Chapter 4:4: More Optimization and Modeling

Calculus I

College of the Atlantic. November 18, 2024

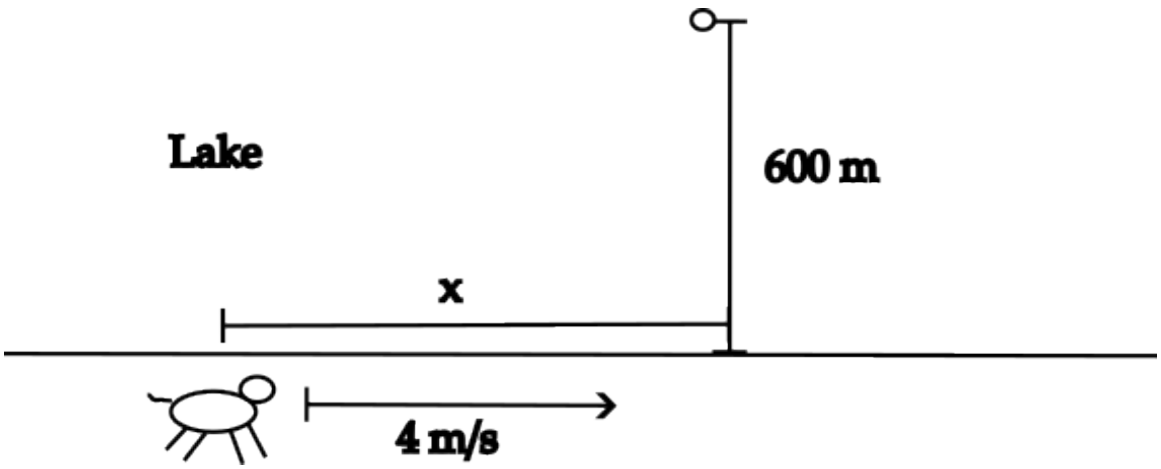

Figure 1: A sidewalk and a snowy field.

1. A dog is running straight along the shore of a lake at a speed of 4 m/s. In the water, 600 meters from the shore, is a ball. Let L represent the distance between the dog and the ball.

   (a) Use implicit differentiation to derive an expression for dL/dt.

   (b) At what x does $dL/dt = 2$?

   (c) Does your answer to the above question look familiar? What is the significance of this?

2. You need to make a cylindrical container that holds 1000 cubic centimeters of liquid. What are the dimensions of such a container that uses the smallest amount of material?

3. In the game League of Legends[1], a player's Effective Health E when defending against physical damage is given by:

$$E = \frac{H(100 + A)}{100}, \tag{1}$$

where H is Health and A is armor. Health costs 2.5 gold per unit and Armor costs 18 gold per unit.

   (a) You have 3600 gold. How much H and how much A should you buy so that you are best able to withstand the attacks of your enemies?

   (b) At some point in the game you now have 1080 H and 10 A, and you have 720 gold to spend. What should you spend the gold on in order to maximize your effective health? Note that you don't want to maximize the effectiveness of what you purchase—you want to maximize the effectiveness E of your *resulting* health and armor.

   (c) A little while later in the game, you have 2000 H and 40 A. You now have 1800 gold to spend. How should you spend your gold to maximize the effectiveness E of your resulting health and armor?

4. When someone ingests a drug[2], it doesn't go into the bloodstream all at once. The concentration $c(t)$ will rise and then fall. In some circumstances[3] the concentration curve is given by:

$$c(t) = \frac{D}{1 - k_b/k_a}\left(e^{-k_b t} - e^{k_a t}\right), \tag{2}$$

where k_a is the absorption rate and k_b is the elimination rate, and D is the dose. (Compare your answer with that given on page 108 of Seng, K-Y., et al. "Population pharmacokinetics of caffeine in healthy male adults using mixed-effects models." *Journal of clinical pharmacy and therapeutics* 34.1 (2009): 103-114.)

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[1]This problem is based very closely on a post by Chris Cunningham on the math educators stack exchange, available at `https://matheducators.stackexchange.com/questions/1550/optimization-problems-that-todays-students-might-actually-encounter`.

[2]Based on a post by Vectornaut on the same state exchange thread referenced above.

[3]Namely, when absorption and elimination are both first-order—meaning the rate of absorption depends on the concentration.