Unit Three Homework

Physics II

Due Sunday, April 19, 2020

College of the Atlantic. Spring 2020

Instructions:

- Do problem three in pairs.
- If you want, you can do the other problems in this assignment in pairs, too. If so, it's fine to hand in only one write-up.
- "Hand in" the problem on google classroom. I'll remember to make an assignment there so you have someplace to submit it.
- In addition to these problems, there are some problems that you should do on Edfinity. There is one Edfinity assignment: Homework 03. You should "hand in" these assignments individually, but of course it's totally fine to collaborate with others.

1. Two clocks, P and Q, leave earth on different spaceships at the spatial origin ($x = 0$) on the earth frame at $t = 0$. Call this event O. Both clocks move along the x-axis. Clock P travels at a speed of $4/5$ and Q travels at a speed of around $1/5$. After a while, however, clock P decelerates, comes to rest, and then begins to move back toward the origin. A short time later, clock P collides with the slower clock Q, which has been moving with constant speed up the x-axis all the while. Let the collision of the clocks be event A.

   (a) Draw a qualitatively accurate spacetime diagram for the situation described above. Label events O and A, and the worldlines for clocks P and Q.

   (b) Assume that clocks P and Q were both synchronized with a clock at the origin on the earth when they departed. Will clocks P and Q read the same time when they collide? Explain briefly.

   (c) An observer on earth measures the time between events O and A with a pair of synchronized clocks (one at $x = 0$ on earth and the other at the location of the clock collision). Clocks P and Q also each register a time between these events.

      i. Which clock(s), if any, measure the coordinate time between events O and A?
      ii. Which clock(s), if any, measure a proper time between events O and A?
      iii. Which clock(s), if any, measure the spacetime interval between events O and A?

2. At $t = 0$ an alien spaceship passes by the earth. Let this be event A. AT $t = 13$ min (according to synchronized clocks on earth and Mars) the spaceship passes by Mars, which is 5 light-minutes from earth at the time: let this be event B. Radar tracking indicates that the spaceship moves at a constant velocity between earth and Mars. Just after the ship passes earth, people on earth launch a probe whose purpose is to catch up with and investigate the spaceship. This probe accelerates away from earth, moving slowly at first, and moving faster and faster as time passes. Eventually it catches up with and passes the alien ship just as it passes Mars. (Ignore the gravity and relative motion of the earth and Mars, which are small on the scale of minutes. Thus, treat the earth and Mars as if they were both at rest in the inertial reference frame of the solar system. Also, assume that both the probe and the alien spacecraft carry clocks.)

(a) Draw a *quantitatively* accurate spacetime diagram of the situation, including labeled worldlines for the earth, Mars, the alien spacecraft, and the probe. Also label events A and B.
(b) Whose clocks measure coordinate times between events A and B? Explain briefly.
(c) Whose clocks measure proper times between these events. Explain briefly.
(d) Does any clock in this problem measure the spacetime interval between the events? Explain briefly.

3. A train is moving due east at a large constant speed on a straight track. Imagine that Anastajia is riding on the train exactly in the middle—halfway between the front and the back. Beowulf is sitting by the tracks only a few feet from the train. Let the event of Anastajia's passing Beowulf be the origin O in both frames. At this same instant, both Anastajia and Beowulf receive the light from lighting flashes that have struck both ends of the train. Anastajia concludes that since she is in the middle of the train and she received the light from the strikes at the same time, the lightning strikes must have occurred at the same time in her reference frame. Is she right? If not, which strike really happened first? Can Beowulf conclude from his seeing the flashes at the same time that the strikes happened at the same time in the ground frame? Why or why not? If not, which strike happened first in his frame? (Hint: Draw spacetime diagrams for the situation in both Ana's and Beowulf's frames.)

4. **Optional. Recommended for folks who want to puzzle through some math. This problem basically has you derive the Lorentz equation for time dilation. It's fun. This is a tricky problem, but by no means impossible.** Consider the figure shown below that we used to derive the result that coordinate time is frame dependent. Let the spatial separation between each side clock and the center clock be $L = 12$ ns. And let the speed of the clocks relative to the at-rest frame be $\beta = 0.4$. Find the time separation $t_B - t_A$ as measured in the at-rest frame. Here are some hints:

   - Work symbolically. Don't plug in numbers for the clock separation L and the speed β until the end.
   - Consider the left clock. In the time between $t = 0$ and $t = t_A$, the clock moves a distance βt_A toward the light flash coming toward it. Thus the total distance that the light flash has to cover in this time interval is not L but instead is $L - \beta t_A$. But since the light flash travels with a speed of 1 in all reference frames, *t*ime that the light flash takes to travel to the clock is equal to the *distance* that it has to travel. Write this statement as an equation and solve for t_A.
   - Carry out a similar analysis for t_B.