Quest 1: Replicator Dynamics

Differential Equations

College of the Atlantic. February 2, 2026

Work on this in a team of four. Be ready to give a 15-20 minute presentation on this on Wednesday 11 February, 2026.

This quest is about **replicator dynamics**. The basic equation is:

$$\frac{dx_A}{dt} = x(f_A - \bar{f}) \tag{1}$$

Here A is a particular strategy in some strategic interaction and x_A is the frequency of that strategy in the population. The quantity f_A is the fitness of strategy A, and $\bar{f}$ is the average fitness of the population.

We'll study this for rock-paper-scissors game. Let

- x_0 = fraction of players playing rock
- x_1 = fraction of players playing paper
- x_2 = fraction of players playing scissors

Since these are fractions, $x_1 + x_2 + x_3 = 1$.

Write down the payoff matrix for rock-paper-scissors. If a player wins, the payoff is 1, if they lose, it's -1, and if they tie, it's 0.

Write down the replicator equations for rock-paper-scissors. This will require thinking through what the average fitness (payoff) is as a function of x_1, x_2, and x_3. Since there are three strategies, there will be three* equations.

Solve the system of equations. What behavior do you see? How can you visualize this behavior? Are there any equilibria? If so, are they stable?

*But not really. Actually kinda two, right?