Unit Four Homework

Physics II

Due Monday, April 27, 2020

College of the Atlantic. Spring 2020

Instructions:

- Do problem one in pairs.
- If you want, you can do the other problems in this assignment in pairs, too. If so, it's fine to hand in only one write-up.
- "Hand in" the problem on google classroom. I'll remember to make an assignment there so you have someplace to submit it. Please don't email me the homework; it's a lot easier for me if you submit it on classroom.
- In addition to these problems, there are some problems that you should do on Edfinity. There is one Edfinity assignment: Homework 04. You should "hand in" these assignments individually, but of course it's totally fine—and in fact encouraged—to work on the Edfinity problems with others.

1. Based closely on a problem from Tom Moore *Six Ideas that Shaped Physics: Unit R (second edition)*, (2003). In 2095 a message arrives at earth from a growing colony at Tau Ceti, which is 11.3 years from earth. The message asks for help in combating a virus[1] that is making people seriously ill. Using advanced technology available on earth, scientists are quickly able to construct a vaccine that confer immunity to the virus. You have to decide how much of the drug can be sent to Tau Ceti. The space probes available on short notice are capable of doing one of the following:

    - Sending 200 g of the vaccine at a speed of 0.95.
    - Sending 1 kg at a speed of 0.90.
    - Sending 5 kg at a speed of 0.80
    - Sending 20 kg at a speed of 0.6

    The catch is that the vaccine degrades such that after five years it is no longer effective.

    (a) Is it possible to send the vaccine to Tau Ceti? If so, how much can you send?

    (b) What is the slowest speed a spaceship could travel at and still have the vaccine be effective when it arrives on Tau Ceti?

    You'll save yourself some trial and error if you answer part b first. But you might need to work through a few of the scenarios in part a before you see how to do part b.

2. **Optional. Recommended for math enthusiasts.** Regular trigonometry is based on the unit circle, $y^2 + x^2 = 1$. Hyperbolic trigonometry is based on the unit hyperbola, $t^2 - x^2 = 1$. You'll explore the hyperbolic trig functions, known as $\sinh, \cosh$, and $\tanh$, and pronounced "sinch", "cosh", and "tanch." This problem is almost verbatim problem R4A.1 from Moore, *Six Ideas that Shaped Physics: Unit R (second edition)*, 2003, p.73.

[1] Yes, Tom Moore really has a problem about a serious virus in his 2003 relativity book. A little spooky, eh?

Just as we can describe the relationship between the hypotenuse of a triangle and the coordinate lengths of its side by using the sine and cosine functions, it turns out that we can describe the relationship between the spacetime interval Δs between two events in terms of the coordinate separations Δt and Δx between those events in terms of the hyperbolic sine and cosine functions, defined as follows:

$$\sinh(\theta) = \frac{1}{2}(e^{\theta} - e^{-\theta}), \quad \cosh(\theta) = \frac{1}{2}(e^{\theta} + e^{-\theta}), \tag{1}$$

(a) Show that $\cosh^2(\theta) - \sinh^2(\theta) = 1$. This means that if the spacetime interval between two events occurring along the spatial x axis is Δs, then the coordinate separations Δt and Δx between these two events can be written $\Delta t = \Delta s \cosh(\theta)$ and $\Delta x = \Delta s \sinh(\theta)$ for some appropriately chosen value of θ, just as in plane trigonometry $\Delta x = \Delta d \cos(\theta)$ and $\Delta y = \Delta d \sin(\theta)$.

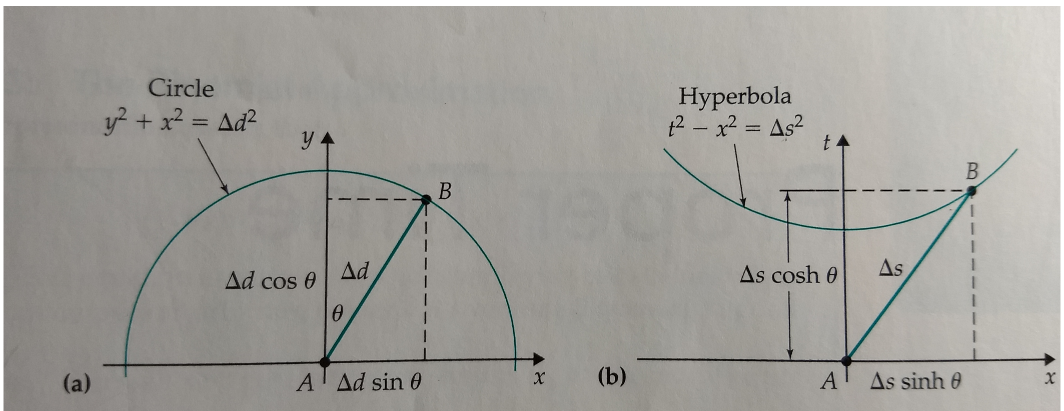

Figure R4.14
(a) Plane trigonometry. (b) Spacetime trigonometry. (Note that $\Delta s \cosh\theta > \Delta s$, even though the "hypotenuse" representing Δs *looks* bigger on the diagram.)

Figure 1: Figure from Moore, *Six Ideas that Shaped Physics: Unit R (2nd edition)*, 2003, p.73.

(b) Argue that θ in the hyperbolic case is *not* the angle that the line AB makes with the t axis in the spacetime diagram in the figure. Argue instead that as $\theta \to \infty$, the angle that AB makes with the t axis approaches 45°.

(c) Argue that if v is the speed of an object that goes from event A to event B at a constant velocity, the "angle" θ is in fact $\tanh^{-1} v$.

(d) When $v = 0.8$, $\theta = 1.10$, and when $v = 0.99$, $\theta = 2.65$. Verify these on your calculator or a computer (wolframalpha) using hyperbolic trig functions. When $v = 0.99$, what are the values of $\cosh(\theta)$ and $\sinh(\theta)$?

(e) Argue that as $\theta \to 0$, $\sinh(\theta) \to 0$, while $\cosh(\theta) \to 0$, just like regular trigonometric functions. This also means that $\tanh(\theta) \to 0$ in this limit. Use this to argue that as $v \to 0$, $\Delta s \to \Delta t$, and $\Delta x \to 0$.

(f) Using the definition of the hyperbolic trig functions given in Eq. (1), show that the second derivative of $\cosh(\theta)$ is $\cosh(\theta)$ and that the second derivative of $\sinh(\theta)$ is $\sinh(\theta)$.