Chapter N1: Introduction

N1.6 & N1.7: The Time-Derivative of a Vector and Velocity

In Unit C, we often wrote

$$v = \frac{dx}{dt} \tag{1}$$

where dx was understood to be a change in position. (Sometimes we used Δx instead of dx.) How small must dx and dt be? We said that dt should be small enough so that dx doesn't change much during dt.

But we can do better than this. We define

$$v(t) = \frac{dx(t)}{dt} \equiv \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} \, . \tag{2}$$

Here $x(t)$ means the value of x at time t. This equation is just "rise over run", where the "run" (Δt) is very very tiny.

The quantity $v(t)$ is the instantaneous velocity—the velocity at the instant in time t. Geometrically, $v(t)$ is the instantaneous slope of $x(t)$.

The derivative of a vector is defined as:

$$\frac{d\vec{r}(t)}{dt} \equiv \begin{bmatrix} \frac{dx(t)}{dt} \\ \frac{dy(t)}{dt} \\ \frac{dz(t)}{dt} \end{bmatrix} \tag{3}$$