Quest 4: Competitive Lotka–Volterra

Differential Equations

College of the Atlantic. February 5, 2026

Work on this in a team of four. Be ready to give a 15-20 minute presentation on this on Wednesday 11 February, 2026.

First, let's modify the Lotka-Volterra Equations so in the absence of the other species, each grows logistically. Recall that logistic growth is:

$$\frac{dP}{dt} = rP(1 - \frac{P}{K}) \ , \tag{1}$$

where P is the population, r a growth rate, and K is the carrying capacity.

However, imagine dividing both sides of the above equation by K, then we have

$$\frac{d\frac{P}{K}}{dt} = r\frac{P}{K}(1 - \frac{P}{K}) \ . \tag{2}$$

Let's define a new variable x, which is the population expressed as fraction of the carrying capacity. That is, $x = P/K$. Then the logistic differential equation takes a simpler form:

$$\frac{dx}{dt} = rx(1 - x) \ . \tag{3}$$

The main point is that we can make the K go away by re-scaling the population.

Then, a two-species Lotka-Volterra system would have the following form:

$$\frac{dx}{dt} = r_x x(1 - x) - axy \ , \tag{4}$$

$$\frac{dy}{dt} = r_y y(1 - y) - bxy \ . \tag{5}$$

Let's further assume that both growth rates are 1. Then, we have

$$\frac{dx}{dt} = x(1 - x) - axy \ , \tag{6}$$

$$\frac{dy}{dt} = y(1 - y) - bxy \ . \tag{7}$$

Note: In this scenario the species are competing with each other. The presence of x negatively impacts y, and the presence of y negatively impacts x. Also, remember that the populations have been rescaled, so x and y are always between 0 and 1.

What does the model do? Is it ever possible for the two species to co-exist? If so, under what conditions? Do the initial conditions matter? Figuring this all out will require a combination of visualizing computational solutions and doing some pencil-and-paper pondering.