Qualitative Analysis of yet another Differential Equation

Differential Equations

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We will again think about an equation of the following form:

$$\frac{dT}{dt} = f(T) \ . \qquad (1)$$

This is of the same form as the logistic equation. The only difference is that now I'm using T instead of N, since we're going to be thinking about temperature T instead of unicorn population P.

You are canning some tomatoes. In the final stage of canning, tomato puree is placed in mason jars which are then immersed in boiling water for around 45 minutes. You then take the cans out of the boiling water and put them on the counter to cool. The temperature of your kitchen is a steady 25 degrees C.

1. After a long time, what is the temperature of the jar of tomatoes?

2. Let's think now about how the jar cools. When is it cooling at the fastest rate? Under what conditions does it stop cooling? Sketch a possible $f(T)$ that could describe how the rate* of cooling dT/dt depends on the temperature T.

3. Sketch $T(t)$ for this situation. Think about the concavity of your graph. How is this

4. At some point during the canning process you take a can of a beverage out of the refrigerator. Your refrigerator's temperature is 5 C. Sketch $T(t)$ for the beverage. (Assume that you don't open it.)

5. Is the behavior of the beverage's $T(t)$ curve consistent with the $f(T)$ curve that you sketched?

6. For this differential equation, find and classify all equilibria.

7. Write down a possible formula for $f(T)$. Your equation should have two terms in it: k, a parameter related to the rate of cooling; and T_e, where T_e is the temperature of the environment. Think carefully about minus signs.

*Note that if the tomatoes are cooling off, the derivative of $T(t)$ should be negative.