Quest 2: SIR with Seasonal Forcing
Differential Equations
College of the Atlantic. February 2, 2026

Work on this in a team of four. Be ready to give a 15-20 minute presentation on this on Wednesday 11 February, 2026.

We'll consider SIR equations with demographics and seasonal forcing

$$\frac{dS}{dt} = \mu - \beta(t)SI - \mu S \ , \tag{1}$$

$$\frac{dI}{dt} = \beta(t)SI - \gamma I - \mu I \ , \tag{2}$$

$$\frac{dR}{dt} = \gamma I - \mu R \ . \tag{3}$$

Here S, I, and R represent the *fractions* of the population that are susceptible, infected, and recovered. We assume that the birth rate and the death rate are both given by μ.

What new is now β is not constant! It's a function of time t, where time is measured in months. Let's say that $\beta(t)$ jumps between two values. From January to June, it is 10, and from July to December, it is 5. (This isn't realistic for seasonal variation, since it probably should be 10 from October to March. But doing it this way would be a bit more difficult to code up, and doesn't matter for the big picture.

BTW, What is R_0 for this model?

Anyway, the main part of your quest is to code up this model. Doing so will require thinking about how to handle $\beta(t)$. What behaviors do you see? How would you describe and visualize this? So parameters to start with: $\gamma = 1/0.25$, and $\mu = 12$. What do these number mean in practical terms?