Logistic Equation with Harvest
Differential Equations
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Let's go back to our old friend the single-species logistic equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \ . \tag{1}$$

Let's say that the growth constant r is 3 and the carrying capacity is 100. We can sketch the right-hand side of this equation and doing so will show us that there are two equilibria. A stable equilibrium at 100 and an unstable equilibrium at 0. Any population greater than 0 and less than 100 grows until it reaches 100. We can summarize this with a phase line.

Now for something new. Let's add a harvest term h:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - h \ . \tag{2}$$

The idea is that fish or whatever are harvested at a constant rate h. E.g., if $h = 10$, that means that 10 fish are harvested every month, regardless of how many fish are present.

The question we will consider is: **How does the phase line for the solutions to the differential equation change as h changes?** I will give each group a few h values. For each value, plot the right-hand side of Eq. (2) and use that plot to make a phase line. Draw the phase line on the strip of paper I give you.

Feel like doing some algebra? Solve for the equilibria of this system. Your answer will depend on h. That is, set the right-hand side of Eq. (2) to 0 and solve for P.

Don't feel like doing algebra? Fair enough. Use WolframAlpha to solve it for you.