue is the *quality* of the energy. Talk about high vs. low quality energy. High quality is orderly energy and low-quality is disorderly. Etc.

4.5 *Exercises*

Exercise *4.1:* In order to dry out some sheetrock "mud", I left a 1500 W heater on for two full days this weekend. How much energy did this use? Answer in both kWh and Joules. How much would this cost in Maine?

Exercise *4.2:* Convert the following to kWh:

1. 1 Joule
2. 1,000,000 J.
3. 7,500 J.

Exercise *4.3:* Convert the following to Joules:

1. 100 kWh
2. 31 kWh
3. 57,160 kWh

Exercise *4.4:* Water is pumped into a tank at the rate of 120 gallons/sec. How much water flows into the tank in one minute? How much in one hour?

Exercise *4.5:* A certain light bulb draws 120 W of power. How much energy does the light bulb use in one minute? How much does it use in one hour? Answer in both Joules and kWh.

Exercise *4.6:* Which of the following are units of energy? Which are units of power? Which units don't make sense?

1. kW
2. kWh

3. Joules
4. kW/h
5. MW
6. kWh/day

Exercise *4.7:* "How much power did you use this month?" Why does this question not make sense?

Exercise *4.8:* The lights in Dave's office draw 120 W. Suppose he leaves them on for three hours a day for a month. How much energy does this use? Express your answer in both kWh and Joules

Exercise *4.9:* If something generates 4.8 kW for one year, how much energy is this? Express your answer in kWh and MWh.

Exercise *4.10:* Suppose you leave a 1000W toaster on for an entire year.

1. How much energy does this use? Express your answer in both kWh and MWh.
2. How much does this cost per month in Maine?
3. Express the power of the toaster in kWh/day.

Exercise *4.11:* Water flows into a reservoir at the rate of 20 gallons/sec. How much water flows into the tank in three hours?

Exercise *4.12:* An appliance draws 20 watts. How much energy does the appliance use in three hours? Express your answer in both kWh and J.

Exercise *4.13:* What wattage light bulb uses 1 kWh in one day?

Exercise *4.14:* Suppose that in a typical day a typical person typically eats around 2500 calories of food.[3] These are dietary calories. Confusingly, 1 dietary calorie equals 1000 "real" calories. (See Appendix A.)

1. How many Joules does a typical person consume in a day?
2. What power is this? Express your answer in kW.
3. Most of the food energy you consume ultimately gets converted to heat. Thus, we can view people as heaters—they convert chemical food energy into thermal energy. How many people would you need to have in a room to have a heating power roughly equivalent to one 1500 W space heater?

[3] The recommended daily caloric intake, according to the U.K. National Health Service is 2000 calories for women and 2500 calories for men (Service). Add citation to wikipedia about reality.

Exercise *4.15:* In the field of hydrology, water flowing into a reservoir at a rate of 1000 gal/s is known as a *kiloCline*.[4] If 2 kiloClines flow into an initially empty reservoir for 3 hours, how much water is in the reservoir? State your answer in kCh and gallons.

[4] Not really.

Exercise *4.16:* A medium-sized electric heater draws around 2 kW. The heater is on for three hours. How much energy has the heater used? State your answer in kWh and Joules.

Exercise *4.17:* The average US home uses 909 kWh of electricity in a month. Estimate how much this is in kWh per person per day. Assume that the average house has three people living in it.

Exercise *4.18:* Very roughly, what size power plant would be needed to generate sufficient electricity for a town 50, 000 in Texas? Consider only the residential electricity needs of the town, not any electricity needed for commerce or industry. Be sure to explain how you arrived at your answer. The average house in Texas uses 1130 kWh of electricity each month.

Exercise *4.19:* The average home in Mexico uses 150 kWh of electricity each month.

1. Express this power in units of kW.
2. A 2 MW power station could provide approximately how many Mexican homes with electricity?

Exercise *4.20:* How long could you afford to keep a 50 W light bulb on in Maine if you had 10 dollars?

Exercise *4.21:* For the following problems assume that "car" corresponds to a speed of 60 mi/hr, as we did in Section 4.3.

1. It is about 500 miles from Toronto, ON to Québec, QC. How many car hours is this?
2. It is 10.5 car hours from San Francisco, CA to Portland, OR. How many miles is this?

5
Electricity

5.1 Charge

Charge is a fundamental property of matter. Electrons have a negative charge and protons a positive charge.[1] Usually macroscopic objects have an equal number of electrons and protons, and so are electrically neutral. But sometimes an object might acquire a surplus (or deficit) of electrons, and thus would be negative (or positively) charged. Opposite charges attract; two differently charged objects exert forces on each other that tend to push those two object apart. Clever people can make use of this fact to make the electric force do useful things, like pumping water from a well, powering a car, or making an LED glow.

[1] I'm worried that this chapter is a bit too physics-ey. Can it be streamlined? Maybe. I think I can emphasize coulombs a lot less. But I'm gonna leave this be for now.

Electrons carry one fundamental unit of charge, usually denoted e and refereed to as the elementary charge. All electrons have a charge of $-e$, never a fraction of an e—always exactly e. And all protons have a charge of $+e$. Measuring charge in fundamental units—i.e., in units e—is useful when thinking about the chemistry of physics of single atoms or molecules, but not so useful when working with macroscopic phenomenon, which typically involve vast numbers of electrons. So an more convenient unit of charge is needed.

That unit is the coulomb, abbreviated C.[2] It take a lot of elementary charges e to make up one coulomb. There are around 6.25×10^{18}, or 6.25 quintillion, electrons in one C. That's a lot. This means that the charge e, measured in C, of a single electron is very small:

[2] In the official SI system of units, all unit names are rendered in lowercase, even those named after people. Abbreviations for units are capitalized if the unit derives from a proper name de la Convention du Mètre (2006), as is the case with coulombs, which are named after Charles-Augustin de Coulomb. I don't really like this capitalization convention, but I'm not up for picking a fight with the Organisation Intergouvementale de la Convention du Mètre, the international organization that oversees unit definitions and naming conventions. **TODO!** Move this note to Chapter 1 when Joules are first referenced.

$$1e = -1.60 \times 10^{-19}\text{coulombs} \,. \tag{5.1}$$

We will use coulombs as our unit of charge in the rest of the book. This is the standard SI unit, and while it is not ideal, it is much more convenient than measuring charge in terms of individual electrons. The standard symbol electric charge is Q.

5.2 Current

An electric current—or, more colloquially, electricity—is moving charge. The rate at which charge flows is known as *current* and is usually denoted by I.[3] The basic unit of current is the *ampere*, usually simply called an *amp* and abbreviated A. An amp is a current in which one coulomb flows every second:

$$1\text{A} = 1\text{C}/s\ . \tag{5.2}$$

An Amp is a rate. Rates tell us how fast something happens. In this case, the thing happening is charges are moving. If something happens at rate r for a time interval t, the amount of thing that has happened is rt.[4] Thus, the amount of charge that has flowed if a current I flows for a time t is given by:

$$Q = It\ . \tag{5.3}$$

The two examples below show several ways to use this equation.

[3] You may wonder why the letter I is used, since "I" is conspicuously absent from the word "current" and plays a supporting role in the word "electricity." The symbol I was used by André-marie Ampère who referred to electric current as *intensité de courant*—current intensity.

[4] Probably the most familiar application of this idea is distance, and you have likely heard the phrase "distance is rate times time." For example, if you are driving at 60 mi/hr for 2 hours, you will have traveled $60 \times 2 = 120$ miles.

Example 5.1. *A current of* 3 *mA (milliamps) flows through a wire into a capacitor for* 10 *minutes. How much charge will have accumulated in the capacitor? A capacitor can be thought of in this context simply as a bucket of charge. The idea is that charge flows through the wire into the bucket (capacitor), and we want to know how much charge has accumulated the bucket in* 10 *min.*

We are looking for Q, the amount of charge that has accumulated in $t = 10$ minutes. The relationship we want is $Q = It$, Eq. (5.3). So

$$Q = (3\text{mA})(10\text{min})\ . \tag{5.4}$$

One milliamp is 0.001 amps, and one amps is a coulomb per second. So

$$Q = (3\text{mA})\left(\frac{0.001\text{A}}{1\text{mA}}\right)\left(\frac{1\text{C}/\text{s}}{1\text{A}}\right)(10\,\text{min})\ . \tag{5.5}$$

Note that the A and mA units cancel, leaving us with:

$$Q = \left(0.003\,\frac{\text{C}}{\text{s}}\right)(10\,\text{min})\ . \tag{5.6}$$

We can't just multiply 3 and 10, since the units aren't right; we need to convert minutes into seconds. There are 60 seconds in a minute, so

$$Q = \left(0.003\,\frac{\text{C}}{\text{s}}\right)\left(\frac{60\text{s}}{1\text{min}}\right)(10\,\text{min})\ . \tag{5.7}$$

Now the units cancel. We multiply out and arrive at our final answer: The charge Q that has accumulated is 1.8 coulombs. By the way, if writing all these steps out seems like overkill, please have a look at our comments in Sec. C.1.

Example 5.2. *Suppose you need to have* 0.4 *C of charge flow through a wire in five minutes. What current would achieve this goal?*

We are looking to solve for the current I. Solving Eq. (5.3) for I, we obtain:

$$I = \frac{Q}{t} \,. \tag{5.8}$$

Plugging in, we have

$$I = \frac{0.4\,\mathrm{C}}{5\,\mathrm{min}} \,. \tag{5.9}$$

There are 60 seconds in one minute, so

$$I = \left(\frac{0.4\,\mathrm{C}}{5\,\mathrm{min}}\right)\left(\frac{1}{\frac{60\,\mathrm{s}}{1\,\mathrm{min}}}\right) \,. \tag{5.10}$$

Simplifying, we obtain

$$I = \left(\frac{0.4\,\mathrm{C}}{300\,\mathrm{s}}\right) = 0.0013\,\mathrm{C/s} = 0.0013\,\mathrm{A} \tag{5.11}$$

If we wanted, we could convert this to milliamps:

$$I = 0.0013\,\mathrm{A}\left(\frac{1000\,\mathrm{mA}}{1\,\mathrm{A}}\right) = 1.3\,\mathrm{mA} \,. \tag{5.12}$$

We will rarely work directly with coulombs, but we will work directly with amps. So it is good to have a feel for what an amp is and what some typical currents are in electrical situations you might be familiar with. Below is a list of approximate currents for a handful of applications.[5,6]

- Current flowing into the pump for a fishtank: 30 mA
- Current flowing into your smart phone when you charge it: 0.02 – 0.05 A
- Current flowing into a 60 W light bulb: 0.5 A
- Current flowing into a cable TV: 2 A[7]
- Current flowing into a hair dryer: 8 A
- Current flowing into a 1500 W electric heater: 12.5 A
- Current flowing into an electric dryer: 17 A
- Current flowing into a hot water heater: 25 A

One final note on current. The current at a point in a wire tells us how much charge is moving past that point each second. But this doesn't tell us how fast the charge is moving. One could get a particular

[5] Should I format this as a table? Probably. But tables are a pain so let's keep it as a list for the time being.

[6] Also, I don't like the list formatting. I think there should be a bit less space between lines and the list should be indented. I can mess around with this later.

[7] why does it matter if it is a cable TV?

current by having a small amount of charge moving quickly, or a large amount of charge moving slowly. An analogy with water in a river may help to make this clearer. If you know the flow rate—i.e., the current—in a river, you know how much water is moving past you each second. It could be the case that the river is narrow and water is moving very quickly. Or the river could be wide and deep, but moving slowly. Both could give rise to the same current.[8,9]

[8] Do we need this paragraph?

[9] At some point we should probably talk—as briefly as possible because it is confusing and doesn't really matter—about how the direction of current is the direction of positive flow and in reality in almost all materials it is negative electrons that move, and thus a current flowing to the right would actually consist of electrons flowing to the left.

5.3 *Voltage*

So far we have talked about current without talking about what makes it go. Why do electrons move down a wire? What pushes them along? The answer is voltage, also known as electric potential. Just like water flows from higher altitude regions to lower ones, charge tends to move from regions of higher potential to lower potential. What matters is the *difference* in potential. The greater the difference in potential from one end of a wire to another, the larger the current will be.

So what *is* voltage? This question turns out to be surprisingly abstract and probably not essential for understanding sustainable energy. It is pretty easy to get a handle on what gravity *does*: it makes things fall down. But what gravity really is, is a much more difficult and abstract question, and knowing the answer to this question doesn't help one stand up or build bridges or airplanes. Similarly, it is fairly easy to say what voltage does—it is something that makes current flow. But it's not easy to say what voltage really is. So we're not going to get too deep into what voltage is and instead will focus on what voltage does.[10]

[10] We will have a bit more to say about what voltage is in Sec. 5.6.

Dave likes to think of voltage as *oomph*. This is a made-up term that conveys the idea that voltage is something that pushes charge. Anna, being a engineer and not a theoretical physicist, doesn't always like to think in terms of oomph, but she tolerates it. Voltage is measured in units of *volts*. In Sec. 5.6 we'll talk about how volts are related to other, more familiar units.[11] It is tempting to think of voltage as a force, but this is not quite correct, as we'll see later on. But viewing voltage as a force gives the right idea.

[11] We can't properly do this now, because volts are related to energy, and we haven't introduced energy yet. But, at the risk of spoiling some of the fun from Chapter 5, one volt is equal to one joule per coulomb.

5.4 *Resistance and Ohm's Law*

So let's just say voltage is oomph. Given a certain amount of oomph, how much current will flow? To answer this question we need to introduce one more physical quantity: *resistance*. Resistance is what it sounds like—it is a property of a material that determines how it resists having charged oomphed through it. The greater the resistance, the less current will flow for a given amount of oomph. Or, the larger the resistance, the more oomph is needed to push a given amount of current.

This relationship is captured in a pleasingly simple equation:

$$V = IR\,. \tag{5.13}$$

This important equation is known as *Ohm's Law*. In this equation V is voltage and R is resistance. The standard unit for resistance is the *Ohm*, abbreviated Ω[12] and defined as follows:

$$1\,\text{ohm} = 1\,\text{V/A} = 1\,\Omega\,. \tag{5.14}$$

[12] This is the capital Greek letter Omega. Dave thinks it looks like a bow-legged cowboy.

Thus, if an object had a resistance of one ohm, a voltage of one volt would be sufficient to push an current of one amp through it. We'll do a few examples with Ohm's law in a moment. First, though, a few words about the the difference between current I and voltage V.

Current is, well, a current: it is moving charge. By analogy, we can picture water that is flowing in a pipe, as in Fig. 5.1. Along the pipe, there are no inlets or outlets, so the current must be the same everywhere. If it wasn't, water would be accumulating somewhere. That is, if at a certain point in the pipe the current flowing in was larger than the current flowing out, then the amount of water have to be increasing at that point, something that isn't possible in a closed, filled pipe. The picture is the same with electric current. We say that current flows through the wire, and the current is the same everywhere in the wire.

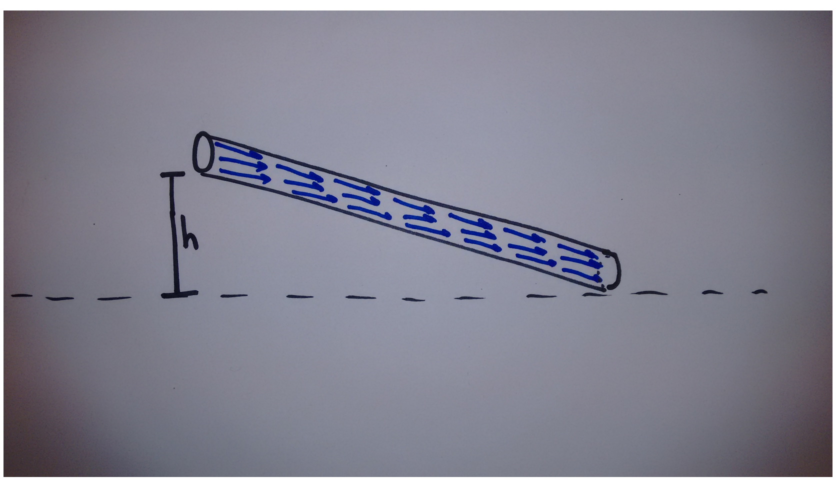


Figure 5.1: Water flowing through a tilted pipe. The current, or rate of flow, is the same everywhere in the pipe. The height h is roughly analogous to the "oomph", or voltage V.

The voltage, or oomph, can be thought of as being like the height difference between one end of the pipe and the the other. We would say that the altitude drops from one end to the other. Similarly, we would say that there is a voltage difference from one side of the wire to the other. One also refers to this as a potential difference. The main point is that the current is the same everywhere in wire or simple circuit without branches, while voltage drops from one side of the wire to the other.

5.5 Ohm's Law Applications

In this section we'll work through a few examples so you can see Ohm's law in action.

Example 5.3. *A certain light bulb has a resistance of* 10 *ohms. It is hooked up to* 1.5 *volt battery. What current flows through the light bulb?*

We start with ohm's law:

$$V = IR\,, \tag{5.15}$$

and solve for the current I:

$$I = V/R\,. \tag{5.16}$$

Plugging in, we obtain:

$$I = \frac{1.5\,\mathrm{V}}{10\,\Omega}\,. \tag{5.17}$$

To make sense of the units, recall that one ohm is one volt per amp. Thus,

$$I = \frac{1.5\,\mathrm{V}}{10\,\frac{\mathrm{V}}{\mathrm{A}}} = 0.15\,\mathrm{A}\,. \tag{5.18}$$

5.6 Energy in Circuits

In Chapter 5, we referred to voltage as *oomph*. Voltage is what pushes current through, say, a light bulb. The greater the voltage across the light bulb, the greater the current. Now that we have introduced the notion of energy, we can say more directly what voltage is: *A voltage of one volt would give one coulomb of charge one joule of energy*. That is

$$1\,\mathrm{volt} = \frac{1\,\mathrm{joule}}{1\,\mathrm{coulomb}}\,. \tag{5.19}$$

This means that if one coulomb were pushed by one volt, the charge would have gained one joule of energy. In general, a voltage is physical quantity measuring "oomphiness". Voltage measures energy per charge.

We can use this fact to come up with a formula for the power associated with an electric current. The power P associated with a current I pushed by a voltage V is given by:

$$P = VI\,. \tag{5.20}$$

It may be helpful to think about the units in this equation:

$$P = VI = \left(\frac{\mathrm{J}}{\mathrm{C}}\right)\left(\frac{\mathrm{C}}{\mathrm{s}}\right) = \mathrm{J/s} = W\,. \tag{5.21}$$

So the formula ends up giving us power in watts, as one would expect.

In Chapter 3 we mentioned that energy comes in different forms, each with its own formula. And we presented such formula: namely, the formula for kinetic energy: $E = (1/2)mv^2$. Equation (5.20) gives us another formula. When considering electricity, one works with power, not energy. Electricity is a flow of current, so it is natural to think in terms of the flow of energy—i.e., power.

Example 5.4. *In the U.S. the voltage of a normal home outlet is* 120 *V. If a lamp with a* 50 *W light bulb is plugged in to this outlet, what current flows through the bulb?*

We need to solve Eq. (5.20) for I. Doing so, we have

$$I = P/V\,. \tag{5.22}$$

Plugging in the values given for P and V, we obtain

$$I = 50\,\mathrm{W}/120\,\mathrm{V} = \frac{50\,\mathrm{J/s}}{120\,\mathrm{J/C}} = 0.41\,\mathrm{C/s} = 0.41\,\mathrm{A}\,. \tag{5.23}$$

5.7 *The Cost of Electricity*

Electric utility companies are responsible for providing your house with electricity. You then use this electricity to make toast, run your refrigerator, light up your house when it is dark outside, and so on. This electricity isn't free, however. The utility company charges you for every kWh or electricity you use. Here in Maine, the cost is 0.17 for on kWh.[13] The average household in Maine uses 520 kWh of electricity in a month. This would cost:

$$520\,\mathrm{kWh}\left(\frac{\$0.17}{1\,\mathrm{kWh}}\right) = \$88.40\,. \tag{5.24}$$

Household electricity use in Maine is well below the national average. The primary reason for this is mostly likely that in most houses in Maine there is no need for air conditioning owing to Maine's coolish climate. Air conditioning is a significant use of residential electricity.[14]

Figure 5.2 shows one of my utility bills. The local utility where we live is a company called Emera. We see that the period of the bill is around one month: from July 15 to August 16, 2014. The total energy my house used during this time period is 215 kWh. This is well below

[13] Include a table with costs for other states. Would also be of interest to list power costs in a few other countries.

[14] Include a few problems about this?

the average for Maine. There are several reasons why I uses so little electricity. First, electric hot water heaters tend to be the largest use of home electricity. However, my hot water heater runs on gas. My clothes direr at the time also ran on gas. But I probably didn't use it much, however, since in the summer I does most of my drying on a clothesline in his backyard. The second reason Dave's utility bill is so small is that only one other person lives in his house, his wife. Dave and his spouse have no kids. They do have three amazing cats.

Emera Maine

Emera Maine Customer Service Center
207-973-2000
1-855-Emera11(363-7211)
Email: custserv@emeramaine.com
Pay Online at www.emeramaine.com

Account Information	
Account Number:	[illegible]
Service Address:	DAVID P. FELDMAN [illegible]
Read Cycle:	14M
Next Planned Meter Read:	13-SEP-2014

Customer Account Summary

Previous Statement Balance	Payments (-) Thank You	Adjustments (+/-)	Balance Before New Charges	New Charges	Current Account Balance	Total Now Due
[illegible]	[illegible]	$0.00	$0.00	$38.48	$38.48	$38.48

Energy Comparison

	This Month	Last Month	One Year Ago
KWH	216	211	256
Service Days	32	30	32
KWH Per Day	6.8	7.0	8.0
Cost Per Day	1.20	1.25	1.28

Residential Service (Service 1)

Meter Number	Units	For Service From	For Service To	Days	Meter Reading Current	Meter Reading Previous	Constant	KWH
007160840	1	07/15/14	08/16/14	32	27701	27485	1	216

Message from Emera Maine

Emera Maine Delivery (Service 1 Rate Code A000)

Distribution Energy	216 kWh @ 0.0762400	$16.47
Transmission	216 kWh @ 0.0261800	$5.65
Balance Forward		**$0.00**
Total Emera Maine Delivery Charges Due		**$22.12**

Message from your Supplier

Your electricity price for Standard Offer service for the period of March 1, 2014 through February 28, 2015 is $0.0757596 per kWh.
For information on buying green power go to www.maine.gov/greenpower

Standard Offer Supply (Service 1 Rate Code 1000 Class S)

Electricity Supply	216 kWh @ 0.0757596	$16.36
Balance Forward		**$0.00**
Total Standard Offer Supply Charges Due		**$16.36**

Figure 5.2: My electricity bill from the summer of 2014.

So I used 216 kWh of electric energy during this 32-day period and was charged $38.48. This charge is broken into two parts, supply and delivery. The supply part of the bill is the charge associated with generating the electricity on Fig. 5.2 toward the bottom, we see that the supply cost per kWh is $0.0757596. Multiplying this cost by the 216 that Dave used, one obtains $16.36.

There is also a charge associated with the delivery service provided by Emera—actually getting the electricity from the power plant to your house. The delivery service is broken down into two parts, *distribution energy* and *transmission*. The rate for distribution energy is 0.762400 per kWh and the rate for transmission is 0.0261800 per kWh. Multiplying these rate by the 216 kWh that Dave used yields \$22.12. The total bill for this month is the sum of the delivery and supply charges.

The difference between transmission and distribution is not an important one, but here are a few words on it anyway. Transmission refers to the transmission of power from the generator to a distribution center. Transmission is often "long-haul"—large power lines that go large distances and perhaps cross state boundaries. Distribution refers to moving the power from a distribution center out through neighborhoods and into homes. I wonder if this discussion shouldn't go in the chapter on the grid.

Add paragraph about how Dave could choose to buy something other than the standard offer electricity. This would mean the energy is generated not by Emera, but Emera's power lines would still deliver the electricity. For example, to buy power from the company Maine Green Power would cost an additional \$0.015, or 1.5 cents, per kWh.[15] Maine Green Power generates their energy from a mix of renewable sources in Maine, including hydro, solar, wind, and biomass.

[15] Actually, it's a bit more complicated than this, since Maine Green Power charges not by the kWh but in bundles of 500 or 250 kWh per month. See `https://megreenpower.com/overview/faq`.

5.8 *Thinking Globally*

Remind readers that residential electrical energy use is a small fraction of overall energy use.

Somewhere in this chapter mention feed-in tariffs and refer readers to Sec. 23.3 for more details.

Do we want this section? At some point we need to connect up with the larger picture. Where does this belong?

5.9 *Exercises*

Exercise *5.1:* A toaster draws 1000 W from a household socket in the U.S. What current is flowing through the toaster?

Exercise *5.2:* Two amps flow out of a 220 V outlet to run an electric dryer. What power is being used by the dryer? If the dryer runs for 3 hours, how much energy has been used? State your answer in kWh.

Exercise *5.3:* If Dave buys an electric heater for his home office and leaves it on for 5 hours per day, 5 days a week for the month of January. How much will it add to his electricity bill assuming he is on an A meter and the heater draws 1kW?

Exercise *5.4:* A piece of plastic happens to have one billion more electrons than protons. What is the net charge, in coulombs, of the piece of plastic?

Exercise *5.5:* The current flowing through a wire is 30 mA.

1. How much charge flows through the wire in 5 minutes?
2. How long must the current flow for 100 coulombs to have flowed through the wire?

Exercise *5.6:* About how many coulombs does it take to charge your phone?

Exercise *5.7:* When charging your phone, how many electrons per second are flowing through your charger into the phone?

Exercise *5.8:* Find the current flow through a light bulb from a steady movement of

1. 60 Coulombs in 4 seconds
2. 15 Coulombs in 2 minutes
3. 3×10^{22} Coulombs in 1 hour.

Exercise *5.9:* Will a current of 25,000 coulombs per hour cause a 5 amp fuse to blow?[16]

[16] A five amp fuse would blow if five or more amps of current flow through it.

Exercise *5.10:* What voltage is needed to push a 0.5 amp current through a 100 ohm load?

Exercise *5.11:* A current of 3 amps flows for 20 minutes. How many coulombs is this?

Exercise *5.12:* What voltage is needed to push 5 amps through a 20 Ω load?

Exercise *5.13:* A current of 5 amps is pushed by a voltage of 60 volts through a light bulb. What is the resistance of the bulb?

Exercise *5.14:* A current of 0.5 amps flows for 2 hours. How many amp-hours is this? How many coulombs?

Exercise *5.15:* How long can a 12 volt, 30 Ah battery deliver a current of 0.5 amps?

Exercise *5.16:* You need a 1.5 volt battery to deliver light up a 5 ohm light bulb for 50 hours. What should be the capacity of the battery?

Exercise *5.17:* You have a battery that has sufficient capacity to light up a 10 ohm light bulb for 30 hours. Suppose that this battery's capacity was doubled, but all other properties of the battery remained the same.

1. For how long could this new battery light up the bulb?
2. What current would the new battery push through the light bulb?
3. What voltage does the new battery deliver?

Exercise *5.18:* A voltage of 120 volts is applied across a heater that has a resistance of 9.6 ohms. How much current flows through the resistor?

6

Generating Electricity

In this chapter we'll talk about generating electricity. One could, we suppose, do sustainable energy without the material in this chapter. But electricity is so fundamental to the world that I think knowing a bit about how electricity is "made" is pretty important. So in this chapter we'll go over the basic physical phenomenon that lies behind almost all modes of electric power generation.

6.1 Electromagnetic Induction

Write this later. Someday.

6.2 Different Ways of Turning Turbines

Discuss steam turbine and gas turbines. Combined cycle, too, I guess. Hydro and wind.

6.3 Carbon Intensities of Electricity Production

Different ways of generating electricity are associated with different CO_2e emissions. Below is a list of average emissions for different electricity-generating technologies. These data are from from Moomaw et al. (2011), available at: `http://srren.ipcc-wg3.de/report/IPCC_SRREN_Annex_II.pdf`. To get these numbers, the IPCC authors assembled many different studies of emissions associated with electricity generation. The numbers below are median values. These emissions numbers are *life-cycle analysis* estimates. That is, they account for emissions associated with all activities (mining, construction, transportation) associated with electricty generation, not only the emissions created when the power plant is operating.

All figures are expressed in gCO_2e/kWh

- Hydropower 4

- Ocean Energy 8
- Wind 12
- Nuclear Energy 16
- Concentrating solar power 22
- Geothermal 45
- Solar photovoltaic 46
- Natural Gas 469
- Oil 840
- Coal 1001

These data can also be found in Appendix A.3.

6.4 Nameplate Capacity and Capacity Factor

The amount of power that can be delivered by a generator under ideal conditions or is known as the *nameplate capacity* of the generator. For a gas or coal plant, the nameplate capacity is the power generated when it is running at "full blast"—the maximum output. For a set of solar panels, the nameplate capactiy would be the power produced on a sunny afternoon.

Generators do not operate at nameplate capacity all the time. For example, a wind turbine might have a nameplate capacity of 2MW. This means that under ideal windy conditions the turbine will generate 2MW of power. But conditions aren't always ideal. So often the turbine is producing less than 2MW. The *capacity factor* measures the actual average power production, expressed as a percentage of the maximum value. An example will make this clearer.

Example 6.1. *A 2MW wind turbine has a capacity factor of* 0.3. *How much energy does it generate in one year?*

If the turbine produced 2MW for one year, the amount of energy produced would be:

$$\text{Energy} = \text{Power} \times \text{Time} = 2\,\text{MW} \times 1\,\text{yr}\left(\frac{365\,\text{d}}{1\,\text{yr}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 17,500\,\text{MWh} = 17.5\,\text{GWh}\,. \tag{6.1}$$

We now account for the capacity factor:

$$\text{Energy Produced} = \text{Capacity Factor} \times 17.5\,\text{GWh} = (0.3)(17.5\,\text{GWh}) = 5.25\,\text{GWh}\,. \tag{6.2}$$

One could, of course, combine the above two equations and do this problem in one step intead of two.

Capacity factors are usually calculated over a one-year period. That is, one looks at the actual energy generated and compares it to how much energy would have been produced if the generator operated at its nameplace capacity for a year. This is illustrated in the next example.

Example 6.2. *The Hoover Dam in the Southwestern US has a nameplate capacity of 2080 MW. The average annual energy generated by the powerplant from 1947 to 2008 was 4.2 billion kWh. (Data source:* `https://www.usbr.gov/lc/hooverdam/faqs/powerfaq.html`*, accessed September 12, 2017.) What is the capacity factor for the Hoover dam for this time period?*

Let's first figure out what the maximum amount of energy the Hoover Dam could produce in a year. This correspondes to one year of producing at a rate of 2080 MW, or 2.08 GW.

$$\text{Energy} = \text{Power} \times \text{Time} = 2.08\,\text{GW} \times 1\,\text{yr} \left(\frac{365\,\text{d}}{1\,\text{yr}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 18220\,\text{GWh} = 18.2\,\text{TWh}\,. \tag{6.3}$$

The actual amount of energy produced in an average year is 4.2 billion kWh. Let's convert kWh to TWh. Recalling that Tera $= 10^{12}$, and one billion $= 10^9$:

$$4.2 \times 10^9\,\text{kWh}\left(\frac{1000\,\text{Wh}}{1\,\text{kWh}}\right)\left(\frac{1\,\text{TWh}}{10^{12}\,\text{Wh}}\right) = 4.2\,\text{TWh}\,. \tag{6.4}$$

We can now calculate the capacity factor:

$$\text{Capacity Factor} = \frac{\text{Actual Production}}{\text{Max Production}} = \frac{4.2\,\text{TWh}}{18.2\,\text{TWh}} = 0.23\,. \tag{6.5}$$

Note that the capacity factor is dimensionless; being a ratio it does not carry any units.

In addition to nameplate capacity, one sometimes also encounters *summer capacity* or *winter capacity*. The nameplate capacity is determined by the manufacturer. It's what would be printed on the nameplate of the power plant or wind turbine or whatever. The summer capacity would be experimentally determined in the summer; it's simply the maximum output that the power plant is capable of in typical summertime[1] conditions.

[1] Thermal power plants tend to be less efficient in the summer, since they are operating under a smaller temperature range because the ambient temperature is warmer. See, for example, `https://www.eia.gov/tools/faqs/faq.php?id=101&t=3`.

6.5 *Exercises*

Exercise *6.1:* Suppose you use a 1500 W electric heater for an average of half an hour a day for an entire year.

1. How much energy has the heater used
2. What CO_2e emissions would be associated with this if the electricity came from windpower?
3. What CO_2e emissions would be associated with this if the electricity came from natural gas?
4. What CO_2e emissions would be associated with this if the electricity came from coal?

Exercise *6.2:* The nameplate capacity of the Three Gorges Dam in China is 22,500 MW. In 2015 it generates 87 TWh of energy. What is its capactiy factor?

7 Thermal Energy

7.1 Heat

In this chapter we explore another type of energy: heat. What we call "heat" is actually hidden kinetic energy—the motion of the molecules that make up an object. For this reason "heat" is usually referred to in physics as internal energy. (For physicists, heat has a slightly restricted meaning: heat refers to any energy that flows from one object to another as a result of a temperature difference between those two object. So, for example, heat flows from a warm woodstove into the air. In so doing, the internal energy of the woodstove decreases and the internal energy of the air increases.) Since this book isn't written for physicists, I will not be militant about the physics meaning of the word heat and will refer to internal energy as heat.[1]

We have an equation for kinetic energy, $E = (1/2)mv^2$, and for the power in an electric current, $P = VI$. We need an equation for heat. Here it is:

$$\Delta U = mc\Delta T \, . \tag{7.1}$$

In this equation ΔU is the change in internal energy of an object and m is the object's mass. The quantity ΔT is the change in object's temperature, and c is the object's specific heat. We don't want to get too deep into thermal physics, but a brief discussion of temperature, internal energy, and specific heat is in order.

*I*nternal energy is the amount of hidden kinetic energy in an object. In a solid, the molecules are bound chemically to other molecules, but molecules can vibrate. The kinetic energy of this motion contributes to the internal energy of the solid. In a gas or liquid, molecules are free to move around, and molecules also can rotate and vibrate. The details of all of this can get a bit involved and is typically studied at some length in a course on thermal physics or statistical mechanics. For our purposes, the key picture is that internal energy refers to the kinetic energy associated with moving, wiggling, and vibrating molecules.

[1] Add citation(s) to physics-ey discussion of heat vs. energy? Perhaps Moore or Schroeder?

We refer to this kinetic energy as hidden, because you can't see it. If someone throws a rock at you, the kinetic energy is immediately apparent, because you can see the rock moving and you could calculate its kinetic energy[2] using $K = (1/2)mv^2$. The rock also has some internal energy: i.e., heat. But you can't see this, because molecules are far too small to be visible. But heat can be experienced: putting ones hands around a pleasantly warm cup of coffee or accidentally touching a hot pan while cooking.

[2] You should duck, first, so the rock doesn't hit you. Then calculate the energy.

Informally, temperature is how hot something it is. But what is temperature more formally? This question can be answered at a number of different levels, some of which are quite abstract and subtle. One way of thinking of temperature is that it is a property of an object that determines if heat will flow from or two it: heat flows spontaneously from hot objects to cold. Equivalently[3], temperature is a measure of the average energy per degree of freedom of an object. Degree of freedom in this context refers to a "place" where kinetic energy could be hidden. Examples of degrees of freedom are the various ways a molecule can vibrate or rotate, and (if not in a solid) the directions in which a molecule can move. Objects that are hot have, on average, a lot of energy per degree of freedom. When a hot object comes in contact with[4] a cold object, which does not have a lot of energy per degree of freedom, the hotter object loses energy while the colder one gains energy.

[3] It's not obvious that these two views of temperature are equivalent.

[4] I.e. exchanges energy with

Finally *specific heat* is a property of an object that tells us how much energy is needed to raise a kilogram of that object by one degree. If an object has very many degrees of freedom per kilogram, then it will take a lot of energy to raise its temperature by one degree. Such an object has many places to hide kinetic energy. So it can absorb a lot of energy without having the average energy per degree of freedom increase much. Different materials have different specific heats. In almost all cases, specific heats are determined experimentally. They are a property of a material, like density or conductivity, that one looks up in a book.

In this book the materials we will be concerned with are air and water. Their specific heats are:

- Specific Heat of Water $\approx$ 4200 J/(kg·K)
- Specific Heat of Air $\approx$ 1200 J/(m^3·K)

Since a liter of water has a mass of one kilogram, the specific heat of water is often expressed in units of Joules per liter per Kelvin. Note that the specific heat for air is given per volume (m^3) and not per mass.

Example 7.1. *How much energy is required to heat* 4 *kg of water from* 20 *to* 100 *degrees Celsius? Answer in both Joules, kWh, and BTUs.*

Using Eq. (7.1), we have

$$\Delta U = mc\Delta T = (4\,\text{kg})(4200\,\text{J}/\,\text{kg}\,\text{K})(80\,\text{K}) \approx 1340000\,\text{J} = 1.34\,\text{MJ}\,. \tag{7.2}$$

Converting to kWh (recall that one kWh equals 3.6 mega Joules), we find:

$$1.34\,\text{MJ} = 1.34\,\text{MJ}\left(\frac{1\,\text{kWh}}{3.6\,\text{MJ}}\right) \approx 0.37\,\text{kWh}\,. \tag{7.3}$$

We convert to BTU by using the fact that one kWh equals 3412 BTU;

$$0.37\,\text{kWh} = 0.37\,\text{kWh}\left(\frac{3412\,\text{BTU}}{1\,\text{kWh}}\right) \approx 1270\,\text{BTU}\,. \tag{7.4}$$

Example 7.2. *Consider a house that is* 2000 *square feet. Assume that the ceilings are* 8 *ft. Let's say that it is* 0 *degrees C outside and you want the air inside your house to be* 20 *degrees. Estimate how much energy it would take to raise the air in this house 20 degrees C. Express your answer in Joules and kWh. If the air in this house changes every two hours, what is the daily energy needed for heat? How much would this cost in Maine if this heat was electric?*

The volume of air in the house is (8ft)(2000ft^2) = $16,000$ ft^3. Let's convert this volume to cubic meters. There are 3.3 feet in one meter. So

$$16,000\,\text{ft}^3 = 16,000\,\text{ft}^3\left(\frac{1\,\text{m}}{3.3\,\text{ft}}\right)^3 \approx 445\,\text{m}^3\,. \tag{7.5}$$

Note that, since we are dealing with volumes, the conversion factor gets cubed. We can now use Eq. (7.1) to determine how much energy it would take to warm up this volume of air by 20 degrees:

$$\Delta U = mc\Delta T = (445\,\text{m}^3)(1200\,\text{J}/(\,\text{m}^3\,\text{K}))(20\,\text{K}) \approx 10,700,000\,\text{J} = 10.7\times 10^6\,\text{J} = 10.7\,\text{MJ}\,. \tag{7.6}$$

So this is the energy needed to warm up the cold air that enters the house. We can now convert to kWh:

$$10.7\,\text{MJ} = 10.7\,\text{MJ}\left(\frac{1\,\text{kWh}}{3.6\,\text{MJ}}\right) \approx 3.0\,\text{kWh}\,. \tag{7.7}$$

In a drafty house the air might turn over every two hours. What this means is that, in effect, every two hours all the warm air has left your house to be replaced by cold air from the outside. This air will need heating. We've seen that it takes around three kWh to heat up the air in your house once. If we need to do it twelve times a day, then heating the air in your house will take 36 kWh/day.

7.2 BTUs

Heat is energy. So one can measure it in Joules, which is the standard metric (SI) unit, or kWhs, which is the standard unit for measuring electrical energy and is a convenient unit for everyday, individual energy consumption. It is common, at least in the US (and Canada?)

to use other units for thermal energy. The most important of these are *British Thermal Units*, or BTUs. A BTU is the amount of energy needed to raise one pound of water by one degree Fahrenheit.

To be honest, I don't have anything nice to say about BTUs. One might think that then I shouldn't say anything at all, but this isn't an option in this instance, because BTUs are the standard unit for thermal energy. British Thermal Units are related to kWhs via the following conversion:

$$3412 \text{ BTU} = 1 \text{ kWh} . \tag{7.8}$$

So like joules, BTUs are annoying small.[5] In case you wanted to convert from BTU to Joules or vice versa, here's the relevant conversion factor:

[5] For example, 250kWh are 853,000 BTU.

$$1BTU = 1055J. \tag{7.9}$$

This seems like a good time remind you that conversion factors and lots of other useful facts are collected in Appendix A.

British Thermal Units are small, so it is common to work with thousand or millions of BTUs. Unfortunately, BTUs don't follow the typical kilo and mega conventions. Perversely, one thousand BTUs are denoted 1 MBTU. Presumably, this is because M denotes 1000 in Roman numerals.[6] What about one million BTUs? It would make sense for these to be abbreviated MBTU, but this is already taken by 1000 BTU so instead MMBTU is used. To summarize:

[6] At least this is what is claimed in an un-cited remark on the Wikipedia page for BTUs.

- 1 MBTU = 1000 BTU ,
- 1 MMBTU = 1,000,000 BTU .

I like to pronounce MMBTU as "mmmmmmmm B.T.U.", but this is not a standard pronunciation. Yet.

Finally, there is one other thing you should know about BTUs. I've saved the worst for last. A BTU is a unit of energy. However, it is also used as a unit of power. The power delivered by furnaces and heaters is very often described in terms of BTUs. For an example of this, see Fig. 7.1. When this is done, what it means is that the furnace is delivering a certain amount of BTUs (energy) per hour (time).

The fact that BTUs are used for both energy and power is really an awful thing—far more so than the fact that a million BTUs is an MMBTU. Power and energy are different physical quantities and so cannot possibly be represented by the same unit. However, the convention of using BTU to mean BTU/h isn't going away any time soon. Usually it is not difficult to tell from the context if one means BTU or BTU/h, so in practice there isn't likely to be much confusion, even if it is unnecessarily ambiguous (and just plain wrong).

Before moving on, there is one commonly used BTU-related unit that I need to mention. A *therm* is $100,000$ BTUs and is roughly the energy

Figure 7.1: A furnace for sale at Home Depot. Its power is listed in BTUs. What this means is that this furnace is capable of producing 60,000 BTUs per hour.

released if 100 cubic feet of natural gas are burned. One hundred cubic feet is sometimes abbreviated CCF. There are slight differences among the definitions of the therm in the US, the UK, and the EU. These differences are less than 0.1 percent, so one needn't worry about them. Depending on your location, natural gas is sold in units of CCF, therms, MJ, or kWh.

7.3 *Properties of Different Fuels*

7.3.1 *Calorific Value of Fuels*

As you are now doubt aware, when you light something on fire it burns and makes things hot. How much thermal energy do you get from this fire? That depends on what you're burning. Different fuels give off different amounts of energy per kg or per liter when burned. These quantities are known as a fuel's *calorific value*.

- Gasoline: 13.0 kWh/kg, 34.7 MJ/L, 120,480 BTU/gallon
- Coal: 8.0 kWh/kg, 19,100,000 BTU/short ton[7].
- Propane: 13.8 kWh/kg, 25.4 MJ/L, 91,600 BTU/gallon
- Natural gas: 14.85 kWh/kg, 0.04 MJ/L, 1,037 BTU/ft^3
- Heating oil: 12.8 kWh/kg, 37.3 MJ/L, 139,000 BTU/gallon
- Kerosene: 12.8 kWh/kg, 37 MJ/L, 135,000 BTU/gallon

[7] A short ton is 2000 pounds

- Wood: ∼4–5 kWh/kg
- Hardwood: 24,000,000/cord
- Pine: 18,000,000/cord

The BTU data can be found at https://www.eia.gov/energyexplained/index.cfm?page=about_energy_units. I need a source for the hardwood and pine. Other data is from MacKay.

TODO! Add discussion of HHV vs. LHV.

7.3.2 *Carbon Intensity of Fuels*

How much CO_2 is released into the atmosphere if you get a kWh worth of thermal energy? Again, it depends on what you're burning. The *carbon intensity* of a fuel is the grams of CO_2 released if you burn enough to produce 1 kWh of thermal energy. Some carbon intensities are listed below.

- Natural gas: 190
- Propane: 217
- Gasoline: 240
- Diesel: 250
- Fuel oil: 260
- Coal: 300

7.3.3 *Costs of Fuels*

Fuel costs are notoriously volatile and fluctuate considerably from year to year and even from month to month. They also vary widely in different parts of the world. Current average fuel prices for Maine can be found at http://www.maine.gov/energy/fuel_prices/. The US Energy Information Administration also tracks weekly fuel prices at: https://www.eia.gov/dnav/pet/pet_pri_wfr_dcus_nus_w.htm

At the present moment[8], average prices in Maine are:

[8] September 2017

- Natural gas: $1.31/therm
- Heating oil: $2.03/gallon
- Wood pellets: $261/ton
- Kerosene: $2.56/gallon
- Propane: $2.36/gallon
- Cord wood: $250/cord

Having just looked up this data, I'm a bit troubled by the fact that I'm currently paying $3.70/gallon for propane. This might be something to look into. The wood price, however, is what I've paid per cord for the last few years.

Example 7.3. *Suppose you burn* 20 *gallons of heating oil. How much energy is this, in kWh and BTUs? How much* CO_2 *has been emitted into the atmosphere?*

I know the calorific value of heating oil in units of MJ/L. So I'll convert to liters first:

$$20\,\text{gal}\left(\frac{3.8\,\text{L}}{1\,\text{gal}}\right) = 76\,\text{L}\,. \tag{7.10}$$

Using the data listed above, we can figure out how much thermal energy we get from burning this amount of heating oil:

$$76\,\text{L}\left(\frac{37.3\,\text{MJ}}{1\,\text{L}}\right) = 2800\,\text{MJ}\,. \tag{7.11}$$

Let's now go to kWH:

$$2800\,\text{MJ}\left(\frac{1\,\text{kWh}}{3.6\,\text{MJ}}\right) = 780\,\text{kWh}\,. \tag{7.12}$$

And next to BTU:

$$780\,\text{kWh}\left(\frac{3412\,\text{BTU}}{1\,\text{kWh}}\right) = 2,660,000\,\text{BTU}\,, \tag{7.13}$$

which is 2.6 MMBTU, recalling that 1 MMBTU = 1,000,000 BTU.

The carbon intensities of fuels are given above in units of grams of CO_2 per kWh of thermal energy. Since we calculated above that 780 kWh of thermal energy have been produced, we can find the CO_2 that has been released:

$$780\,\text{kWh}\left(\frac{260\,\text{gal}}{1\,\text{kWh}}\right) = 203000\,\text{gal} = 203\,\text{kg}\,. \tag{7.14}$$

7.4 *Efficiencies*

When one burns something to heat ones home, not all of the thermal energy released by combustion goes into the air or water that you're trying to warm up. Some of it inevitably goes up the chimney. The fraction of the thermal energy that actually goes into heating what you want to heat is given by the efficiency:

$$\text{Energy used} \times \text{Efficiency} = \text{Energy gained}\,. \tag{7.15}$$

The larger the efficiency, the more heat you get and the less is wasted. Equation (7.15) can be re-arranged as follows:

$$\text{Energy used} = \frac{\text{Energy gained}}{\text{Efficiency}}\,. \tag{7.16}$$

Be careful using these two formulas. It is easy to multiply by the efficiency when you should be dividing, and vice versa. Efficiencies are discussed in Appendix 8.1.

Example 7.4. *Suppose you want* 100 *kWh of heat and you have a furnace that is* 90% *efficient. How much fuel would you need to burn to get this amount of thermal energy*

You would need to burn enough fuel to release

$$\frac{100\,\text{kWh}}{0.9} \approx 111\,\text{kWh}\,. \tag{7.17}$$

What happens is that you burn enough fuel to release 111 kWh. Ten percent of this amount of energy (11 kWh) are wasted and go out your chimney. The other 90% (100 kWh) is used to heat up your house.

7.5 Exercises

Exercise *7.1:* A 1kW electric heater corresponds to how many BTU/hr?

Exercise *7.2:* Suppose you take a ten-minute shower every day using a shower with a flow rate of 2.5 gallons per minute. Estimate how much energy, in kWh, you use for one shower. Is this a little or a lot?

Exercise *7.3:* The dining hall at College of the Atlantic is roughly 90 ft by 45 ft. The ceiling is 12 feet tall.

1. What is the volume of the air in the dining hall?
2. Suppose it is zero degrees outside and we want to heat the air inside the dining hall to a comfortable temperature of 20. How much energy would be required to heat up all the air in the dining hall from 0 to 20? Express your answer in kWh.
3. The air in the dining hall changes over every hour. How much energy would it take to heat the air in the dining hall for one day? Is this a little or a lot?

Exercise *7.4:* Cottage House, a smallish house at College of the Atlantic, uses around 80 MMBTU per year of heating oil.

1. How many gallons of fuel is this?
2. Convert this to kWh.
3. How many tons of CO_2 does burning this oil release into the atmosphere?
4. How many people live in Cottage House?
5. How many tons of CO_2 is this per person? How many kWh of energy is used per person per day? Are these numbers large or small? Put them into perspective.

Figure 7.2: Cottage House at College of the Atlantic. Image source: College of the Atlantic.

Exercise *7.5:* The Katherine W. Davis Center on COA's campus is heated with fuel oil. In one year, 368 MMBTU of fuel were used.

1. How many gallons of fuel is this?
2. Heating oil in Maine currently costs $2.70/gallon. How much does it cost to heat Davis for one year? Is this a lot or a little? Put this number in perspective.
3. How many tons of CO_2 does burning this oil release into the atmosphere? Is this a lot or a little?

Exercise *7.6:* Suppose you want 100 kWh of heat to keep your house warm on a cold Maine day.

1. If you generate this heat with a traditional electric heater, how much CO_2 is released as a result? How much would it cost?
2. If you generate this heat with a furnace burning heating oil and the efficiency of the furnace is 83%, how much CO_2 would be released? How much would it cost?

Figure 7.3: The Seafox dormitory at College of the Atlantic. Photo credit: College of the Atlantic.

Exercise *7.7:* In 2010–11 the Seafox dormitory at the College of the

Atlantic used 4120 gallons of heating oil. Around 25 students live in Seafox.

1. How much thermal energy is this in BTUs? In kWh?
2. Express this rate of energy use in kWh/p/d. Put this number in context. Is it a little or a lot?
3. How much carbon dioxide was be released into the atmosphere by burning this fuel? Put this number into context. Is this a little or a lot?

Exercise *7.8:* 1. How much energy does it take to heat the water for a typical bath? Answer in joules, BTUs, and kWh.
2. If you used an electric hot water heater that was 90% efficient[9], how much would this cost in Maine? How much CO_2 would be released into the atmosphere as a result.
3. If you used a propane hot water heater that was 70% efficient, how much propane would you need to use? How much would this cost? How much CO_2 would be released into the atmosphere as a result?

[9] This is apparently the minimum efficiency for new hot water heaters. See http://smarterhouse.org/water-heating/replacing-your-water-heater.

8
Thermodynamics, Energy Quality, and Efficiencies

Goals of this chapter:

1. Talk about efficiencies, do some example problems.
2. Introduce idea of energy quality/entropy and its consequences.
3. Namely, low quality energy can't be converted to high quality energy with 100% efficiency.

8.1 Conversion Efficiency

The term *efficiency* is used to denote several related, but not identical, concepts. The most common context in which the notion of an efficiency arises is when converting energy from one form to another. The idea is that we start with an amount of energy E_{in} and we wish to convert this to some other form of energy. The amount of this other form of energy is denoted E_{out}. There is almost always some loss in this process—the conversion isn't 100% efficient. Some energy is lost in the conversion process.

This is illustrated schematically in Fig. 8.1. Algebraically, this situation is represented with the following equation:

$$E_{\text{out}} = eE_{\text{in}} \, . \tag{8.1}$$

Where the efficiently is denoted by e. The Greek letter η (eta) is also used for efficiency, but in this book I'll always use e.

In an energy conversion process the total amount of energy does not change. After all, energy can be neither created nor destroyed; it only changes form. In other words, referring to Fig. 8.1,

$$E_{\text{in}} = E_{\text{out}} + \text{Energy Loss} \, . \tag{8.2}$$

The issue is that during a conversion process usually not all of the energy ends up in a useful form. For example, in a traditional car

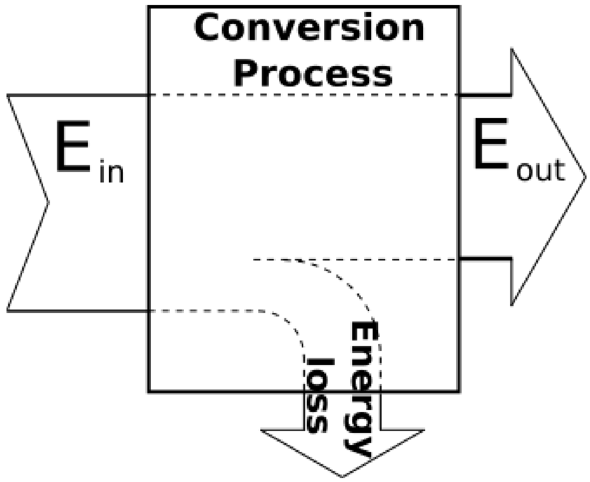


Figure 8.1: Efficiency diagram. Energy is converted from one form to another. An amount of energy E_{in} is converted into an amount E_{out} in some other form.

engine, gasoline is ignited, producing thermal energy. Only about 25% of this thermal energy is converted into kinetic energy—the motion of the car. The rest of the thermal energy is not converted into kinetic energy and instead leaves the car through the tailpipe.

Note that Eq. (8.1) can be written as:

$$e = \frac{E_{\text{out}}}{E_{\text{in}}} . \tag{8.3}$$

In words, the efficiency e is the ratio of useful energy output to the energy input. Eq. (8.3) makes is clear that e dimensionless: it has no units. The energy units on the top and bottom of the fraction in Eq. (8.3) cancel out, leaving a unitless quantity.

Not unlike unit conversions, when one is tired or working too quickly, it is easy to make a mistake and do an efficiency calculation "upside-down". Keeping either Eq. (8.1) or Fig. 8.1 in mind when dealing with efficiencies can help. I'll illustrate this process with several examples.

Example 8.1. *A home electric generator works by converting the the thermal energy obtained by burning gasoline into electrical energy. The Westinghouse WGen7500 has a fuel capacity of 6.6 gallons of gasoline, which produces 233 kWh of thermal energy when burned. The generator's efficiency is* 18%. *How much electrical energy can the generator generate from* 6.6 *gallons of gasoline?*

We know the input energy E_{in}—233 kWh—and we would like to figure out the output energy: how much electrical energy we can get from the generator. This state of affairs is represented in the diagram on the left of Fig. 8.2. To solve for E_{out} we use Eq. (8.1):

$$E_{\text{out}} = eE_{\text{in}} . \tag{8.4}$$

Here E_{in} is the 233 kWh of thermal energy, and we are given that the efficiency is 18%, or 0.18. We then plug in and solve for E_{out}, the electrical energy produced by the generator:

$$E_{\text{out}} = (0.18)(233\,\text{kWh}) \approx 42\,\text{kWh} . \tag{8.5}$$

So we get only 42 kWh of electrical energy from the 233 kWh of thermal energy. The rest of the thermal energy, 191 kWh, remains as thermal energy. This energy would be considered lost or wasted, since there we don't get anything useful from it—the extra heat is just sent into the atmosphere as warm exhaust.

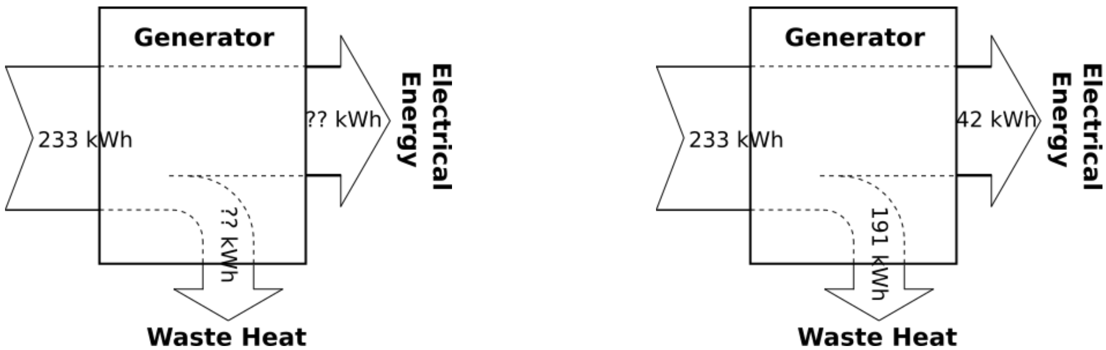


Figure 8.2: The efficiency diagram for Example 8.1.

Example 8.2. *Suppose you wish to use a propane heater such as the Rinnai shown in Fig. 8.3 to heat the first floor of your house. You estimate that on a cold winter day your heater will need to supply* 400, 000 *BTUs of thermal energy to keep things at a comfortable temperature. The efficiency of this heater is* 0.8*. How much propane will you burn per day? Burning one gallon of propane releases roughly* 90, 000 *BTUs of thermal energy.*

In this case we know what the output energy is: it's 400, 000 BTUs. And we know that the efficiency e is 0.8. We need to figure out the input energy is—i.e., the amount of thermal energy we would need to get from burning propane so that 400, 000 BTUs of thermal energy end up in the house. This state of knowledge is represented in the left diagram in Fig. 8.4.

So we need to solve for E_{in}. To do so, we start with Eq. (8.1):

$$E_{\text{out}} = eE_{\text{in}} \,. \tag{8.6}$$

We solve for E_{in} and plug in:

$$E_{\text{in}} = \frac{E_{\text{out}}}{e} = \frac{400,000\,\text{BTU}}{0.8} = 500,000\,\text{BTU} \,. \tag{8.7}$$

So we need to burn enough propane to yield 500,000 BTUs of thermal energy. Of this thermal energy, 400,000 remains in the house providing warmth, while 1000,000 BTUs are wasted—they leave the house through the heater's exhaust vent.

Note that we calculated that E_{in} is larger than E_{out}. This makes sense. Since some of the thermal energy leaves through the vent, we have to start with more than 400,000 BTUs of thermal energy to end up with 400,000 BTUs of thermal energy in the house. When doing an efficiency calculation such you did in this example or the previous one, it's a good idea to pause and check that your work makes sense, by thinking through whether your expect your answer to be larger or smaller than the number you started with.

To complete this problem, we need to figure out how many gallons of propane we need to burn to get 500, 000 BTUs of thermal energy.

$$500,000\,\text{BTU}\left(\frac{1\,\text{gal}}{90,000\,\text{BTU}}\right) \approx 5.6\,\text{gal} \,. \tag{8.8}$$

Figure 8.3: Left: The Westinghouse WGen7500 generator. Image from https://www.amzn.com/B01N80F68E/. Right: The Rinnai EnergySaver 20,700 BTU Propane Furnace, Model #EX22CTWP. Image from https://www.homedepot.com/p/Rinnai-EnergySaver-20-700-BTU-Vented-Propane-Fur 306653491.

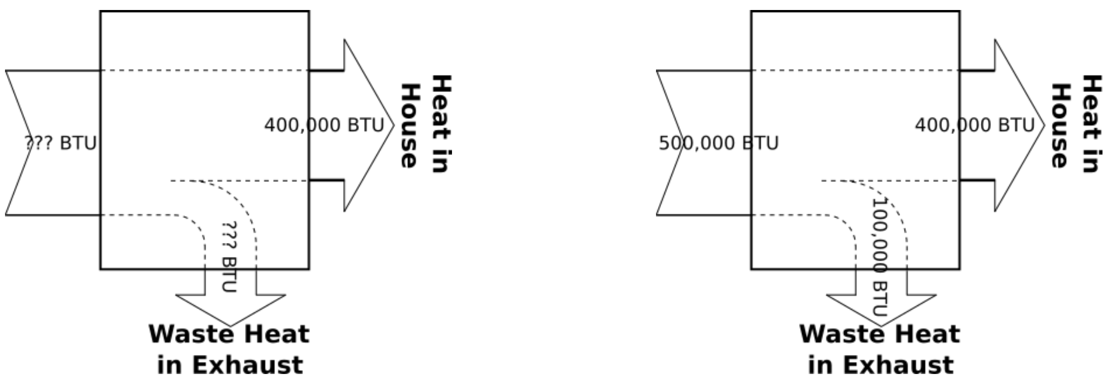


Figure 8.4: Efficiency diagrams for Example 8.2.

8.2 The Second Law of Thermodynamics

First, mention that the first law is the conservation of energy.

Conservation of Energy... but can't always convert energy from one form into another with perfect efficiency. Sometimes, the reason for this is specific to the particular types of energy involved. But there are some general principles that imposed limits on converting thermal energy to or from other types of energy. The study of these limits is part of the field of physics known as thermodynamics.

8.3 Implications

8.4 An Example: Steam Turbines and the Rankine Cycle

Or should I do a steam turbine to generate electricity? I think yes.

Conservation of energy says that there's nothing impossible about converting all of the thermal energy into electrical energy. After all, such a transformation certainly would conserve energy. However...

Example 8.3. *Statement of example*

Solution goes here

TODO! Move efficiency sections from appendix to here.

8.5 *Exercises*

For some of these exercises you'll need to know the calorific values of various fuels. These can be found in Appendix A.3.3.

Exercise *8.1:* A fire burns in the a woodstove. Over the course of a day, the burning wood releases 30,000 BTU. Of this, 23,000 BTU remains in the house, warming it up. What is the efficiency of the woodstove?

Exercise *8.2:* To heat 400 liters of water from 10 to 40 degrees C requires about 48,000 of thermal energy. Suppose this energy is provided by a propane heater that is 90 % efficient.

1. In order to heat up the water, how many gallons of propane need to be burned by the heater?
2. Complete an efficiency diagram for this situation.

Part III

Consumption

9
Lighting and Appliances

blah blah

9.1 Incandescent Lights

9.2 Fluorescent Lights and LEDs

9.3 Other Appliances

9.4 Phantom Loads

more blah blah.

Example 9.1. *Statement of example*

Solution goes here

9.5 Exercises

Exercise *9.1:* A questions

1. part a
2. part b

10
Cars and Transportation

10.1 Some Basic Car Facts

Cars convert the chemical energy of gasoline into kinetic energy. Later in this chapter we'll explore this process in more detail. For now, let's just focus on some basic facts.

Cars use gasoline. How much? It depends on the car and how one drives it. Are you driving at a constant speed on the highway or are you starting and stopping frequently in a city or town? We'll start by keeping things simple and just use 30 miles per gallon for the typical rate of fuel consumption.[1] There is a lot of variation, but 30 mpg seems like an ok average. In the US the average is more like 25 mpg[2] but 30 is probably a better figure for Europe. Outside of the US fuel efficiencies are typically expressed in terms of how many liters of gasoline are needed to drive 100 km. For more, see Question 10.1, below.

[1] How are fuel efficiencies measured outside the US? Liters per km?

[2] http://www.autonews.com/article/20150604/OEM05/150609925/average-u.s.-mpg-edges-up-to-25.5-in-may

Figure 10.1: My old car, a 2003 Toyota Corolla. According to www.fueleconomy.gov, its combined city/highway mileage was 31 mpg. It was a good car; I miss it.

Gasoline, when ignited, produces thermal energy. How much? One liter of gasoline produces 10 kWh of energy. If you prefer gallons: igniting one gallon of gasoline produces 38 kWh. Lastly, gasoline, when burned, produces CO_2, along with some other unpleasant pollutants.

How much CO_2? One kWh of energy from gasoline yields 240 grams of CO_2.

You may initially be surprised that one kWh of energy from gasoline yields only 240 grams of carbon dioxide, while generating a kWh or electricity produces, on average in the US 500 grams of carbon dioxide. Why is this? First, recall that 500 grams is an average over different ways of generating electricity: primarily nuclear, hydro, and various fossil-fuel power plants. For comparing with burning gas in cars, let's focus on fossil-fuel-generated electricity. The average carbon intensity is: 1001 (coal), 840 (oil), and 469 (natural gas)[3] in units of grams of carbon dioxide per kWh of electricity. All these figures are a lot less than the carbon that results from getting one kWh of thermal energy from gasoline.

[3] Figures from Table A.II.4 of Moomaw et al. (2011).

The reason is that when we burn gas we are getting thermal energy, which is of a much lower quality than electrical energy.[4] Thermal energy is not of high quality—it is not all readily available to us. We can use the hot air produced by burning a bit of gasoline to charge our cell phones. Electricity, on the other hand, is very high quality energy. Almost all of the energy in electricity is available to us to do useful things.

[4] Where should I talk about energy quality? Probably somewhere in part I, maybe even in the very first energy chapter.

In any event, let's use these facts to do a simple example so we can start to get a feel for these numbers:

Example 10.1. *Suppose you drive a typical car for two hours at 60 mph. How much energy does this take? How much carbon dioxide is emitted by the car during these two hours? How much carbon dioxide would be produced if you did this amount of driving every day for one year?*

First lets figure out how far the car goes.

$$2\,\text{h} = 2\,\text{h}\left(\frac{60\,\text{mi}}{1\,\text{h}}\right) = 120\,\text{mi}\,. \tag{10.1}$$

Note that I'm essentially using the speed to convert from hours to miles. How much fuel does this use? Assuming that the car gets 30 miles to the gallon:

$$120\,\text{mi} = 120\,\text{mi}\left(\frac{1\,\text{gal}}{30\,\text{mi}}\right) = 4\,\text{gal}\,. \tag{10.2}$$

Next, using the fact that 38 kWh are produced if we burn on gallon of gasoline:

$$4\,\text{gal} = 4\,\text{gal}\left(\frac{38\,\text{kWh}}{1\,\text{gal}}\right) = 152\,\text{kWh}\,. \tag{10.3}$$

Let's now turn our attention to carbon dioxide. One kWh of thermal energy from gasoline is responsible for 240 g, or 0.24 kg, of CO_2. So, the 152 kWh we use when driving yields:

$$152\,\text{kWh} = 152\,\text{kWh}\left(\frac{0.24\,\text{kg}}{1\,\text{kWh}}\right) = 35\,\text{kg}\,. \tag{10.4}$$

If we did this every day for a year:

$$35\,\frac{\text{kg}}{\text{d}} = 35\,\frac{\text{kg}}{\text{d}}\left(\frac{365\,\text{d}}{1\,\text{yr}}\right) = 13300\,\text{kg}\,. \tag{10.5}$$

So this amount of driving would result in around 13 tons of CO_2 being emitted into the atmosphere in one year.

Whoa. That's a lot of energy and carbon. Equation (10.3) tells us that this amount of driving uses 152 kWh. This is more energy than the average UK resident uses in a day for everything. The 13 tonnes of carbon dioxide is also huge. The average emissions worldwide is around 5.5 tonnes of CO_2 per person per year. So this amount of driving already puts you at twice the world average. And remember that our goal is not to make US residents average but to reduce worldwide emissions to essentially zero.

This has been a fairly rough calculation, and not everyone in the US drives this amount daily. But we hope the point is clear. Driving conventional gasoline cars uses a lot of energy and produces a lot of carbon dioxide. The numbers add up quickly.

10.2 A Basic Model for Car Energy

The above discussion gave some basic facts about cars: how much gasoline is needed to make them go a certain distance (miles per gallon), how much energy is released when burning gasoline (kWh per gallon), and how much Co_2 is released when doing so (kg of CO_2 per kWh). But where does this energy go? In this section we'll develop a simple model of car physics: why does driving a car take the amount of energy it does, and what (if anything) could be done to make cars use less energy.

The chemical energy in the gasoline is converted into thermal energy when it is ignited inside an engine's pistons. This thermal energy is then converted into kinetic energy and the car moves. Physics tells us that objects in motion tend to say in motion. So once the car is rolling along, we should be all set, right?

As you know, it's not so simple. From time to time we need to stop our car to avoid hitting someone or because there is a red light or a stop sign. Also, friction is constantly exerting a force on your moving car, continually sucking away kinetic energy—energy that has to be replaced by your car's engine if you want to keep moving. There are two main types of friction: air resistance and rolling resistance. The latter is the friction associate with your wheels and tires. This tends to be much smaller than air resistance, so we'll ignore it for now.

So let's build up a simple model of the energy use of a car. We'll look at the energy lost do to stopping and starting, and the energy lost due to air resistance. This analysis closely follows that in Chapter A of MacKay (2009).

We'll start by thinking about stopping and starting. Let's picture that you wish to drive your car at an average speed of v and that the average distance between stops is d. This situation is illustrated in Fig. 10.2. What, then, is the average time between stops? Distance equals rate times time, $d = vt$, so

$$\text{Ave. time between stops} = t = d/v\,. \tag{10.6}$$

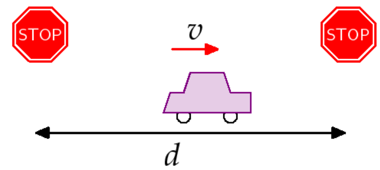


Figure 10.2: A car drives a distance d between stops. (Image source: David J.C. MacKay 2009, Fig. A.2, p. 254.)

Every time you stop, all of the car's kinetic energy goes into the brakes. The kinetic energy of the car is $(1/2)mv^2$, where m is the mass of the car. So, the rate at which energy flows into the brakes is

$$\text{Power going to brakes} = \frac{\text{kinetic energy of car}}{\text{time between stops}} = \frac{\frac{1}{2}m_c v^2}{d/v} = \frac{\frac{1}{2}m_c v^3}{d}\,. \tag{10.7}$$

This equation is the power needed to make the car go, given that you wish to travel at speed v and that the average distance between your stops is d. This is just part of the power that your car needs. The additional power is needed to overcome air resistance, which we discuss next.

As your car moves through air, air resistance exerts a force on the car. This force acts to slow the car down, requiring additional energy from the engine to keep the car moving at a constant speed. We'll think about air resistance in terms of energy and not forces. Suppose you get your car moving nice and fast down a flat highway and then put the car neutral. What happens? You'll gradually coast to a stop. Where has that energy gone? Not into the brakes, since you haven't used the brakes. The answer is that the kinetic energy of your car has been turned into the kinetic energy of the air left swirling in the car's wake.

Here's a simple way to think about this. As the car moves through the air, it needs to push air out of its way to make room for itself. Similarly if you are running through a very crowded room—perhaps you are in a busy train station and are running to catch a train—you need to push people out of your way so you can make it through the

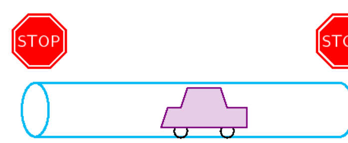


Figure 10.3: A car drives a distance d between stops. In so doing, it needs to displace a volume of air equal to Avt. (Image source: David J.C. MacKay 2009, Fig. A.4, p. 255.) I should perhaps modify this to include vt and A.

crowd. How much air does the car have to move out of its way during some time interval t? During this time interval the car moves a distance vt. So it displaces a volume of air equal to Avt, where A is the frontal cross-sectional area of the car. What is the mass of this air? We convert from mass to volume via the density ρ; the density ρ of air is around 1.225 kg/m^3. So, the mass of the air displaced by the car is $m_{\text{air}} = \rho Avt$.[5]

This is the mass of the air that is moved by the car. How fast is this air going? We'll need this in order to come up with an expression for

[5] Maybe talk about effective area of car versus actual area of car; drag coefficient. Probably should do this here or somewhere.

the kinetic energy of the swirling air. We'll make the assumption that the speed of the swirling air left behind the car is equal to the speed of the car. This is only an approximation, but it seems reasonable. Think about standing next to a road while a car zips by. The faster the car is moving, the faster the wind you'll experience roadside.

With this assumption, we can now write down an expression for the kinetic energy of the swirling air left behind by the car:

$$\frac{1}{2} m_{\mathrm{air}} v^2 = \frac{1}{2}\rho A v t v^2 = \frac{1}{2}\rho A t v^3 \,. \tag{10.8}$$

This is the kinetic energy lost by the car to swirling air every time interval t. So the power associated with this is

$$\text{Power going to swirling air} = \frac{\frac{1}{2}\rho A t v^3}{t} = \frac{1}{2}\rho A v^3 \,. \tag{10.9}$$

We can now combine Eqs. (10.7) and (10.9) and come up with an expression for the total power the car is losing to stopping-and-starting and air resistance:

$$\text{Power lost by car} \approx \frac{1}{2} m_c v^3/d + \frac{1}{2}\rho A v^3 \,. \tag{10.10}$$

This equation gives the rate at which the car is losing energy. The car's engine needs to replenish this energy at the same rate in order to keep the car moving. To do so, the engine converts the chemical energy in the gasoline into thermal energy (heat) and then kinetic energy. This conversion chain is only around 25% efficient. This means that if you burn gasoline and release 100 kWh of thermal energy, only 25 kWh of this can be converted into kinetic energy by the engine. Thus, we can write the power used by the car as:

$$\text{Power used by car} \approx 4\left[\frac{1}{2} m_c v^3/d + \frac{1}{2}\rho A v^3\right] \,. \tag{10.11}$$

This equation gives the rate at which the car uses gasoline.

This is a very simple picture of a car's energy use, but it gives reasonable results. If you plug reasonable numbers into Eq. (10.11) you'll get numbers consistent with the empirical figures discussed in the previous section. You can explore this in Question 10.8 if you wish.

10.3 *Implications of the Car Model*

To do better in city driving:

- Make car less massive: decrease v_c.
- Drive more slowly
- Stop less often

- Recover the energy lost to braking.

To do better for highway driving:

- Drive more slowly
- Decrease the cross-sectional area of the car.
- Decrease the drag on the car.

10.4 Electric Cars

How Green are Electric Cars? https://www.nytimes.com/2021/03/02/climate/electric-vehicles-environment.html?action=click&module=Top%20Stories&pgtype=Homepage.

This resource from Jessika Trancik's research group at MIT is ridiculous: https://www.carboncounter.com/#!/explore?taxfee_state=KY&price_Gasoline=2.2&price_Diesel=2.5&price_Electricity=9&electricity_ghg_fuel=500.

10.5 Planes

What about planes. Planes use gasoline? How much? This varies, but a good average is a plane uses roughly 40 kWh to move one person 100 km. This is also written as 40 kWh/100person-km. This figure takes into account the fact that planes are mostly full. That is, 40 kWh is the single-person share of the total energy needed to move the plane 100 km.

Example 10.2. *Suppose you make two round-trip flights from New York City to San Francisco each year. Approximately how much energy does this take, in kWh/day?*

It is approximately 3000 miles from New York to San Francisco. So for two round trips, the total distance flown will be 12,000 miles. Converting to kilometers:

$$12000\,\text{mi} = 12000\,\text{mi}\left(\frac{5\,\text{km}}{3\,\text{mi}}\right) = 20000\,\text{km}\ . \tag{10.12}$$

We can now figure out the energy used using the fact that planes use 40 kWh to move one person 100 km:

$$20000\,\text{km} = 20000\,\text{km}\left(\frac{40\,\text{kWh}}{100\,\text{p km}}\right) = 8000\,\text{kWh}\ . \tag{10.13}$$

Dividing this by the number of days in the year to get the energy use per day, we find:

$$\frac{8000\,\text{kWh}}{365\,\text{d}} \approx 22\,\text{kWh/ d}\ . \tag{10.14}$$

This is a lot of energy—almost ten percent of the total 250 kWh/day used by the average American.

Here is another way to think about how much energy these two flights are. Let's take kWh/day and convert to kW:

$$22\,\text{kWh/ d} = \frac{22\,\text{kWh}}{1\,\text{d}}\left(\frac{1\,\text{d}}{24\,\text{h}}\right) \approx 0.92\,\text{kW}\,. \qquad (10.15)$$

So, rounding up a bit, flying across North America twice per year corresponds to a power of one kW. One kilowatt is the power drawn by a typical toaster. So taking these two round-trip flights uses as much energy as running a toaster 24 hours a day for an entire year.

Figure 10.4: Taking two transcontinental flights uses as much energy as leaving this toaster on non-stop for an entire year. (Image source: Donovan Govan, posted on wikipedia https://en.wikipedia.org/wiki/File:Toaster.jpg, licensed under the GNU Free Documentation License https://en.wikipedia.org/wiki/en:GNU_Free_Documentation_License.

How much carbon dioxide is produced by flying and what are its effects? It turns out that there is not quite universal agreement this question. To read more about where some of the disagreements lie, a good starting point is the article by Wihbey 2015, available at http://tinyurl.com/j4yclnd. Despite some a lack of unanimity about the details of how to account for the greenhouses gasses due to flying, the big picture is clear: Flying takes a lot of energy, this energy is produced from fossil fuels, and so flying is a large source of greenhouse gases. Estimates seem to be that flying is responsible for two to four percent of all anthropogenic climate change.[6]

[6] See http://www.grida.no/publications/other/ipcc_sr/?src=/climate/ipcc/aviation/index.htm and https://en.wikipedia.org/wiki/Environmental_impact_of_aviation#Total_climate_effects.

A little bit of Plane Physics

So planes are a lot like cars, except they fly through the air instead of roll along the highway.[7] But this flying business makes a difference in our basic model of transportation. Planes need to use a certain amount of energy just to stay up in the air. This is in addition to the energy they need to use to move forward. It turns out that there is no energetic advantage to flying slowly. If a plane goes faster it takes more energy to move it forward, but the plane gets more lift so less energy is needed to stay up. The velocity-dependence of these two things—lift and drag—tend to cancel out, so that the energy consumption of planes are essentially constant. What this means in practical terms is that we couldn't double the fuel efficiency of planes by flying half as fast.

[7] Add more later from MacKay Chapter C? Not sure how much we want to get into this. Or maybe just keep this to a paragraph.

This article: https://www.yaleclimateconnections.org/2015/09/evolving-climate-math-of-flying-vs-driving/ looks like a good overview of the climate and energy impacts of flying and driving.

10.6 Transporting Stuff

Energy intensity of different forms of shipping:[8]

- Road: 1 kWh per ton-km
- Container Ship: 0.015 kWh per ton-km

[8] I think shipping maybe should be its own chapter? Or maybe should be part of the Making Stuff chapter?

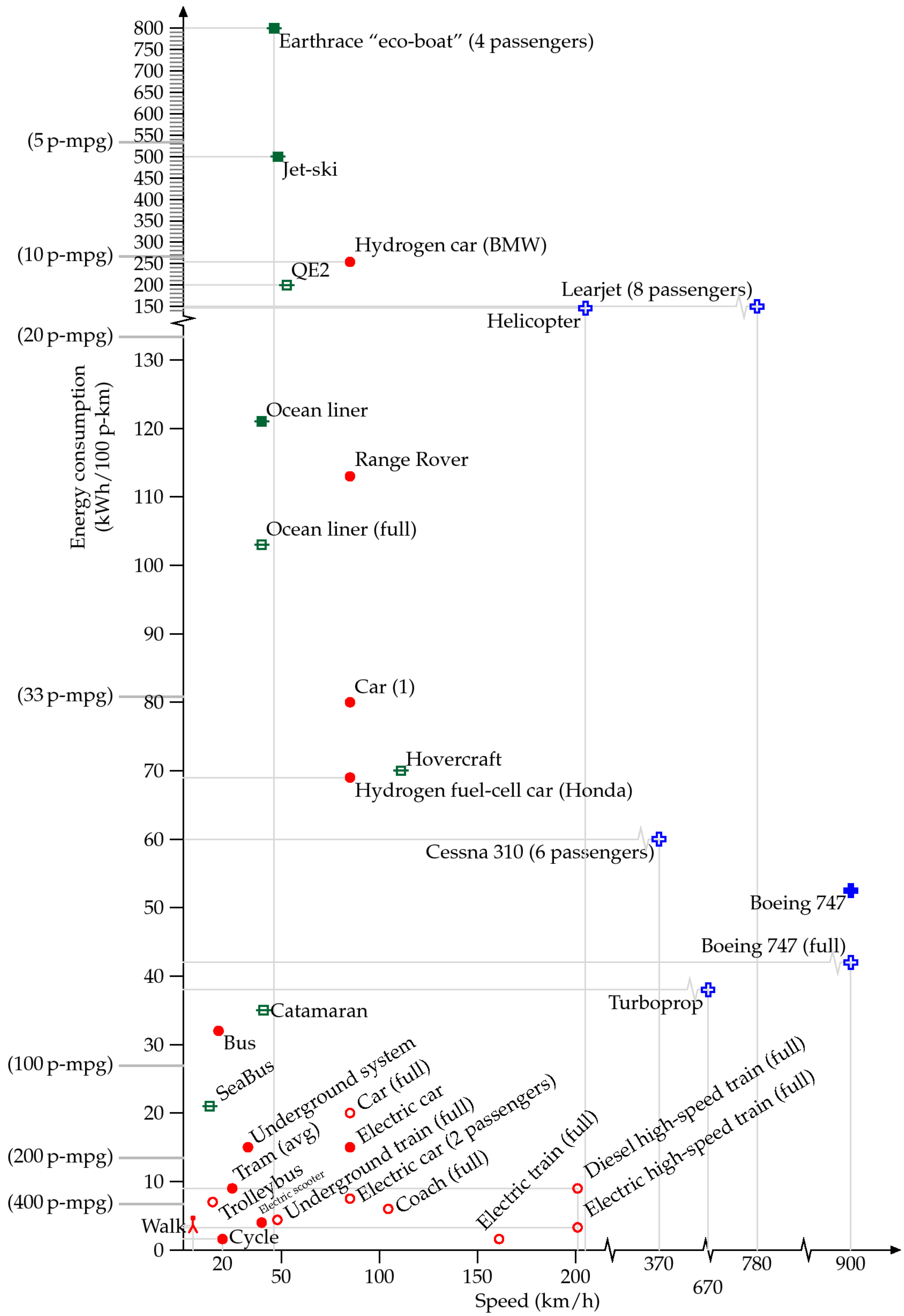


Figure 10.5: Figure 20.23 from MacKay (2009). Comparing the energy use and speed of different forms of transportation.

- Plane: 1.6 kWh per ton-km
- Rail: 0.1 kWh per ton-km

This article looks like a decent overview: https://www.vox.com/2015/12/23/10647768/shipping-environmental-cost.

10.7 Conclusion

- Gasoline: 10 kWh per liter or 38 kWh per gallon
- Typical gas mileage for car: 30mph, but this ranges considerably.
- Flying uses roughly 40 kWh per 100 p-km.
- A typical car (with one passenger) uses roughly 80 kWh per 100 p-km.
- An electric car uses roughly 15 kWh per 100 p-km.
- Carbon intensity of gasoline: 240g per kWh.
- Carbon intensity of electricity generation in the US: 613 g per kWh.

10.8 Exercises

Exercise *10.1:* Outside of the US, it is common to express fuel efficiencies in units of liters per 100-kilometer instead of miles per gallon. Convert 30 mpg to L/100-km.

Exercise *10.2:* The average US driver drives 13, 500 miles a year.[9]

1. How many kWh of energy does this use per day? Is this a little or a lot?
2. How much CO_2 is emitted into the atmosphere in one year as a result of driving this distance. Is this a little or a lot?

[9] https://www.fhwa.dot.gov/ohim/onh00/bar8.htm

Exercise *10.3:* Suppose you make one round trip flight a year from New York to Beijing. About how much energy does this take? Is this a little or a lot?

Exercise *10.4:* Suppose it takes a certain amount of time, traveling at a certain speed, to go a certain distance via car. Now suppose you travel twice as fast.

1. What happens to the duration of your trip?
2. What happens to the power used by your car when you are driving?

3. What happens to the total energy used by your car to complete the trip?

Exercise *10.5:*

Suppose someone invents a super-strong plastic that can be used to make cars that have half the mass as traditional metal cars. Would this improve the power consumption of city driving, highway driving, or both? Briefly explain.

Figure 10.6: A Toyota Prius with two sea Kayaks on top. Shown for scale is Dr. Doreen Stabinsky, who is roughly 5.5 feet tall.

Exercise *10.6:* You are driving from Mount Desert Island to Rivière-du-Loup, Québec, in a Toyota Prius that usually gets 45 mpg driving on highways. For this trip you have two sea kayaks attached to the roof of the car, as shown in Fig. 10.6. Very approximately what gas mileage do you think you will get under these conditions? (Briefly explain the reasoning behind your estimate.)

Exercise *10.7:* You need to go from Boston to Minneapolis to attend a Prince memorial concert. Approximately how much energy would you use if you:

1. Drove by yourself?
2. Took an airplane?
3. Took a train?

To answer these questions, refer to the data on this web page `https://ourworldindata.org/travel-carbon-footprint`.

Exercise *10.8:* Let's see what happens if we plug some reasonable numbers in to Eq. (10.11). Use 70 miles per hour for the car's speed. At this speed, the air resistance term is much larger than the braking term. So ignore braking. Use 1 m^2 for the frontal area of the car, and 1.3 kg/m^3 for the density of the air.

1. Plugging in to Eq. (10.11), what do you get for the power used by a car?
2. If you drive for two hours, how much energy would the car use?
3. How does this answer compare to the energy use for a car that we found in Example 10.1?

Exercise *10.9:* In this problem you will investigate the transition from city to highway driving.

1. Show that if $m_c > \rho A d$, then the braking term is larger than the air resistance term in Eq. (10.11).
2. Using the same numbers that you did in Exercise 10.8 show that if the distance d between stops is less than approximately 750 meters, then the braking term in Eq. (10.11) is larger than the air resistance term.

Exercise *10.10:* Go to `https://flowcharts.llnl.gov/commodities/energy` and select the most recent energy flowchart for the US.

1. How much energy is used for transportation in the US in one year? Express your answer in kWh/p/day and also in Giga Watts.
2. Suppose we wanted to use electricity for this energy. How much electric power would we need to do so? Assume that gas engines are 25% efficient and the electric engines are 90% efficient.
3. If we wanted to generate this amount of electricity using wind power, how much land would be needed? Express this area in a meaningful way
4. If we wanted to generate this amount of electricity using solar PV how much land would be needed? Express this area in

a meaningful way.

5. If we wanted to generate this amount of electricity using nuclear power, how many 2-GW nuclear generating stations would we need to build?

11
Heating I: Leakiness

When it is cold outside we need to heat our homes. The reason is that heat escapes from the inside of our house to the outside. In this chapter we'll think about how and why heat leaves a house. It is this rate of heat loss that determines how much heat we need to provide to a house to keep it at a comfortable temperature. In the next chapter we'll look at different ways of generating that heat—woodstoves, furnaces, heat pumps, and so on.

Heat leaves a home via two mechanisms: warm air escapes through holes in the house, and heat conducts through the walls, windows, and roof. We'll start by looking at escaping warm air.

11.1 Heat Loss Due to Air Leakiness

Let's consider a house that holds a volume of air V, and let ΔT be the different between the outside temperature and our desired inside temperature. For example, it if is -5 degrees Celsius outside and you want the air in your house to be 20 degrees C, then $\Delta T = 25$. How much energy is needed to increase the temperature of a volume V of air by ΔT? To answer this question, we use Eq. (7.1):

$$\text{Heat needed} = mc\Delta T\,, \tag{11.1}$$

where c is the specific heat of air, whose value is approximately 1200 J/m^3 C. In this case, however, we use V instead of m, because the specific heat of air is expressed per volume instead of per kilogram. So I'll rewrite Eq. (11.1) as:

$$\text{Heat needed} = Vc\Delta T\,, \tag{11.2}$$

Warm air is continually leaking out of the house and being replaced by cold air from outside—air that needs to be heated so that its temperature increased by ΔT. The rate at which air leaves the house will be larger for leakier and more poorly sealed homes. We'll express this leakiness in term of the number of air changes that occur every hour. Although this is often abbreviated ACH[1], I'll use N. The leakier the house, the larger N will be. An N of 0.5 means that half of the air in a house changes over every hour. Or, in one day you'll have to heat up the air in your hour 12 times. So smaller air exchange rates are good. But there is a limit to how small we can make N for a house. If N is too small then the occupants of the building won't have enough fresh air to breathe. For typical residences N is between 1 and 2. You might want this to be a bit higher for bathrooms and kitchens, however. I think anything 0.5 or lower would start to be considered unhealthful, both for the house and for the creatures living in it.

[1] Somewhere mention that N can be indirectly measured by a blower door test.

Using the air changes per hour N, I can express the rate at which energy leaves house.

$$\begin{aligned} \text{Rate of heat loss} &= \text{No. of changes per hour} \times \text{Energy to heat air} \\ &= NCV\Delta T \,. \end{aligned} \tag{11.3}$$

Note that the rate of heat loss, being a measurement of the flow rate of energy, is a power.

Let's futz around with units a bit so we can get everything in terms of meters, kilograms, and seconds. This means that our rate of heat loss will come out in units of Watts. We'll need to convert N to exchanges per second. Doing so:

$$N = \frac{N}{1\,\mathrm{h}}\left(\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right) = \frac{N}{3600\,\mathrm{s}} \,. \tag{11.4}$$

Using this, Eq. (11.3) becomes:

$$\text{Rate of heat loss} = 1200\frac{\mathrm{J}}{\mathrm{m}^3\,\mathrm{K}}\frac{N}{3600\,\mathrm{s}}V(\mathrm{m}^3)\Delta T(\mathrm{K}) \,. \tag{11.5}$$

This then simplifies to:

$$\text{Rate of heat loss} = \frac{1}{3}NV\Delta T \,. \tag{11.6}$$

This equation gives the rate of heat loss, in units of Watts, due to warm air leaking out of the house being replaced with cold air from the outside. It is important to remember that this equation is only true if the volume V is measured in cubic meters and the temperature difference is measured in Celsius.

11.2 *Heat Loss Due to Conduction*

Houses also lose heat because it flows out through the walls, roof, and windows. This type of heat transfer is called *conduction*, and is a different mechanism than hot air directly leaving the house through holes in the walls. In conduction, the heat is flowing through the wall itself, just as heat might flow from a hot frying pan into its handle.

The rate of conductive heat loss is given by the following formula:

$$\text{Rate of heat loss} = \frac{A}{R}\Delta T \,. \tag{11.7}$$

In this equation A is the surface area of the house and ΔT is again the temperature difference between the inside and the outside of the house. Let's think about this equation term by term. The larger the area of the wall[2], the more area there is for the heat to flow, and so the greater the rate of heat loss. The ΔT in Eq. (11.7) tells us that the rate of heat loss depends on the temperature difference between the interior and exterior of the wall. In practical terms, the colder it is outside, the faster heat will flow out through the wall.

[2] Eq. (11.7) applies to all surfaces, not just walls. But for the sake of concreteness, I'll refer to the surface as "wall."

The quantity R in Eq. (11.7) is the thermal resistivity. It is a measure of how well insulated the house is. The larger R is, the better the insulation, and the smaller the heat loss. To see this mathematically, note that in Eq. (11.7), R is on the bottom; this means that increasing R will make the fraction smaller, leading to less heat loss. Different materials will have different R values. Metal, which conducts heat very well, will have a small R value. Styrofoam or wool, which conduct heat poorly, have large R values.

What are the units of R? Good question. In the friendly metric SI system, R has units of square meters times degrees Celsius divided by Watts. That is:

$$R = \frac{\mathrm{m}^2\,\mathrm{C}}{\mathrm{W}} \,. \tag{11.8}$$

One way to see this is to note that the rate of heat loss in SI units must be Watts. Then R has to have units of $\mathrm{m}^2\ \mathrm{C/W}$ in order for Eq. (11.7) to be dimensionally consistent. What about American/imperial units? Here things start to get a bit ugly. In imperial units, R is a mess:

$$R = \frac{\mathrm{ft}^2\,\mathrm{F}^\circ\,\mathrm{h}}{\mathrm{BTU}} \,. \tag{11.9}$$

If you go to buy insulation in the US and Canada, its resistance will be given in these units. Since feet-squared-Fahrenheit-hour-per-BTU doesn't exactly roll of the tongue, one almost always refers to an R-value simply with a number. The unwieldy units are understood and need not be spoken. Outside of the US and Canada, the units for R are usually the metric ones: $\mathrm{m}^2\ \mathrm{C/W}$.

Another way of measuring the insulative value of a material is via its conductivity or thermal transmittance. This is usually denoted U and is just the reciprocal of R. That is, $U = 1/R$. So materials with high thermal resistance have low conductivity. And conversely, materials with low thermal resistance have a high conductivity. In much of the non-North American world, u values are more common the R values. I'm going to proceed in a North American centric way, and use imperial R values. We're not going to do much, if anything, with numerical values for R, so the units aren't going to be crucial.

11.3 Total Heat Loss

Summing up and stepping back, we've got two things going on. Your house is getting colder because hot air escapes and because heat leaves via conduction through the walls and other outward-facing surfaces. These flow rates are captured, respectively, by Eqs. (11.6) and (11.7). Since both of these phenomena are occurring, to get the total rate of heat loss in a house we need to add them together:

$$\text{Total rate of heat loss} = \frac{1}{3}NV\Delta T + \frac{A}{R}\Delta T\,. \tag{11.10}$$

Significant care must be taken when using this formula. The first term, for the heat loss due to escaping air, is true only if N is the number of air exchanges per *hour*, V is measured in *cubic meters*, and ΔT is in *Kelvin*, or, equivalently, *Celsius*. The second term of this equation involves R-values, which in the US are going to be in messed-up imperial units. Rather than transform your R-value to SI units, it is usually easier to turn other quantities into imperial. That is, one has to convert the surface area of the house into square feet and use Fahrenheit for the inside–outside temperature difference.

We would like to minimize the rate at which heat flows out of our house, because then we will need to generate less heat in order to stay warm. This will save us money and reduce greenhouse gas emissions. So how can we make the total rate of heat loss, Eq. (11.10), smaller? There are several possibilities.

- Live in a smaller house. A smaller house will have a smaller surface area A and a smaller volume V, leading to a smaller rate of heat loss. Also, you can make the effective size of your house smaller if you can control heat room-by-room. There is no need to fully heat rooms that you are not in.

- Air seal your house. Make your house less leaky so that hot air escapes less quickly. Mathematically, this corresponds to decreasing N in Eq. (11.10).

- Turn down the thermostat. If you keep your house a bit less warm, then ΔT will be smaller and you will lose heat at a slower rate.

This last point is worth saying a bit more about. Following MacKay, note that both of the terms on the right-hand side of Eq. (11.10) are of the following form:

$$\text{Rate of heat loss} = \text{leakiness} \times \text{temperature difference} . \quad (11.11)$$

Which is larger, heat lost due to escaping air or conducing heat? The answer seems to be that there is not a single answer.[3] There does seem to be agreement that doing some basic air sealing if usually the easiest way to save money and energy. The money needed to do do air-sealing is not large, and the payback time is quick. Improving the insulation of an already-built house can be more expensive and, depending on how your house is built, can be tricky.

[3] http://www.greenbuildingadvisor.com/blogs/dept/qa-spotlight/air-leaks-or-thermal-loss-what-s-worse

11.4 Degree Days and Heating Loads

Do I need this section? I'm not sure.

11.5 A bit more about R Values

Key point: R values combine like resistors in parallel. So a small region of a wall with a low R can "undo" lots of good insulation (high R) elsewhere. Discuss thermal bridging.

My impression is been that insulating homes—either new homes or retrofitting old homes—requires a good bit of experience and specialized knowledge to do well. It's pretty easy to do a poor job of insulating; doing it really well is hard.

11.6 Exercises

Exercise *11.1:* Consider a house that is 2000 square feet. Assume that the ceilings are 8 ft. Let's say that it is 0 degrees C outside and you want the air inside your house to be 20 degrees.

1. Estimate how much energy it would take to raise the air in this house 20 degrees C. Express your answer in Joules, kWh, and BTU.
2. If the air in this house changes every two hours, what is the daily energy needed for heat? How much would this cost in Maine if this heat was electric?

Exercise *11.2:* A 1kW electric heater corresponds to how many BTU/hr?

Exercise *11.3:* Cottage House uses around 80 MMBTU per year of heating oil.

1. How many gallons of fuel is this?
2. Convert this to kWh.
3. How many tons of CO_2 does burning this oil release into the atmosphere?
4. How many tons is this per person? How many kWh of energy is used per person per day?

Exercise *11.4:* For heating in Maine, the average ΔT is 11 degrees C. Suppose the occupants of the Davis Center (see Exercise 7.5) turned down their thermostat by one degree C.

1. How much less oil would Davis use in a year?
2. How much money would Davis save in a year?
3. How much less CO_2 would be released into the atmosphere?

12
Heating II: Sources of Heat

In the previous chapter we looked at the rate and which heat escapes a home. This is important, because the rate at which heat escapes is the rate at which you must supply heat to the house if the temperature is to remain constant. Heat leaves a structure both because hot air escapes and because heat flows through the walls and windows. The contribution from these two effects was capture in Eq. (11.10). Or, more simply, we wrote this in Eq. (11.11) as:

$$\text{Rate of heat loss} = \text{leakiness} \times \text{temperature difference} . \tag{12.1}$$

We considered some strategies for reducing the rate of heat loss smaller: make your house less leaky and turn down the thermostat. In this chapter we'll consider different ways of providing heat to a house.

12.1 A bit about Efficiency

Before we look at particular heating technologies, let's think some about efficiencies. The efficiency e of a process can be thought of as follows:

$$\text{Output} = e \times \text{Input} . \tag{12.2}$$

For example, suppose you want to get 2000 kWh out of a furnace that has an efficiency of 0.8. How much heat would you have to create in the furnace? We can use Eq. (12.2) to obtain:

$$\text{Input heat} = \frac{\text{Output heat}}{e} = \frac{2000\,\text{kWh}}{0.8} = 2500\,\text{kWh} . \tag{12.3}$$

The picture here is that in order to get 2000 kWh of heat in your house, you have to burn enough fuel to create 2500 kWh of heat in the furnace. The difference, 500 kWh, goes up the chimney and does not heat your home. When dealing with efficiencies, as we will be doing often in this chapter, it is easy mistakenly multiply when one should be dividing, or vice-versa. So treat efficiencies with a bit of care.

Another thing to be mindful of is that in this chapter we will be dealing with multi-step processes. For example, we might want to analyze converting natural gas to electricity (in a generator) and then converting this electricity to heat (via any of a number of technologies). We'll have to think about the the efficiencies of *both* processes in the chain. When working with multiple efficiencies, it is helpful to keep in mind that to specify the efficiency of a process, we need to know both the initial and final stages of the process. For example, it makes sense to speak of the efficiency of converting coal into electricity. Simply talking about the efficiency of coal can be ambiguous.

12.2 *Traditional Electric Heaters*

Electric heaters take several forms, but all operate on the same basic principle. An electric current is sent through a material with high electrical resistance—something that doesn't conduct electricity very well. The resulting "friction" that occurs when electrons are pushed through this material making frequent collisions with the material's molecules, creates heat which then warms up the house. So an electric heater is basically just a resistor. It's a simple, inexpensive technology. As we'll see, however, they are not great from an economic or CO_2 perspective.

Figure 12.1: An electric space heater for sale at Home Depot. (Figure source: http://www.homedepot.com/p/Lasko-23-in-1500-Watt-Electric-Portable-Ceramic-100669059Anelectricheater.)

As noted above, there are several different forms of electric heat commonly found in houses. Many houses have electric baseboard heat. These are long, rectangular protuberances running along a wall a few inches above the floor. An example of such a heater is shown in Fig. 12.2. This heater is six feet long and provides 1500 Watts of power. In May of 2016 they were for sale at Home Depot for $54.98. A heater like this would need to be hard-wired into the house's electric system. It requires a 240 volt circuit instead of the North American standard of 120 volts.

Figure 12.2: An electric baseboard heater sold at Home Depot. (Image source http://www.homedepot.com/p/Cadet-72-in-1-500-Watt-240-Volt-Electric-Baseboa100080894, accessed May 16, 2016.)

Electric space heaters are quite common and come in a variety of shapes and sizes. One spaceheater is shown in Fig. 12.1. This space heater is also 1500 Watts. It plugs into a standard all socket. This heater comes with a remote control, saving on the bother of getting up, walking across the room, and pressing a button if one is too cold or too hot. This heater costs $49.99 at Home Depot in May of 2016. Electric heaters are cheap.

What is the efficiency of an electric heater? One hundred percent! All of the electrical energy the flows through the circuit is converted to heat. When converting from a high-quality form of energy like electricity to a low-quality form like heat, 100 percent efficiency is possible. This seems incredible—what could be better than 100 percent? We'll see, however, that there are technologies that are more than 100 percent efficient at turning electricity into heat.

By the way, an old-fashioned incandescent light bulb is essentially a heater. In these bulbs, current is pushed through a filament with a high resistance. The filament warms up and get so hot that it begins to glow, emitting light. Most of the energy goes into heat; only a small fraction of the energy is turned into light. And this light eventually becomes heat once it hits a surface and is absorbed.

In any event, electric heaters are 100% efficient. But what about the efficiency of taking natural gas and converting it into electricity? MacKay (2009, pp. 150–1) uses 53% for the state of the art natural gas plant and then discounts this by 8% to account for grid losses, arriving at 49%[1]. To keep the calculations simple, I'll round this up to 50%.

[1] $0.53 \times 0.92 \approx 0.49$

So conclusion to draw here is that 100 kWh of chemical energy in natural gas can be turned into 50 kWh of electrical energy in a modern power plant. This 50 kWh can then be turned into 50 kWh of heat in your house, since electric heaters are 100% efficient.

12.3 *Fossil-Fuel Furnaces*

Modern natural gas furnaces can provide hot air at efficiencies between 80 and 95%. The furnace shown in Fig. 12.3 is known as the Kelvinator KG7SM and has an efficiency of 95%. It can deliver up to 118,000 BTU/hour. In the fall of 2017 you could purchase it from home depot for $1,88.57.

Figure 12.3: A gas furnace for sale at home depot. (Figure source: http://www.homedepot.com/p/Kelvinator-95-AFUE-118-000-BTU-Downflow-Resident 206511962.)

Older boilers will have an efficiency a good bit less than 95%—perhaps 80% is typical. These are nameplate efficiencies, however, so in practice it seems likely that the efficiency is a bit less, since the furnace might not be operating optimally. Nevertheless, we'll use 80% as a guess at the average efficiency of a natural gas furnace, understanding that some old ones will be much less efficient, while newer models will have efficiencies as high as 95%.

So suppose again that we have 100 kWh of chemical energy in natural gas and we want to heat our house. If we used that chemical energy in a furnace in our house, we could get 80 kWh of heat. Note that this is a lot better than turning the gas into electricity and then using that electricity in an electric heater. Doing this only got us 50 kWh of heat.

12.4 *Combined Heat and Power (CHP)*

TODO! This should go somewhere else, I think. This chapter is going to get large. Yes. But where? I'm worried about finding places for bigger/systemic things.

12.5 *Heat Pumps*

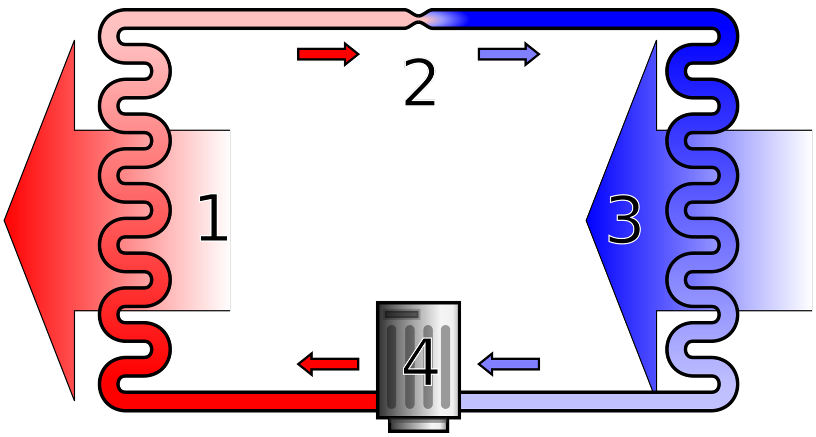


Figure 12.4: A schematic diagram of a heat pump. Image source: https://en.wikipedia.org/wiki/File:Heatpump2.svg. Image released into the public domain by its author, Ilmari Karonen.

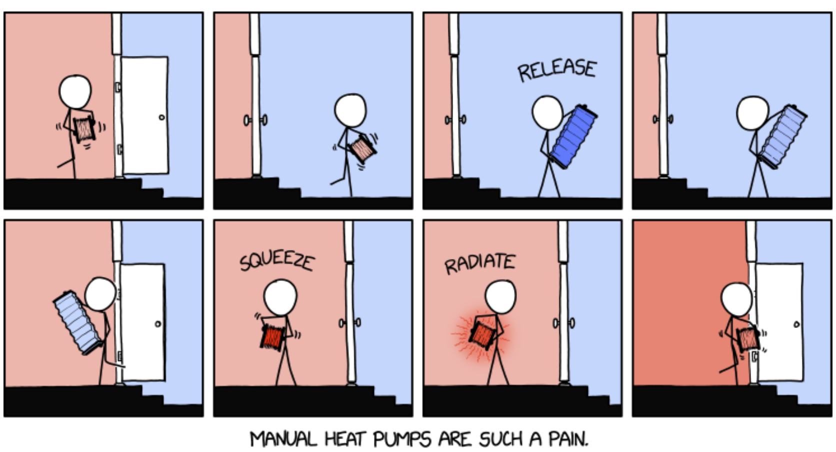


Figure 12.5: "Manual Heat Pumps Are Such A Pain", xkcd cartoon 2780 by Randall Monroe. Image source: https://xkcd.com/2790. Image is licenxed under a Creative Commons Attribution-NonCommercial 2.5 License: https://creativecommons.org/licenses/by-nc/2.5/.

The Efficiency for a heat pumps is called the *coefficient of performance* or COP. COPs can be greater than 1. Typical COPs for modern heat pumps are around 4. This means that to get 40 kWh of heat in your house would require only 10 kWh of electricity. So this is a much better

Figure 12.6: The interior portion of a heat pump in my living room. The heat pump is the rectangular contraption near the ceiling in between the two windows. Photograph by the author.

Figure 12.7: The compressor of my heat pump. Photograph by the author.

thing to do with electricity than generating heat by running it through an electric heater.

COPs depend on the temperature, however. The colder it is outside the harder your heat pump has to work to pump heat inside, and thus its efficiency drops.

Heat pumps are "future proof" in that they run on electricity, and so will work for any method of generating electricity.

12.6 Wood

- Is heating with wood carbon-free? Definitely not. Carbon is released due to cutting and transport.
- Is wood low-carbon? Yes. Although how low is in some dispute. The main thing is that burning wood doesn't add any new carbon to the atmospheric cycle the way burning fossil fuels does.
- Can wood be scaled up to heat the world? No.
- Does wood heating made sense for some regions? Almost definitely.
- Efficiencies for woodstoves seem to range from 65% to 85%. Efficiencies for pellet stoves are higher.
- Burning wood in woodstovers may lead to problematic particulate matter. But I suspect this is less of an issue with modern, efficient woodstoves. I think that particulate matter polution is not much of an issue with pellets, since they are burned at a higher temperature and so combustion tends to be more complete. **TODO!** get some references for this.

See `https://www.gov.uk/government/publications/an-assessment-of-the-carbon-impacts-of-using-different` and `https://www.gov.uk/government/publications/life-cycle-impacts-of-biomass-electricity-in-2020`

12.7 Some Stuff about Natural Gas*

Some of the details around natural gas are a little involved. If you're going to be working with natural gas, these details are probably important. But if you're not going to work with natural gas you can certainly skip this chapter.

Natural gas is a mixture of gases—mostly ethane, propane, and butane—and some impurities, such as carbon dioxide or nitrogen. This means that the calorific value of natural gas varies, since not all natural gas has the exact same mixture of gases.

Like other fuels, when burned, natural gas produces thermal energy—heat. This thermal energy is typically measured in a unit called a *therm*.

One therm equals 100,000 BTU or 29.3 kWh. Energy is energy. There's no good reason why thermal energy from natural gas is often measured with a different unit than other types of energy. But as we've seen, energy units don't always make sense.

Natural gas is a gas[2], and quantities of it are typically measured in terms of its volume at a standard temperature and pressure. Liters, gallons, and cubic meters are all unites of volume. But in the US the usual way that volumes of natural gas are measured is in units of 100 cubic feet. This is abbreviated *CCF* for centum cubic feet:

[2] In contrast, gasoline, often simply called gas, is actually a liquid.

$$CCF = 100\,\mathrm{ft}^3 \,. \tag{12.4}$$

Cenutum is 100 in Latin.

It just so happens that if one burns 1 CCF of natural gas one gets almost exactly 1 therm of thermal energy. However, this relationship isn't exact. It needs to be corrected with a conversion factor known as the *therm factor*.

$$\text{Therma energy from one CCF of natural gas} = \text{Therm factor} \times 100{,}000\,\text{BTU} \,. \tag{12.5}$$

The therm factor has units of Therms/CCF. The therm factor is also sometimes called the BTU factor.

The therm factor is a number that is very close to one. What number is it? Well, it varies depending on the exact makeup of the natural gas. Natural gas suppliers in the US typically measure the therm factor for their gas supply weekly or monthly. Customers are charged per therm, not per CCF. The gas utility figures out how much natural gas you have used, measured in CCF, and then convert this to therms using the therm factor. An example will make this clearer.

Example 12.1. *Suppose in one month you use 48 CCF of natural gas. The cost of natural gas is $0.479 per term, and the therm factor is 1.015. How much will you pay for this gas?*

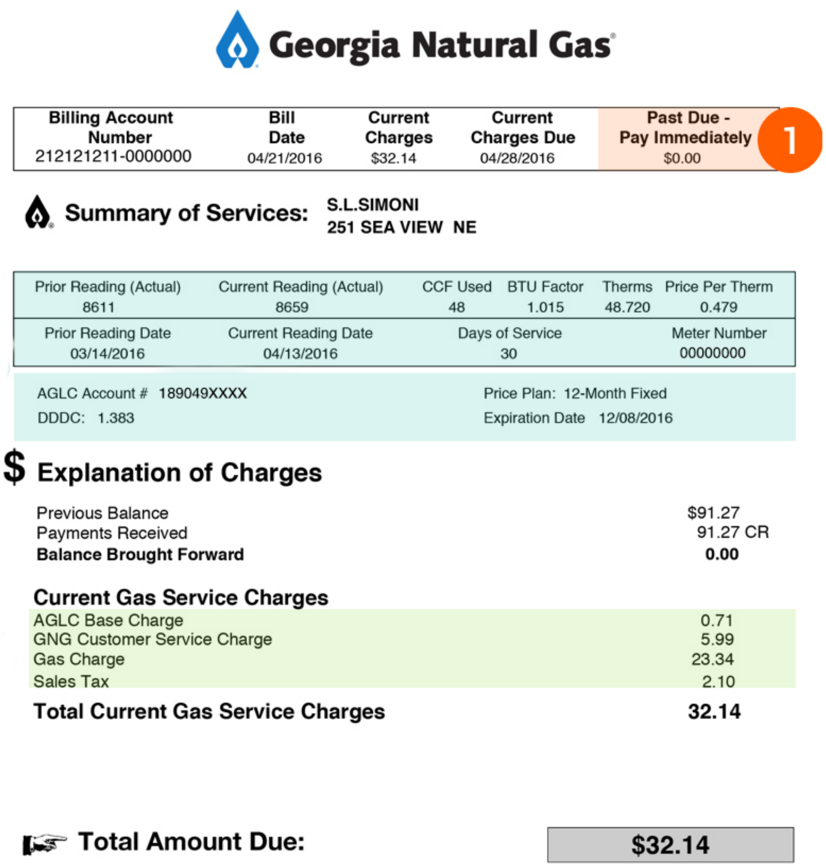
We start with 48 CCF and convert to therms:

$$48\,\text{CCF} = 48\,\text{CCF}\left(\frac{1.015\,\text{Therm}}{\text{CCF}}\right) = 48.72\,\text{Therm} \,. \tag{12.6}$$

We then convert from therms to dollars:

$$48.72\,\text{Therm} = 48.72\,\text{Therm}\left(\frac{\$0.479}{\text{Therm}}\right) \approx \$23.34 \,. \tag{12.7}$$

The above example was taken from a sample gas bill provided by the Georgia Natural Gas company, as shown in Fig. 12.8. Note that this gas bill shows a gas charge of $23.34, as we calculated. Added to this are additional charges. There is a base charge of $0.71 that is charged to every customer, regardless of how much gas is used. This is the "AGLC Base Charge" in Fig. 12.8. AGLC is the Atlantic Gas Light Company. They own the systems of pipes that delivers the gas to customers. The Georgia Natural Gas company also charges a monthly customer service charge of $5.99 per month. On top of this there is a seven percent sales tax.


Georgia Natural Gas

Billing Account Number	Bill Date	Current Charges	Current Charges Due	Past Due - Pay Immediately
212121211-0000000	04/21/2016	$32.14	04/28/2016	$0.00

Summary of Services: S.L.SIMONI
251 SEA VIEW NE

Prior Reading (Actual)	Current Reading (Actual)	CCF Used	BTU Factor	Therms	Price Per Therm
8611	8659	48	1.015	48.720	0.479

Prior Reading Date	Current Reading Date	Days of Service	Meter Number
03/14/2016	04/13/2016	30	00000000

AGLC Account # 189049XXXX
DDDC: 1.383
Price Plan: 12-Month Fixed
Expiration Date 12/08/2016

$ Explanation of Charges

Previous Balance	$91.27
Payments Received	91.27 CR
Balance Brought Forward	**0.00**

Current Gas Service Charges

AGLC Base Charge	0.71
GNG Customer Service Charge	5.99
Gas Charge	23.34
Sales Tax	2.10
Total Current Gas Service Charges	**32.14**

Total Amount Due: **$32.14**

Figure 12.8: A sample natural gas bill from Georgia Natural Gas. Image from `https://gng.com/my-gng/understand-bill`, accessed October 12, 2020.

12.8 Air Conditioning

Does this need to be its own chapter? Maybe. Let's see how long this gets with just heating.

12.9 Exercises

Exercise *12.1:* The Kelvinator furnace shown in Fig. 12.3 produces 118000 BTUs per hour. Convert this power to kW.

Exercise *12.2:* Your house needs 75 MMBTU of heat in order to stay warm throughout the entire winter. You have a furnace that burns heating oil. The efficiency of your furnace is 80%.

1. How many gallons of fuel will you need to burn over the winter?
2. How many tons of carbon dioxide will be released into the atmosphere as a result of burning this fuel?

Exercise *12.3:* You have a heat pump with a COP of 3.65. You would like to use your heat pump to get 1100 kWh of thermal energy. How much electrical energy would you need to use to do so?

Figure 12.9: My Jøtul wood stove and three cats. Photograph by the author.

Exercise *12.4:* The primary source of heat in my house is a Jøtul 500 Oslo woodstove. In a typical winter, I burn 4 cords of wood. The efficiency of the stove is around 74%.[3] The actual efficiency is surely a bit below this, since the stove isn't operated in the ideal temperature ranges all the time. Let's assume that the actual efficiency is 70%. To answer these questions you will need to use some of the data in Appendix A.

[3] http://jotul.com/us/products/wood-stoves/jotul-f-500-oslo--50528#technical-area

1. How much does this wood cost?
2. How much thermal energy is released by burning 4 cords of wood? We use well-dried hardwood.
3. How much of this thermal energy goes into heating my

house?

4. How much fuel oil would I need to burn to get the same amount of thermal energy in my house? Assume that the efficiency of the oil furnace would be 80%.

5. How much would this fuel oil cost?

6. How much CO_2 would be emitted by burning this much fuel oil? Put this number in perspective. Is this a little or a lot?

Figure 12.10: Cottage House at College of the Atlantic. Image source: College of the Atlantic.

The next several problems concern Cottage House, a single-family home that is now used as a small dormitory at College of the Atlantic. We will use Cottage as a case study for residential heating. Here are some facts and assumptions.

- The average fuel consumption of Cottage House from 2010 to 2014 was 875 gallons. However, the house is unoccupied during December, which is a cold month that would require a lot of heating. So let's say that the average fuel use is 950, so that its consumption is closer to that of a "normal" house and not a dorm.[4]
- I believe that up to six people live in Cottage. But let's say that the occupancy is four, which again makes it closer to a "normal" house and not a dorm.
- The furnace in Cottage is a Milwaukee Thermoflo purchased in 1997. Its nameplate efficiency is 83%. Let's assume that because it is old it isn't operating at full efficiency. Use an efficiency of 80%.
- The average number of degree days per year was 7211 (using a base of 65 degrees). This means that the average difference between

[4] The State of Maine claims that an average, well-insulated, home of 1500 square feet will use approximately 540 gallons of fuel oil per year. http://www.maine.gov/energy/fuel_prices/heating-calculator.php. Cottage is approximately 960 ft^2.

the inside and outside temperature was a ΔT of around 20 degrees Fahrenheit, assuming an inside temperature of 65 degrees Fahrenheit.

Other useful information can be found in Appendix A.

Exercise *12.5:* **Oil Heat:**

1. How much does it cost per year to heat Cottage per year, assuming a cost of $2.20/gallon for heating oil?
2. How much thermal energy from the heating oil is released when Cottage burns 950 gallons of fuel? Answer in kWh and both kWh per day per person. Is this a little or a lot?
3. How much CO_2 is released into the atmosphere as a result of burning the 950 gallons of fuel oil needed to heat Cottage? Answer in tons of CO_2 per year per person. Is this a little or a lot?
4. Given the furnace's 80% efficiency, how much heat (in kWh) was delivered to the inside of Cottage? Answer in kWh per year and kWh per person per day. This quantity is the *heating load* of Cottage—the amount of heat we need to add to Cottage so it is a comfortable temperature.

Exercise *12.6:* **Resistive Electric Heating.** Suppose that you wanted to generate the heat for Cottage using traditional electric resistive heating.

1. How much would this cost?
2. How much CO_2 would be released into the atmosphere? Assume that we're using average US electricity, that released about 600 grams per kWh of electricity generated. Express your answer in tonnes of CO_2 per person per year.

Exercise *12.7:* **Heat Pumps.** Suppose that you want to generate heat for Cottage by using an electric heat pump with a COP of 4.

1. How much electricity would you need to use in one year to meet the heating load of Cottage? Express your answer in kWh and kWh per person per day.
2. How much would this electricity cost?
3. How much CO_2 would be released into the atmosphere as a result of generating this electricity?

Exercise *12.8:* Comment briefly on the three options you analyzed in the three previous problems: Using an oil furnace, using traditional resistive heating, and using an electric heat pump. Which is the best financially? Which produces the least CO_2?

Exercise *12.9:* In one month you use 70 CCF of natural gas and burning it produced 68.5 therms of thermal energy. What is the therm factor for this natural gas?

Exercise *12.10:* In one week you use 40 CCF of natural gas. The therm factor for this gas supply is 1.084, and the cost of the gas is $0.519/therm. How much will you pay for this gas?

13
Hot Water and Solar Thermal

13.1 Hot Water

Estimate for daily hot water use are all over the place.

13.2 Solar Thermal Hot Water

- Some argue that it is cheaper to heat water with an electric heater powered by solar thermal: `http://www.greenbuildingadvisor.com/blogs/dept/musings/solar-thermal-really-really-dead`. On its face, this doesn't quite seem right, since thermal energy is of such a lower quality than electric energy. But economics and costs don't always make sense.

- I think that a lot depends on whether or not the system is being retrofitted into an existing house, which is expensive and difficult, or if it is built into a new home.

- I've seen installation costs estimated between $8000 and $10,000. I think this is for a retrofit.

- If this was part of a 30-year mortage, it would be a very small addition to your monthly payment.

- We make it easy (via car and home loans) to pay for some things we can't afford. How can we do this for other things, like solar panels and solar thermal?

Example 13.1. *Statement of example*

Solution goes here

13.3 *Exercises*

Exercise *13.1:* A questions

1. part a
2. part b

14 The Energy of Making Things

14.1 Life Cycle Assessment

Let's break down the lifecycle of something into four phases.

1. **Raw materials.** The energy to get materials out of the ground and convert them to a useful form so we can use them as ingredients to make something.

2. **Production.** The energy required to build/make/assemble the thing.

3. **Use.** The energy associated with using a thing. For a car, which needs to be fed gasoline, this could be a lot. For a book, which doesn't need to be fed regularly in order to be used, this would be almost nothing.

4. **Disposal.** How much energy does it take to get rid of the thing? This could be a lot, if we recycle or need to process or dispose of the thing carefully. In a few cases it could be negative; if we get rid of a book by burning it, we could get a bit of energy back.

The total energy costs associated with all four phases of making an object is known as the *embedded energy* of that object. For example, the stainless steel bottle on the right weighs 0.32 pounds, or around 0.145 kilograms. LCA folks have determined that the embodied energy of stainless steel is 56.7 MJ per kg, or 15.75 kWh per kg. So, the embodied energy in the water bottle is:

$$0.145\,\mathrm{kg}\left(\frac{15.75\,\mathrm{kWh}}{1\,\mathrm{kg}}\right) \approx 2.3\,\mathrm{kWh}\,. \tag{14.1}$$

Figure 14.1: A stainless steel water bottle that will hold 18 fluid ounces of your favorite beverage. http://www.amazon.com/dp/B00SA2ZKKU/ref=twister_B00X6ZKQA0?_encoding=UTF8&psc=1

So it took around 2.3 kWh of energy to make the steel that was used to make the water bottle. The total embodied energy of the bottle would be higher, because we'd need to account for the energy used when taking the steel and converting it into something bottle-shaped. We'll also need to account for the energy needed to make the bottle purple, to make the lid, to ship all those materials around, and so on.

A few thoughts on LCA before launching into some examples.

1. LCA is limited by the quality of available data.

2. LCA give a number that represents an average. Thus, LCA ignores potentially large differences in the way that things are made. Of particular importance seems to be assumptions about whether or not ingredients are recycled or freshly-mined, requiring large energy costs to mine and process.

3. LCA reduces environmental impact to a single number, which may give one a false sense of simplicity.

4. On the other hand, it is surely very very important to think about the energy costs of making stuff, and LCA is probably the best place to start.

14.2 *LCA for Different Materials*

The LCAs for a handful of materials are shown in Fig. 14.1.

Material	Energy	Carbon
Stainless Steel	56.7	6.15
Steel	20.1	1.37
Polyurethane insulation (rigid foam)	101.5	3.48
Aluminum (general & incl 33% recycled)	155	8.24
Plywood	15	1.07
PVC	77.2	2.41
Iron	25	1.91
Glass	15	0.85

Table 14.1: Embodied energies and carbon for different materials. Energies are in units of MJ/kg. Carbon is in units of kg of CO_2 per kg. From the Circular ecology database, http://www.circularecology.com/embodied-energy-and-carbon-footprint-database.html, cited on https://en.wikipedia.org/wiki/Embodied_energy.

14.3 *LCA for Generating Electricity*

The results of an IPCC meta-study are shown in Table 14.2. Note the relatively large ranges for estimates of carbon intensity.

I think this should go earlier in the book, in a chapter on generating electricity.

Here is another reference for LCA emissions for electricity generation: https://www.factcheck.org/2018/03/wind-energys-carbon-footprint/. And here's another: https://www.saskwind.ca/blogbackend/2016/1/14/carbon-and-energy-payback-of-a-wind-turbine.

Table A.II.4 | Aggregated results of literature review of LCAs of GHG emissions from electricity generation technologies as displayed in Figure 9.8 (g CO_2eq/kWh).

Values	Bio-power	Solar		Geothermal Energy	Hydropower	Ocean Energy	Wind Energy	Nuclear Energy	Natural Gas	Oil	Coal
		PV	CSP								
Minimum	-633	5	7	6	0	2	2	1	290	510	675
25th percentile	360	29	14	20	3	6	8	8	422	722	877
50th percentile	18	46	22	45	4	8	12	16	469	840	1001
75th percentile	37	80	32	57	7	9	20	45	548	907	1130
Maximum	75	217	89	79	43	23	81	220	930	1170	1689
CCS min	-1368								65		98
CCS max	-594								245		396

Note: CCS = Carbon capture and storage, PV = Photovoltaic, CSP = Concentrating solar power.

Figure 14.2: The results of a meta-study of LCAs for different methods of generating electricity. From the IPCC Annex II Moomaw et al. (2011).

14.4 *LCA for Cars*

See the article by Xiaoyu Yan 2009. He assembles the results of a number of different studies that attempt to estimate the embodied energy and carbon of a car. The differences in estimations for the US are likely due to different assumptions about how much of the materials are recycled. I think Yan seems to believe the larger estimate. The large GHG for China is due to the fact that much of their energy is derived from coal. The energy or carbon cost of making a car is a fairly significant fraction of its overall climate impact.

Country	Energy	Carbon
USA	119,150	10,480
USA	76,206	5511
USA	75,300	5390
Japan	61,500	5500
China	69,108	6575

Table 14.2: Different estimates of embodied energies and carbon for automobiles. Energy is in units of MJ; carbon is in units of kilograms of CO_2e. Data cited in Yan (2009).

Somewhere, either as an example or an exercise, consider whether it is good to buy a Prius right away or wait until your conventional car dies.

14.5 *Cement*

There are a lot[1] of emissions associated with cement. There are two reasons for this. A key ingredient in cement is calcium oxide, CaO. Calcium oxide is produced by starting with a substance such as limestone or sea shells that contains a lot of calcium carbonate, $CaCO_3$. Calcium carbonate can be converted into calcium oxide via the following

[1] **TODO!** how much?

reaction:

$$CaCO_3 + \text{heat} = CaO + CO_2 \,. \tag{14.2}$$

It take a lot of thermal energy to break apart $CaCO_3$ so it forms CaO and CO_2. The calcium carbonate is heated to around 1500 C. Doing so requires burning a lot of fossil fuels, often coal, which leads to CO_2 being emitted into the atmosphere.

But also note that that carbon dioxide is a by-product of the chemical reaction shown in Eq. (14.2). This is another source of CO_2 from cement production. Roughly 40% of the emissions from cement are due to the fossil fuels burned to produce the heat necessary to make the reaction in Eq. (14.2) go, and 60% of the emissions are from the CO_2 that results from breaking down calcium carbonate. We produce a lot of cement [2], and emissions from cement are significant.

[2] An accessible overview is the article by Maddie Stone, "Cement has a carbon problem. Here are some concrete solutions." Grist.org. https://grist.org/politics/cement-has-a-carbon-problem-here-are-some-concre

See also https://medium.com/energy-impact-partners/making-concrete-a-climate-solution-7ccc360dbe65. Looks like a good overview?

14.6 References

- http://www.theguardian.com/environment/datablog/2011/apr/28/carbon-emissions-imports-exports-trade.
- http://www.pnas.org/content/107/12/5687.full.

14.7 Exercises

Exercise *14.1:* Beverage containers..

1. Calculate the embodied energy and CO_2 of a 15 gram aluminum can.
2. Calculate the embodied energy and CO_2 of a 192 gram glass bottle.

Exercise *14.2:* A 2MW wind turbine requires around 80 tons of steel.

1. How much energy would such a turbine produce every month?
2. How much CO_2 is saved by the turbine, assuming that its electricity displaces electricity generated in the U.S?
3. What is the CO_2 cost of the steel in the materials?
4. What is its carbon payback time? That is, how long does the turbine have to run before the CO_2 savings from displacing traditionally generated electricity equals the CO_2 associated with the production of the steel used to make the turbine?

This is, of course, only a small part of the carbon dioxide associated with the raw materials and production of a wind turbine.

Exercise *14.3:* The Commonwealth Scientific and Industrial Research Organisation (CSIRO) in Australia has estimated that there are around 1000 GJ of embodied energy in the materials in a new house[3].

[3] http://www.yourhome.gov.au/materials/embodied-energy

1. Convert this energy into kWh.
2. If two people live in the house and the house lasts 100 years, how much energy is this in kWh per person per day?
3. Burning how many gallons of fuel oil would yield 1000 GJ?
4. How many year's worth of heating could you get from this amount of oil? Assume that the house uses 600 gallons of oil a year for heating.

Exercise *14.4:* Mike Berners-Lee [4] cites an estimate that the carbon cost of building a new, two-bedroom house is 80 tons. Let's round this up to 100 tons.

[4] Berners-Lee, Mike. How bad are bananas?: the carbon footprint of everything. Greystone Books, 2011.

1. Assume the house lasts for 100 years. How much carbon dioxide is this per year?
2. How much fuel oil, per year, would generate the same amount of carbon dioxide? Put this number in perspective.

Exercise *14.5:* It takes the energy equivalent of 260 gallons of gasoline to make a typical car. A hybrid car like the Toyota Prius takes the energy equivalent of 325 gallons of gasoline to make. [5] For the calculations below, assume that the typical car has a gas mileage of 30 mpg and the hybrid gets 50 mpg. Assume that each car is owned for ten years and is driven 13,500 miles annually.

[5] These data are from a study by Argonne National Lab that is referred to in this article: https://www.sierraclub.org/sierra/green-life/2013/10/ask-mr-green-how-much-energy-make-new-car.

1. How much gasoline does the typical car use in its lifetime?
2. How much gasoline does the hybrid car use in its lifetime?
3. Compare the total energy use of the two cars. (By total energy use, I mean the energy needed to make the car and the energy needed to drive it.)

15 Agriculture

This chapter is just some mostly unorganized notes right now.

15.1 How much Food do People Need?

Go through basic food example, discuss dietary calories versus actual calories. kcal to MJ example? Yes, as this is often confusing.

For measuring the energy/climate impacts of agriculture, we will use our usual units: tonnes of CO_2e per person per year or kWh per person per year. But it is less clear how to measure the benefit of agriculture? Mostly we'll think in terms of the kilograms of food produced or the kilocalories of food produced. Sometimes energy/climate impacts are expressed per dollar spent or person or household. Also, sometimes the impacts are expressed per acre or hectare. This maybe doesn't make the best sense, because what we're interested in is thinking about to feed people without frying the planet.

It's also worth noting that agricultural efficiencies can be measured in different ways. In general, we can think of the efficiency e as:

$$\text{Output} = \text{Efficiency} \times \text{Input} . \tag{15.1}$$

Just as there are different ways of measuring outputs (kcal, kg of food, dollar value of food), there are different ways of measuring inputs: area of land, amount of human labor, dollar value of all inputs, energy of all inputs, and so on. This is definitely something to be mindful of.[1]

[1] Cite Levins article(s). What else?

15.2 Overall Estimates of GHG associated with Agriculture

Agriculture is responsible for GHG emissions for two reasons: energy inputs and land-use impacts (LUC). Energy inputs include fertilizer, tilling soil, moving crops and fertilizer around, etc. Land-use impacts involves changes in C and N cycles as the result of bringing land in and out of production and changing agricultural practices. LUC is a complex topic that is beyond the scope of this book. But it is significant—likely at least as significant as energy inputs. Also, mention that energy inputs are broken down into primary and secondary.

- Pimentel (cited in (Pelletier et al. (2011))) estimated that the American diet is underpinned by 2000 liters of oil equivalents per year, accounting for 19% of the total US energy use.
- In terms of primary energy inputs, it seems that organic ag uses only a little bit less (perhaps around ten percent) than conventional ag. See Table 1 in Woods et al. (2010).

15.3 Transportation Costs

Data from Weber and Matthews (2008):

Mode	Energy	Carbon
rail	0.3	18
truck	2.7	180
air	10	680
container ship	0.2	14

Table 15.1: Energy and carbon dioxide associated with different modes of transportation. Energy is in units of MJ per ton-kilometer. Carbon is in units of tonnes of CO_2e per 10^6 ton-kilometers. Data taken from Table 1 of Weber and Matthews (2008).

15.4 Estimates of GHG Associated with Different Foods

From Weber and Matthews (2008), GHG emissions for different types of food, in units of kg of CO_2e per kilogram of food.[2]

[2] I eyeballed some of these of Fig. 2 from the Weber paper. Are the actual numbers published somewhere?

- Red Meat: 22
- Chicken/Fish/Eggs: 5.7
- Cereals/Carbs: 2.9
- Fruits/Vegetables: 2.2
- Oils/Sweets/Condiments: 2.2

A totally local diet would save 0.36 tCO_2e per year per household (Weber and Matthews, 2008).

This looks like a really good review article: `https://thebreakthrough.org/index.php/issues/the-future-of-food/the-future-of-meat`.

15.5 Why is Agriculture So GHG Intensive?

This is complicated and there are different answers for different sectors. Primary inputs can be large. These are things like diesel to plow fields and pumps to pump water. But the largest is probably nitrogen fertilizer.

Nitrogen

Plants need nitrogen to grow. Nitrogen is a constituent of most proteins and enzymes, and so plants need a source of nitrogen in order to make plant matter. They get this nitrogen from the soil. And so in order to maintain or increase yield, one very often has to add nitrogen to the soil. The significant (vast?) majority of nitrogen fertilizer is made via the Haber–Bosch process. This is an industrial process that takes nitrogen from the air (N_2), and turns it into ammonia (NH_3). Nitrogen is air is quite stable, so a lot of energy has to be added to the system to break it apart and form ammonia. The HB process is usually carried out at temperatures of $400 - 500$ degrees Celsius and pressures of $150 - 250$ atmospheres. It takes a lot of energy to maintain pressures this high, and producing nitrogen in this way is quite energy intensive. This translates into a lot of CO_2e. One kilogram of nitrogen fertilizer is responsible for the release of 6.69 kg of CO_2e (cited in Woods et al. (2010), Table 5).

Some estimate[3] that worldwide around five percent of all natural gas burned is used to make nitrogen fertilizer. Eighty percent of the N fertilizer in China is produced by coal.

[3] Need citation(s).

Nitrogen is about 5 (or more?) times more GHG intensive to produce than P or K fertilizer.

Ruminants

Another big issue when thinking about GHG and agriculture is the effect of ruminants. Ruminants are mammals that have a rumen—a funny sort of pre-stomach in which they kinda ferment their food before digesting it. This fermentation processes produces methane, which is a very powerful greenhouse gas. The large greenhouse gas emissions associated with "red" meat is in large part due to the methane emissions from the animals before they are eaten. Cows are the most famous and notorious of the ruminants, but sheep, goats, and deer are also ruminants.

Some facts quoted in Ripple et al. (2014):

- 14.5% of all anthropgenic emissions are from livestock sector.
- of this 14.5%, 44% is due to enteric CH_4.
- 27% is from land-use change and fossil-fuel use
- and 29% is from N_2O from manure and fertilizer applied to feed-crops

- Total area used for grazing cattle is 26% of the terrestrial surface of the earth.
- livestock sector accounts for 70% of the global agricultural land
- And 33% of the world's total arable land is used to grow feed-crop—grain that is fed to cattle. This is land that could be used directly to feed people, or to grow plants that could be used for generating electricity or fuel.

15.6 *What is to be Done?*

List based on Garnett 2011.

- Eat less meat and dairy, especially less red meat. (high priority)
- Eat less. (high priority)
- Reduce waste. (medium/high)
- Eat seasonal food and/or foods that store well. (medium)
- Accept variability of supply. I.e., you can't eat tomatoes or mangoes in Canada all year long. (medium)

Other thoughts: eat lower on the food chain, eat less processed foods, eat in season. We need to expand our notion of local further down the food-processing chain. That is, not only should be farm from which food comes be close to us, but the plant that provides materials and machines to the farm should be close to the farm, and so on. But even then, we should bare in mind that transport, in toto, is still a small part of the overall energy/GHG cost of agriculture.

15.7 *Misc References*

Some review papers Pelletier et al. (2011); Garnett (2011); Pimentel et al. (2008); Woods et al. (2010). See also the Eshel, et al paper in PNAS `http://www.pnas.org/content/111/33/11996`.

See also the "Beans for Beef paper" Harwatt et al. (2017). And Nijdam et al. (2012). `https://link.springer.com/article/10.1007%2Fs10584-017-1969-1`: "Substituting beans for beef as a contribution toward US climate change targets."

See the fun article, "LED salad and Jevon's paradox" by David Keith[4] Looks like some great info and a fun sample calculation about lettuce.

Perhaps see also "What's the Climate Ipact of Your Diet?" and references therein. `https://makewealthhistory.org/2016/05/06/whats-the-climate-impact-of-your-diet/`.

Also `http://www.pnas.org/content/113/15/4146.full`. "Analysis and valuation of the health and climate change cobenefits of dietary change." from PNAS.

[4] `http://www.keith.seas.harvard.edu/blog/led-salad-and-jevons-paradox`.

15.8 Exercises

Exercise *15.1:* The cost of shipping

1. How much energy does it take to ship 3 metric tons of corn from Iowa to Bar Harbor via truck? How many tons of carbon dioxide does this emit?
2. How much energy does it take to ship 3 metric tons of potatoes from Belfast, Ireland to Boston via container ship? How many tons of carbon dioxide does this emit?
3. How much energy does it take to ship 3 metric tons of bananas from Ecuador to Boston via plane? How many tons of carbon dioxide does this emit?

Exercise *15.2:* Suppose you live in Boston and are interested in buying 20 kg of canned tomatoes.

1. One option would be to buy imported tomatoes from Italy. What is the CO_2e associated with shipping the tomaotes from Italy to Boston via container ship?
2. Another option would be to buy canned tomatoes shipped to Boston via truck. How far from Boston would these tomatoes have to be so that driving them to Boston would be responsible for the same amount of CO_2e as shipping the tomatoes from Italy?

Exercise *15.3:* Swapping red meat for beans and rice...

1. The total beef consumption in the US in 2015 was 24.8 billion pounds.[5] Convert this to kilograms of beef per person per day, and kilograms of beef per person per year.
2. What is the per person CO_2e assoiated with this rate of beef consuption? Answer in tons per person per year. Put this number in perspective.
3. Suppose you replace this beef consumption with beans. How much CO_2 have you prevented from being emitted in one year? Put this number in perspective.
4. How much driving in an average car would emit an amount of CO_2 equivalent to the amount of CO_2e saved by this dietary switch?

[5] https://www.ers.usda.gov/topics/animal-products/cattle-beef/statistics-information.aspx, accessed October 30, 2017.

For more on this idea, see Harwatt et al. (2017)

Exercise *15.4:* David Pimentel and collaborators 2008 in a paper in the journal *Human Ecology* have estimated that the typical American requires the equivalent of 2000 liters of oil to supply their food.

1. How much energy is this in kWh/day?
2. How much CO_2 is this in tonnes per year?

Exercise *15.5:* Suppose you are going to ship 3 tons of corn from Iowa to Boston via truck.

1. How much energy does it take to transport the 3 tons of corn from Iowa to Boston?
2. What are the emissions associated with transporting 3 tons of corn from Iowa to Boston?
3. What are the total lifecycle emissions associated with 3 tons of corn?
4. Transport costs are what fraction of the total corn emissions?

To answer these questions, use the information in Table 1 and Figure 2 from the Weber and Matthews paper.

Exercise *15.6:* Repeat Question 15.5, but for three tons of beef instead of three tons of corn.

Exercise *15.7:* Repeat Question 15.5, but for three tons of mangos shipped from Quito, Ecuador to New York City, via plane.

Exercise *15.8:* Repeat Question 15.5, but for three tons of pasta shipped from the port of Gioia Tauro, Italy to New York City, via ship.

16
Computing and the Internet

16.1 What is the Internet?

16.2 How much Energy is used by Computing?

16.3 Bitcoin and Cryptocurrencies

more blah blah.

https://www.nytimes.com/2023/04/09/business/bitcoin-mining-electricity-pollution.html

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Example 16.1. *Statement of example*

Solution goes here

16.4 Exercises

Exercise *16.1:* A questions

1. part a
2. part b

17
Consuming Less?

blah blah

17.1 Rebounds

You buy a Prius, which is more efficient than your previous car.

1. You use it more, because it uses less gas.
2. You use less gas, so you save money, spending it instead on other stuff. And emissions are associated with that stuff.
3. You use less gas, the demand for gas falls, the price for gas falls, other people use more gas because it is cheaper.

The Rebound Effect or Jevon's paradox limits the benefits of conservation.

Considerable disagreement as to how large this effect is and when/how it applies.

But again and again, consumption (and hence GHG emissions) grow in spite of efficiency gains.So efficiency gains are not straightforward.

See Gillingham, Kenneth, David Rapson, and Gernot Wagner. "The rebound effect and energy efficiency policy." *Review of Environmental Economics and Policy*. 10.1 (2016): 68-88.

Example 17.1. *Statement of example*

Solution goes here

17.2 Exercises

Exercise *17.1:* A question

1. part a

2. part b

Part IV

Generating Electricity

18
Windpower

Figure 18.1: Wind turbines in Holderness, UK. (Image course: Tom Corses (www.tomcorser.com), Licensed under the Creative Commons Attribution-Share Alike 2.5 Generic license, https://creativecommons.org/licenses/by-sa/2.5/deed.en.

Add a paragraph or two about how wind works by turning a turbine which generates electricity.

18.1 How Much Power is Blowing in the Wind?

How much power is in the wind? A little or a lot? With what efficiency can we extract that power and generate electricity?

Let's picture a wind turbine with diameter D. Air is flowing past the wind turbine with a velocity of v. Let us denote by A the frontal area of the turbine. In a time t how much air moves by the turbine?

$$\text{volume of air in time t} = Avt\ . \tag{18.1}$$

What is the mass of this air? Let ρ denote the density of air.[1] Then the

[1] The density of air is about 1.3 kg per m^3. It varies slightly depending on temperature and humidity.

mass is given by:

$$\text{mass of air in time t} = \rho A v t \,. \tag{18.2}$$

The kinetic energy of this air is given by the usual kinetic energy formula, $E = \frac{1}{2}mv^2$, as discussed in Section 3.3. Thus,

$$\text{kinetic energy of air in time t} = \frac{1}{2}mv^2 = \frac{1}{2}(\rho A v t)v^2 \,. \tag{18.3}$$

We are ultimately interested in power, not energy. To get power, we divide by time, since $P = E/t$. This gives

$$\text{power in wind} = \frac{\frac{1}{2}(\rho A v t)v^2}{t} \,. \tag{18.4}$$

Simplifying, this becomes:

$$P = \frac{1}{2}\rho A v^3 \,. \tag{18.5}$$

This formula tells us the power in wind of speed v flowing into a wind turbine of area A; the density of air is given by ρ.

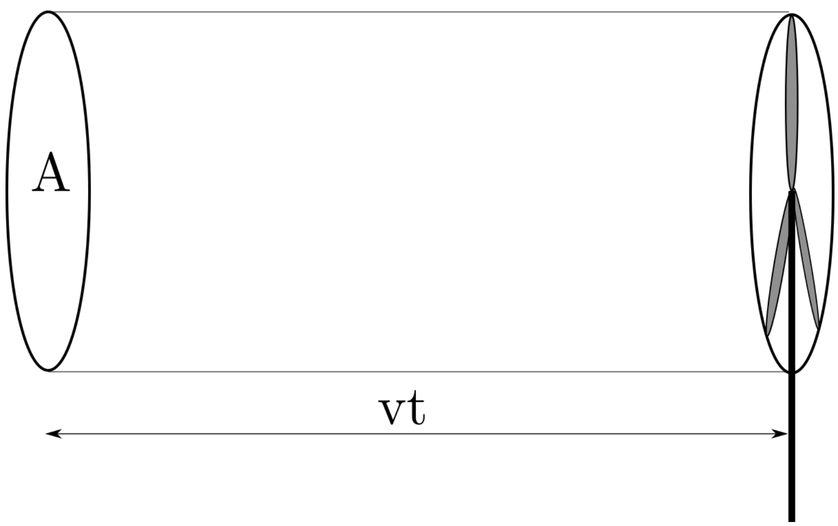

Figure 18.2: A wind turbine with an area of A. In time t the wind in the cylinder flows past the wind turbine. The volume of this cylinder is Avt.

Let's do one more bit of algebra. I'd like to express the area A of the turbine in terms of the diameter d of the blades.[2] The area of a circle is given by πr^2. And the the radius is half the diameter; $d = r/2$. Thus,

$$\text{Area of circle} = \pi r^2 = \pi(\frac{d}{2})^2 = \frac{\pi r^2}{4} \,. \tag{18.6}$$

Putting it all together, we arrive at the following equation:

$$P = \frac{1}{2}\rho\frac{\pi r^2}{4}v^3 = \frac{\pi\rho d^2 v^3}{8} \,. \tag{18.7}$$

This is an expression for the power in wind of speed v that hits a wind turbine that has a diameter of d.

[2] It seems to me like the radius would be more natural to use here, but stating the diameter is a more common way of specifying turbine size. Perhaps this is because the diameter is larger than the radius, and wind turbine companies like big numbers.

18.2 How Much of that Power can we Get?

18.2.1 Betz Limit

How much of the power in the wind can the turbine convert into electrical energy? Surely not all of it, for if that was the case—i.e. if all the kinetic energy of the wind was converted to electricity—then the speed of the wind would drop to zero immediately behind the turbine. But this is impossible. If the wind had zero velocity, the still air would accumulate behind the turbine, blocking the wind. As a result, there would be no wind and the turbine would stop. So it is definitely not possible to convert all of the energy in the wind into electrical energy.

It turns out that the best one can do is to extract 16/27, or around 59% of the kinetic energy in the wind. This result is known as the *Betz limit* or Betz's law. This law follows from basic physics definitions of work, energy, force, and power, as well as conservation of mass.[3,4] So a perfectly engineered turbine could convert at most fifty-nine percent of the power in the wind, given by Eq. (18.7), into electrical power. In practice, turbines might be able to turn 40–50 percent of the wind's power into electrical power. For simplicity's sake, we'll assume, a bit optimistically, that 50 % of the power in wind is converted to electricity.

[3] In a flowing fluid, mass is conserved—for every small region of space, whatever mass flows into that region must also flow out. This type of relationship is often referred to as a *continuity equation*.

[4] A derivation of the Betz result can be found on Wikipedia: `https://en.wikipedia.org/wiki/Betz's_law`. Find a better citation. Perhaps Erlich's new physics of energy book?

With this assumption, Eq. (18.7) becomes:

$$P = \frac{1}{2}\frac{\pi\rho d^2 v^3}{8} \, . \tag{18.8}$$

This expression is the power delivered by a turbine of diameter d in a location with average windspeed v, whereas Eq. (18.7) is the power in the wind that hits the turbine.

Let's step back and think about what this tells us. The power delivered by a turbine depends on three things:

- Power is proportional to the density of the air.
- Power is proportional to the square of the diameter.
- Power is proportional to the cube of the wind speed.

The fact that power is proportional to v^3 is super important, as the following example shows.

Example 18.1. *Suppose a certain wind turbine produces a certain amount of energy per year. What would happen to the yearly energy produced by the turbine if it was moved to some place where it was* 30 *percent windier? What would happen to the energy output if the diameter of the turbine blades was doubled?*

If v increases by 30%, then we replace v by $1.3v$. Since the power from the wind turbine is proportional to the cube of the wind speed, $P = kv^3$, then an increase of 30 % has the following effect:

$$v \to 1.3v\,, \text{ so } P \to k(1.30v)^3 \approx 2.20kv^3 = 2.20P\,. \tag{18.9}$$

In other words, a 30% increase in windspeed leads to an increase in energy production of around 120%; the energy production more than doubles! For a discussion and some examples of working with proportionalities, see Appendix E.

The power produced by a turbine is proportional to the square of the diameter: $P = kd^2$. So if the diameter doubles, we replace d by $2d$. Thus

$$d \to 2d\,, \text{ so } \; P \to k(2d)^2 \approx 4kd^2 = 4P\,. \tag{18.10}$$

So, a turbine that is twice as large (as measured by blade diameter) will produce four times as much energy.

18.2.2 Capacity Factors

Turbines have a maximum rating. This is the maximum amount of power they can produce under ideal conditions. This is often called called *nameplate capacity*. It is also known as the *rated capacity*, *nominal capacity*, *installed capacity*, or *maximum effect*.[5] Note that "load factor" is the term used by MacKay 2009.

[5] https://en.wikipedia.org/wiki/Nameplate_capacity

The *capacity factor* refers to the actual average power delivered by the turbine. For example, suppose a turbine with a nameplate capacity of 10 MW produced an average power over a year of 2.5 MW. Then the capacity factor would be

$$\text{capacity factor} = \frac{\text{actual average power}}{\text{nameplate capacity}} = \frac{2.5\,\text{MW}}{10\,\text{MW}} = 0.25\,. \tag{18.11}$$

One could also do this calculation in terms of energy. In this case, we would take the actual energy generated in a year and divide that by the maximum possible energy that could be generated in a year—the energy that would be generated if the turbine was operating at full capacity the entire year. But, as always, take care to not mix up energy and power. In the ratio in Eq. (18.11), either both terms should be energy, or both terms should be power. The capacity factor is a ratio; it is a unitless quantity.

A typical capacity factor might be around 1/3, although this can vary. Capacity factors tend to be larger for newer and larger turbines.[6] Capacity factors also tend to be larger for off-shore installations.

[6] http://cleantechnica.com/2015/08/04/wind-could-replace-coal-as-us-primary-generation

When the power of a turbine, or any other generating system, is stated without qualification—i.e., a 100 MW solar installation or a 2 GW coal plant—these powers are the nameplate power.

18.3 Where is it Windy?

The wind is stronger higher up. How exactly wind speed depends on height can be a bit complicated.[7] MacKay writes, "as a ballpark figure, doubling the height typically increases wind-speed by 10% and thus increases the power of the wind by 30%. (MacKay, 2009, p. 266)"

[7] Add an experimental graph? **TODO!**

From Ehrlich (2013, Section 7.2.3): Let v_{10} be the windspeed at a height of 10 meters. Then the speed v_h at a height h is given by:

$$v_h = v_{10} \left(\frac{h}{10} \right)^{\alpha} , \tag{18.12}$$

where α is known as the Hellmann exponent. This is an empirical relationship. Different terrains will have different values of α. Ehrlich says that for a flat, unimpeded terrain the value of $\alpha = 0.14$ is often used. In less flat areas where there are house, hills, or trees, αs in the range of 0.3–0.6 are used. **TODO!** come up with an example? I'm not sure how far down this rabbit hole I want to go. I should probably at least mention the Hellmann equation but I don't think I want to do a lot with it.

Additionally, wind is stronger whereever it is unimpeded. This fact is not surprising.

- Ridge-tops are often good.
- Open plains are often good.
- Offshore is often good.
- Taller turbines are better.

18.4 Wind Farms

It turns out that it is best, according to MacKay, to use a spacing of $5d$ between turbines. So far as I know, there is no obvious or simple argument in suggesting why this spacing is best. It is something that was arrived at empirically via trial and error. At issue is the following tradeoff. On the one hand, it is good to have a lot of wind turbines, since each turbine generates electricity. However, the wind speed immediately behind a turbine is slower than the wind speed in front of the turbine. The turbine takes kinetic energy out of the wind. After doing so, the kinetic energy, and hence the speed, of the wind is lower. So if you put turbines too close together, the downwind one will be operating in lower-speed wind and hence will generate less electricity—it would be like putting a solar panel in the shadow of another panel.[8]

In any event, a spacing of $5d$ between turbines leads to:

$$\frac{\text{power per turbine}}{\text{area needed for each turbine}} = \frac{\frac{1}{2}\rho \frac{\pi r^2}{4} v^3}{(5d)^2} . \tag{18.13}$$

This simplifies to:

$$\text{wind farm power density} = \frac{\pi}{200} \frac{1}{2} \rho v^3 . \tag{18.14}$$

[8] **TODO!** Clearly a diagram and a couple of photos are in order here. MacKay has written some about this on his blog. It may be that the spacing of $5d$ that he originally cited is too small.

Note, again, the v^3 dependence. Note also that this doesn't depend on the diameter d. For windspeeds of 6 m/s, the above expression evaluates to a power density of around 2.2 W/m^2.

18.5 Other Issues with Wind

18.5.1 Aesthetics

Some people don't like how wind turbines look. This is a matter of opinion, but they certainly do alter the landscape. But lots of things alter the landscape. For example, agriculture has radically changed the landscape. There are a lot less forests and prairies on the planet than there used to be. This is the price we pay so we can feed the earth's human population. And we think that farms aren't necessarily ugly. Farms can be viewed as enterprises that are essential for providing life for us. So it is with wind turbines. Maybe we can see them as peaceful, life-giving machines.[9]

[9] Add references to Kennedys and others opposing offshore wind turbines because they are visible from shore? Also mention how some people call them farms and others call them industrial wind.

18.5.2 Birds

Do wind turbines kill birds. Yes. Unarguably. The question is not whether or not turbines kill birds, but how many birds are killed. What is the right way to think about this? Let's first talk about a wrong way to contemplate bird deaths.

It is commonplace to compare turbine-induced bird mortalities with the number of birds estimated to be killed every year by housecats. Typically these estimates indicate that cats are a much greater threat to birds than windpower. This is perhaps amusing, but is not a helpful analysis. For one, if we build more and more turbines, then the number of birds lost to turbines will increase. Are we to conclude from the cat/wind analysis that wind turbines are ok until they kill as many birds as cats, and then wind turbines are bad? This makes no sense.

Figure 18.3: A cat eating a bird on a lawn. (Image source: dr_relling posted on Wikipedia https://commons.wikimedia.org/wiki/File:Domestic_cat_eating_bird_on_lawn-8.jpg, licensed under Creative Commons Attribution-Share Alike 2.0 Generic https://creativecommons.org/licenses/by-sa/2.0/deed.en.

But more crucially, the issue is that cats do not generate electricity, so it not meaningful to compare cats and wind turbines as sources of bird mortality. A theme of this book is choosing among alternatives. The choice before us is not one of cats versus windpower. Cats and windpower are not in competition with each other. Rather, the issue is, say, do we invest in windpower over solar or nuclear energy. So the sensible way is to compare the harmful effects of windpower (or any other form of power) in terms of mortalities per amount of energy produced.

Two papers by Sovacool (2009; 2013) take a careful look at avian[10] mortality due to wind turbines. By reviewing hundreds of studies, he estimated the number of avian deaths per GWh of electricity generated by nuclear, wind, and fossil fuel. His estimates are shown in Table

[10] This includes bats as well as birds.

18.1. There is some uncertainty in these estimates, and surely more research is needed. But the overall picture seems clear: windpower is not particularly harmful to birds compared to other sources of energy.

Energy Type	Mortality Rate
Windpower	0.269
Fossil Fuels	5.18
Nuclear power	0.416

Table 18.1: Avian mortality rates (fatalities per GWh). Data from Sovacool (2013).

That said, it's worth noting that there is significant variation in avian mortality from wind farm to wind farm, so extrapolating from the analysis of just one or two wind farms can be misleading. A poorly sited wind farm can be a hazard to birds. Some of the first wind farms were not located well, and had a significant impact on birds. Most recent wind farms have been better located and have much less of an impact. Additionally, newer turbines are much safer for birds. They rotate less quickly, and taller turbines tend to pose less of a threat than shorter ones.[11] (The Royal Society for the Protection of Birds (UK) has a sensible statement about wind power that might be worth referencing: `http://www.rspb.org.uk/forprofessionals/policy/windfarms/index.aspx`)

[11] Citation needed. Sovacool?

Finally, it might be worth remembering that the stakes for global climate change are high. If we continue more-or-less with business as usual, most (many?) scientists think we are headed toward a truly catastrophic situation.[12] How many bird deaths are justifiable to lessen the chance of catastrophic climate change? How much of the landscape can we alter in order to prevent N million people from losing their homes due to sea level rise?[13] These aren't scientific questions, of course, but as always we'd argue that these sorts of large, ethical questions need to be informed by facts.

[12] Add citation(s) to how bad the news might be.

[13] Add citation.

18.5.3 Human Health and Annoyances

Are wind turbines annoying? Probably if you live close to one. Are they unhealthy? Highly doubtful.

18.5.4 Intermittency

Critics of wind power like to point out that it is not windy all the time.[14] True. It's not windy all the time. So this is why the grid is important. How much intermittent wind can the grid accommodate? See Chapter 23.

[14] They often point out in the same breath that the sun does not shine all time. It's fine to point this out, but please don't act like you're the first person to think of this.

18.6 Conclusions

Here are the main takeaways from this chapter:

- Power is proportional to v^3. A small increase in average wind speed can make a big difference.
- For terrestrial wind farms: power density ≈ 2 W/m^2.
- For off-shore wind farms: power density ≈ 3 W/m^2.

There is considerable variability in these 2 and 3 W/m^2 figures.[15]

[15] Citation(s) needed.

One important conclusion to draw from the analysis in this chapter is that *bigger is better* for wind turbines. Wind turbines need to be tall, since that is where the wind is windiest. And since the power depends on the cube of the windspeed, this makes a big difference. Put another way, there are economies of scale for wind. A tall wind turbine generates more energy than two turbines half as tall. (See Exercise 18.5).

18.7 Exercises

Exercise *18.1:* Consider Eq. (18.14), which is an expression for the power density of a windfarm. It tells us how much power in watts we could get per meter of a windfarm. What is the power density we would expect from a windfarm in a location where average windspeed is 6 m/s? (The density of air is $\rho = 1.225$ kg/m^3.)

Exercise *18.2:* How much area would it take to generate all of the energy needs of one American person from electricity generated on a terrestrial wind farm?

Exercise *18.3:* Suppose that a certain wind turbine generates a certain amount of energy per month. What would happen to the energy generated per month if:

1. The diameter of the blades was increased by 20%?
2. The turbine was re-located someplace where the average wind speed was 30% higher?
3. The turbine was re-located someplace where the density of the substance flowing around it was twice as large?

Exercise *18.4:*

In this problem we'll think some about off-shore wind in the Gulf of Maine.[16]

[16] This exercise makes clear that the spacing in the list environment is highly suboptimal. Dave will work on this someday.

1. First, let's collect some facts: Write down the following values. (All of these figures should be in your class notes. They're also in MacKay and also my book.)

   (a) The average total energy consumption per person in the US, in units of kWh per person per day.
   (b) The worldwide average emissions per person per year, in tons of CO_2 equivalent.
   (c) The average emissions per person per year of the average American, in tons of CO_2 equivalent.
   (d) The average amount of CO_2 released per kWh of electricity generated.
2. What power would be needed to provide the total energy needs of everyone in Maine?
3. Would would be the area needed for an offshore windfarm that could deliver this power. Assume that the offshore windfarm generates 3 Watts per square meter. Express your answer in km^2 and mi^2.
4. What is side of a square whose area is equal to the area you found in the above problem?
5. Find and print out map of New England that includes the Gulf of Maine. Be sure this map includes a scale. Draw on this map a square that is the size of the square you calculated in the previous question. Be reasonably careful when you draw the square, but don't stress out about getting the size super accurate.
6. Assuming that this electricity from wind replaced "average" US electricity, how much CO_2 has been prevented from being released into the atmosphere? Express your answer in terms of tons of CO_2 per Mainer per year. Is this a lot or a little?

Exercise *18.5:*

Suppose that certain wind turbine generates 100 MWh in one year. Approximately how much energy would you expect from two turbines that are identical to the first one, but are half as tall? Explain. Assume that the windspeed increases by 10% when the height doubles.[17]

Exercise *18.6:* According to Windustry[18], A 10 kW wind turbine might cost $50,000 and a 2 MW turbine might cost 3 million dollars.

1. How many 10 kW wind turbines would you need to have a total power of 2 MW?

[17] **TODO!** Re-write this problem so that the diameters of the short turbines are smaller.

[18] http://www.windustry.org/how_much_do_wind_turbines_cost

2. How much collectively would these 10 kW turbines cost? How does this compare to the cost of one 2 MW turbine?

Exercise *18.7:* The Vestas V80 wind turbine has a diameter of 80 meters. Let's assume that it operates in a windy location where the typical windspeeds are around 12 m/s. Use Eq. (18.2.1) to estimate the power produced by the turbine. How does the power you calculate compare with the nameplate capacity of 2 MW? More information about this turbine can be found at http://www.4coffshore.com/windfarms/turbine-vestas-v80-2.0-mw--tid53.html.

Exercise *18.8:* A 3MW (nameplate) wind turbine generates 6.6 GWh in one year. What is the turbine's capacity factor? How does this capacity factor compare to typical capacity factors for modern terrestrial turbines?

Exercise *18.9:* The Bingham wind farm will be a 185 megawatt (capacity) wind farm near Bingham, Maine. An article in the *Boston Globe* states that the project is expected to generate enough electricity to power 65,000 homes. [19] Does this seem about right? Explain.

[19] https://www.bostonglobe.com/business/2015/07/01/maine-wind-project-breaks-ground/9ghiUvfC3iDdbzoY7zYImL/story.html

Figure 18.4: Plant Bowen, a coal-fired power plant in Euharless, Georgia. It is one of the largest coal-fired power plants in North America. Figure source: Sam Nash, https://en.wikipedia.org/wiki/File:Plant_Bowen.jpg. Licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license, https://creativecommons.org/licenses/by-sa/3.0/deed.en.

Exercise *18.10:* The Bowen coal-fired power plant has a nameplate capacity of 3.5 GW. In 2006 it generated 22,600 GWh of electricity.[20]

[20] https://en.wikipedia.org/wiki/Plant_Bowen

1. What is the plant's capacity factor?

2. Suppose we wanted to replace the electricity delivered by this coal plant with electricity generated by terrestrial wind power. How large a wind farm would be needed to do so? Come up with a useful way to visualize or conceptualize this area.
3. If this coal plant were shut down, approximately how much CO_2e would be prevented from being released into the atmosphere? The lifecycle greenhouse gas emissions for electricity from coal is 1001 g CO_2e /kWh. For electricity from wind, it is 12 g CO_2e /kWh. [21]

[21] Add citation to IPCC annex.

Exercise *18.11:* This problem concerns the Bull Hill wind project near Eastbrook, Maine. Find the Bull Hill wind project on https://www.eia.gov/state/maps.php.

1. What is the nameplate capacity of the wind project?
2. How much total energy did the wind project generate in 2015? To find this information you'll need to click on "View Data in the Electricity Data Browser."
3. In 2015 what was the capacity factor of the Bull Hill Wind Project?
4. What was the average power that the project delivered in 2015.
5. Find the Bull Hill wind project on google maps. What area does the Bull Hill project take up? Consider only the 19 western turbines.
6. Compute the power density of the Bull Hill project in W/m^2.

Exercise *18.12:* This problem concerns the Pisgah Mountain Wind Power plant located, unsurprisingly, on Pisgah Mountain. Find the Pisgah Mountain wind project on https://www.eia.gov/state/maps.php.

1. What is the nameplate capacity of the wind project?
2. How much total energy did the wind project generate in 2017? To find this information you'll need to click on "View Data in the Electricity Data Browser."
3. In 2017 what was the capacity factor of the Pisgah Mountain Wind Project?
4. What was the average power that the project delivered in 2017.
5. Find the Pisgah Mountain wind project on google maps. What area does the facility take up?
6. Compute the power density of the Pisgah Mountain project

in W/m^2.

Exercise *18.13:* Go back to the https://www.eia.gov/state/maps.php and find the Maine Independence Station in Veazie, which is just north of Bangor. This is a natural gas power plant.

1. What is the nameplate capacity of MIS?
2. How much energy did MIS generate in 2017?
3. What was the capacity factor for MIS in 2017?
4. Find the MIS on google maps. What area does the facility take up?
5. Compute the power density of the MIS natural gas plant.
6. In 2017, approximately how much CO_2e was emitted by MIS?
7. Suppose you want to shut down MIS and replace it with a terrestrial wind facility that will produce an equivalent amount of energy. Approximately what area you need for such a wind farm? Express your answer in square kilometers.
8. If the windfarm was a square, what would be its dimensions?
9. Draw such a square somewhere on a map of Maine.

Exercise *18.14:* Repeat problem 18.11 for the Block Island Wind Farm, located south of Rhode Island.

In the next series of problems you will derive the Betz limit, which says that, at maximum, a wind turbine can obtain 16/27 of the power that is in the wind. This derivation follows Ehrlich (2013, Section 7.3).

To begin, consider the air flowing past a wind turbine, as illustrated in Fig. 18.5. We will view air as incompressible. So whatever flows into a region must flow out; air is not created or destroyed, nor can it be compressed or stretched. What this means is that when a column of air slows down it must spread out. This is shown in Fig. 18.5. Air approaches the turbine with a speed v_1. It slows down and flows through the turbine at speed v and continues to slow down and leaves the vicinity of the turbine at a speed v_2.

The same amount of air flows through the turbine (whose area is A) and through the imaginary circles A_1 and A_2. It may help to think of a portion of a river that has no streams flowing into it. In such a river, the flow rate must be the same everywhere. If it wasn't water would accumulate somewhere and it wouldn't be flowing. Thus, where the river is wider the water is flowing slowly, and where the river is narrower the river is flowing more quickly.

We calculated the rate at which air flowed through an area A at speed v back in Eq. (18.2). The rate at which mass flows through A_1, A,

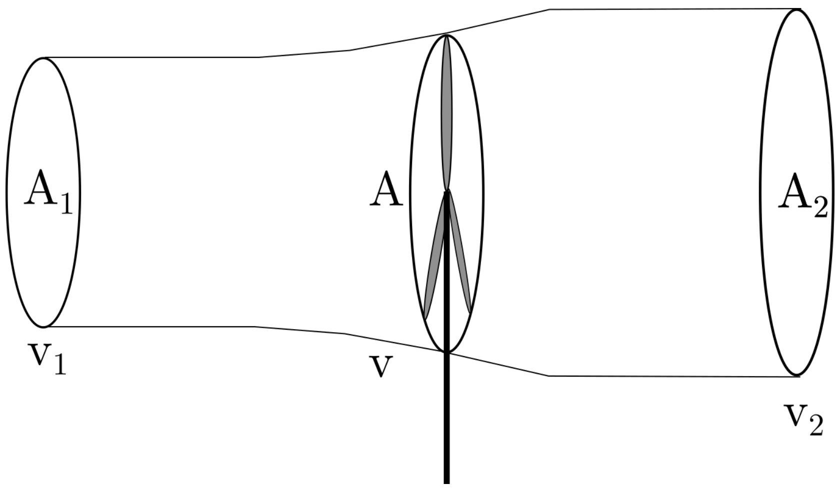

Figure 18.5: Air flowing past a wind turbine. Figure based on Ehrlich (2013, Fig. 7.12). The wind is flowing left to right.

and A_2 must be the same. Thus:

$$\text{mass flow rate} = \frac{dm}{dt} = \rho A_1 v_1 = \rho A_2 v_2 = \rho A v\ , \qquad (18.15)$$

where, as before, ρ is the density of air.

Let's now think about forces. Why does the air slow down? It must be that something is exerting a force on the air—Newton's second law[22] tell us that forces are what cause velocities to change. It is the turbine blades that exert force on the wind. We can calculate the Force using Newton's second law

[22] $F_{\text{net}} = ma$.

$$F = \text{mass flow rate}(v_1 - v_2) = \rho A v(v_1 - v_2)\ . \qquad (18.16)$$

In the last equation I've used the the fact that the mass flow rate is equal to $\rho A v$, from Eq. (18.15).[23]

[23] **TODO!** Improve derivation of Eq. (18.16).

Now, Power is energy E per unit time. An object's energy changes by the amount of work W done on it, and work is force times distance x, so

$$P = \frac{E}{\Delta t} = \frac{W}{\Delta t} = \frac{Fx}{\Delta t} = Fv\ , \qquad (18.17)$$

where I've used the fact that velocity v is $x/\Delta t$. Combining Eqs. (18.17) and (18.16), we have

$$P = \rho A v^2 (v_1 - v_2)\ . \qquad (18.18)$$

This is the rate at which energy flows out of the wind into the turbine. Equation (18.18) is thus the power that the turbine gets from the wind.

We can come up with another expression for the power that the wind turbine gets from the wind. As argued above, the power lost by the wind is the power gained by the turbine. The power lost by the wind is equal to the difference the rates at which kinetic energy flows through A_1 and A_2. Using Eq.(18.5), we have:

$$\text{Power lost by wind} \;=\; \frac{1}{2}(\rho A_1 v_1)v_1^2 - \frac{1}{2}(\rho A_2 v_2)v_2^2 \,. \tag{18.19}$$

We now have the ingredients in hand to derive the Betz limit.

Exercise *18.15:* Show that one can use Eq. (18.15) to express Eq. (18.19) as:

$$P \;=\; \frac{1}{2}\rho A v(v_1^2 - v_2^2) \,. \tag{18.20}$$

Exercise *18.16:* We now have two expressions for power, Eq. (18.20) and Eq. (18.18). Equate the right-hand sides of these two equations and derive the following result:

$$v \;=\; \frac{v_1 + v_2}{2} \;;. \tag{18.21}$$

Exercise *18.17:* We next plug Eq. (18.21) into Eq. (18.20). Do so, simplify, and show that one can write

$$P \;=\; C(x)\frac{1}{2}\rho v_1^3 \,, \tag{18.22}$$

where

$$C(x) \;=\; \frac{1}{2}(1 - x^2 + x - x^3) \,, \tag{18.23}$$

and

$$x \;=\; \frac{v_2}{v_1} \,. \tag{18.24}$$

Note that Eq. (18.22) for the power delivered to the wind turbine from the wind is of the following form:

$$P \;=\; C(x) \times \text{power in the wind} \,, \tag{18.25}$$

since we know that the power in the wind is $(1/2)\rho A v_1^3$. Thus, $C(x)$ is the fraction of the power in the wind that goes into the turbine. We'd like to make $C(x)$ as large as possible so as to have the most efficient wind turbine.

Exercise *18.18:* Determine the maximum of $C(x)$. To do so, take the derivative of $C(x)$, set it equal to zero, and solve for x. You'll get two values for x, only one of which is physically meaningful. Plug that value for x into $C(x)$ and you should find that the maximum value for $C(x)$ is $16/27$.

19
Solar Photovoltaic

19.1 How much Energy is in Sunlight?

The sun delivers energy to the earth. At what rate? On a clear day at noon near the equator, the sun delivers around 1000 W per square meter. On average, though, the power hitting the earth is a good bit less. Why? For several reasons:

1. We do not all live on the equator. As one gets closer to the poles, the amount of sun hitting a square meter of earth gets smaller.[1]

2. Most of us live places where it is cloudy at least part of the time.

3. All of us lives places where it is dark for much of the day.[2] This is a phenomenon known as *night time*.

[1] Add simple figure for this?

[2] Except for summers for those who live above or near the Arctic circle.

When we account for all this, we get a quantity known as *insolation*, which is the average power in sunlight actually hitting the earth, per square meter.

Location	Insolation (W/m^2)
London, UK	109
Boston, MA	149
Madrid, ES	177
San Francisco, CA	204
Nairobi, KE	234
Addis Ababa, ET	243
Honolulu, HI	248
Djbouti, DJ	266

Table 19.1: Insolations. Data taken from (MacKay, 2009, p. 46).

So now we know how much power is available to us from sunlight.

19.2 *How much of that Energy can we Get?*

Solar PV works via the photoelectric effect. An single electron in a semiconductor interacts with a single photon—a particle of light. The electron is loosely bound in the solid. If the photon has enough energy, it can set the electron free. In physics parlance, the electron is promoted to the conduction band. What this means is that it is now free to flow through the material, like an electron is a metal. For conventional silicon solar cells, the amount of energy needed to free one electron is around 1.1 eV.[3]

[3] An "eV" is an *electron volt*. It is a small unit of energy that is commonly used when considering energies for single particles. One electron volt is approximately 1.602×10^{-19} joules.

The spectrum from the sun contains photons of a range of energies, as shown in Fig. 19.1. Any photon that that has an energy less than 1.1 eV cannot set the electron free. Thus, all of the energy from sunlight that is in photons of energy 1.1 eV or less cannot be harvested by the solar cell. Energy in photons whose energy is larger than 1.1 eV can be harvested, but only the part that is left over after the 1.1 eV is used to set the electron free. The net result is that even in a maximally efficient solar cell, not all the energy in sunlight is available to be harvested.

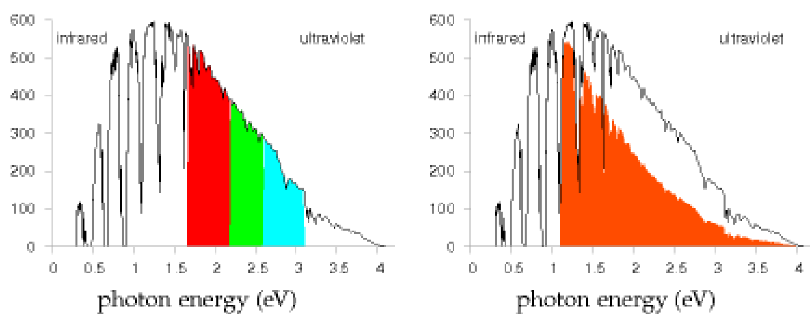

Figure 19.1: The electromagnetic spectrum of sunlight hitting the surface of the earth. The units on the vertical axis are W/m^2 per eV of spectrum interval. For a conventional solar panel with a 1.1 eV band-gap, the orange area on the right figure is the energy that can be captured by the solar panel. Figure source: David MacKay, 2009, p. 47. I don't know if I'm actually allowed to use this figure.

This result is known as the *Shockley–Queisser* limit.[4] For a system with a 1.1 eV cost of freeing electrons, the maximum theoretical efficiency of a solar cell is approximately 32%. In practice, standard solar cells are around ten percent efficient.[5] There are more efficient solar panels, but they are much more expensive, and of these more efficient types of solar are on the market. The vast majority of solar PV presently in are polycrystalline solar panels have an efficiency of around 10%. That is, ten percent of the energy from the sun that lands on the solar cell can be converted into electricity.

[4] Their paper establishing this result was published in 1961 in the Journal of Applied Physics.

[5] Citation(s) needed.

19.3 *Capacity Factor and Power Density*

The insolations in Table 19.1 range from roughly 100 to 250 W/m^2. If we accounted for the fact that typical solar cells are roughly 10%

efficient, we would expect that the power density of solar PV would be between 10 and 25 W/m^2. In reality, it is a bit lower than this, in large part because in a large solar installation one needs to put some space between the the solar cells so one can access them for maintenance. The highest power densities for solar PV are around 20 W/m^2 in desert PV farms. In Germany, solar installations average around 5 W/m^2. [6] I often use 10 W/m^2 when doing rough calculations.

Average US capacity factor for utility-scale solar PV in 2015 was 28.6%.[7] Typical capacity factors for solar PV in Maine range from 15 to 20%. The higher end range applies to well-sited utility-scale installations.

Let's get a feel for some of these numbers by working through an example.

[6] http://www.theenergycollective.com/robertwilson190/257481/why-power-density-matters, accessed September 21, 2017.

[7] https://www.eia.gov/electricity/monthly/epm_table_grapher.cfm?t=epmt_6_07_b.

Example 19.1. *A ten-panel solar array on the roof of a small barn at College of the Atlantic's Beech Hill Farm has a nameplate capacity of* 2.3 *kW. In 2016 it generated* 3467 *kWh. What is the capacity factor of this array? The area of the barn is approximately* 33 *m*^{2}. *What is the power density of the array in W/m*^{2}*?*

To determine the capacity factor we'll calculate how much energy the array would make in one year if it operated at 2.3 kW of power for the entire year:

$$\text{Energy} = 2.3\,\text{kW} \times 365\,\text{d} \left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 20,160\,\text{kWh}\,. \tag{19.1}$$

The capacity factor is the actual energy generated in a year divided by the maximum possible energy generated in a year:

$$\text{Capacity factor} = \frac{3467\,\text{kWh}}{20,160\,\text{kWh}} \approx 0.172 = 17.2\%\,. \tag{19.2}$$

The power density is expressed in W/m^2. So I need to express the average power of the array in W.

$$\frac{3467\,\text{kWh}}{1\,\text{yr}} \left(\frac{1\,\text{yr}}{365\,\text{d}}\right) \left(\frac{24\,\text{h}}{1\,\text{d}}\right) \approx 0.4\,\text{kW} = 400\,\text{W}\,. \tag{19.3}$$

So the solar cells on the 33 m^2 barn produce power at an average rate of 400 Watts. So the power density is:

$$\text{Power density} = \frac{400\,\text{W}}{33\,\text{m}^2} \approx 12.5\,\text{W/ m}^2\,. \tag{19.4}$$

Not bad. This is toward the high end of the power densities for solar PV at our latitude.

19.4 Rooftop Solar

A natural place to put solar PV is on the top of roofs. This is unused space[8], so putting panels on our roofs is not a sacrifice or inconvenience. How much electrical energy can we get from our roofs? Would it be enough to power the typical house? Let's do a very rough calculation and see.

[8] Unless you spent a lot of time on your roof, I guess.

The mean size of the average home built in the US in 2015 was 2, 687 ft^2 (of Housing & Urban Development, 2015, p. 345). Let's assume that this is a two-story house, so the footprint of the house—and thus the area of the roof—is half of 2, 687. Thus, the area of the roof is 1340 ft^2. Of this, let's estimate that half of the roof area would be available for solar panels. Most roofs are slanted in two directions, only one of which would face south and thus be appropriate for solar. So the effective area of our solar panels will be 670 ft^2. Let's convert this to square meters:

$$670\,\text{ft}^2 \left(\frac{1\,\text{m}}{3.3\,\text{ft}}\right)^2 = 62\,\text{m}^2 \,, \tag{19.5}$$

where I've used the fact that there are around 3.3 feet in a meter. Note that I need to square the unit conversion factor in parentheses so that the units come out right.

For the power density, I'll use 10 W/m^2. This yields a power of 620 Watts. Let's figure out how much energy these 620 Watts would generate in a month. First, note that 620 W are 0.62 kW. Then,

$$0.62\,\text{kW} \times 30\,\text{d} \left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 430\,\text{kWh} \,. \tag{19.6}$$

What to make of this number? To put things in perspective, recall that the average Maine home uses 520 kWh of electricity a month, while the average home across the US uses 950 kWh a month. So the result in Eq. (19.6) suggests that it will be hard to generate sufficient electricity on your roof in order to meet your home electricity needs. That said, the above estimate was perhaps not very generous. Many locations will be able to achieve a power density of more than 10 W/m^2, and some houses will have more rooftop real estate than the 620 I've allotted.

If I were more generous and assumed that there are 800 ft^2 we can devote to solar panels and that the power density is 15 W/m^2, then I find that the panels would produce almost 800 kWh/month. So this is in the ballpark of what a typical house would use. But in the larger picture, it's worth remembering that residential electricity use is a small fraction of our overall energy use. In the US, we use around 250 kWh per person per day. This includes residential, commercial, industrial, and transportation energy uses of all forms. 800 kWh per month is 26 kWh/day. That's a lot, but still only a bit more than a tenth of our total energy needs.

The picture that emerges is that rooftop solar PV is a significant source of electrical energy. But in order for solar PV to really take a bite out of our total consumption we'll need to do more than just put solar panels on our houses.

19.5 Solar Farms

19.6 Conclusion

1. Solar farms produce around 10 W/m^2. There is considerable variance, however. Locations with higher insolation are clearly better.

2. If you use land for solar, you can't use it for anything else. In contrast, wind turbines can be placed above cropland without affecting the crop yield. An exception, though, is rooftop solar. You can have a perfectly nice house underneath solar panels. But as we've seen, rooftop solar alone isn't going to cut it.

3. Unlike wind, there are no economies of scale. Lots of small installations sprinkled about are ok. For wind, though, this isn't a good idea, due to the fact that the power depends on v^3, and wind is faster as one goes higher up.

19.7 Exercises

Exercise *19.1:* A small town-house style dormitory[9] at College of the Atlantic has a solar array with a nameplate capacity of 8.7 kW. In 2016 it generated 10.4 MWh. The area of the south-facing roof on the house is approximately 125 m^2.

[9] Known locally as Eno House

1. What is the capacity factor of the array?
2. What is the average power produced by the array, in units of Watts?
3. What is the power density of the array, in units of W/m^2?
4. Suppose this solar electricity is displacing electricity generated with a carbon intensity of 500g CO_2 /kWh. How much less carbon is emitted into the atmosphere each year as a result of this array? Is this a little or a lot?

Exercise *19.2:* In October for 2020, College of the Atlantic president Darron Collins tweeted enthusiastically about a new building being built on campus that will have 323 solar panels on its roof. Based on the calculations Darron made (see Fig. 19.2), what is the expected capacity factor of the solar cells. Does this capacity factor seem reasonable, given that the solar cells will be located in Maine?

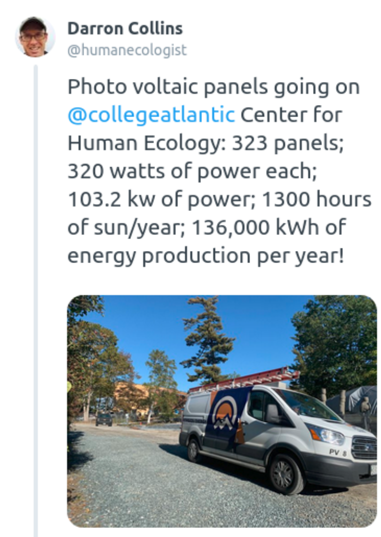

Figure 19.2: A tweet by College of the Atlantic president Darron Collins on October 7, 2020: https://twitter.com/humanecologist/status/1313888984192385025.

Exercise *19.3:* The Limerick nuclear power plant, shown in Fig. 19.3, is a pair of nuclear power stations. Taken together, the two plants

Figure 19.3: The Limerick Generating Station in Limerick, Pennsylvania, USA. Figure source: Arturo Ramos, https://commons.wikimedia.org/wiki/File:LimerickPowerPlant.JPG. Licensed under the Creative Commons Attribution-Share Alike 2.5 Unported license, https://creativecommons.org/licenses/by-sa/2.5/deed.en.

generate around 19,000 GWh of electricity per year.[10]

[10] https://en.wikipedia.org/wiki/Limerick_Generating_Station

1. Express the power of the Limerick stations in watts.
2. What area of solar PV would be needed to generate this amount of power? Assume a power density of 10 W/m^2. Express this area in a sensible way—probably this means figuring out the side of a square whose area is equal to the area of the solar installation.

Exercise *19.4:* In this exercise you'll calculate some energy statistics for Japan. You can get some basic energy facts about Japan on the EIA website: https://www.eia.gov/beta/international/country.cfm?iso=JPN.

1. The total energy consumption is given in terms of a truly horrible unit: Quadrillions of BTUs. These are often referred to as *quads*. One quad is 10^{15} BTUs. BTUs are British Thermal Units, about which the less said, the better. Let's get out of quads and into better units. One quad $= 293 \times 10^9$ kWh. What is the primary energy consumption of Japan per year in units of kWh?
2. What is the *per capita* energy consumption for Japan in units of kWh/person/day? How does this compare to the average *per capita* consumption of the United States?
3. What is the power consumption of all of Japan expressed

in watts? This will be a very large and not very meaningful number.

4. What is the power consumption per unit area of Japan? Your answer should be expressed in units of watts per square meter. To obtain this, take your answer to Question 3 and divide by the area of Japan.
5. How does your answer to Question 4 compare to the W/m^2 for solar farms? What fraction of Japan's landmass would have to be devoted to solar farms if it wanted to get all of its energy from solar? Does this seem feasible?

Exercise *19.5:*

There is a solar installation a few miles northeast of Amalia, New Mexico, USA. In 2014 this plant generated 2997 MWh of electricity.[11] The nameplate capacity of the plant is 1.3 MW.

[11] http://www.eia.gov/electricity/data/browser/#/plant/58240

1. What is the capacity factor of the plant?
2. What is the average power delivered by the plant?
3. Find the plant on google maps. Look at Ventero Road as it heads northwest out of town. Use google maps to estimate the area of the installation.
4. Use your answer to the above two questions to estimate the power density of the solar array, in units of watts per square meter.

Exercise *19.6:* This problem concerns a *Guardian* article[12] about the African Renewable Energy Initiative (AREI).

[12] http://www.theguardian.com/global-development/2015/dec/07/africa-plans-renewable-energy-initiative-solar-h

1. The article states that the average annual per person consumption of electricity is around 600 kWh. Convert this to kWh per person per day.
2. The article also states that the worldwide average per capita electricity use is around 3000 kWh per year. Convert this to kWh per person per day.
3. The per capita yearly electricity use in the US is around 13000 kWh.[13] Convert this to kWh per person per day.
4. The *Guardian* article states that AREI has a goal of adding 300 GW[14] of renewable electricity generation by 2030. How much electricity is this in kWh per person per day? The population of Sub-Saharan Africa is, very roughly, 1 billion.
5. If these 300 GW of power were entirely from solar PV, how much land would this be? Express your answer in an understandable way; perhaps find the size of a square that has this area, or express the area as a fraction of one of the countries in

[13] http://www.iea.org/statistics/statisticssearch/report/?year=2013&country=USA&product=Indicators

[14] It's not quite clear to me if this is nameplate or actual power, but I think it's the latter. So let's assume that this is 300 GW of actual power.

Sub-Saharan Africa.

Exercise *19.7:* You are considering buying a solar PV system that would have 20 panels, each with a nameplate capacity of 250 Watts. You expect a capacity factor of 0.15. Purchasing and installing the panels will cost around three dollars per Watt (nameplate). You expect the system to last for 20 years.

1. How much will this system cost?
2. How much electricity will the system generate over its lifetime?
3. If this electricity was replacing "normal" electricity generated with a carbon cost of 0.5 kg of CO_2 per kWh of electricity, how much carbon dioxide would you have prevented from being emitted?
4. Suppose that there was a price on carbon of $15 per ton.[15] Suppose that this means[16] that you could get a $15 credit for every ton of carbon that you prevented from being emitted. How much money could you get for carbon credits for your solar PV installation? How does this compare to the cost of the solar panels?

[15] The minimum price per ton of CO_2e was around $14.50 in 2018 in California. In the Northeastern US the RGGI (Regional Greenhouse Gas Initiative) has a minimum carbon price of around three dollars per ton. (https://rhg.com/research/the-footprint-of-us-carbon-pricing-plans/)

[16] The carbon market in California actually involves selling credits that would allow one to emit, and applies only to regulated industries, not individuals. But the point of this problem isn't to go into the details of carbon markets, but to think some about what a price of $15/ton would mean.

Exercise *19.8:* Table 19.2 shows the electricity generated by a solar array on the roof of one of the barns at Beech Hill Farm on Mount Desert Island, Maine. The data shown is for 2017. The installation has ten panels, and thus a total capacity of 2.3 kW. The area of the barn is around 40 m^2. (This includes the entire barn; panels are only installed on the south-facing roof.)

1. What is the capacity factor of the system?
2. What is the power density of the system, in W/m^2?
3. What is the power density of the system for the month of December?
4. The average energy delivered by the system is 260 kWh per month. (To see this, divide the total energy produced in a year by 12.) How large (in square meters) would the system need to be in order to produce 260 kWh in December?

Month	Energy Generated (kWh)
January	170
February	185
March	220
April	210
May	248
June	333
July	361
August	364
September	321
October	296
November	225
December	183
Total	3117

Table 19.2: Energy generated by a solar installation in 2017 on the roof of one of the barns at College of the Atlantic's Beech Hill Farm.

20
Hydropower

This chapter was jointly authored with Sara Löwgren.

20.1 Introduction

The idea behind hydropower is to use the kinetic energy in moving water to turn a turbine which generates electricity. Hydropower is similar to windpower, in that both involve using a moving fluid (air or water) to turn a turbine. We shall see, however, that the physics of hydropower is different from wind power.

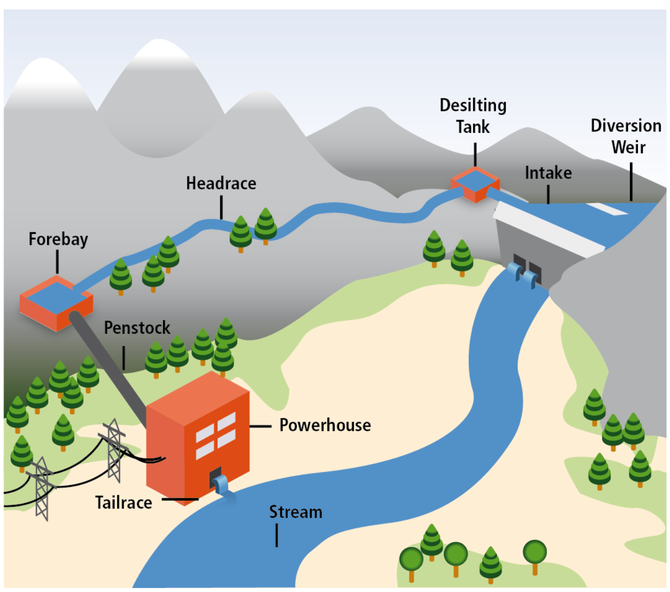

Figure 20.1: A schematic diagram of a run-of-river hydroelectric dam. Image source: (IPCC, 2011, Figure 5.5).

There are two main type of hydropower plants. The first type is called a *run-of-river* plant and is shown in Figure 20.1. In such a power plant a portion of the river is diverted and sent through a turbine. In the figure, the turbine is located inside the powerhouse. Most run-of-river hydropower plants have a dam, but the dam does not create a large reservoir. Thus, the power from the plant is dependent on the flow of the river. A run-of-river power plant, the Chief Joseph plant on the Columbia River, in Oregon, USA, is shown in Fig. 20.2.

Figure 20.2: The Chief Joseph dam on the Columbia river in Oregon, USA. Image by the US Army Corps of Engineers, in the public domain. Available at: `https://en.wikipedia.org/wiki/File:Chief_Joseph_Dam.jpg`.

The second type of hydropower is called conventional hydropower or storage hydropower. These power plants have dam that creates a large reservoir that might hold months or even years' worth of river flow. A schematic diagram of a conventional hydropower plant is shown in Fig. 20.3. Such a station is thus not dependent on the flow of the river in the way that a run-of-the-river hydropower plant is. A conventional hydropower plant can have water flow through its turbine at a faster rate than water flows into the reservoir, since the reservoir is so large. Similarly, a conventional hydropower plant can also have water run through its turbines at a slower rate than water flows into the reservoir. In this case, assuming that the reservoir is not already at capacity, the water level of the reservoir will rise. In contrast, in a run-of-river hydropower plant, if it were to have water flowing through its turbines at a slower rate than the river itself, the small reservoir would soon fill up and the extra water would have to be sent over a spillway. This water is wasted, in the sense that it does not get used to generate electricity.

The difference between the two types of hydroelectric power is somewhat arbitrary. The difference is a matter of time-scales. Typically run-of-river hydro plants have a reservoir that can store a few days worth of the river's flow, while conventional hydroelectric power plants have reservoirs that can hold months or even years worth of a river's flow.

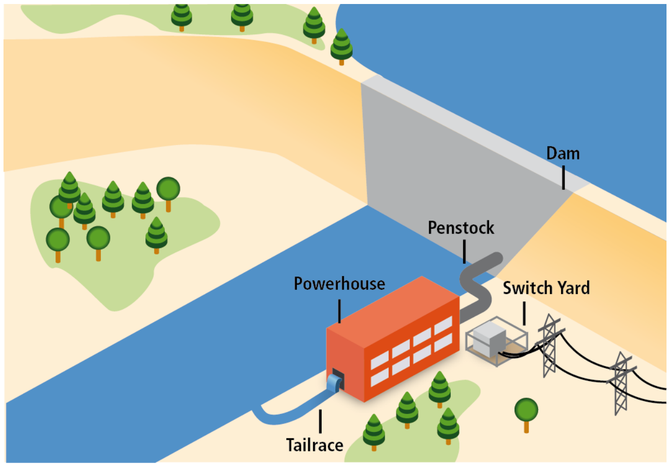

Figure 20.3: A schematic diagram of a conventional hydroelectric plant with a reservoir. Image source: (IPCC, 2011, Figure 5.6).

20.2 *Gravitational Potential Energy*

Both run-of-river and conventional hydropower generate power by converting the kinetic energy of water into electricity in a turbine. What features of the river and reservoir determine how much power can one get from a hydropower station? To answer this question, we need to introduce a type of energy we haven't considered yet: *gravitational potential energy*. This is the energy associated with changing the altitude, or height, of an object. Imagine dropping a 10 kg box out of a window that is five meters above the ground. The box will accelerate on its downward journey. It gains speed, and thus it gains kinetic[1] energy. Where did this energy come from? Gravitational potential energy. By virtue of being five meters above the ground we say the box has some potential energy; it has the potential, if released, to gain kinetic energy.

[1] Recall that kinetic energy is the energy associated with motion. The kinetic energy of an object of mass m moving at a speed v is $E = (1/2)mv^2$.

The formula for gravitational potential energy of an object of mass m at an height of h is

$$E = mgh \,. \tag{20.1}$$

In the above equation g is the *acceleration due to gravity*. Near the surface of the earth g is approximately 9.8 m/s^2. It is often sufficient to approximate g with 10 instead of 9.8.

Example 20.1. *A* 20 *kilogram rock is at an altitude of* 100 *feet. What is the rock's gravitational potential energy?*

Let's first convert 100 feet to meters.

$$100\,\text{ft}\left(\frac{1\,\text{m}}{3.28\,\text{ft}}\right) = 30.5\,\text{m} \,. \tag{20.2}$$

We can now plug values in to Eq. (20.1) to solve for the gravitational potential energy:

$$E = mgh = (20\,\text{kg})(9.8\,\text{m/s}^2)(30.5\,\text{m}) = 6000\frac{\text{kg}\,\text{m}^2}{\text{s}^2} = 6000\,\text{J} \,. \tag{20.3}$$

So, if we dropped the rock from a height of 100 feet, it would have 6000 J of kinetic energy right before it landed. (This calculation ignores the small amount of energy that would be lost to air resistance as the rock fell.)

Add short paragraph about the need to set a zero level for h and how what really matters is Δh.

20.3 Power from Rivers

How much power can a hydroelectric dam generate? This will depend on the energy in the flowing water. However, rather than work directly with the water's speed, as we did when thinking how much energy is in the wind in Section 18.1, we will instead consider the potential energy of water at rest in a reservoir behind a dam. The amount of potential energy in the water depends mainly on two things: the height of the dam, and the rate at which water flows through the turbines.

We'll start by thinking about flow rate. The *flow rate* of a river is a measure of the volume of water that flows per second. In most of the world, the units for flow rate are cubic meters per second. In the US, flow rates are usually expressed in cubic feet per second, sometimes abbreviated cfs. We will use the symbol Q to represent flow rate:

$$Q = \text{volume flow rate of river} . \tag{20.4}$$

To get an expression for energy, we'll need an expression for the rate at which *mass* is flowing, not volume. We can obtain such an expression by multiplying the volume flow rate by the density[2] of water, denoted by ρ, the Greek letter "rho.":

$$\rho = \text{density of water} = 1000\,\frac{\text{kg}}{\text{m}^3} . \tag{20.5}$$

[2] A handy fact to remember is that one cubic meter of water has a mass of 1000 kilograms, or one metric ton. This means that $\rho = 1000\,\text{kg}/\,\text{m}^3$.

So the flow rate of the mass is:

$$\rho Q = \text{mass flow rate of river} . \tag{20.6}$$

The units for the mass flow rate ρQ are usually kg/s.

The other factor relevant to the power available from water is the height h through which the water falls before it hits the turbine. This distance is sometimes called the height of the head or the *hydraulic head*. Note that the height that the water falls is not necessarily the same as the height of the dam. The hydraulic head is the difference in elevation between the top of the water in the reservoir and the turbine. This number is typically close to, but not the same as, the height of the dam. Putting it all together, we can arrive at an expression for the power in a river:

$$\text{Power in River} = \text{mass flow rate} \times g \times h = \rho Q g h . \tag{20.7}$$

This equation is almost identical to Eq. (20.1), the equation for gravitational potential energy. Here, however, the formula gives us power (energy per time), since rather than mass m in kg we have the mass flow rate ρQ in kg/s.

Equation 20.7 tells us the power associated with the moving water in a river. This moving water turns a turbine which creates electricity. As usual, we'll use e to denote the turbine's efficiency. So we can now write down a formula for the power from a hydroelectric plant.

$$\text{Power from Hydroelectric Plant} = e\rho Q g h \,. \tag{20.8}$$

Efficiencies for modern turbines in large dams are around 90 percent (IPCC, 2011, Chapter 5).

Let's work through an example using Eq. (20.8)

Example 20.2. *Consider a dam that has a hydraulic head of* 26 *meters and a flow rate of 4580* m^3/s. *Approximately how much power could such a hydroelectric plant produce? Assume the turbines are 90 percent efficient. These are roughly the specifications for the Long Spruce Generation Station on the Nelson River in northern Manitoba, Canada.*

We can plug directly in to Eq. (20.8):

$$P = 0.9\left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(4580\frac{\mathrm{m}^3}{\mathrm{s}}\right)\left(10\frac{\mathrm{m}}{\mathrm{s}^2}\right)(26\,\mathrm{m}) \approx 1.1\times 10^{10}\,\mathrm{W}\,. \tag{20.9}$$

To see how the units work out, recall that

$$1\,\mathrm{W} = 1\,\mathrm{J/s} = \frac{\mathrm{kg\,m^2/s^2}}{\mathrm{s}} = \frac{\mathrm{kg\,m^2}}{\mathrm{s^3}}\,. \tag{20.10}$$

Let's convert the power output of the hydroelectric plant into Megawatts:

$$1.1\times 10^{10}\,\mathrm{W}\left(\frac{1\,\mathrm{MW}}{10^6\,\mathrm{W}}\right) = 1{,}100\,\mathrm{MW}\,. \tag{20.11}$$

TODO! Two details to discuss:

1. Why the Betz limit doesn't apply to hydropower.
2. Why we can use the mgh formula even though the water isn't actually falling—the turbine pulls in water from the bottom of the reservoir.

20.4 *Optimizing Hydropower*

Some rivers are more suitable for hydropower than other rivers. To start thinking about how we can assess which rivers would generate more power, let's look at Eq. (20.8), which gives the power output from

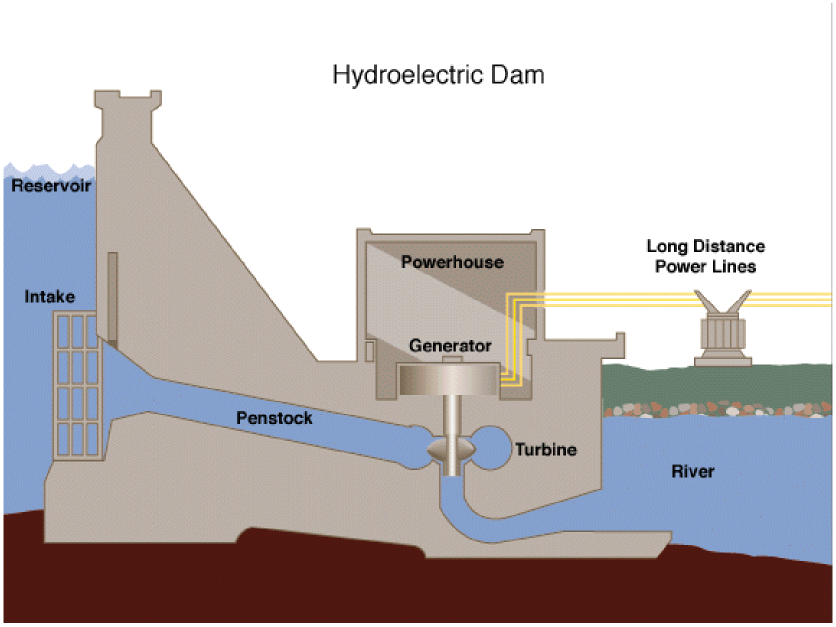

Figure 20.4: A schematic diagram of a hydroelectric dam. Image source https://commons.wikimedia.org/wiki/File:Hydroelectric_dam.png, originally produced by Tennessee Valley Authority. In the public domain. **Do I need this figure? I think the two IPCC figures are probably sufficient?**

a hydroelectric dam. To get as much power as possible, we would like all terms on the right-hand side of this equation to be as large as possible. The density of water ρ and the g acceleration due to gravity are constants, and thus are the same for all hydroelectric dams. But the efficiency η, the flow rate Q, and and the hydraulic head h will be different on different rivers.

The amount of water passing through the turbine Q depends on how much water is in the river and how quickly it is moving. This is affected by how large the river is (width and depth of the river channel), how much water comes into the river (precipitation, springs feeding into the river), and how much water leaves the river. Rivers lose water to the air (evaporation), to nearby plants (transpiration), to the ground below (infiltration), and to human structures taking water out of the river (diversion). Discharge varies significantly throughout the year. The more water in the river, the better for hydropower production.

The hydraulic head h depends on how tall the dam is. Some landscapes are more suitable for tall dams than others. For example, picture a narrow canyon with tall walls and a river at the bottom. A dam in this kind of canyon could be very tall and the canyon walls could still keep the water together. Picture instead a flat and expansive floodplain. A dam on this part of a river could not be very tall before the water would burst into the floodplain and flow around the dam instead of through it. Mountainous areas with narrow valleys or steep canyons are better for hydropower production than flat floodplains.

The efficiency of the turbine η, of course depends on the turbine and not the river. However, the height of the head determines which kind of turbine can be used in the dam. Generally, turbines that are made for low-head dams have lower efficiency[3] than turbines made for dams with higher head. This explains why building a long, low-head dam does not generate the same amount of energy as a tall, high-head dam, and why most large rivers in the mountains and canyon of the United States have been dammed.

[3] **TODO!** find reference(s) for this.

20.5 *Benefits of Hydropower: Storage and Dispatchability*

There are many benefits to hydropower. For one, hydropower can produce a lot of electricity. A large hydropower plant typically has a capacity of several GW. There are five hydroplants[4] that have a capacity of over 10 GW. Worldwide hydropower generation[5] in 2020 was around 500 MW. This is almost 17% of the electricity generated worldwide and 6.5% of the total energy use. Hydropower is, by far, the largest source of non-fossil-fuel electricity. In 2020 around 6% of the world's electricity came from wind and only 3.3% came from solar photovoltaic.

[4] Three Gorges Dam (China), Itaipu Dam (Brazil and Paraguay), Xiluoku Dam (China), Guri Dam (Venezuela), and the Belo Monte Dam (Brazil).

[5] The source for this and the rest of the statistics in this paragraph is `https://ourworldindata.org/renewable-energy`, accessed June 21, 2021.

In addition to providing a lot of electricity, a great thing about hydropower is that it is easily dispatchable. It is a simple matter to turn hydropower on and off—doing so basically involves opening or closing a faucet. In contrast, it is very hard, say, to turn a nuclear power plant on or off quickly. This means that hydroelectric dams can compensate for the intermittency of solar and wind power. The power output of solar and wind fluctuates on multiple time scales. There may be variations from minute to minute on partly cloudy day or in blustery and unsettled weather. Cloudy or calm stretches of weather can persist for several days. And there are also seasonal variations. Away from the equator it is less sunny during the winter, and in many locations some seasons are much windier than others.

Because hydropower is easy to turn on and off, it can respond quickly to sudden increases or decreases in wind or solar power. And large reservoirs can compensate for longer stretches of time when solar or wind power lulls. For these reasons hydroelectric dams are sometimes described as batteries.

Some countries have taken this idea of a hydroelectric battery one step further and are experimenting with pumped-storage facilities. These facilities are systems of dams and reservoirs that, when there is more electricity in the grid than the consumers need, use excess energy to pump water uphill into reservoirs at higher elevation. A diagram of a pumped storage facility is shown in Fig. 20.5. When there is too little electricity in the grid, the pumped-storage facilities can release the water back down and regain the energy from the water in the form of electricity from a hydroelectric turbine. Because there are no perfect machines, this process loses some[6] energy, but the loss is small

[6] **TODO!** get citation(s). Would be nice also to get an estimate for the losses incurred when using a pumped storage facility to store energy.

compared to the value of having a reliable supply of renewable energy.

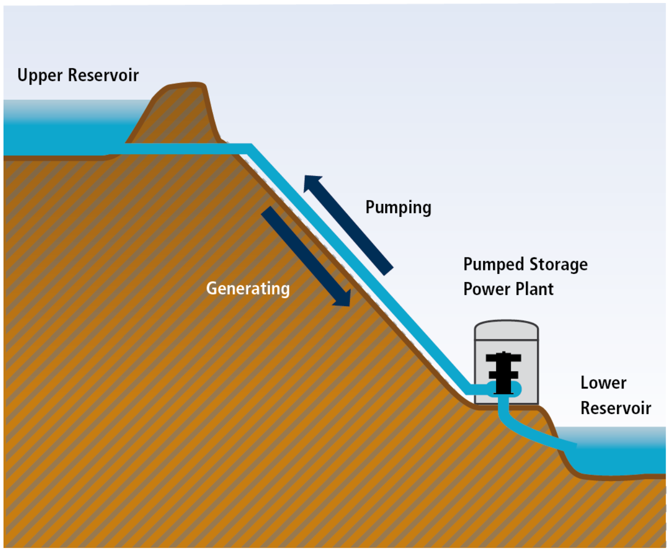

Figure 20.5: A schematic diagram of a pumped storage hydroelectric plant with a reservoir. Image source: (IPCC, 2011, Figure 5.7).

20.6 *The Carbon Footprint of Hydropower*

Hydropower is often celebrated as the cleanest source of renewable energy. The IPCC report by Moomaw, et al Moomaw et al. (2011) finds that the life-cycle CO_2e emissions for hydropower are just 4 grams per kWh of electricity generated. This should be compared to 12 g/kWh for wind, 46 g/kWh for solar photovoltaic, and 469 g/kWh for electricity generated by natural gas. To arrive at these numbers the authors considered emissions estimates from many different published sources. The numbers they reported are the median values. For hydropower, the minimum emission estimate was 0 g/kWh and the maximum was 43 g/kWh.

Unfortunately, the CO_2e emissions from hydropower may be significantly higher than the number Moomaw arrived at in 2011. New and ongoing research is suggesting that some reservoirs emit vast amounts of methane, which is a very potent greenhouse gas. The principal reason for this is that biological material deep in a reservoir decays anaerobically—in an environment without oxygen. Anaerobic decay produces methane. This methane then bubbles up through the surface of the lake and into the atmosphere.

In an extensive literature review published in 2016, Deemer et al. estimate that globally, annual emissions from reservoirs account for approximately[7] 0.8 Gigatons of CO_2e . The IPCC estimated that in 2017, global CO_2e emissions including land use change were 53.5 Gt CO_2e . This would mean that emissions from reservoirs may account

[7] Deemer, et al, report an uncertainty of 0.3 Gt on their estimate.

for approximately 1.5% of human greenhouse gas emissions.

In another 2016 review paper, Scherer and Pfister estimated the average CO_2e emissions for hydroelectric dams to be 273 grams of CO_2e per kWh of electricity—much higher than the estimate of 4 g/kWh by 2011. Scherer and Pfister note that the carbon intensity of hydropower varies widely from reservoir to reservoir. A more recent paper by Ocko and Hamburg 2019 emphasize both emissions variations from reservoir to reservoir, but also over time. The rate of methane emissions is not constant over a reservoir's lifetime.

Methane is a potent greenhouse gas. It is also a potential source of energy. It may be possible to capture a large portion of the methane produced by reservoirs Lima et al. (2008). If so, this could be used as a low-carbon source of energy. The methane emitted by reservoirs is from decaying botanical matter. The carbon is not new carbon being dug up from underground and added to the carbon cycle (as is the case with fossil fuels), but is already part of the carbon cycle.

So what are the greenhouse gas emissions associated with hydropower? It is surely higher than 4 g of CO_2e per kWh reported in Moomaw et al. (2011). How much higher is an open question that appears to not have a simple answer. It seems to be the case that more bio-productive reservoirs produce more emissions. This means that larger reservoirs tend to emit more than smaller ones, and that reservoirs in bio-productivetropical regions emit more than those in boreal regions. For example, a recent paper (Levasseur et al. (2021)) estimates the emissions associated with hydropower in Québec, Canada, to be 34.5 g CO_2e per kHw.

Need conclusion.

20.7 Other Issues with Hydropower

Ecology, hydrology

Rivers are complex systems. They are home to countless aquatic organisms–from macroinvertebrates and plants to fish and birds–that all depend on the river for migration, food, and habitat. Dams cut off the migration routes and the food supply, and they alter the habitat's temperature, nutrients, flooding regimes, and water quality. When dams hold back sediment, the downstream river tends to erode the river banks more quickly than wild rivers. The dams also withhold important food sources for fish, including microorganisms, plant material, and nutrients attached to sediment. Dammed rivers typically have fewer rapids, which means there are less opportunities for oxygen to dissolve in the water. In some cases, dams have created is slow-moving rivers where water-living species starve and suffocate.

Communities

Over the thousands of years of human civilization, humans have tended to settle along coasts or rivers. Rivers supply fish, transportation routes, freshwater for drinking and irrigation, and fertile soils as a result of major flood events. Damming rivers means removing fish (and hence livelihoods for fishing communities) and preventing the natural floods with regenerate soil fertility (stopping large floods can also be seen as a good thing, depending on where in the floodplain you have built your house and what types of crops you grow along the river).

Dams can balance intermittency and be switched on and off easily because they have massive reservoirs. Because humans tend to settle along rivers, creating the reservoirs often means flooding human settlements and forcing people to move away from their homes, livelihoods, and spiritual sites. National Geographic recently reported that dams worldwide have displaced over 400 million people. In this group, a disproportionate number are impoverished and indigenous people who rarely get to enjoy the benefits of hydropower.

20.8 How to think about hydropower

Hydropower currently makes up 70% of the world's renewable energy. Because it can balance intermittency and generate vasts amount of electricity on demand, it seems to be a crucial part of transitioning away from fossil fuels. At the same time, damming rivers has devastating impacts on aquatic species and the people who depend on them. Reservoirs have destroyed invaluable indigenous sites and displaced millions of people.

If we continue to burn fossil fuels, we know climate change will harm and displace millions of people. But we are not able to give up consuming electricity, so we have to find alternatives for generating renewable, low-carbon energy. While dams have severe impacts on rivers and people, hydropower generates large amounts of electricity and can enable countries to increase the share of solar power and wind power in their energy mix. A crucial aspect in this transition is to develop policies that allow the communities burdened by hydroelectric developments to receive an equitable share of the benefits.

20.9 Exercises

Exercise *20.1:* Calculate the gravitational potential energy of the following objects:

1. A 50 kg object at a height of 12 meters.

2. A 12 kg object at a height of 50 meters.
3. A 100 pound object at a height of 10 meters.
4. A 200 kg object at a height of 100 yards.

Exercise *20.2:* The Hudson River discharges into the Lower New York Bay between Manhattan, NY and Jersey City, NJ. The average flow rate of the Hudson at its discharge is 21,900 cubic feet per second.

1. Convert this flow rate to cubic meters per second.
2. What is the mass flow rate, in kg per second, of the water flowing out of Hudson river?

Exercise *20.3:* In 2009, the total hydropower capacity worldwide[8] was 926 GW and the total amount of electricity generated was 3,551 TWh. What was the worldwide capacity factor for hydropower in 2009?

[8] (IPCC, 2011, Chapter 5)

Exercise *20.4:* Consider a hydroelectric plant with a hydraulic head of 30 meters and which dams a river that has a flow rate of 300 cubic meters per second. Assume an efficiency of 90 percent and a capacity factor of 60 percent.

1. What maximum power would you expect this hydropower plant to be able to deliver?
2. Accounting for the capacity factor, about how many homes could this plant supply electricity to, assuming that this plant is in a location where the average home uses 700 kWh of electricity in a month?

Exercise *20.5:* Consider again the Long Spruce power station discussed in Example 20.2. Assume that capacity factor[9] for the dam is 0.7.

[9] See `www.pub.gov.mb.ca/pdf/cos_review/exhibits/mh-43.pdf`, accessed June 20, 2021.

1. How much energy does the hydro station generate in one year?
2. Suppose an equivalent amount of energy was produced by a modern natural gas plant that emits 400 grams of CO_2 for every kWhof electricity generated. How much CO_2 would be emitted in one year?
3. This amount of CO_2 is equal to the yearly per capita emissions of approximately how many Canadians? In 2018 Canada's emissions were around 725 Mt CO_2e and its population was around 37 million.

Exercise *20.6:* This problem concerns the Chief Joseph Dam, shown in Fig. 20.2. The dam creates a reservoir, known as Rufus Woods Lake, with a capacity of $636 \times 10^6\,\mathrm{m}^3$. The average flow rate of the Columbia River is around 3000 m^3/s. Suppose the river stopped flowing, but water was still allowed to flow through the turbines at a rate of 3000 m^3/s. How long would it take the reservoir to empty?

Exercise *20.7:* This problem concerns the Hoover Dam, a dam in the Colorado River in the US. The Hoover dam creates a reservoir known as Lake Mead. The volume[10] of Lake Mead is approximately $3.5 \times 10^{10}\,\mathrm{m}^3$ and the average inflow is approximately $450\,\mathrm{m}^3/\mathrm{s}$. How long would it take to fill up Lake Mead?

[10] https://www.nps.gov/lake/learn/nature/overview-of-lake-mead.htm, accessed June 21, 2021.

Exercise *20.8:* The Hoover Dam[11] hydropower station has a capacity of 2,080 MW and the capacity factor of 0.23. Hoover Dam creates Lake Mead, whose area is approximately 640 km^2. What is the power density of the Hoover Dam, in W/m^2?

[11] https://en.wikipedia.org/wiki/Hoover_Dam, accessed June 22, 2021.

Exercise *20.9:* Lake Mead, the reservoir created by the Hoover Dam, has a volume of roughly $3.5 \times 10^{10}\,\mathrm{m}^3$. Suppose that an equivalent amount of water fell as rain over the state of Maine. How many inches of rain would this be?

21
Nuclear Power

21.1 Chemical Reactions

Before we look at nuclear reactions, let's talk some about chemical reactions. A chemical reaction is one in which chemical bonds are rearranged: molecules are typically broken apart and new molecules form, often releasing energy. A familiar example is combustion. As an example, here is the chemical reaction that occurs when one burns methane, the main component of natural bas:

$$\mathrm{CH_4} + 20_2 \ \rightarrow \ 2\mathrm{H_2}0 + \mathrm{CO_2} + \mathrm{energy}\ . \tag{21.1}$$

The methane, CH_4 combines with oxygen and is converted to carbon dioxide, water vapor, and energy. Burning one kilogram of methane will produce around 50 MJ of thermal energy. This energy can be used to heat your house, cook food, or produce steam that can turn a turbine and generate electricity.[1]

As we know, some initial heat is needed to begin combustion.[2] Once started, the burning process continues on its own. Some of the heat released by the chemical reaction in Eq. (21.1) is used to combust other methane molecules. The 50 MJ per kilogram that we get out is the net energy released—the energy that does not go into starting other molecules burning.

Once the combustion process starts, it is important to know how to make it stop. Fortunately, we know three ways to put out fire. First, we can remove the fuel. If, for example, we are gradually releasing methane into a hot boiler, we can stop releasing methane and, after the remaining fuel has burned, the fire is over. Second, we could cut off the supply of oxygen. Third, we could remove heat from the fuel. When we put water on on fire, it typically has the effect of both denying oxygen and cooling off the fire so the burning stops.

[1] Consulting a number of different sources yields a range of values for the heat of combustion of methane. Why is this? Perhaps different assumptions are being made about temperature or pressure?

[2] You can't start a fire without a spark Springsteen (1984).

21.2 Nuclear Fission

In the next section we'll look at nuclear fission and compare and contrast it to a chemical reaction. In a chemical reaction, chemical bonds are rearranged; in a nuclear reaction, nuclear bonds are rearranged. However, there are some important differences between these processes that will be discussed below.

Consider the isotope of Uranium with 235 nucleons: the total number of neutron and protons is 235. This atom is called Uranium-235 and is denoted by ^{235}U. The atomic number of Uranium is 92. This means that it has 92 protons. It thus has $235 - 92 = 143$ neutrons.

Uranium-235 is only a small fraction of naturally occurring Uranium, in which for every ^{235}U atom there are 140

The nucleus of an atom is made up of protons and neutrons (except for hydrogen, whose nucleus consists of a single proton).

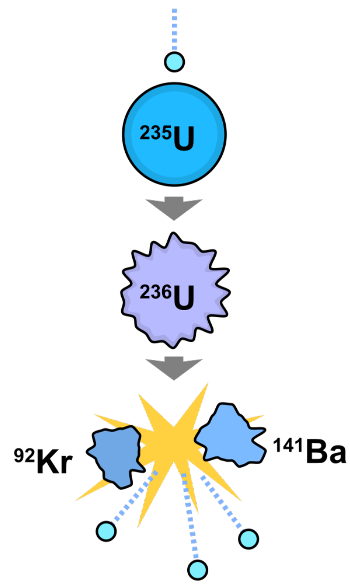

Figure 21.1: Nuclear fission that occurs when a ^{235}U atom under goes fission after absorbing a neutron. The large atom splits into two pieces, releasing energy and three additional neutrons. (Figure Source: Wikimedia Commons, released into the public domain by its author, Fastfission. https://en.wikipedia.org/wiki/File:Nuclear_fission.svg.)

21.3 Meltdowns and Accidents

21.4 Nuclear Waste

21.5 Nuclear Weapons and Terrorism

21.6 Exercises

Figure 21.3: The Seabrook Nuclear Power Plant Image source: Jim Richmond. Licensed under the Creative Commons Attribution-Share Alike 2.0 Generic license. https://creativecommons.org/licenses/by-sa/2.0/deed.en.

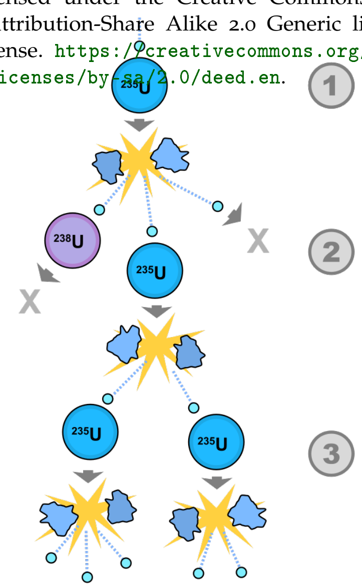

Figure 21.2: A schematic illustration of a nuclear chain reaction. Nuclear fission that occurs when a ^{235}U atom under goes fission after absorbing a neutron. The large atom splits into two pieces, releasing energy and three additional neutrons. (Figure Source: Wikimedia Commons, released into the public domain by its author, Fastfission. https://commons.wikimedia.org/wiki/File:Fission_chain_reaction.svg.)

Exercise *21.1:* The Seabrook Nuclear Power Plant, in Seabrook New Hampshire, is the second largest nuclear power plant in New

England. In 2016 it generated 10,800,000 MWh of electricity. The nameplate capacity of the plan is 1,240 MW.

1. What is the capacity factor of the plant?
2. How much area does the power plant take up? Answer this by finding the power station on Google maps.
3. What is the power density of the Seabrook plant. That is, how many Watts of electricity does it generate per square meter?
4. How much area would a solar PV installation need to be to produce as much power as the Seabrook plant? Express your area in a meaningful way.
5. The average home in New Hampshire uses 620 kWh of electricity per month. About how many homes could the Seabrook plant power?

22
Biomass

This chapter is just some mostly unorganized notes right now.

The idea of biofuels is to grow plants—corn, sugar cane, beets, whatever—and then use the plants to meet some part of our energy needs. There are three main ways this is done:

1. Plants are converted to diesel or ethanol, which is then used to make cars and trucks go.

2. Plants are burned directly to create thermal energy to make our houses warm in the winter.

3. Plants are burned (or products derived from plants) to make electricity.

Of course another traditional thing to do with sugars from plants is to feed them to humans and/or animals.

22.1 *Basic Considerations*

The process through which plants turn carbon dioxide[1] and sunlight into energy is *photosynthesis*. How efficient can plants do this? The range seems to be between 3%–6% Ehrlich (2013). This is just the efficiency for converting sun energy into chemical energy in the plants. We'd then have to figure out how efficiently we can do other things with this chemical energy: turn it into ethanol, burn it to heat our homes, or turn it into electricity.

[1] Plants eat CO_2 from the atmosphere.

Part of the reason that this efficiency is so low is that plants make use only of light in a fairly narrow band of wavelengths. Also, efficiencies tend to drop in higher light levels. When it is very sunny, plants can harness only a fraction of the photons that hit it. A good overview of biochemical and biophysical limits biofuel efficiency is the article by Sinclair in *American Scientist* Sinclair (2009).

Typical efficiencies for crop plants seem to be around one or two percent. Zhu et al Zhu et al. (2008) estimate that the maximum possible efficiency is 4.5% for C3 plants and 6% for C4 plants. They report that the highest efficiencies reports are 2.4% for C3 crops and 3.7% for C4 crops.

The basic efficiencies for biofuel are going to come out very low compared to photovoltaics, which produce very roughly 10 W/m^2. However, one important thing that biofuels have going for them is that they are easy to store, whereas electricity is very difficult to store—doing so requires batteries of some sort which are expensive.

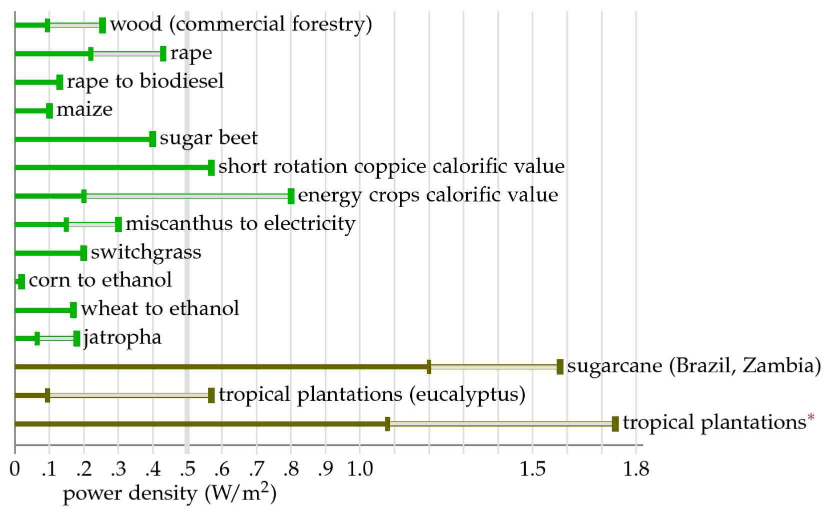

Figure 22.1: Efficiencies for various forms of bio-energy. Figure 6.11 from MacKay (2009).

Figure 22.1 (MacKay, 2009, Fig. 6.11) shows the efficiencies of various forms of bio-energy in units of W/m^2. All of these efficiencies are puny compared to solar PV. The statistics shown in green are for northern Europe. Based on this, it seems very unlikely that bioenergy could be a reasonable contributor to energy in northern Europe or similar climates. There is some hope that bioenergy grown in tropical regions could contribute non-negligibly.

There are two challenges with biomass. Biomass uses land and takes energy to make—energy has to be input to transport and process the biomass. These considerations will be discussed in the next two chapters.

22.2 *Lifecycle Greenhouse Gas Emissions*

Figure 22.2 shows the lifecycle GHG emissions for a variety of different fuels. Before talking about biofuels, let's look at ordinary gasoline—a fossil fuels. The EPA estimates that the lifecycle GHG emissions for gasoline are 98.2 kg CO_2e per MMBTU. Converting MMBTU to kWh, this is: **355 g** CO_2e /kWh. This includes the 240 grams of CO_2 emitted

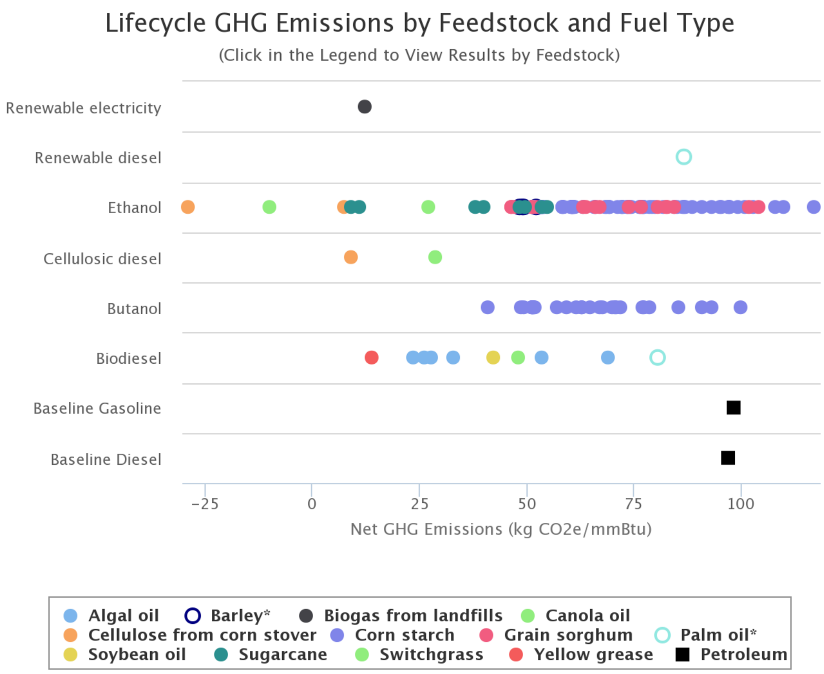

Figure 22.2: Lifecycle GHG emissions for various biofuels. Figure from https://www.epa.gov/fuels-registration-reporting-and-compliance-help/lifecycle-greenhouse-gas-results, accessed 10 November, 2019. The live version on the webpage is interactive; hovering over each data point reveals the numerical data.

when the gasoline is burned, and also the CO_2e emitted when fuel is mined, processed, transported, etc.

Looking at Fig. 22.2, we see that some biofuels are associated with sizable emissions. Some ethanol production leads to more CO_2e per kWh than gasoline. Yikes. Major yikes.

Why would anyone think biofuels are a good idea in the first place? After all, burning plants (or plants processed into biofuels) certainly releases CO_2 into the atmosphere. The difference is that the carbon in plants was already part of the atmospheric carbon cycle. So burning those plants does not increase the total carbon in atmospheric carbon cycle. In contrast, burning fossil fuels does increase the carbon in the atmospheric carbon cycle. See Fig. 22.3.

However, there is the matter of time scales. Carbon released from burning plants will stay in the atmosphere for a while before it gets re-absorbed by growing plants. So if we suddenly start burning down forests, that's going to put a lot of additional CO_2 into the atmosphere that won't revert to plants for several decades.

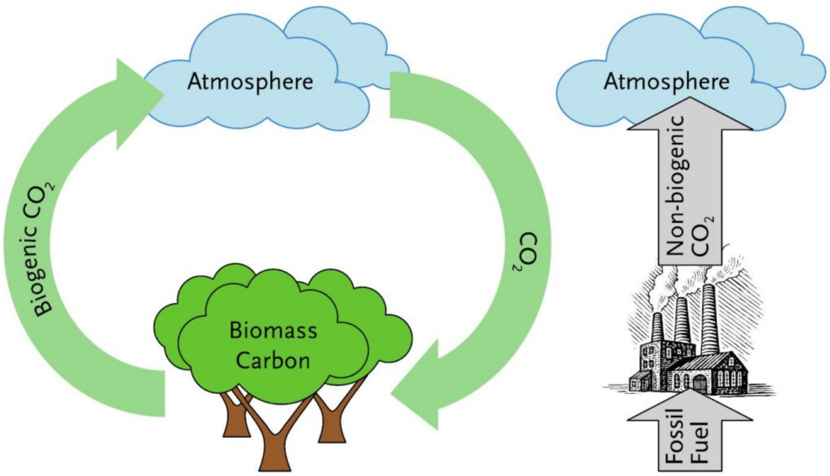

Figure 22.3: A schematic view of the carbon cycle. Figure from http://www.wfpa.org/our-forest-today/climate-change/, accessed 10 November, 2019.

22.3 Land Use

22.4 Some Facts about Ethanol

One thing we can do with chemical energy (sugars) from plants is to turn them into ethanol. Doing so requires some processing which takes energy. Some ways to measure the efficiency of bioethanol:

- *Net energy ratio* or NER. This is the ratio of the energy supplied by a biofuel to the energy required to create it. For example, an NER of 1.5 means that to get 1.5 kWh of energy would require an input of 1 kWh.
- Fuel produced per unit area of land. This doesn't account for the energy used to process the plants into whatever form we want.
- GHG reduction. This measures how much less GHGs are produced by the biofuel compared to the comparable non-bio fuel. This usually[2] takes into account the changes in land use associated with the production of the plants.

[2] Is this standard?

The two biggest producers of bio-ethanol are the US and Brazil. But the two countries use different crops to do so, with very different results. In the US, we turn corn into ethanol; in Brazil they use sugar cane. Some facts (Ehrlich, 2013, p. 140):

- Brazil: 1798 gallons of ethanol per hectare. NER is $8.3 - -10.2$. GHG reduction is 61%.
- US: 900 gallons of ethanol per hectare. NER[3] is 1.3. GHG reduction is 19%.

[3] This seems high. Elsewhere I've seen estimates that are a bit lower.

Ethanol can be used in cars. Per gallon (or liter), ethanol produces less thermal energy than gasoline. Burning one gallon of gasoline produces around $120,000$ BTU while igniting a gallon of ethanol produces only around $84,000$ BTU. What this means is that one needs to burn 1.42 gallons of ethanol to produce the same thermal energy obtained by burning a gallon of gasoline.

22.5 Outlook

- Unlikely to increase efficiencies of plant photosynthesis, as they are already pushing against biophysical and biochemical limits.
- Biofuels probably have a role to play, but a small one, in a sustainable energy future. Biofuels can be used to replace fossil fuels when electrification isn't possible. A prime example is jet fuel.
- Corn ethanol subsidies in the US are insane. We spend a lot of money propping up a technology that is only marginally better from a carbon point of view than gasoline, while taking land out of food production. This is great for corn farmers, but not-so-great for the rest of us.
- Biofuels or bioenergy is probably not a useful term, as it is so broad. It encompasses a very wide range of stuff.
- There's some reason to believe that algae technology could be developed further and could be feasible. (But does this all depend on growing the algae in an enhanced-CO_2 environment?)

22.6 Misc References

- Another source for lifecycle GHG emissions for biofuels: https://www.gov.uk/government/publications/renewable-transport-fuel-obligation-rtfo-guidance-2019.

22.7 Exercises

Exercise *22.1:* The average miles driven by a car annually in the US is $14,425$. In so doing the average car uses 733 gallons of fuel. There are $221,000,000$ cars in the US.[4]

1. What is the mpg of the average car in the US?
2. How much fuel is used in the US by all cars in one year?
3. Suppose we wanted to replace all of this gasoline with ethanol derived from US corn? How much ethanol would we need?

[4] All figures from https://www.fhwa.dot.gov/ohim/onh00/onh2p11.htm. I think this data is from 1990.

4. How much land would be required? Express this area in some way that makes sense.

(Note, btw, that this doesn't include the fuel used for trucks and planes.)

Exercise *22.2:* To heat an average home in Maine requires approximately 540 gallons of fuel oil per year.[5]

1. How much thermal energy does this fuel oil produce? Answer in BTUs and kWh.
2. What power does this correspond to?
3. Suppose that you decide to heat your house with wood. Assume that the efficiency of your woodstove will be the same as the efficiency of your oil furnace.[6] How much land would you need to get this amount of power? Refer to Fig. 22.1 from the textbook.
4. There are very roughly half a million homes in Maine[7] How much land would be needed if all of Maine was to heat with wood? Put this number in perspective. Is this a little or a lot? What fraction of Maine is this?

[5] https://www1.maine.gov/energy/fuel_prices/heating-calculator.php, accessed November 6, 2017.

[6] This might not be a super assumption, but since in what follows we're interested in getting a very rough estimate, I think this assumption is ok.

[7] http://www.census-charts.com/HF/Maine.html, accessed November 6, 2017.

Exercise *22.3:* This exercise is based on problem 4 on page 153 of Ehrlich (2013). Suppose a farmer has 10 pigs and wants to use the pigs' waste to generate some electricity. Assume that each pig generates 1 kg of solid waste per day, from which one can obtain 0.8 m^3 of methane. Burning one cubic meter of methane will produce 38 MJ of thermal energy.

Figure 22.4: A pig in a bucket. Ben Salter, licensed under the Creative Commons Attribution 2.0 Generic license, https://creativecommons.org/licenses/by/2.0/deed.en Available at https://commons.wikimedia.org/wiki/File:Pig_in_a_bucket.jpg.)

1. Let's suppose that we can turn the thermal energy from burning methane into electricity with a 25% efficiency. How much electric power would the farmer get? Express your answer in kW.
2. Put this amount of energy into perspective. Could this amount of power be sufficient to provide electricity to the farmer's home?

Exercise *22.4:* In the US, one gets around 900 gallons of corn ethanol from one hectare of land every year.

1. Convert this into Watts per square meter. (The calorific value of ethanol is 84,000 BTU/gallon.)
2. Put this number into perspective. How does it compare to the power density of solar PV and terrestrial wind turbines?

Exercise *22.5:* Repeat the above question, but for ethanol from Brazilian sugar cane, which produces around 1800 gallons of ethanol per hectare.

Figure 22.5: Sugarcane being harvested in São Paulo state, Brazil. Image source: https://commons.wikimedia.org/wiki/File:Caminh%C3%A3o_Carregado.jpg, released into the public domain by its author, Edrossini.

Exercise *22.6:* The total amount of jet fuel used by US airlines in 2016 was roughly 17 billion gallons.[8]

[8] https://www.eia.gov/todayinenergy/detail.php?id=31512, accessed November 6, 2017.

1. Suppose we want to replace this fuel with ethanol derived by corn. How much ethanol would be needed to do this. Note that the energy content of ethanol is about 1/3 less than that of gasoline. Since ethanol has 2/3 the energy of gasoline (and jet fuel), we would need to replace every gallon of gasoline with 1.5 gallons of ethanol.
2. How much land in the US would be needed to produce this amount of ethanol from corn?
3. Put this area in perspective. Express it a meaningful way.

Exercise *22.7:* Ho et al. (2014) have collected data for the yields of a number of different types of biomass. For example, they find that eucalyptus yields 10–12 tons of biomass per hectare per year, and that the thermal energy resulting from burning a ton of eucalyptus ranges from 0.2 to 1.2 toe, where *toe* stands for tons of oil equivalent. This is yet another unit for energy (see Section A.2.6).

1. What is the power density for eucalyptus in W/m^2? Calculate low and high values using the ranges given by Ho, et al.
2. Put this power density into perspective. How does it compare to wind or solar?

Part V

Systems

23
The Grid

23.1 The Grid

23.2 Overview

23.3 Feed-in Tarrifs

This article from NREL might be a good primer http://www.nrel.gov/tech_deployment/state_local_governments/basics_tariffs.html.

23.4 Smart Grids

Say what is meant by this. Why smart grids matter.

23.5 Smart Grids and Electric Vehicles

Another reference to check out http://erpuk.org/project/managing-flexibility-of-the-electricity-sytem/. Looks like

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Example 23.1. *Statement of example*

Solution goes here

23.6 Batteries

Some battery facts. Batteries store energy. How much?

- AAA batteries store around 1.5 Wh of energy (or 0.0015 kWh).
- Prius (hybrids) have a battery capacity of around 1.5 kWh
- The powerwall by Tesla has a capacity of 13.5 kWh and costs around $6000, but this does not include installation. With installation, costs are $8,200-$14,200.[1]

[1] https://www.energysage.com/solar/solar-energy-storage/tesla-powerwall-home-battery/

- The second-generation Chevy Volt has an 18.4 kWh battery.

All facts are from wikipedia pages unless noted.

There are 250 million vehicles in the US. If they all had a 15 kWh battery, similar to the Chevy Volt, what is the total storage capacity? Let's see

$$250 \times 10^6 (15\,\mathrm{kWh}) \left(\frac{1\,\mathrm{GWh}}{10^6\,\mathrm{kWh}} \right) = 3750\,\mathrm{GWh}\,. \qquad (23.1)$$

I suspect that storage and batteries will eventually become their own chapter. Time will tell.

23.7 *Exercises*

Exercise *23.1:* The Raccoon mountain pumped storage facility, one of the largest in the US, can store around 35 GWh. For how approximately long could this "battery" provide electrical power to all the homes in Maine?

Exercise *23.2:* The Tesla powerwall battery has a capacity of 13.5 kWh. The typical US home uses 900 kWh of electricity per month. For how many hours could the powerwall batter provide power to such a house?

Exercise *23.3:* Each Tesla powerall battery has a capacity of 13.5 kWh and costs around $10,000 (including installation).

1. How many Tesla powerwall batteries would be needed to get 35 GWh of storage?
2. How much would all these batteries cost? Compare this to the cost of the Raccoon Mountain pumped storage facility (1.2 billion dollars).

Exercise *23.4:* The batterics in electric cars vary, but roughly 30 kWh seems to be a reasonable average for newer all-electric cars. How many such batteries would be needed to get a storage capacity of 6 GWh?

24
Carbon Capture and Negative Emissions

blah blah

24.1 How to Capture Carbon

24.2 How to Store Carbon

Some notes to myself:

- See the power set of slides that I made for class
- Check out this article. It looks good: http://www.carbonbrief.org/in-depth-experts-assess-the-feasibility-of-negative-emissions.
- Maybe also http://www.carbonbrief.org/carbon-capture-essential-for-climate-friendly-fracking-in-uk-say utm_content=buffer88eb2&utm_medium=social&utm_source=twitter.com&utm_campaign=buffer?
- This looks good: https://www.carbonbrief.org/beccs-the-story-of-climate-changes-saviour-technology
- And http://www.pnas.org/content/109/14/5185.full
- Another thing: https://makewealthhistory.org/2016/06/10/turning-waste-co2-into-stone/.
- http://www.carbonbrief.org/beccs-the-story-of-climate-changes-saviour-technology
- Biophysical and economic limits to negative CO_2 emissions. Article from Nature Climate Change in 2015: http://www.nature.com/nclimate/journal/vaop/ncurrent/full/nclimate2870.html
- Biofuels: http://www.climatecentral.org/news/study-biofuels-worse-for-climate-than-gasoline-20634.

24.3 Exercises

Exercise *24.1:* In this exercise you'll work through some calculations to get a sense of the scale needed for CCS. We're just interested in a rough estimate, so round to one or two significant digits throughout.

1. How much CO_2 is emitted per year by the US?
2. Suppose we want to put one tenth of this CO_2 underground. To do so, we would need to liquefy the CO_2. What is the volume of the CO_2 we would need to put under the earth? The density of liquid CO_2 is around 1100 kg per cubic meter.
3. Now suppose that we wanted to put the liquid CO_2 deep underground somewhere in a saline aquifer or old oil well. Let's imagine that this is an empty cavity that has a height of 100 meters. What would be the floor area of such a cavity sufficient to store this CO_2? Are you surprised by the answer?

Part VI

Financial Mathematics

25
The Time Value of Money

Add a paragraph introducing this section. Why are we talking about finance? Well, the hard part of most renewable energy projects is paying for them. There are lots of great things one can do with currently existing, standard technology. Many of these technologies will save a lot of money, but not all at once. They may have a large up-front cost and then produce savings over a period of many years. Should you put solar panels on your roof? Improve the insulation in your house? Get a better water heater or an electric car? Ideally you would do all of these these. But alas, time and money are limited, so we have to choose? How can we decide among several appealing options?

The goal of this part of the book is to teach you some financial mathematics that can help you make these decisions. You'll learn a framework that will enable you to determine the financial value of different investments. This framework and vocabulary is one that is commonly used by investors and CFOs. You'll need to speak this language if you want to go to banks or boards of directors to seek funding for your projects.

25.1 A Thought Experiment

Example from class: a machine that produces $1000 every year, guaranteed, forever. How much would you pay for this machine? Write a few paragraphs about ways to think about this and what issues come up when doing so.

25.2 Time Value of Money

One of the key issues that arises when thinking about the example in the previous section is that money in the future is not as valuable as it is today. This general phenomenon is known as the *time value of money*. If given the choice between being given $100 today or $100 two years from now, everybody would select the first option. What if the two options were $100 today or $120 two years from now. There obviously isn't a right or wrong answer to this question—it depends on your circumstances. How badly do you need that money now? And what would you do once you got it? In this section we introduce some important tools that can help us make decisions like this.

Suppose you got the $100 today and invested it in a bank account that earned 5% interest every year. After one year, you would have

$$\text{Value after one year} = 100(1.05) = 105 . \tag{25.1}$$

After two years you would have,

$$\text{Value after two years} = 105(1.05) \approx 110 , \tag{25.2}$$

where we've rounded to the nearest dollar. The 105 in this equation arose because we multiplied 100 by 1.05 in Eq. (25.1). So we can write

$$\text{Value after two years} = 100(1.05)^2 \approx 110 . \tag{25.3}$$

After two years, the initial 100 has been multiplied by 1.05 twice. And after t years, the initial 100 gets multiplied by 1.05 t times:

$$\text{Value after } t \text{ years} = 105(1.05)^t . \tag{25.4}$$

We'll return to this equation in a moment. Let's first go back to the the conundrum: would you rather have $100 today or $120 in two years. According to this analysis, you would be better off waiting two years and taking the $120. Why? Because we saw in Eq. (25.3) if you accepted the $100 now and deposited it in the bank account it would grow to $110 in two years—less than the $120 you could get by simply waiting.

Of course this analysis assumes that if you got the $100 you would put it in the bank, or do something with it that would make its value grow by five percent a year. If you were starving, you would use the $100 to buy food right away, and not invest it. The point is that this sort of financial analysis makes sense in the context of investment, not survival.

25.3 *Future Value of a Payment Today*

We return now to Eq. (25.4). It tells us how much $100 will be worth in t years if we invest those hundred dollars in a bank account or some investment that grows at 5% a year. We can generalize this result as follows:

$$\text{FV} = \text{PV}(1+r)^t . \tag{25.5}$$

In this equation PV is the present value of a payment and FV is the future value—how much it would be worth t years into the future assuming an interest rate of r.[1] Let's do a few examples to see how this equation can be used.

[1] Does using FV and PV as variable names bother you as much as it bothers us? We're sorry. It seems to us to be perverse to use *two* letters for *one* variable. Unfortunately, this notation is quite standard. And this sort of thing is a common occurrence in economics, where combinations of letters are frequently used for single variable.

Example 25.1. *Suppose you invest \$1000 in an investment that earns* 4% *a year. How much money would you have in* 10 *years. What if the interest rate was* 5% *instead of* 4%*?*

Using Eq. (25.5), we have

$$FV = 1000(1.04)^{10} \approx 1480 \, . \tag{25.6}$$

If the interest rate was 5%, then:

$$FV = 1000(1.05)^{10} \approx 1629 \, . \tag{25.7}$$

Note that a fairly small difference in the interest rate r makes a large difference in the future value.

Example 25.2. *In* 25 *years you wish to have \$100,000 to use to help your daughter go to college. How much should you deposit in a bank today in order to do this? Assume an interest rate of* 5%.

In this problem we are given the future value; we want to have a bank account that is worth \$100,000 twenty-five years in the future. What we are looking for is the present value. We can solve Eq. (25.5) for PV. To so, we divide both sides of the equation by $(1+r)^t$, obtaining

$$PV = \frac{FV}{(1+r)^t} \, . \tag{25.8}$$

Then, plugging in, we find

$$PV = \frac{100,000}{(1+0.05)^{25}} \approx 29,530 \, . \tag{25.9}$$

So \$29.530 deposited in a bank account today would grow to \$100,000 in twenty-five years, if you were fortunate enough to have a bank account that paid 5% interest annually.

25.4 Present Value and the Discount Rate

Equation (25.5) tells us the future value of a pile of money (or whatever) that has a value of PV today. As we saw in Example 25.2, above, we can take this equation and rearrange it to isolate PV. Doing so, we obtain:

$$PV = \frac{FV}{(1+r)^t} \, . \tag{25.10}$$

We use this equation to tell us the present value of a payment that we receive t years in the future and which has a value of FV.

Equations (25.5) and (25.10) look quite similar, and mathematically are equivalent, in that one equation can be algebraically transformed into the other. But they represent different points of view. The equation for future value is thinking forward, from the current moment into the future t years away. In this context, the rate r is usually thought of as the interest rate on an account or the rate of return on an investment.

Equation (25.10) moves in the opposite direction. It takes a future payment of FV and transforms it backwards through time to the present. In this context, the rate r is usually referred to as the *discount rate*. That is, r is the amount by which future payments are discounted.

There are two important things to bear in mind about the discount rate. First, the present value is an exponential function of time, where $1 + r$ is the base of the exponent. This means that small changes in the interest or discount rate can make a very big difference. The same is true about the future value. The future value of your bank account depends quite strongly on the interest rate r.

The second important thing about the discount rate is that it is a made-up quantity. Where does the discount rate come from? Why might we sometimes use five percent and other times use ten percent? The answer is that the discount rate is, in large part, a heuristic—a quantity that we use to help us think through how we made tradeoffs between the present and the future. [2]

Cite (Richter, 2014, Chapter 7) comments about discount rate.

[2] Use the term *productivity of capital* and discuss. **TODO!**

Example 25.3. *You are to receive a payment of* 4000 *dollars five years from now. What is the present value of that payment if the discount rate is five percent? What is the present value of the discount rate is ten percent?*

We will use Eq. (25.10):

$$PV = \frac{FV}{(1+r)^t} = \frac{4000}{(1+r)^5}, \tag{25.11}$$

where we've plugged in $FV = 4000$ and $t = 5$. For $r = 0.05$, I get $PV \approx 3134$. If the discount rate is ten percent, then $r = 0.10$ and we obtain $PV \approx 2484$.

25.5 Doubling Time

How long does it take for an investment to double? That is, when does $FV = 2PV$? Let's plug this in to the formula for future value:

$$2PV = PV(1+r)^t . \tag{25.12}$$

We need to solve this equation for t. First, we divide both sides of the equation by PV to obtain

$$2 = (1+r)^t . \tag{25.13}$$

Next, take the natural logarithm of both sides.[3] Doing so, we obtain

$$\ln(2) = \ln((1+r)^t) . \tag{25.14}$$

Using the properties of logarithms, we solve for t:

$$t = \frac{\ln(2)}{\ln(1+r)} . \tag{25.15}$$

[3] If you're not familiar with logarithms, it's not the end of the world. We'll end up with an approximate formula that doesn't involve a log.

This is an equation for the doubling time of an investment. The variable t is how long it takes an investment to double in value, given an interest rate of r compounded annually.[4] This equation is exact.

[4] We should briefly mention different types of compounding somewhere.

There are some approximate forms for Eq. (25.15) that are very useful. We will mention one of them here. The doubling time t can be well approximated by

$$t \approx \frac{72}{R} . \tag{25.16}$$

This formula is known as the *rule of* 72. In this formula, we've used a capital R to indicate that the rate is plugged in as a percent. So if the interest rate was 5 percent, $R = 5$, not $r = 0.05$. The use of this formula and the exact result, Eq. (25.15) is illustrated in the following example.

Example 25.4. *How long would it take for your investment to double with an annual interest rate of* 6%*? Calculate the doubling time using the exact formula, Eq. (25.15) and the rule of* 72*.*

First, let's use the exact formula for the doubling time t:

$$t = \frac{\ln(2)}{\ln(1+0.06)} \approx 11.9 . \tag{25.17}$$

So using the exact value we see that the investment doubles in around 11.9 years. Let's see what we get using the rule of 72, Eq. (25.16):

$$t \approx \frac{72}{6} = 12 . \tag{25.18}$$

So we see that the two equations give very similar results.

Add discussion of rule of 72 versus rule of 70. **TODO!**

25.6 Non-financial Discounting

Just a placeholder for now. Discuss the social cost of carbon.

- `http://grist.org/article/discount-rates-a-boring-thing-you-should-know-about-with-otters/`. Looks like an excellent article.
- `https://www3.epa.gov/climatechange/EPAactivities/economics/scc.html`.
- `http://green.blogs.nytimes.com/2012/09/18/the-social-cost-of-carbon-how-to-do-the-math/?_r=0`
- Johnson and Hope, The social cost of carbon in U.S. regulatory impact analyses: an introduction and critique. `http://link.springer.com/article/10.1007%2Fs13412-012-0087-7`.
- Stern vs. Nordhaus on cost of carbon and different discount rates.
- Would be interested to see what Frank Ackerman has written or has to say about this.

25.7 Exercises

Exercise *25.1:* In 15 years you wish to have \$200,000 to put toward buying a house. How much money should you deposit in a bank today to do so? Assume an interest rate of 5 percent. How much money would you need to deposit if the interest rate is 7 percent?

Exercise *25.2:* Suppose that in fifty years someone will give you a million dollars. What is the present value of this gift?

Exercise *25.3:* You deposit 1,000 Euro in a bank account with an interest rate of 1 percent. How much money do you have in six years? How long does it take to double your money?

Exercise *25.4:* Would you rather receive \$1000 today or \$1300 four years from now? Explain briefly.

Exercise *25.5:* You are considering spending some money today that will save you 1,000,000 pesos in four years. What is the value of this savings today? Assume a discount rate of 5 percent.

Exercise *25.6:* You invest \$15,000. Assume your money grows at 4% annually.

1. How much money do you have in ten years?
2. How long would it take your money to double?

Exercise *25.7:* Someone will give you \$10,000 in ten years.

1. What is the present value of this payment? Answer this question using discount rates of 5, 10, and 15%.
2. Repeat the above analysis, but assume that the payment comes in 100 years and not 10 years.

Exercise *25.8:* Fill in the missing steps between Eq. (25.14) and (25.15).

Exercise *25.9:* Calculate the doubling time using both Eq. (25.15) and the rule of 72 for each of the following interest rates:

1. 2%
2. 4%
3. 6%

4. 8%

Exercise *25.10:* 1. In one year you will receive a payment of $2000. What is the present value of this payment?

2. In two years you will receive another payment of $2000. What is the present value of this payment?

3. In three years you will receive yet another payment of $2000. What is the present value of this payment?

4. What is the total present value of all three of these payments?

Exercise *25.11:* Let's return to the example that began this chapter—the value of a machine that gives you $1000 every year forever. Use a spreadsheet or write a program to determine the value of:

1. The first 10 payments
2. The first 100 payments
3. The first 1000 payments
4. The first 10000 payments

What do you think happens to the present value of the machine as we account for more and more payments?

Exercise *25.12:* Continuing Exercise 25.11, use math to come up with an exact formula for the net present value of a machine that gives you $1000 every year forever. To do so, write the NPV as a geometric series and use the forumal for the sum of an infinite geometic series. (This is a standard result often taught in the second term of calculus classes.)

Exercise *25.13:* Add problem where students derive the rule of 70.

26
Valuing and Comparing Investments

In this chapter we will learn about different ways of valuing or comparing investments.

It is very easy to come up with good ideas for generating renewable energy or decreasing energy consumption. The hard part is choosing among several good options and then securing funding so you can carry out these projects. Should you install solar cells on your roof, insulate and seal your house, buy an electric car? If you're like us, you can't afford to do all of these things at once, even if you'd like to. How can you choose?

Surely one set of considerations will be financial. If you insulate your house you will lower your energy bill. But by how much? You will enjoy these savings forever—or at least as long as you own your house. But as we have seen in the previous chapter, money in the future is not as valuable as money today. How can we account for this?

There are a number of common metrics that are used to evaluate and compare investments. In this chapter we will introduce these metrics and give you ways to think about what they mean. It is our opinion that these metrics each have strengths and weaknesses, and we will lay these out. The terms and techniques we introduce here are very standard in the world of investing, and are used not just when working with sustainable energy. In order to get a project funded, you will inevitably need to talk to funders—bankers and venture capitalists and the like. To do so effectively, you will need to be able to carry out these analysis and speak their language.

26.1 Two Examples

Here are two examples of businesses that we will use to motivate the analysis that follows. These examples are obviously too simple and not quite realistic, but they will be useful for illustrating the financial metrics that we'll develop below.

Demeo's Donuts

Anna is thinking of opening a donut business. The business will cost $20,000 to start up. She'll need to buy donut-making machines and such. Once the business is up and running, she expects to make $5000 a year for five years. She'll thus make a total of $30,000.

Figure 26.1: mmmm...Donuts. (Figure source: Rob Boudon. https://www.flickr.com/photos/robboudon/824752133, licensed under Creative Commons-Attribution-NonCommercial-ShareAlike 2.0 https://creativecommons.org/licenses/by-nc-sa/2.0/.)

Brooklyn Bagels

Dave grew up in New York City[1] and has a fondness for bagels. So he is keen to open a bagel business. He expects to make $2000 in revenue in year one and that his revenue will increase by $3000 each year after that. So Dave anticipates a revenue of $5000 in year two, $8000 in year three, $11,000 in year four, and $14,000 in year five. He expects sales to start off slow, because Dave lives in Maine, and Mainers don't necessarily appreciate fine bagels. So it will take a while for word to spread and for his business to take off. Starting the bagel business costs as much as starting the donut business: $20,000.

[1] He's from Manhattan, not Brooklyn, but Brooklyn is trendy now, so he thought it would be a good name for his bagel business. Also, it alliterates nicely.

The projected revenue streams for the two businesses are shown in Table 26.1.

Table 26.1: Projected revenues for the donut and bagel businesses. Both businesses can be started for an initial investment of $20,000.

Year	Donut Revenues	Bagel Revenues
1	$6,000	$2,000
2	$6,000	$5,000
3	$6,000	$8,000
4	$6,000	$11,000
5	$6,000	$14,000
Total:	$30,000	$40,000

TODO! Check the numbers in the Brooklyn Bagels example. See emails from Kobi Eng and Yoi Ashida, October 2019.

Figure 26.2: mmm...Bagels. (Figure source: Ezra Wolfe, https://www.flickr.com/photos/ezraw/86861363 licensed under Creative Commons-Attribution-ShareAlike 2.0 https://creativecommons.org/licenses/by-sa/2.0/.)

26.2 Payback Time

When considering buying a donut or bagel business—or investing in renewable energy or energy conservation—one important consideration is the *payback time*. This is basically just what sounds like. The payback time is the time needed to earn back the original investment. For the donut business, the payback time is a bit more than three years. After three years, the total revenue Anna will have earned from her business is 18. After four years, she will have earned 24. So between year three and four her revenue will cross $20,000 and she will have made as much money as she spend. (Recall that the cost of the donut business was $20,000.

What about the bagel business? Let's see. After three years Dave will have made \$15,000 in revenue and after four years he'll have made \$26,000 in revenue. So as was the case for the donut business, the payback time is somewhere between three and four years. years you'll have \$18,000 in revenue, and revenue will reach \$30,000 after five years. So the payback time is a bit more than four years. As one would suspect from looking at Table 26.1, the donut business pays for itself a bit faster than the bagel business.

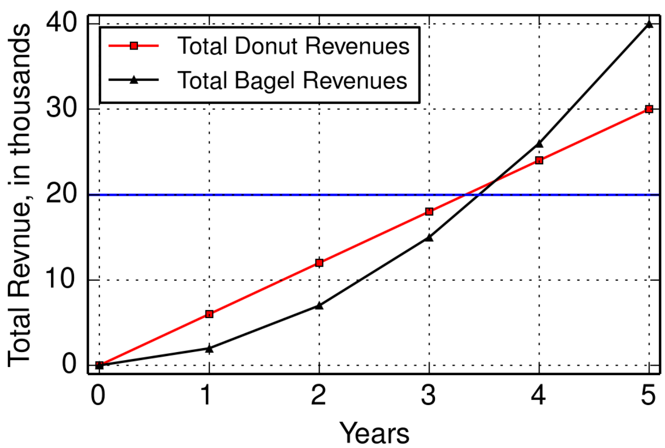

Figure 26.3: The cumulative revenue for the donut and bagel businesses. The revenues were given in Table 26.1. We see that both businesses have a payback time between three and four years.

This situation is shown graphically in Fig. 26.3. The cumulative revenue for the donut business is shown in red and while the bagel business's cumulative revenue is shown in black. The horizontal blue line is the break-even point: \$20,000. We can see that the red and black lines both cross the the blue line between years three and four. The red line crosses a bit sooner, but for all practical purposes they cross at the same time. Remember that the revenues in Table 26.1 are just estimates; they're statements about the future and, of course, the future hasn't happened yet, so we can't be sure about it. So the bottom line is that these two investments, the bagel and donut businesses, have essentially the same payback time.

In any event, the payback time is one very common way to evaluate an investment. Benefits of the payback time are that it is simple to calculate, intuitive, and easy to understand. However, it does not take into account the time value of money. Equally important, it doesn't tell you anything about what happens after you have paid back your original investment. After you've broken even, do you make any more money? If so, how much? Or do you expect the investment to stop making money for you shortly after you've broken even, perhaps because the equipment has reached its lifetime? Put another way, the payback time tells you how long it will take you to break even (without discounting) but says nothing about how much money you will make, if any, after the break-even point.

26.3 ROI: Return on Investment

A metric complementary to payback time is the *return on investment*, or *ROI*. Like the payback time, the ROI is basically just what it sounds like: how much money an investment makes divided by the cost of that investment.

$$\text{ROI} = \frac{\text{Gain from Investment} - \text{Cost of Investment}}{\text{Cost of Investment}} \tag{26.1}$$

For the donut business, the cost of the investment is \$20,000 and the return—the total revenue—is \$30,000. Thus the ROI for the donut business is:

$$\text{ROI} = \frac{\$30,000 - \$20,000}{\$20,000} = 0.5\ . \tag{26.2}$$

The ROI is a percent. In this case we would say that the return on investment is 50%. The profit for the donut shop is \$10,000, which is 50% of the cost of the investment. That is, fifty percent of \$20,000 is \$10,000.

The ROI for the bagel business is higher. The bagel shop pulls in more revenue than the donut business, yet both businesses cost the same. The ROI for the bagel business is:

$$\text{ROI} = \frac{\$40,000 - \$20,000}{\$20,000} = 1.0\ . \tag{26.3}$$

So the ROI is 1, or 100%, indicating that Dave's bagel shop will double his money in five years. So if we look only at ROIs, it would seem that the bagel business is a much better investment than the donut shop.

But we hope this seems a bit too simple. The bagel shop makes more money in the long run, but in the first two years, the donut store does better than the bagel shop during the first two year. And due to the time value of money, revenue a few years out is not worth as much as revenue today or next year. So those big bagel-store revenues in years four and five ought to be discounted. The ROI does not account for the time value of money. All that matters in a calculation of ROI is how much revenue you get total, not when you get it. The next two investment metrics we'll encounter account for the time value of money. Before we turn our attention to them, however, a few additional remarks about ROI.

First, the notion of ROI can be extended to account for things other than money. Of note is the energy ROI, in which the accounting is done purely in energy units. For example, suppose we were interested in a wind turbine. To calculate the energy ROI one would measure the cost by determining the amount of energy used to make, transport, install, and maintain the turbine. The gain would be the total energy that the wind turbine would generate over its lifetime

Second, people say ROI and mean all sorts of different things. At issue is what gets counted as a cost and what gets counted as a gain. Also, often ROI is expressed per year, but other times it isn't.

26.4 NPV: Net Present Value

The *net present value*, or NPV, of an investment is the total present value of all future revenue. For example, for the donut investment the NPV would be calculated as follows. First we need to figure out the present value of the revenue of $6,000 that is received one year in the future. To do we need to choose a discount rate. We'll use five percent. Using Eq. (25.10), this is:

$$PV = \frac{\$6,000}{(1+0.05)^1} = \$5,710 \,. \tag{26.4}$$

Next, we need to determine the present value of the $6,000 that is received in year two. Doing so, and again assuming a five percent discount rate, we obtain:

$$PV = \frac{\$6,000}{(1+0.05)^2} = \$5,440 \,. \tag{26.5}$$

Year	Donut Revenues	Present Value	Bagel Revenues	Present Value
1	$6,000	$5,714	$2,000	$1,900
2	$6,000	$5,440	$5,000	$4,540
3	$6,000	$5,180	$8,000	$6,910
4	$6,000	$4,940	$11,000	$9,050
5	$6,000	$4,700	$14,000	$10,970
Total:	$30,000	$25,980	$40,000	$33,370

Table 26.2: Projected revenues and their present values for the donut and bagel businesses. These calculations were done assuming a five percent discount rate. For this discount rate, the NPV of the donut business is $25,980; for the bagel shop it is $33,370.

We could continue on, using Eq. (25.10) to calculate the present values of the donut revenues for years three, four, and five. The result of doing this is shown in the third column of Table 26.2. The sum of all of the present values of the donut revenues is the *net present value*, or NPV, of this investment. For the donut business, the NPV is $25,980.

The NPV is, as the name suggests, the total present value of all future revenues. It is a way of putting a value on an investment. For the donut example, the NPV of \$25, 980 is the value of the business, assuming a discount rate of five percent. If someone offered to sell us the business for \$22, 000, this would be a pretty good deal; we could buy the business for a good bit less than it is worth.

We can also calculate the NPV for bagel business. The present values of all the future revenues are shown in the rightmost column of Table 26.2. Adding up all of these present values yields the net present value of \$33, 370 for the bagel business. Note that in both cases the NPV is smaller than the total revenue. The reason for this is that the NPV discounts revenue that we receive in the future. For example, \$8, 000 three years from now only has a present value of \$6, 910 today, assuming a five percent discount rate.

The NPV depends on what one chooses for the discount rate. The larger the discount rate, the smaller the NPV will be. We will explore this phenomenon in the next section. Doing so will lead us to another important metric that is used for valuing investments.

26.5 IRR: Internal Rate of Return

In the previous section, we saw that if we assumed a discount rate of 5%, the net present value (NPV) of Anna's donut business is \$25, 980. Different discount rates give different NPVs. If the discount rate is smaller, the NPV will get larger. The reason for this is that a smaller discount rate means that we are discounting the future payments less, and so the total present value is more. For example, if the discount rate is 3%, the NPV increases to \$27, 480. And we were to increase the discount rate, the NPV would decrease. A larger discount rate means we apply more of a discount to future payments, decreasing the total present value of the investment. If, for example, we used a discount rate of 7%, the NPV decreases to \$24, 601.

These NPV values, and many more, are shown in Table 26.3. We've calculated the NPV for many discount rates r for both the donut and the bagel investments. We see that as the discount rate increases the NPV decreases.

The cost of both of these investments is \$20, 000. For a discount rate of 5%, the NPVs for the donut and bagel businesses are \$25, 980 and \$33, 370, respectively. In both cases, the NPV is larger than the system cost. The discount rate "punishes" future payments by devaluing them. As we apply a greater and greater punishment to future payments, eventually the NPV decreases until it is equal to the system cost. This defines a quantity known as the *internal rate of return* (IRR). The IRR is the discount rate at which the NPV of the investment equals the cost.

Looking at Table 26.3, we see that for the donut shop, the IRR is between 15 and 16 percent. The NPV at a discount rate of 15% is

Discount Rate	Donut NPV	Bagel NPV
0	$30,000	$40,000
1	$29,121	$38,538
2	$28,281	$37,148
3	$27,478	$35,826
4	$26,711	$34,568
5	$25,977	$33,370
6	$25,274	$32,228
7	$24,601	$31,140
8	$23,956	$30,103
9	$23,338	$29,112
10	$22,745	$28,167
11	$22,175	$27,264
12	$21,629	$26,401
13	$21,103	$25,575
14	$20,598	$24,786
15	$20,113	$24,030
16	$19,646	$23,306
17	$19,196	$22,613
18	$18,763	$21,948
19	$18,346	$21,311
20	$17,944	$20,700
21	$17,556	$20,113
22	$17,182	$19,550
23	$16,821	$19,009
24	$16,472	$18,489
25	$16,136	$17,989

Table 26.3: A listing of the net present value (NPV) for the donut and bagel business for different values of the discount rate r. As the discount rate increases, the NPV decreases. The internal rate of return (IRR) is the discount rate at which the NPV equals the system cost (in this case $20,000).

$20,110 and the NPV at a discount rate of 16% is $19,650. The IRR is the discount rate at which the NPV equals $20,000. So the IRR must be between 15 and 16. For the bagel business, we see from Table 26.3 that the IRR is between 21 and 22%. A more accurate calculation, experimenting with different discount rates, find that the IRR for the donut business is 15.24% and that the bagel's IRR is 21.2%.

Here are a few ways to think about what the IRR tells us. The IRR is a yearly rate of return. If you have an investment with an IRR of, say, 7 percent, then this is an indication that this investment is equivalent to a bank account that earn interest at 7%.[2] The IRR is thus like a yearly ROI, except that it accounts for the time value of money. The larger the IRR, the better the investment. A large IRR indicates a large annual return on your investment. The IRR is probably the most commonly

[2] This is illustrated in Exercise 26.8.

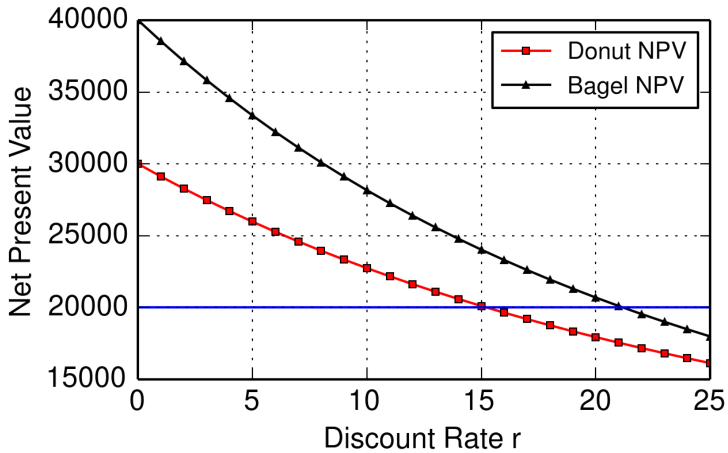

Figure 26.4: A plot of the net present values (NPVs) for the donut and bagel businesses as the discount rate r is varied. The NPV values were given in Table 26.3. As the discount rate is increased, the NPV decreases. The system cost for both the donut and bagel businesses is $20,000. This value is indicated by the horizontal blue line in the figure. The IRR is the discount rate at which the NPV equals the system cost. Graphically, this occurs when the NPV crosses the horizontal blue line. We see that the IRR for the donut business is almost exactly 15% and the IRR for the bagel business is around 21%.

used metric for evaluating investments.

To sum up, here is how one calculates the IRR. Choose a discount rate and calculate the NPV for the investment. Experiment with different discount rates until the NPV equals the system cost.[3] The discount rate at which this occurs is the IRR. In all but the simplest cases the IRR cannot be calculated by hand; there is not a simple formula for it. One needs to develop a spreadsheet that calculates the NPV for you, and then

[3] This is a somewhat restricted definition of the IRR. More generally, the IRR is the discount rate at which the present value of all costs and expenses are zero. The idea is that we view the initial expense of the system as a cost—a negative revenue or a negative cost flow. In addition, there may be other negative cost flows, perhaps due to maintenance or labor costs, at any point in the project.

26.6 *Cost of Capital and the Hurdle Rate*

Determining the discount rate is a subjective thing.[4] Different organizations or people will use different rates depending on their needs. A company (or town or individual) figures out or estimates the rate of return that an investment of similar risk would be expected to make. This rate is often called the **cost of capital**. It is a measure of how expensive it would be to pull capital away from your typical investment. For example, if I am making a rate of return of r by keeping my money in a bank, and then I take this money out of the bank to invest somewhere else, I have lost that interest, and thus would view that lost interest as a cost.

[4] Where does this section go? Maybe in the previous chapter? I'm not sure.

A company will want the rate of return on an investment to be greater than its cost of capital. For this reason the cost of capital is also sometimes referred to as the **hurdle rate**. The rate of return on an investment must be higher (hurdle over) the cost of capital in order for it to be worthwhile.

26.7 Discussion

All of this is nowhere near as objective and scientific as it sounds. There is the matter of risk, which is fundamentally unknowable, especially for new technologies. Risk is often accounted for by using very large discount rates.

How far out should we do our calculations? Five years? Twenty years? Forever? Also, it is not always clear what should count as a cost, even from the point of view of the investor. There are also costs that are born by others. E.g., The burning of fossil fuels creates particulate matter in the atmosphere, leading to asthma. The costs of increased asthma rates—medical costs and lost productivity— are not born by the investor. This is an *externality*—a cost that is not reflected in what the investor has to pay to build and operate the power plant.

26.8 Exercises

Exercise *26.1:* An investment that costs $15,000 has an ROI of 0.8. How much revenue will you get from this investment?

Exercise *26.2:* Verify that the rightmost column of Table 26.2 is correct.

Exercise *26.3:* Suppose that the donut business whose revenue is described in Table 26.1 lasted for seven years instead of five. (Perhaps the donut-making machine is higher quality and will last longer.) Without doing any calculations, determine if the following quantities will increase, decrease, or stay the same. Explain briefly.

1. The payback time
2. The ROI
3. The NPV
4. The IRR

Exercise *26.4:* Suppose that the bagel business described in Table 26.1 has a year-five revenue of $5,000 instead of $14,000. (Perhaps a rival bagel business opens, depriving you of some revenue.) Without doing any calculations, determine if the following quantities will increase, decrease, or stay the same. Explain briefly.

1. The payback time
2. The ROI
3. The NPV

4. The IRR

Exercise *26.5:* You are considering an investment that will cost \$20,000. If one uses a discount rate of 5%, the NPV of this investment is \$18,000. Is the IRR of this investment greater or less than 5%? Briefly explain.

Exercise *26.6:* You are considering purchasing a scone business. You have good reason to believe that you will make \$3000 in profit at the end of the first year, \$4000 in profit at the end of the second year, and \$5000 in profit the third year. You can buy the business for \$10000.

1. What is the payback time of this investment?
2. What is the return on investment?
3. What is the total present value of the three payments if you use a discount rate of 0.05?

Figure 26.5: mmm...scones! (Figure source: OctopusHat, https://commons.wikimedia.org/wiki/File:Pile_of_scones.jpg licensed under Creative Commons Attribution-Share Alike 2.0 Generic https://creativecommons.org/licenses/by-sa/2.0/deed.en.

Exercise *26.7:* What is the IRR of the scone business described in the previous problem?

Exercise *26.8:* This exercise will lead you through a series of calculations designed to help you understand what the internal rate of return (IRR) means. We will analyze an investment that will pay you \$3000 in one year and \$5000 in two years. You can buy this investment for \$7000. It turns out that the IRR for this investment is 8.6%.

1. Verify that 8.6% is indeed the IRR for this investment. To do so, calculate the NPV of the investment and show that it is approximately \$7000.
2. If you buy the investment, how much money will you have in two years? To figure this out, you will need to take the value of the \$3000 that you get in year one and project it forward, using a discount rate of 8.6%, to determine its value in year two.
3. Suppose that you decide against buying the investment and instead opt to deposit your \$7000 in a bank account that makes 8.6% interest. How much money would have in two years?
4. The answers to the above two questions should be the same. Use this observation to write a sentence or two about the meaning of IRR.

Part VII

Appendices

A
Data

This is an appendix in which we can collect a bunch of data and conversions an other useful things. For now, let's just do lists. I can make pretty tables when the time comes.

A.1 Fundamental Definitions

A.1.1 Definitions of Units

- $1e = -1.60 \times 10^{-19}$ Coulombs.[1]
- Amp (electric current) = A = 1 C/s.
- Joule (energy) = J = kg·m^2/s^2.
- Watt (power) = W = J/s = kg m^2/ s^3.
- Volt (electric potential, aka voltage) = V = J/C.

[1] e is the electric charge of one electron. The Coulomb is the standard metric unit of charge.

A.2 Conversion Factors

A.2.1 Prefixes

- micro = μ = 10^{-6}
- milli = m = 10^{-3}
- kilo = k = 10^{3}
- mega [2] = M^2 = 10^{6}
- giga = G = 10^{9}
- tera = T = 10^{12}
- peta = P = 10^{15}
- exa = E = 10^{18}

[2] Except for BTUs, where M = 10^3 and MM = 10^6. Seriously. See Section 7.2.

A.2.2 Lengths

- 5 km = 3.1 miles, 1 mile = 1.61 km, 1 km = 0.62 miles.
- 1 ft = 12 inches.
- 1 yard = 3 feet = 0.914 meters.

- 1 meter = 3.28 feet.
- 1 mile = 5280 feet.

A.2.3 Areas

- 1 acre = four rods $\times$ one furlong = 4047 m^2
- 1 hectare (ha) = 10000 m^2 = 2.47 acres[3]

[3] Usually one can round and use 1 ha = 2.5 acres, and 1 acre = 4000 m^2.

A.2.4 Volumes

- 1 gallons = 3.8 liters

A.2.5 Mass & Weight

- 1 kg = 2.2 pounds
- 1 kg of carbon (C) = 3.67 kg of carbon dioxide (CO_2)
- 1 metric[4] ton = 1000 kg
- 1 short[5] ton = 2000 pounds
- 1 liter of water[6] has a mass of 1 kg.

[4] A metric ton is often written tonne. Outside the US, the metric ton is just referred to as a ton.

[5] Short tons are used only in the US, where they are often just referred to as tons.

[6] This is only true for water. Other substances have different densities, and so one liter may have a mass higher or lower than 1 kg.

A.2.6 Energy

- 1 kWh = 3,600,000 J = 3.6 MJ
- 1 kWh = 3412 BTU
- 1 BTU = 1055 J
- 1 dietary calorie = 1000 calories
- 1 calorie = 4.184 Joules
- 1 MBTU = 1000 BTU
- 1 MMBTU = 1,000,000 BTU
- 1 therm = 100,000 BTU = 29.3 kWh
- 1 Quad = 10^{15} BTU = 293×10^9 kWh
- 1 ton of oil equivalent (toe) = 11,630 kWh = 41.868 GJ

A.3 Carbon Intensities and Other Data

A.3.1 Carbon Intensity of Electricity by Form of Generation

Below is a list of the lifecycle carbon intensity of different forms of electricity generation. These numbers are averages, taking into account many different studies. Compiled from Moomaw et al. (2011), available at: `http://srren.ipcc-wg3.de/report/IPCC_SRREN_Annex_II.pdf`. All figures are expressed in gCO_2e/kWh

- Solar photovoltaic 46
- Concentrating solar power 22

- Geothermal 45
- Hydropower 4
- Ocean Energy 8
- Wind 12
- Nuclear Energy 16
- Natural Gas 469
- Oil 840
- Coal 1001

(Note to self: I might also want to pull data from the report available on the IEA webiste here: http://goo.gl/yPBQdu. It's a bit curious that the IEA's numbers are a bit different from the IPCCs. That said, the differences don't appear to be significant. (For example, the IEA has 400 for the carbon intensity of natural gas.) Check to see if the IEA uses CO_2 or CO_2e .)

A.3.2 Carbon Intensity of Electricity by Country

The the data below are from Ritchie et al. (2022), and were current as of 2022. These numbers will change as countries change the mix of technologies they use to generate electricity. The full dataset can be found at https://ourworldindata.org/electricity-mix#carbon-intensity-of-electricity. All figures are in units of grams of CO_2e per kWh of electricity.

- Sweden: 12
- France: 58
- Kenya: 112
- Canada: 119
- Brazil: 142
- Columbia: 193
- United Kingdom: 265
- Germany: 325
- Australia: 486
- China: 541
- Indonesia: 625
- India: 626
- South Africa: 665
- Poland: 728

A.3.3 Calorific Value of Fuels

The calorific value of a fuel is the amount of thermal energy (heat) that is released if a certain about of that fuel is burned. The numbers listed below are all known as *low heating values* (LHV), which is the thermal energy that directly results from burning the fuel.

When fuel is burned one gets not only heat and carbon dioxide, but also water. This water is in gaseous form—it's steam—since the temperature of combustion is higher than the boiling point of water. It is possible to extract some of the thermal energy in the steam via condensation: turning it into a liquid. When the water undergoes a phase transition from gas to liquid, it releases a lot of heat[7]. The *high heating value* of a fuel is the thermal energy directly released by combustion and the thermal energy that results from condensing steam.

[7] This quantity of heat is known as the latent heat of vaporization

In almost all settings it is the lower heat value that is relevant. An exception is condensing boilers, which, as their name suggests, pull heat from the steam by condensing it. Many new high-efficiency home furnaces are condensing boilers.

Calorific values for fuel can vary for at least two reasons. First, the composition of fuels are not standard. For example, the particular mix of chemicals in gasoline varies around the world. For example, countries have different requirements for how much ethanol gasoline contains. The calorific value of natural gas varies, depending on the particular mix of gases in it. This mix changes over time, as discussed in Sec. 12.7. The calorific value of wood depends on its density and water content. There are different types of coal. And so on.

The second reason that calorific values can vary is because combustion conditions vary. Calorific values are usually reported for combustion that takes place at 0 Celsius and one bar[8] of pressure. In furnaces and generators, fuel is usually not burned at 0C, so the calorific value will vary slightly from the value listed in tables such as the one below.

[8] One bar is approximately equal to typical atmospheric pressure

- Gasoline: 13.0 kWh/kg, 34.7 MJ/L, 120,480 BTU/gallon
- Coal: 8.0 kWh/kg, 19,100,000 BTU/short ton[9].
- Ethanol: 84,000 BTU/gallon, 21.2 MJ/L
- Propane: 13.8 kWh/kg, 25.4 MJ/L, 91,600 BTU/gallon
- Natural gas[10]: 14.85 kWh/kg, 0.04 MJ/L, 1,037 BTU/ft^3
- Heating oil: 12.8 kWh/kg, 37.3 MJ/L, 139,000 BTU/gallon
- Kerosene: 12.8 kWh/kg, 37 MJ/L, 135,000 BTU/gallon
- Wood: ∼4–5 kWh/kg
- Hardwood (maple): 24,000,000 BTU/cord
- Softwood (Norway pine): 17,000,000 BTU/cord

[9] A short ton is 2000 pounds

[10] Natural gas is a mixture of a number of different gases, and the calorific value of natural gas depends on the exact mixture of gases, which is not completely standard. Many utilities add a small correction factor—often called a therm factor or a BTU factor—to adjust for the particular mix of gases in their natural gas. This mixture tends to vary over time, so these correction factors are regularly recalculated.

The BTU data can be found at https://www.eia.gov/energyexplained/index.cfm?page=about_energy_units. Lots of firewood data can be found at http://worldforestindustries.com/forest-biofuel/firewood/firewood-btu-ratings/

For a very comprehensive but also somewhat awkward list of data, see https://www.engineeringtoolbox.com/fuels-higher-calorific-values-d_

169.html. Is there a single source I can cite? Maybe the MIT Energy book.

A.3.4 Carbon Intensity of Fuels

Carbon intensity of fuels (gCO_2/kWh). This this the amount of CO_2 released if one burns a sufficient quantity of fuel to release 1 kWh of thermal energy.

- Natural gas: 190
- Propane: 217
- Gasoline: 240
- Diesel: 250
- Fuel oil: 260
- Coal: 300

A.3.5 Cost of Fuels

Fuel costs are notoriously volatile and fluctuate considerably from year to year and even from month to month. They also vary widely in different parts of the world. Current average fuel prices for Maine can be found at http://www.maine.gov/energy/fuel_prices/. The US Energy Information Administration also tracks weekly fuel prices at: https://www.eia.gov/dnav/pet/pet_pri_wfr_dcus_nus_w.htm

At the present moment[11], average prices in Maine are:

[11] November 2022

- Natural gas: between $1.35/therm and $3.58/therm
- Heating oil: $5.71/gallon
- Wood pellets: $312/ton
- Kerosene: $7.23/gallon
- Propane: $2.48/gallon
- Firewood: $350/cord

A.4 Some Useful Resources

- UNDP Human Development Reports. Tons of country-level data here, can be displayed in tables or downloaded in a well formatted spreadsheet. http://hdr.undp.org/en/data. Even more amazing is the UNDP's public data explorer: http://goo.gl/M9CvEQ. You can make graphs and animations of all UNDP data.

- The CAIT Climate Data Explorer http://cait.wri.org seems to be one of the most user-friendly places to get climate data, country-level GHG emissions, historical emissions, and so on. CAIT is a project of the World Resources Institute. **TODO!** Update link.

- The US Energy Information Administration `http://www.eia.gov` is another good resource for data. Has state-level info on consumption.
- The Lawrence Livermore National Laboratory makes some super-fun and interesting energy flow charts. You can access them at `https://flowcharts.llnl.gov/commodities/energy`.

B
Working with Numbers

Here are some issues

1. *what words do we use for large numbers?*
2. *How can we compactly represent large numbers, and how can we do calculations with them?*
3. *How do we understand large numbers (like millions and billions) for which we don't have a point of reference? How do we communicate large numbers to others in a way that is meaningful and intellectually honest?*

A central goal of this text is to help you learn to think critically about numbers and to learn to present numerical information in a understandable and intellectually honest way. The numbers that arise in discussions and debates about energy and climate can be vexing for several reasons. First, the numbers are often huge: teraJoules of energy, gigatons of CO_2e , megawatt hours of electrical energy, and quadrillions of BTUs. Second, the units on these numbers are often not well understood. Few people have a good feel units such as kWh, BTUs, or MW. Lastly, people often communicate these numbers in a way doesn't provide any context or frame of reference. Sometimes this is done intentionally; large numbers can be used to intimidate or obscure. I think this happens on both "sides." Both environmentalists and anti-environmentalists have been known to present numbers in potentially misleading ways.

Here's a silly example. I am 1700 millimeters tall. Wow! That's a lot of millimeters. I must be very tall. I am also 0.0017 kilometers tall. Wow! That's almost no kilometers. I must be very short. I can also express my height in a more conventional way; I am five feet, seven inches tall. This is about two inches shorter than the height of the average male in the US. So I'm a bit shorter than average.

This example is silly, because you are familiar with millimeters, kilometers, and the heights of men. So playing games like this seems daft and a bit puerile. But if you didn't know about these units, and you weren't familiar with men, you might be given the wrong impression by seemingly large or small numbers.[1]

I begin this Appendix by discussing scientific notation, a technique for working with very large and very small numbers. Then in Section B.3 I present some guidelines and ideas for communicating large numbers in ways that are understandable and intellectually honest. In Section B.4 I discuss some particular challenges associated with thinking about and visualizing large areas.

[1] **TODO!** Use one or two actual examples where numbers are used in a misleading way?

B.1 Non-Standard Vocabulary vs. Standard Prefixes

In much of the English-speaking world, there are standard words to describe large numbers:

$$\text{One million} = 1,000,000 = 10^6 \,, \tag{B.1}$$

$$\text{One billion} = 1,000,000,000 = 10^9 \,, \tag{B.2}$$

$$\text{One trillion} = 1,000,000,000,000 = 10^{12} \,, \tag{B.3}$$

and

$$\text{One quadrillion} = 1,000,000,000,000,000 = 10^{15} \,. \tag{B.4}$$

Straightforward[2] enough, I guess. It is important to know that this scheme for naming large numbers is far from universal.

[2] Although maybe not perfectly straightforward. Based on the roots of the words, it sounds like trillion should be three times as large as million, but a trillion is actually a million times larger than a million.

Until the mid 1970's, different words were used for large numbers in the United Kingdom. In the UK: million, milliard, billion. In the US: million, billion, trillion. So a UK billion is 1000 times larger than a US million. Confusing. Nowadays, in almost all forms of English, the meanings of million, billion, trillion are those given in Eqs. (B.1–B.3).

Many other languages use a naming system similar to that of the old UK system. For example, in French, "million, billion, trillion" translates to "*um million*, *un millard*, *un billion*." In other words:

$$\textit{un billion} = \text{one trillion} \,, \tag{B.5}$$

which is definitely confusing. The situation is similar in Spanish, in which "million" is " *un millón*", and "trillion" is "*billones*". What is a "billion" in Spanish? *Mil milones*, which translates as one thousand million. Systems in which a billion is 10^9, as is the case in most current forms of English, are called *short scales*. Systems in which a billion is 10^{12}, such as French and Spanish, are called *long scales*.

And there are some widely-used systems of naming large numbers that are neither the short nor long scale. In India, there are names for every second power of ten, starting at 10^5:

$$\text{One lakh} = 10^5 \,, \tag{B.6}$$

$$\text{One crore} = 10^7\,, \tag{B.7}$$

$$\text{One arab} = 10^9\,, \tag{B.8}$$

and so on. In modern Chinese, there are names for every fourth power of ten, starting at 10^4:

$$\text{One w/'an} = 10^4\,, \tag{B.9}$$

$$\text{One yì} = 10^8\,, \tag{B.10}$$

$$\text{One zhào} = 10^{12}\,, \tag{B.11}$$

and so on.

The point of all this is that names for large numbers are not standard and are often confusing, especially for people who speak multiple languages. Is there any way out of this linguistic mess? Yes! In science here are completely standard prefixes used for large (and small) numbers. In particular, mega-, giga-, and tera- are used to denote millions, billions, and trillions (as per Eqs. (B.1–B.3)):

$$\text{One mega ton} = 10^9 \text{ tons}\,, \tag{B.12}$$

$$\text{One giga ton} = 10^9 \text{ tons}\,, \tag{B.13}$$

and

$$\text{One tera ton} = 10^{12} \text{ tons}\,, \tag{B.14}$$

B.2 *Scientific Notation*

We first need to talk about scientific notation, which is a super useful way of representing and carrying out calculations with very large or very small numbers. It is likely you have seen scientific notation in school before. Many folks encounter it in chemistry classes in high school. If you are reading this part of the appendix, perhaps this is because your first attempt at establishing a positive relationship with scientific notation didn't go so well. I urge you to stick with. You'll get the comfortable with scientific notation with a little bit of practice, and you'll come to appreciate how helpful it is.

Let's start with an example. The population of the US state of Maine in 2019 was approximately $1,300,000$. If you were to say this number aloud, you might say "One point three million," or possibly "One million, three hundred thousand." You would most likely not say "One-three-zero-zero-zero-zero-zero." This statement wouldn't be wrong, but it's not helpful. It would be hard for your listener to parse the sequence of "zero"s.

Scientific notation is a way of representing large numbers that avoids long strings of repeated zeros. Here is how to write $1,300,000$ in scientific notation. First, note that

$$1,300,000 = 1.3 \times 1,000,000 \,. \tag{B.15}$$

Then, we re-write 1,000,000 using exponents:

$$1,000,000 = 10^6 \,. \tag{B.16}$$

The expression on the right-hand side of the above equation is ten raised to the sixth power. This means that ten is multiplied by itself six times:

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000 \,. \tag{B.17}$$

We can use this to re-write Eq. (B.15) as follows:

$$1,300,000 = 1.3 \times 1,000,000 = 1.3 \times 10^6 \,. \tag{B.18}$$

B.3 Making Large Numbers Understandable

Two techniques:

- Divide a large number by the population size to arrive at a per capita quantity. We've done this when thinking about CO_2e emssions.
- Divide a large annual number by the number of days in a year. We've done this when thinking about energy consumption.

  Guiding principles
- Choose units and scales so that numbers are between 1 and 100 or perhaps 1000. I think most people have a good feel for numbers in this range.[3]
- Use units that are common and understood: miles, kilometers, pounds. Units such as kW and MWh are not commonly understood and so you will need to give you audience some sense of what these units mean.

[3] **TODO!** citations?

Clearly and honestly communicating areas poses some additional challenges which are discussed in the following section.

B.4 Communicating Areas

B.5 Representing Numbers Visually

Some thoughts on good and bad or deceptive graphs and figures? Not sure if I need this section. Probably I don't.

B.6 Estimation

Maybe this should be its own Appendix? This is how I feel now, but the future may be different. Or maybe this doesn't need to be here at all.

B.7 Significant Digits

This definitely needs to be here.

B.8 Exercises

Exercise *B.1:* Convert the following numbers into scientific notation:

1. 1234
2. 430,000
3. 6,130,000
4. 14 trillion

Exercise *B.2:* In January 2016 the US national debt was 13.6 trillion dollars. Turn this into an understandable number by calculating the national debt per capita. Having done so, put this number into perspective, perhaps by comparing the per capita national debt to the amount of debt people sometimes go into to pay for college or a house.

Exercise *B.3:* The 2015 annual budget of the US National Endowment for the Arts is 146 million dollars. Turn this into an understandable number by expressing this number per capita. Put this number into perspective; is it large or small?

Exercise *B.4:* Convert one trillion seconds into an understandable number.

Exercise *B.5:* In 2019, collectively people in the US consumed 27 billion pounds[4] of beef. Express this consumption as a per-capita rate: the pounds of beef per person per day.

[4] `https://www.ers.usda.gov/topics/animal-products/cattle-beef/statistics-information.aspx`, accessed August 3, 2020.

C
Unit Conversions

C.1 On the Importance of Developing a System

Talk about how good style is important. That you want to develop a solid, comfortable system for performing unit conversions. You might not need the full machinery of your systems for simple problems, although if you have a good system then you'll almost never make a mistake on simple problems. But when you really need a unit-conversion system—or, more generally, a systematic approach to solving problems—is on harder problems. You'll want to have a comfortable, familiar system in place before you get into more complex terrain.

C.2 Basic Conversions

I'll illustrate basic unit conversion with an example. Suppose you weigh 160 pounds and want to know how much that is in kilograms. The first thing we need to know is how kilograms and pounds are related:[1]

[1] A bunch of unit conversions are listed in Appendix A.

$$1\,\text{kg} = 2.2\,\text{lbs}\,. \tag{C.1}$$

We then use this fact to carry out the conversion:

$$160\,\text{lbs}\left(\frac{1\,\text{kg}}{2.2\,\text{lbs}}\right) = 73\,\text{kg}\,. \tag{C.2}$$

Let's dissect the above equation. We start with your weight of 160 pounds. You want to re-express this fact in kilograms. In so doing, we don't want to change the physical fact of your weight; we just want to express the same thing in a different way.

We know that we can multiply a number by 1 and the number is unchanged. In particular, we know that $160 \times 1 = 1$. The term in parentheses in Eq. (C.2) is just the number one in disguise. To see this, note the top and bottom of the fraction are the same. For example, $\frac{3}{3} = 1$, since 3 goes into 3 one time. Similarly, $\frac{1\,\text{kg}}{2.2\,\text{lbs}} = 1$, since 2.2

pounds go in one kilogram. So multiplying 160 kilograms by the term in parentheses doesn't change its value, since the parentheses term really just has a value of 1. So the result of Eq. (C.2) is to change units, but not the physical value of quantity.

A common issue when doing unit conversion is whether or not to divide or multiply. I.e., which term should go on top? Should it be $\frac{1\,\mathrm{kg}}{2.2\,\mathrm{lbs}}$ or , $\frac{2.2\,\mathrm{lbs}}{1\,\mathrm{kg}}$? We answer this question by looking at the units on all the terms in Eq. (C.2). We need pounds on the bottom to cancel the pounds that we started with attached to the 160. Then the units cancel:

$$160\,\cancel{\mathrm{lbs}}\left(\frac{1\,\mathrm{kg}}{2.2\,\cancel{\mathrm{lbs}}}\right) = 73\,\mathrm{kg}\,. \tag{C.3}$$

The strategy is to write out your conversion factor (the term in parentheses) with units attached. Choose which quantity goes on top so that the units cancel, leaving you with the new units that you desire. To make sure that you've done things correctly—it's super easy to insert an inversion factor upside down by mistake—make sure that the units cancel, as I've done in Eq. (C.3).

Let's try this technique out on a slightly more complex example.

Example C.1. *How many seconds are in one year?*

The key facts are: there are 365 days in a year, 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute. Putting this all together, we have:

$$1\,\mathrm{yr}\left(\frac{365\,\mathrm{d}}{1\,\mathrm{yr}}\right)\left(\frac{24\,\mathrm{h}}{1\,\mathrm{d}}\right)\left(\frac{60\,\mathrm{min}}{1\,\mathrm{h}}\right)\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) = 31536000\,\mathrm{s}\,. \tag{C.4}$$

Note that the units cancel, leaving us with seconds, as desired:

$$1\,\cancel{\mathrm{yr}}\left(\frac{365\,\cancel{\mathrm{d}}}{1\,\cancel{\mathrm{yr}}}\right)\left(\frac{24\,\cancel{\mathrm{h}}}{1\,\cancel{\mathrm{d}}}\right)\left(\frac{60\,\cancel{\mathrm{min}}}{1\,\cancel{\mathrm{h}}}\right)\left(\frac{60\,\mathrm{s}}{1\,\cancel{\mathrm{min}}}\right) = 31536000\,\mathrm{s}\,. \tag{C.5}$$

This large number of seconds is more conveniently expressed in scientific notation: $31536000\,\mathrm{s} = 3.15 \times 10^7$. Another way to say this is that there are around 32 million seconds in a year.

When I am assembling the conversion factors in Eq. (C.4), I say to myself, "There are 365 days in a year, there are 24 hours in a day," and so on. This makes it clear that what goes in each parenthetical factor is one in disguise; it's the same thing on the top and the bottom.

For all but the simplest conversions, I recommend writing out the conversion factors as I've done in Eq. (C.4). Some students learn to do unit conversions by slotting the conversion factors in a grid rather than parentheses. The two methods are, of course, equivalent. Use whichever makes you happier.

C.3 Areas and Volumes

Converting areas and volumes follow the same general procedure as the conversion described above, but there is an added subtlety. I'll illustrate this with an example showing how to convert from square kilometers to square miles. First, a word about these square units. One square kilometer, 1 km^2, is an area[2] equal to the area of a square whose sides both have a length of one kilometer. A surface that has an area of 1 km^2 is not necessarily square in shape. It could be a triangle or a circle or a blob that happens to have an area the same as that of a square whose sides have a length of one kilometer. The unit 1 km^2 would be read aloud as either "one kilometer squared" or "one squared kilometer." Sometimes 1 km^2 is written as 1 sq km. I prefer 1 km^2, since it is easier to work with algebraically.

[2] **TODO!** make figure

Now for the example:

Example C.2. *The area of the US state of New Mexico is 315,000 km^2. What is the area of New Mexico in square miles?*

I'll use the fact that 5 km = 3.1 miles. Then,

$$315,000\,\text{km}^2 \left(\frac{3.1\,\text{mi}}{5\,\text{km}}\right) = \text{Ummmm... err....No.} \tag{C.6}$$

Wait. This isn't going to work. There is a km^2 on top with the 315,000 but only a km downstairs with the 5 km. So the kilometers don't cancel: km^2/km $\neq 1$. We need another km downstairs. We can get one by squaring the conversion factor—the term in parentheses in Eq. (C.6). Can we just do that? Sure. The term in parentheses is just one in disguise. And $1^2 = 1$. So we can square one and still get one. So our unit conversion proceeds as follows:

$$315,000\,\text{km}^2 \left(\frac{3.1\,\text{mi}}{5\,\text{km}}\right)^2 = 315,000\,\text{km}^2 \left(\frac{3.1^2\,\text{mi}^2}{5^2\,\text{km}^2}\right) = 315,000\,\text{km}^2 \left(\frac{9.61\,\text{mi}^2}{25\,\text{km}^2}\right) = 121,000\,\text{mi}^2\,. \tag{C.7}$$

Note that in the above equation *everything* inside the parentheses get squared: 3.1, 5, mi, and km.

Volume conversions proceed similarly to area conversions. Volume units are lengths cubed: meters^3, cm^3, and so on. In order to make the units work for volume conversions, we need to cube the conversion factor. I'll illustrate this with a short example

---

Example C.3. *A cube whose sides are* 10 *cm has a volume of one liter. How many cubic meters (m^3) equal one liter?*

The volume of cube of side s is equal to s^3. So

$$\text{Volume of one liter} = (10\,\text{cm})^3 = 10^3\,\text{cm}^3 = 1000\,\text{cm}^3\,. \tag{C.8}$$

Then, using the fact that 1 cm = 100 m

$$\text{Volume of one liter} = 1000\,\text{cm}^3 \left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^3 = = 1000\,\text{cm}^3 \left(\frac{1^3\,\text{m}^3}{100^3\,\text{cm}^3}\right) = 0.001\,\text{m}^3\,. \tag{C.9}$$

Alas, this is not the end of the story. There are two other area units that are commonly used and which you should know about. We'll start with *acres*, which are an old farming unit that for some reason is still used today in the US, the United Kingdom, and some of the UK's former colonies. One acre is the area of a rectangle that is one chain (66 ft) by one furlong (660 ft) which happens to be equal to 1/640 of a square mile. Ug. An acre also turns out to be 4047 m^2, which is a more useful fact for conversions.

How can one picture an acre? One acre is just about the size of one American football field, without the endzones. An acre is also a bit more than half of an full-size actual football (soccer) field. If you lived in medieval times, it might be useful to know that an acre is about how much land a team of oxen could plow in one day. Large forest fires in the US are often described with acres: usually tens or hundreds of thousands of acres. This seems completely daft to me; does anyone other than forest fire experts have a feel for what 10,000 acres means? Acres are commonly used to express land areas in the US. For example, the land around my house that my spouse and I own happens to be around 2.47 acres. For this usage acres seem not unreasonable.[3]

[3] Well, there's still the unreasonableness of non-metric units, but in the US, that's sort of the way it goes.

Another special area unit is the *hectare*, defined as the area of a square with a side of 100 meters. So one hectare = 10,000 m^2. One hectare equals 2.47 acres—so by pure coincidence I happen to own one hectare of land. A hectare is about two and a half US football fields. The abbreviation for hectare is ha. Like acres, hectares are used commonly for measuring land areas, hectares being used in parts of the world where acres aren't.

It is important to remember that acres and hectares are already area units; they do not need to be squared. I'll illustrate this with an example.

Example C.4. *As I am writing this, there is a a wildfire in the Kenow Mountains in Alberta, Canada that has burned* 33,000 *hectares. What is this area in square kilometers?*

$$33,000\,\text{ha}\left(\frac{10,000\,\text{m}^2}{1\,\text{ha}}\right)\left(\frac{1\,\text{km}}{1000\,\text{m}}\right)^2 = 33,000\,\text{ha}\left(\frac{10,000\,\text{m}^2}{1\,\text{ha}}\right)\left(\frac{1^2\,\text{km}^2}{1000^2\,\text{m}^2}\right) = 330\,\text{km}^2\,. \tag{C.10}$$

Note that the first conversion factor, 1 ha = 1000 m^2, does not need to be squared. The second conversion factor, however, does need to be squared so that the units work out right.

Some strategies for visualizing and clearly communicating areas are presented in Section B.4. A handful of conversion factors for areas are included in Appendix A.

C.4 Temperature

The units for temperature are unlike the units for other quantities such as length, time, and energy. As such, converting from one temperature unit to another is different than the other unit conversions discussed in this appendix.

What is *Temperature??*

Before we can think about units, we need to think about temperature itself. What is temperature? At first, the answer seems simple: temperature is a measure of how hot something is. This is true, but what does this mean? What is hotness? One way to approach this is to note if two objects at different temperatures are put in contact with each other, heat will flow from the hot object (higher temperature) to the cooler object (lower temperature). This will continue until the two temperatures are equal. This is all well and good, but it still doesn't tell us exactly what temperature is.

Here is another way to think about temperature. An object's temperature is related to its internal energy. I think of this as microscopic kinetic energy. Think about the molecules in a glass of water sitting on your desk. Macroscopically, the water is at rest, so we'd say that it has zero kinetic energy. But microscopically, the water molecules are moving; they are not at rest. So the water molecules, being in motion, have kinetic energy. This kinetic energy is what we call heat[4].

Figure C.1: A bowl of miso soup. Image by Douglas Perkins, released into the public domain. Image source: https://commons.wikimedia.org/wiki/File:A_bowl_of_miso_soup.jpg.

[4] Physicists use the term heat in a slightly different way. To them, heat refers only to an energy transfer from one object to another due to a temperature difference between the two objects. As an example, consider a hot bowl of soup. Normal people would say there is a lot of heat in the bowl of soup. Physicists would say that the soup has a lot of internal energy. If an ice cube is placed in hot bowl of soup, physicists would call the energy that flows from the soup to the ice cube heat.

So the internal energy of an object is hidden kinetic energy. This hidden kinetic energy is related to temperature. Temperature is not the total amount of hidden kinetic energy in an object, rather it is a measure of the *average* amount of hidden kinetic energy. Thus temperature measure something independent of the size of an object. For example, consider a small cup of of soup and a large bowl of soup at the same temperature. The large bowl of soup, being larger, has more total hidden kinetic energy than the cup of soup.

So temperature is a measure of average hidden kinetic energy. But an average with respect to what? Is it an average per kilogram? Per molecule? It turns out that temperature can be thought of as the average hidden kinetic energy per hiding place. Here's what I mean by this. There are different "places" energy can hide microscopically. The easiest to picture is the energy associated with the motion of the molecules. In a gas the molecules move around in straight lines, occasionally bumping in to another molecule or the walls of a container. For math/physics accounting reasons, this kinetic energy is actually considered three hiding places, since there are three directions the molecule can move, left-right, front-back, and up-down.

Molecules consisting of more than one atom, such as O_2 and H_2O, can rotate in addition moving in straight lines. Math/physics accounting tallies up this rotation as another two hiding places. There are other places where energy can hide in objects. Some molecules vibrate in addition to rotate. Molecules in solids are locked in place. These molecules can't roam around like molecules in liquids or solids, but they do vibrate in three directions and can also hide energy in the bonds that lock them in place. Counting hiding places can be complicated and abstract. But the key takeaway is:

$$\text{Temperature is a Measure of Average Hidden Energy}\ , \tag{C.11}$$

where the average is per hiding place.[5]

[5] Some physics details: This statement about temperature is a version of the equipartition theorem from physics, which states that an object's internal energy U is given by the following expression:

$$U = \frac{1}{2} f N k T\ ,$$

where N is the number of molecules, k is a constant known as Boltzmann's constant, and T is the temperature in Kelvin. The quantity f is what I'm calling energy "hiding places" per molecule. The standard term is quadratic degree of freedom. This means any form of energy in which the thing that can change (e.g. velocity for the kinetic energy) appears quadratically, i.e. as a square of the degree of freedom. For the kinetic energy, the degree of freedom is the velocity and the kinetic energy is given by $\frac{1}{2}mv^2$. So this is a quadratic degree of freedom, since the energy formula has a v^2 in it.

One can view the equipartition theorem as telling us what temperature means. The quantity fN is the total number of energy hiding places in an object. So

$$\text{Ave E per hiding place} = \frac{U}{fN} = \frac{1}{2} k T\ .$$

So, except for the $1/2k$ in front of T, the temperature is the average internal energy per hiding place. However, not all forms of energy are quadratic, so the equipartition theorem does not hold for all objects. Thus, this view of temperature as average internal energy is not perfect general, but for our purposes it gives a clear and relatively concrete way to think about temperature.

Temperature Scales

We are now finally ready to think about temperature scales, using the idea of temperature as average hidden energy. The two temperature scales which you are probably familiar with are Celsius and Fahrenheit. In Celsius, water freezes at zero degrees and boils at 100 degrees. In Fahrenheit, water freezes at 32 degrees and boils at 212 degrees. These facts lead to the following formulas that can convert from Fahrenheit F to/from Celsius C:

$$C = \frac{5}{9}(F - 32)\ , \tag{C.12}$$

$$F = \frac{9}{5}C + 32\ . \tag{C.13}$$

Here are two examples illustrating how to use these formulas

Example C.5. *What is 82 degrees Fahrenheit in Celsius:*

We plug F=82 in to Eq. (C.12):

$$C = \frac{5}{9}(82 - 32) = \frac{5}{9}(50) = 27.8\,\text{C}\ . \tag{C.14}$$

Note that 82 F is approximately 28 C. I find this a useful fact to remember when attempting to translate from one temperature scale to another.

Example C.6. *What is 40 degrees Celsius in Fahrenheit?*

We plug C=40 in to Eq. (C.13):

$$F = \frac{9}{5}(40) + 32 = 104 \,. \tag{C.15}$$

A 40 degree Celsius day is a very hot day.

Equations (C.12) and (C.13) are straightforward to use. You've likely encountered them before in physics or chemistry classes. But they're weird. In normal conversions, one uses a conversion factor, like 60 seconds equal one minute, or one yard equals 0.914 meters. Why isn't temperature like this?

The answer is that both Celsius and Fahrenheit are messed up systems of units. That a strong, judgmental statement, but I think it's justified. Here's why. Let's think about what zero Celsius means. This is the temperature at which liquid water freezes into ice. According to our definition of temperature as average hidden energy, Eq. (C.11), a temperature of zero would mean that the average hidden energy in the water would be zero: all the molecules would be perfectly at rest. But we know that this isn't true. After all, it's possible for ice to be colder than zero. For example, ice can be -5 Celsius. According to the view that temperature is a measure of average kinetic energy, a negative temperature is nonsense, since kinetic energy can never be negative.

The issue is that zero Celsius does not mean zero temperature, in the sense of zero average hidden kinetic energy. Zero kilograms means there is no mass. Zero meters means there is no length. But zero Celsius does not mean that there is no average hidden kinetic energy. So Celsius (and Fahrenheit) are both scales where zero doesn't truly mean zero. Moreover, their zero points are different. Zero degrees Celsius is not the same as zero degrees Fahrenheit.[6] So this is why the conversions between Celsius and Fahrenheit, Eqs.(C.12) and (C.13), are different than other conversions: neither temperature scale has a true zero, and their non-true zero points are different.

[6] In contrast, something with no mass has zero mass in all mass units: zero kilograms, zero pounds, zero tons, zero ounces, etc.

Let me say a little bit more about this idea of a true zero temperature. True zero is a temperature at which the average energy of the molecules that make up an object is zero. This would mean that the molecules are all perfectly at rest. What I'm calling true zero is more commonly called *absolute zero*. True zero turns out to be around -273 Celsius. This is the temperature at which the average kinetic energy of molecules in an object is zero. This idea forms the basis for the *Kelvin* temperature scale. Kelvin is Celsius shifted up by 273 so that zero Kelvin is absolute zero. To convert from Celsius to Kelvin, just add 273:

$$K = C + 273 \,. \tag{C.16}$$

For example, 20 Celsius is 293 Kelvin.

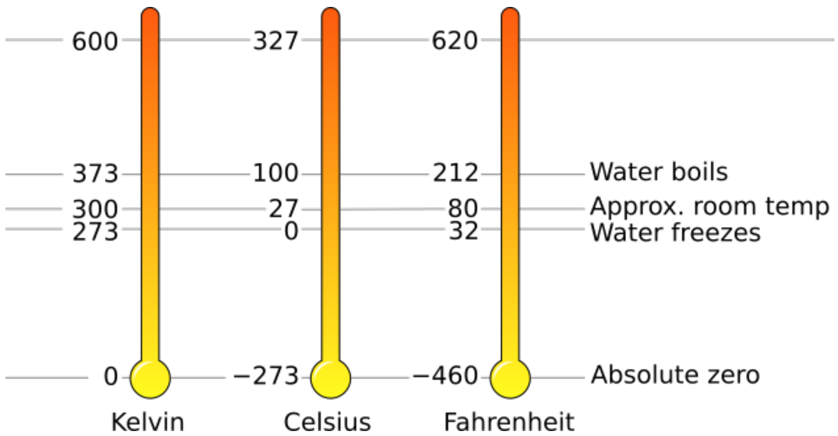

Figure C.2: A comparison of three temperature scales: Kelvin, Celsius, and Fahrenheit. This is a modified version of an image made by Steven Baltakatei Sandoval and licensed under the Creative Commons Attribution-Share Alike 4.0 International license (https://creativecommons.org/licenses/by-sa/4.0/deed.en). Original image at: https://commons.wikimedia.org/wiki/File:Temperature_scales_comparison_(K,R,C,F).svg.

So we have three ways of measuring temperature: Kelvin, Celsius, and Fahrenheit. The relations among these three temperature scales are shown in Fig. C.2. A temperature scales begin at absolute zero, just as all length scales begin a zero. Kelvin, being an absolute temperature scale, begins at zero. Celsius and Fahrenheit are not absolute temperature scales; they begin at -273 and -460, respectively. Note that Kelvin and Celsius are the same, just shifted by 273. So the difference between the boiling and freezing points of water are the same in Kelvin and Celsius: in both, $\Delta T = 100$. The difference between boiling and freezing in Fahrenheit is $\Delta T = 180$.

Kelvin is the preferred temperature scale in most scientific settings. It is an absolute temperature scale, since zero Kelvin corresponds to zero temperature, the temperature at which the average hidden kinetic energy is zero. As a result, ratios can be formed with temperatures in Kelvin. For example, 600 Kelvin is twice as hot (in the sense of having twice the average hidden kinetic energy) as 300 Kelvin, just as 600 meters is twice as far as 300 meters, or 600 seconds is twice as long as 300 seconds.

By the way, you may have wondered why temperature units are sometimes referred to as degrees. For example, one might say that it is 40 degrees outside (Fahrenheit or Celsius being implied). Why is the word “degree” being used here? Degree in this context indicates that something is being measured with respect to an arbitrary zero point. Degrees are also used to describe angles: e.g., a right triangle has a ninety degree angle. As with temperature, the zero point for measuring angles is arbitrary. For example, if I walk in a direction that makes a 30 degree angle, you don’t know which way I’m going unless I tell you how I’m measuring angles: am I walking 30 degrees east of north, or 30 degrees south of west, etc? In contrast, if I say that a whale is 30 meters long, that's an unambiguous statement.

In any event, since Celsius and Fahrenheit are temperature scales with a non-true zero, temperature measurements in those units are often referred to as degrees: e.g., a very hot day is 40 degrees Celsius, written 40°C. It's fine to leave off degrees. There's nothing wrong with simply saying that it's 40 Celsius out. Kelvin, however, is an absolute temperature scale, and thus it does not need to be referred to as degrees. One would say that room temperature is approximately 300 Kelvin, not 300 degrees Kelvin.

One final note about temperature. The only difference between Kelvin and Celsius is that they have different zero points. So an increment of one Kelvin is the same as an increment of one Celsius. For example, an increase of 10 Celsius is the same as an increase of 10 Kelvin. It is often the case in physics that what is relevant is a change in temperature. One example is the formula for changes in internal energy, Eq. (7.1) from Chapter 7: $\Delta T = mc\Delta T$. For this equation one can work in either Kelvin or Celsius, since ΔT will have the same value in either system of units.

C.5 *Exercises*

Exercise *C.1:* Convert the following to meters

1. 5 miles
2. 1 kilometer
3. 1 inch
4. 100 yards
5. 3959 miles (The radius of the earth)
6. 6.2 miles

Exercise *C.2:* Some time conversions:

1. One billion seconds is how many years?
2. How many days in 80 years, a typical lifespan?

Exercise *C.3:* About how many years is $\pi \times 10^7$ seconds?

Exercise *C.4:* Convert the following quantities to kilometers

1. 6 miles
2. 100 yards
3. 60 miles

Exercise *C.5:* Convert the following speeds to meters per second

1. 100 km/hr
2. 55 mi/hr
3. 560 mi/hr[7]

[7] The cruising speed of a Boeing 787 aircraft (Boeing).

Exercise *C.6:* Convert the following to miles per hour:

1. 80 km/hr
2. 100 m/s
3. 3×10^8 m/s (The speed of light.)

Exercise *C.7:* Look up the area of a football field.[8] The following areas are equivalent to how many football fields?

[8] Use either an American football field or a football football field—what we in America call soccer—as you prefer. But be sure to state which football field you're using.

1. 1 acre
2. 1 hectare
3. 1 km^2
4. 1 mi^2

Exercise *C.8:* The area of the state of Massachusetts is $10,565$ mi^2.

1. Convert this to square kilometers.
2. If Massachusetts was a square, how long would the side of the square be?

Exercise *C.9:* Mount Desert Island (MDI) is an island on the coast of Maine. It is home to Acadia National Park. The area of MDI is 108 mi^2.

1. How many acres is MDI?
2. What is the area of MDI in square kilometers?
3. If MDI was a square, how long would the side of the square be?

Exercise *C.10:* The College of the Atlantic's campus is 37 acres. Convert this to:

1. Hectares
2. Square kilometers
3. Square miles

Exercise *C.11:* In 2022, Cascadian Farm cereals boast that they are

Figure C.3: Mount Desert Island, Maine, USA. Its shape reminds some of a walnut. Others think it looks like a pair of lungs. Figure source: https://commons.wikimedia.org/wiki/File:NPS_acadia-map.jpg. Image produced by the US National Park service and edited by Matt Holly.

restoring 25 million square feet. Wow! That's a lot of area. Or is it?

1. The area that Cascadian Farm is restoring is how many square kilometers?
2. A square with a side of how many kilometers has an area equal to the area that Cascadian Farm is restoring?

Figure C.4: A Cascadian Farms "Buzz Crunch" cereal box. Image source: https://www.cascadianfarm.com/products/cereal-granola/, accesses September 2, 2022.

By the way, Cascadian Farm is a brand owned by General Mills, a US multinational food corporation. (Thanks to Suzanne Morse for calling this example to my attention.)

Exercise *C.12:* A cup of coffee cools off by ten degrees Celsius. So $\Delta T = -10$.

1. What is ΔT expressed in Kelvin?
2. What is ΔT expressed in Fahrenheit?

Figure C.5: A cup of black coffee. (I prefer mine with half and half.) Image by Jon-Isac Lindberg, released into the public domain. Image source: https://commons.wikimedia.org/wiki/File:White_cup_of_black_coffee.jpg.

Exercise *C.13:* The Paris Agreement, made as part of the United Nations Framework Convention on Climate Change (UNFCCC), puts for the goal of keeping average global warming below 2 Celsius. If the planet warms by 2 C, how much is this in Fahrenheit?

Exercise *C.14:* Let $T_1 = 20\,\mathrm{C}$ and $T_2 = 40\,\mathrm{C}$.

1. Convert T_1 and T_2 to Fahrenheit.
2. In Celsius , $T_2 = 2T_1$. Is this also true in Fahrenheit? Discuss.

D

Averages and Typicality

TODO! Re-write introduction.

TODO! Mention per-capita measure as a type of average.

TODO! Carbon footprints and problems thereof.

TODO! Add something about the importance of thinking on the margins.

TODO! I feel like this is important enough that maybe it should be the last chapter in the Energy Basics Part?

In this appendix we'll explore two ideas: *averages* and *efficiencies*. On the surface, these seem like simple, uncontroversial mathematical constructs: both an average and an efficiency is just one number divided by another. However, we shall see that there are some interesting and important subtleties associated with these quantities. In particular, although averages and efficiencies seem like neutral, objective constructions, this is not always so. Averages and efficiencies often embody a point of view or perspective. Understanding this can be important when working in sustainable energy.

I think I should start with efficiencies and then have the averages stuff? Yes, the more I think about this, the more I think averages should go later.

D.1 Introduction to Averages

The average is a way to measure the typical value of a group of numbers. It is defined as follows:

$$\text{Average Value} = \frac{\text{Sum of all measurements}}{\text{Number of measurements}}\,. \tag{D.1}$$

Figure D.1: Three veggie tacos. (Image by Siddhantsahni28 (Own work) [CC BY-SA 4.0 (http://creativecommons.org/licenses/by-sa/4.0)], Image available at https://commons.wikimedia.org/wiki/File:Veggie_Tacos.jpg).

This type of average is known as the *mean*. I'll illustrate the use and meaning of Eq. (D.1) via an example. Suppose you are planning a meeting and have generously decided to get tacos for everyone.

You send a note to the ten people you're inviting to the meeting asking them how many tacos they want, and here are their answers:

$$1, 2, 2, 2, 2, 2, 3, 3, 5, 6 \; . \tag{D.2}$$

That is, there is one person who wants one taco, five people who want two tacos, two people who want three tacos, and so on.

What is the average number of tacos desired by your meeting participants? We can calculate this using Eq. (D.1):

$$\begin{aligned} \text{Average no. of tacos} &= \frac{1+2+2+2+2+2+3+3+5+6}{10} \\ &= \frac{28}{10} \\ &= 2.8 \end{aligned} \tag{D.3}$$

So the average meeting participant will consume 2.8 tacos. Here is a way to think about what this means. Instead of thinking of all the different taco consumption rates among your colleagues, you can just imagine that everyone as your meeting is average, and you will buy the right amount of tacos. Ten average people each of whom eat 2.8 tacos means you need to get $10 \times 2.8 = 28$ tacos.

Stating that the average taco consumption is 2.8 tells us nothing about how much taco consumption varies from person to person. The average disregards variation—indeed, that's the point of an average. Sometimes that's a reasonable thing to do, sometimes not.

Here's a simple example to illustrate this point. Consider two towns, A and B, each consisting of four homes. Suppose the monthly electricity consumption of the homes in each town is as follows:

$$\text{Town A} : 550\,\text{kWh},\ 600\,\text{kWh},\ 600\,\text{kWh}, 650\,\text{kWh} \; . \tag{D.4}$$

$$\text{Town B} : 200\,\text{kWh},\ 200\,\text{kWh},\ 200\,\text{kWh}, 1800\,\text{kWh} \; . \tag{D.5}$$

In both towns, the average home uses 600 kWheach month. Looking only at the average consumption, the towns appear identical. But the story of energy consumption in the two towns definitely isn't the same. In town A, there is not much variation from home to home. In town B, however, almost all of the consumption is due to the fourth house—the one that uses 1800 kWh every week.

If you only know the average of set of numbers, that tells you nothing about the variation around the average, as the previous example shows. Indeed, this is the point of an average. We can

Mention that averages obscure variation. Indeed, that's the point of averages!

Also mention median and (briefly) the mode.

D.2 Averages: The Class-size Paradox

There's a subtlety associated with averages known as the *class size paradox*. I'll introduce this by using an example from the college at which I teach, College of the Atlantic, where I teach the average class size is 12. Average class size is a standard statistic that US colleges report. A smaller average class size is seen as more desirable, since this is taken as a proxy for more engaged and interactive classes. The average class size is a widely known fact. When I ask my students what our college's average class size is, they almost always know the answer.

I then ask my students another question about class sizes. Suppose I went around the room and asked everyone to list the size of the classes the they are currently enrolled in and then averaged those answers. What number would I get? Most students[1] answer 12, since that's the college's average class size. Is 12 the answer? Let's see.

[1] Most are a little wary, though, wondering why I'd ask such a question and suspecting a trick.

Suppose there are only two classes being offered, one with an enrollment of 6 and the other with an enrollment of 18. This scenario does indeed lead to an average class size of 12, as we can check:

$$\text{Average Class Size} = \frac{18+6}{2} = \frac{24}{2} = 12\,. \tag{D.6}$$

This is a direct application of Eq. (D.1); there are two measurements which, when averaged together, yield 12.

But what about from the students' point of view? Supposed I asked all the students in the school to tell me the size of their class[2], and averaged those results. There are 18 students in the 18-person class, so 18 times I would get an answer of 18. Similarly, there are 6 students in the six-person class, so 6 times I would get an answer of 6. There are 24 total students. So the average class size is:

[2] In this simple example, let's suppose that each student takes one and only one class.

$$\begin{aligned}\text{Average Class Size for Students} &= \frac{(18\times 18)+(6\times 6)}{24} \\ &= \frac{324+36}{24} = 15\,.\end{aligned} \tag{D.7}$$

So the average class size experienced by students is 15, but we found in Eq. (D.6) that the average class size is 12. What is going in?

As you have probably noted, each way of calculating the average embodies a different point of view. In Eq. (D.6) we take a class-centric view: classes are the things we're analyzing. In Eq. (D.7) we adopt the student's perspective and calculate the average class size experienced by students. Introducing a bit of jargon, at issue is what we average *over*. In Eq. (D.6), we say we're averaging over the classes, while in Eq. (D.7), we say we're averaging over the students.

The phenomenon described above is known as the *class size paradox*[3]. The student average will always be higher than the class average, since there are more students in the larger classes.

[3] To my knowledge, this term was first used in this context by Feld and Grofman in 1977. This effect appears in a number of different contexts and goes by a number of different names. A fun overview is Downey (2019).

So the notion of an average embodies a point of view. Are we averaging according to the point of view of the students or the point of view of a college administrator counting class enrollments? This general issue comes up fairly often whenever there are individuals and groups. We'll get different answers depending on how we carry out the average.

Add individual vs. group averaging example, preferably relevant to energy.

D.3 Other Efficiencies

D.4 Exercises

Exercise *D.1:* What is the median value for the taco consumption amounts listed in Eq. (D.2)?

Exercise *D.2:* Suppose the physics department of a small college has classes of the following sizes: 40, 25, 14, 12, 8, 7, 7, 6.

1. What is the average class size if one averages by class?
2. What is the average class size if one averages by student?

Exercise *D.3:* Suppose that ten people have the following yearly incomes, in thousands of dollars:

$$20, 20, 20, 20, 20, 20, 20, 20, 20, 100\ . \tag{D.8}$$

That is, of the ten people, nine make 20 thousand dollars a year and one makes 100 thousand dollars a years.

1. What is the mean value of the incomes?
2. What is the median value of the incomes?
3. Suppose the person making 100 thousand dollars has their salary doubled to 200 thousand dollars. How would this change the mean and median values?

Exercise *D.4:* This problem is based on an example from Kramer (2020). Suppose in a small town there are two households. In one household, there is only one person, and nine people reside in the other household.

1. What is the average household size?
2. What is the average number of people a person lives with?

Exercise *D.5:* Something about the average business having a surprisingly small number of employees, but most employees working at much larger than average firms. Another version of the class size paradox.

E
Proportional Reasoning

This probably doesn't need to be its own chapter. Likely it is better as a chapter in a larger appendix? Or maybe not. We'll see what develops.

In thinking about renewable energy, what often is most important is to know how two quantities change with each other. If we increase driving speed by ten percent, what would happen to the car's energy use (and hence fuel consumption)? If we double the length of a wind turbine's diameter, what happens to the energy generated by the turbine? Learning how to efficiently think through questions like these is know at *proportional reasoning*.

E.1 Proportionality

Quite often two quantities vary in proportion to each other. For example,

TODO! Add a bunch more exercises and some examples. Maybe see if there are any good problems in Arons's book on intro physics teaching?

E.2 Not Every Relationship is Proportional

E.3 Exercises

Exercise *E.1:* Suppose you have a pizza of a certain area. If you triple the radius of the pizza, what happens to the pizza's area?

Exercise *E.2:* Suppose you have a pizza of a certain area. If you increase the radius of the pizza by ten percent, what happens to the pizza's area? What happens to the area if the radius is decreased by ten percent?

Exercise *E.3:* Which is more pizza: two pizzas of radius a or one pizza of radius $2a$? Explain.

F
Details and Rabbit Holes

This appendix contains some details that generally aren't important to the big picture of sustainable energy and climate, but might be very important in some particular applications or context, and/or which are sufficiently arcane or annoying that I didn't want to clutter up the main text with them.

F.1 Efficiency Measures: COP, AFUE, HFUE

The efficiency e is a conceptually clean quantity. It is just the ratio of useful energy E_{out} to the input energy E_{in}. Since energy is conserved—it is neither created nor destroyed—E_{out} cannot be larger than E_{in}. That is, you cant get more energy out than you put in. This means that the efficiency e cannot be greater than 1.

$$e = \frac{E_{\text{out}}}{E_{\text{in}}} \leq 1 \, . \tag{F.1}$$

As noted above e is unitless. The energy units in the fraction above cancel, leaving a number without units.

There are a number of variants of the basic efficiency that I'll describe in this section.

COP: Coefficient of Performance

As discussed in Sec. 12.5 are a form of heating that is different from most other forms of heat. A heat pump uses electrical energy, but rather than converting that electrical energy directly to heat, the electrical energy is used to power a fan and a compressor that pumps heat from the outside into your house. So this is not a conversion from one form of energy to another, as depicted in Fig. 8.1. Instead, the situation for the heat pump is shown in Fig. F.1.

The amount of energy E_{out} pumped into the house is almost always larger than E_{in} the energy used to run the heat pump. This means that

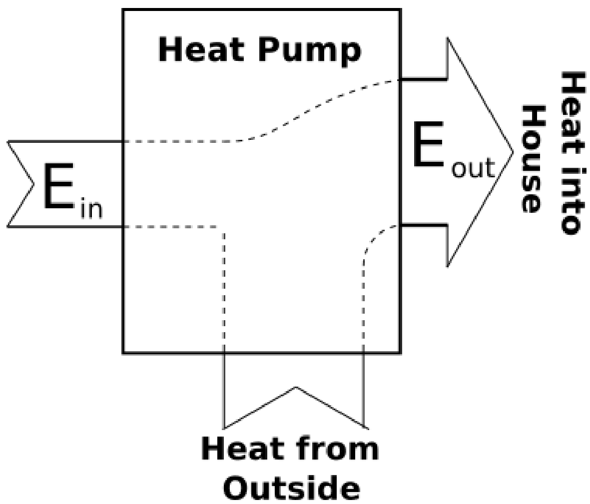

Figure F.1: A diagram for energy flows in a heat pump. Note that the energy E_{out} that flows into the house is greater than the electrical energy E_{in} that is used to run the heat pump. This means that the "efficiency" is greater than one. **Do I need a small amount of waste energy flowing out? Probably.**

if we were to calculate the efficiency of the heat pump using Eq. (8.1) we would get a number larger than one. This is possible because the process in a heat pump is not a direct conversion: we are not converting E_{in} into E_{out}. Because this is not a conversion process, one usually uses a different term to describe efficiency, the *coefficient of performance*, usually abbreviated COP:

$$\mathrm{COP} = \frac{E_{\mathrm{out}}}{E_{\mathrm{in}}} \,. \tag{F.2}$$

Note that Eqs. (8.1) and (F.2) are identical. What is different is the context: Eq. (F.2) applies to a heat pump.

Example F.1. *As in Example 8.2, you need to provide* 400,000 *BTU in one day to keep your house comfortable. Suppose you provide this heat with a heat pump that has a COP of* 3*. How much energy would it take to run the heat pump? Express your answer in both BTUs and kWh.*

We know that E_{out} is 400,000 and that the COP Is 3. We can rearrange Eq. (F.2) to solve for E_{in}:

$$E_{\mathrm{in}} = \frac{E_{\mathrm{out}}}{\mathrm{COP}} \,. \tag{F.3}$$

Plugging in, we get

$$E_{\mathrm{in}} = \frac{400,000\,\mathrm{BTU}}{3} \approx 133,000\,\mathrm{BTU} \,. \tag{F.4}$$

Note that E_{in} is less than E_{out} as expected. It takes only 133,000 BTU of electrical energy to pump 400,000 BTU of thermal energy into your house.

Electrical energy is usually measured in kWh. So let's convert using the conversion factor 1 kWh = 3412 BTU:

$$133,000\,\mathrm{BTU} = 133,000\,\mathrm{BTU}\left(\frac{1\,\mathrm{kWh}}{3412\,\mathrm{BTU}}\right) \approx 40\,\mathrm{kWh} \,. \tag{F.5}$$

AFUE: Annual Fuel Utilization Efficiency

The efficiency of a heater is the maximum possible efficiency that the heater is capable of—the efficiency under ideal conditions. However, we do not live in an ideal world, and heaters do not always operate under ideal conditions. Usually it takes a heater a little while to "warm up" and operate at peak efficiency, and efficiencies may vary across the seasons.

To account for this, there is an alternative measure of efficiency known as the *annual fuel utilization efficiency* or AFUE. This measure is just the average efficiency over an entire year:

$$\text{AFUE} = \frac{\text{thermal energy delivered to home during one year}}{\text{energy used by heater during one year}} \,. \tag{F.6}$$

So to determine AFUE, just figure out the total thermal energy delivered to your house in one year and divide it by the total thermal energy produced by all the fuel you burned in one year. In practice, this is not so straightforward, since it isn't easy to measure the actual amount of heat that ended up in your house. Heater manufacturers determine the AFUE of their products through a complicated[1] test procedure.

[1] In 2015 when the US Department of Energy proposed changed to the AFUE measurement procedures, the document specifying the changes was 183 pages. (See `https://energy.gov/sites/prod/files/2015/02/f19/2014_FB_TP_NOPR.pdf`, accessed October 11, 2020.)

In any event, heaters and furnaces usually have an AFUE listed instead of an efficiency. This seems reasonable, since the AFUE is the actual average efficiency one would expect over the course of one year. I worry, though, that most people browsing furnaces on home depot or other websites won't know what AFUE stands for or what it means.

F.2 Various Efficiency Ratios: SEER, HSPF, and CEER

There is a family of efficiency measurements used in heating and cooling which, although commonly used, have some annoying features. These efficiency measures are, like the efficiency e in Eq. (8.1), defined as follows:

$$\text{Efficiency Measure} = \frac{E_{\text{out}}}{E_{\text{in}}} \,. \tag{F.7}$$

Consider a heat pump. In this case, the E_{in} is in the form of electrical energy and E_{out} is in the form of thermal energy. The coefficient of performance is just the ratio of these two quantities.

When calculating the ratio, one measures both energies in Eq. (F.7) in the same units. This seems to me like the only sensible thing to do. However, it is common in some circles to use different units for energy. For example, one could measure the electrical energy in Watt-hours and the thermal energy in BTUs. Doing so yields a measure known as the *Heating Seasonal Performance Factor* or HSPF. In other words:

$$\text{HSPF} = \frac{E_{\text{out}}\ (\text{in BTU})}{E_{\text{in}}(\text{in Wh})} \,. \tag{F.8}$$

The HSPF is often reported without units, but this is misleading, since HSPF has units of BTU/Wh. The efficiency of a heat pump depends on the inside and outside temperatures, and thus the efficiency varies from day to day. The HSPF is the efficiency averaged over a full heating season.

For example, Fig. F.2 shows the efficiency information for a Bosch heat pump, model No. 8733954442. This device is capable of delivering 18,000 BTU/hr. The HSPF falls between 9.40 and 10.30.

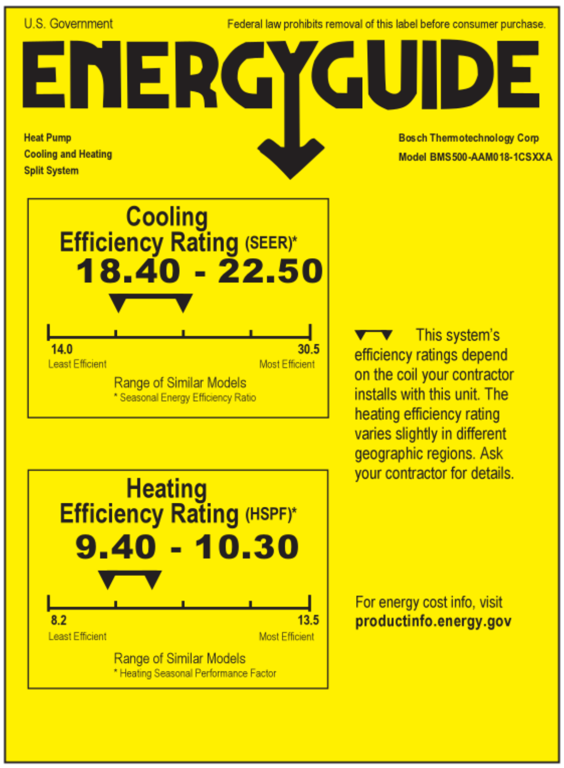

Figure F.2: Energy efficiency information for a Bosch heat pump.

Cooling efficiency is measured similarly. One forms a ratio as in Eq. (F.7), except E_{out} is now the amount of cooling—the amount of thermal energy removed from the building, measured in BTUs. And as with the HSPF, the electrical energy E_{in} is measured in Watt-hours. The efficiency is then averaged over an entire cooling season, to yield a quantity known as the *seasonal energy efficiency ratio*, or SEER.

To get a feel for HSPF and SEER, let's do an example where we convert an HSPF into a coefficient of performance (COP).

Example F.2. *What is the coefficient of performance (COP) of a heat pump that has a HSPF of 10?*

Recall that the HSPF has units of BTU/Wh. To turn this into a COP, we want the same units on top and on bottom. Let's convert both BTU and Wh into the more familiar kWh.

$$\frac{10\,\mathrm{BTU}}{1\,\mathrm{Wh}} = \frac{10\,\mathrm{BTU}}{1\,\mathrm{Wh}} \left(\frac{1000\,\mathrm{Wh}}{1\,\mathrm{kWh}}\right) \left(\frac{1\,\mathrm{kWh}}{3412\,\mathrm{BTU}}\right) \approx 2.93\,. \tag{F.9}$$

So this heat pump has a COP of 2.93. This means that for every kWh of electricity the heat pump uses, 2.93 kWh of thermal energy will be pumped inside the building.

There's one more energy efficiency measure of note: the *Combined Energy Efficiency Ratio* or CEER. This measure, which like the HSPF and SEER, has units of BTU/Wh, is used for window air conditioners. The CEER accounts for the electrical energy used when the air conditioner is on standby, not only when it is in operation.

Finally, these ratios are sometimes expressed in terms of power and not energy. Doing to doesn't change the value of the efficiency measure. To see this, note that these efficiencies have units of BTU/Wh. We can divide top and bottom by hours:

$$\frac{\mathrm{BTU}}{\mathrm{Wh}} = \frac{\frac{\mathrm{BTU}}{\mathrm{h}}}{\frac{\mathrm{Wh}}{\mathrm{h}}} = \frac{\mathrm{BTU/\,h}}{\mathrm{W}}\,. \tag{F.10}$$

The result is a ratio of two powers: The rate that heat is delivered to the building (in BTU/hr) and the rate that electrical energy is used (measured in Watts).

more blah blah.

Example F.3. *Statement of example*

Solution goes here

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*F.3 Batteries and Amp·hours**

* This chapter can safely be skipped. It is only needed for Chapter N on storage. Or will it be needed at all?

A battery is a device that maintains a voltage difference across its two terminals. If a wire is attached to be battery, connecting one terminal to the other, then a current will flow through the wire. How much current depends on the voltage of the battery and the resistance of the wire. Different batteries have different voltages. For example, in Fig. F.3 is a familiar AAA battery. The potential difference between the two terminals is 1.5 volts.

Figure F.3: The AAA battery that Dave used to power his noise-canceling headphones.

Almost all commonly used round batteries such as AA, AAA, and D batteries are 1.5 volts. The rectangular battery whose terminals are "snaps" are 9-volt batteries. An example is shown in Fig. F.4. The batteries in most conventional cars are 12-volt batteries.

Sadly, as you have no doubt experienced, batteries do not last forever. As noted above, a battery is a device that maintains a voltage difference.

If you hook the battery up to something, current flows. This current is moving charge—and the charge comes from the battery. Eventually, the battery runs out of charge. So an important characteristic of a battery, in addition to the voltage it maintains, is how much charge can flow out of it before it poops out. This amount of charge is known as the battery's *capacity*.

Capacity is an amount of charge. As discussed in Sec. 5.1, the standard SI unit for charge is the coulomb. So one would expect that one would measure capacity in coulombs. This is certainly reasonable and logical, but actually another unit is used instead: the *amp-hour*. An amp-hour is the amount of charge that has flowed through a circuit if a current of an amp flows for one hour. Let's figure out how to convert from amp-hours to coulombs.

Figure F.4: A 9 volt battery. (Image by Lead holder (Own work) [CC BY-SA 3.0 (http://creativecommons.org/licenses/by-sa/3.0) or GFDL (http://www.gnu.org/copyleft/fdl.html)], via Wikimedia Commons.)

Example F.4. *How many coulombs are in one amp-hour?*

We will start with the relationship $Q = It$ and recall that one amp is one coulomb per second:

$$Q = It = 1\,\mathrm{A} \times 1\,\mathrm{h} = \left(1\frac{\mathrm{C}}{\mathrm{s}}\right) \times 1\,\mathrm{h}\,. \tag{F.11}$$

Converting hours into seconds, we obtain

$$Q = \left(1\frac{\mathrm{C}}{\mathrm{s}}\right)(1\,\mathrm{h})\left(\frac{60\,\mathrm{min}}{1\,\mathrm{h}}\right)\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) \tag{F.12}$$

Multiplying out, we find that one 1 amp-hour $= 3,600$ coulombs.

Amp-hours are perhaps a bit clumsy, but they are the standard unit for battery capacity, so we need to get comfortable with them. To that end, let's do one more example.

F.4 Residential vs. Commercial Meters

Maine takes a different approach to pricing electricity. There are three different rate structures, referred to as different meter types.

TODO! These figures need updating. Also, I think other sorts of metering should go later in a separate chapter or maybe even in an appendix. Yes. Definitely not here! Maybe as part of the grid chapter? Or just after the grid chapter?

F.4.1 Class A

This is the type of meter for homes with low or average electricity demand. Dave's home, and almost all homes in Maine, are on a type A meter. The rates are:

1. Supply Energy: 0.0757596$/kWh
2. Distribution Energy: 0.0762400 $/kWh
3. Transmission Energy: 0.0261800 $/kWh

So the total rate is: 0.1781796 $/kWh.

F.4.2 Class B

These meters are for small commercial buildings or homes with a large electricity demand. Customers are billed based on how much energy they use, and are also charged an additional monthly fee. The rates are

1. Supply Energy: 0.07720$/kWh
2. Distribution Energy: 0.05089 $/kWh
3. Transmission Energy: 0.02512 $/kWh

So the total rate is: 0.15021 $/kWh

The monthly fee is $13.08.

Note: The info for Class B and Class C meters is a bit over a year old. For your projects, you'll want to get updated rate information from Emera. See `http://www.emeramaine.com/business/rates/rates-schedules/`**.**

F.4.3 Class C

Class C meters are for medium-sized commercial buildings with a high electricity demand and whose peak power demand is at least 25 kW. Recall that power companies don't like big surges in demand. They would much rather you use energy at a steady rate rather than use a lot of it all at once. The reason for this is that large peak demands require additional power capacity in the grid, which is expensive.

So class C meters have a low per kWh charge and a high penalty for peak power. This provides business-owners with a strong incentive to even out their energy consumption and avoid large peak demand. The details of this type of meter are:

1. Distribution Demand: $9.04/kW
2. Transmission Demand: $8.38/kW
3. Distribution Energy: $0.00896/kWh

4. Supply Energy: $0.07420/kWh

Note that the demand charges are in units of power (kW), whereas the energy charges are in units of energy (kWh).

So the total energy charge is 0.08316 $/kWh and the total demand (power) charge is $17.42/kW. In addition, there is a monthly charge of $40.27.

F.5 Demand Pricing

Electricity use is not constant throughout the day. This poses challenges to utility companies, who by law are required to meet electricity demand of consumers. If lots of people return home after work around 6:00 PM, and everybody wants to turn on their air conditioners and plug in the electric cars they just drove home, the utility company has to meet that demand. It cannot make the customer wait. This surge in electricity poses a challenge to the utility company. It needs to have power in reserve to meet a sudden spike in demand. Doing so is difficult from engineering perspective and is costly.

So the utility company would like to even out demand—have less people use electricity during the peak times of 5:00–7:00pm and more people use electricity during low-demand times late at night. To do so, many utility companies offer different prices for electricity depending on the time of day. Electricity is more expensive in the late afternoon and is cheaper late at night.

An example of this is the tiered base plan offered by PG&E (Pacific Gas and Electric Company) in northern California. For more, see: http://www.pge.com/en/myhome/myaccount/explanationofbill/etoub/index.page. We should likely say a bit more about this and work through an example or two.

Figure F.5: A MRCOOL 45,000 BTU gas furnace. Model number MGM95SE045B3XA. Image from https://www.homedepot.com/p/MRCOOL-45-000-BTU-95-AFUE-Upflow-Horizontal-Mult 309173334, accessed October 11, 2020.

F.6 Exercises

Efficiency Ratios

Exercise *F.1:* A MRCOOL[2] gas furnace has an AFUE of 95%. See Fig. F.5. Keeping your house warm over the course of a year requires a total of 96,000,000 BTU.

1. How much natural gas will the furnace burn in order to produce 96,000,000 BTU?
2. How much CO_2e will be emitted into the atmosphere as a result of burning this much natural gas?

[2] Yes, there really is a furnace with the brand name MRCOOL. And yes, it really is in all caps. I'm not yelling.

Exercise *F.2:* A heat pump, when used in cooling mode, has an average COP of 3.14. What is this heat pump's SEER?

Exercise *F.3:* The EnergyGuide for a heat pump is shown in Fig. F.6. Convert its HSPF into a COP.

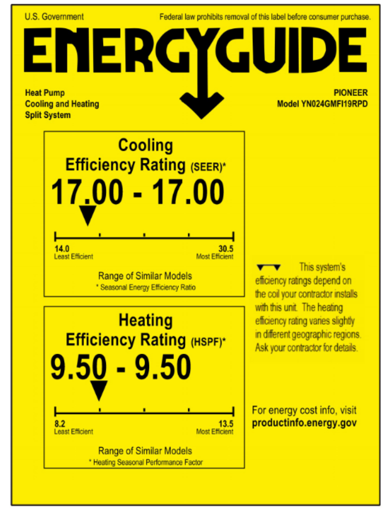

Figure F.6: EnergyGuide for the Pioneer Model #WYS024GMFI19RL-16 24,000 BTU Heat pump.

Exercise *F.4:* A resistive electric heater has a COP of 1. Convert this into an HSPF.

Batteries and amp-hours

Exercise *F.5:* How many coulombs are 200 mAh?

Exercise *F.6:* An average car battery has a capacity of approximately 48 Ah. Headlights draw about 10 A. How long will it take a fully charged battery to discharge completely if the headlights are left on after the car is turned off?

Exercise *F.7:* How long can a 4.5Ah, 1.5V flashlight battery deliver 100mA?

Exercise *F.8:* A small 1.55 v battery a capacity of 200 mAh.[3] Suppose you use this battery to power a microphone that has a resistance of 775 ohms.

1. What current flows through the microphone?
2. How long could you power the microphone with this battery?

[3] Dave uses this battery to power a small external microphone that he uses when recording math videos. See `http://www.amazon.com/Enercell-200mAh-Button-Battery-3-Pack/dp/B007TV7V0U`.

Residential vs. Commercial Meters

Exercise *F.9:* The March 2013 electricity bill for Turrets as shown here: `https://sites.google.com/a/coa.edu/college-of-the-atlantic-archive-of-sustanable-energy-projects/home/class-c000-meters` shows charges of \$836.13 from Bangor Hydro (including balance forward) for Transmission and Distribution and \$786.52 from Constellation Energy for supply (commonly referred to as generation) bringing the total monthly expense to \$1622.65.

1. What would the difference in cost be if Turrets were an A meter rather than a C meter?
2. In southern Maine CMP has increased C-meter energy rates to \$.12/kwh (leaving peak power demand rates at current level).

What would be the impact on this total if the same rate hike were levied against this bill?

3. What would have offered more savings: (1) if energy consumption had been reduced by 50% or (2) if peak power demand had been reduced by 50%?

Exercise *F.10:* An Emera customer discovered that her meter has been incorrectly classified as a B meter rather than an A meter for the past year. If she used 622 kWh per month for 12 months was she over or under charged by the company and by how much

Exercise *F.11:* A business uses 2000 kWh in a month and has a peak draw of 30 kW.

1. What is the business's power bill if it is on a C meter?
2. If the business reduced its peak draw and so was reclassified to a B meter (but still used 2000 kWh of energy), what would its power bill be?

Exercise *F.12:* A business uses 20000 kWh in a month and has a peak draw of 30 kW.

1. What is the business's power bill if it is on a C meter?
2. If the business reduced its peak draw and so was reclassified to a B meter (but still used 20000 kWh of energy) what would its power bill be?

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