Homework Four
Thermodynamics
College of the Atlantic
Due Friday, Feb 7, 2025

All problems are from the textbook, unless otherwise stated.

1. Deriving a useful approximation.
   (a) What is the derivative of $\ln(1+x)$?
   (b) Evaluate this derivative for $x = 0$.
   (c) Figure out the equation of the line tangent to $\ln(1+x)$ at the point $x = 1$. You should find that the equation of the tangent line is simply $y = x$.
   (d) You have thus derived the approximation we've used in class repeatedly over the last several days:
   $$\ln(1+x) \approx x \,, \tag{1}$$
   which is valid for $|x| \ll 1$.
   (e) Check the accuracy of the approximation in Eq. (1) for $x = 0.1$, $x = 0.01$, and $x = 0.001$. I.e., for each value of x evaluate the left-hand side of Eq. (1) using a calculator, and compare it two the right-hand side.

2. Suppose you flip 1000 coins.
   (a) Write down an expression for the multiplity of the macrostate for 500 heads and 500 tails.
   (b) Write down an expression for Ω_{all}, the total number of microstates. (I.e., the total number outcomes that can occur if you flip 1000 coins.
   (c) Determine the probability of the macrostate with 500 heads and 500 tails. Do so by using Sterling's approximation:
   $$N! \approx N^N e^{-N} \sqrt{2\pi N} \,. \tag{2}$$

3. 2.21 (Use WolframAlpha or desmos or whatever you're used to using to make plots.)

4. 2.26

5. **Optional:** 2.17. In this problem you'll determine an expression for the multiplicity of an Einstin solid for $q << N$. Good practice using Sterling's approximation and the Taylor expansion for the natural log, if that's your thing.