Linear Algebra
Background for Strang's "Elimination with Matrices" Lecture

In this lecture Strang tells us how to solve $Ax = b$ for x, where x is a column vector. In the examples in the first lecture and first assignment, things were sufficiently nice that the solution x could be discovered without any computation.

Strang begins the lecture by considering a 3×3 system and doesn't quite explain what he's doing before he starts. These notes should provide background so you can jump into Strang's lecture.

I'll do a simple example. Suppose we want to solve the following system:

$$\begin{aligned} 2x - y &= 8 \qquad (1) \\ 4x + y &= 10\ . \qquad (2) \end{aligned}$$

There are lots of ways to solve this system of two equations and two unknowns. One way is to get rid of the x in the second equation. To do so, we take row two and subtract two times row one from it. This gives:

$$\begin{aligned} 2x - y &= 8 \qquad (3) \\ 0 + 3y &= -6\ . \qquad (4) \end{aligned}$$

We now see that the second equation must have $y = -2$ as a solution. We can then substitute $y = -2$ in to the first equation. This gives:

$$2x - (-2) = 8\ . \tag{5}$$

Thus, $2x = 6$, so $x = 3$.

We can also think of this in terms of the matrices. We don't need to keep writing x and y. Then, ignoring the right hand side for a moment, we start with:

$$\begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix} \tag{6}$$

We subtract two of row one from row two and obtain

$$\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \tag{7}$$

The matrix is now *upper triangular*. Everything on the bottom right, below the diagonal, is zero. This process is called *elimination*. One then can quickly solve for x and y, starting with the bottom equation and working up the rows. This part of the process is called *back substitution*.

In the lecture Strang jumps in to a 3×3 system of equations and starts the elimination process using the matrix form. I think if you take a moment to work through the 2×2 example here, you'll then be able to follow Strang's lecture.