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Feldman, David P.

Chaos and fractals. An elementary introduction. (English)

Oxford: Oxford University Press. xxi, 408 p. £ 55.00, $ 99.99/hbk; £ 27.50, $ 54.99/pbk (2012). ISBN 978-0-19-956643-3/hbk; ISBN 978-0-19-956644-0/pbk

This is an interesting and unconventional text book aimed at introducing students with modest mathematical background (in North American terminology one might optimistically pitch it at exceptionally energetic students whose major is not in the sciences with mathematical courses prior to but not including 'calculus'). This locates the main topic – dynamical systems, chaos, and insight into the way that simple mathematical models can exhibit complex dynamical behaviour – much lower down in the mathematical tree than is usual. The preface opens with a well-know quotation from May's influential Nature paper of 1976 arguing that the way in which nonlinear models of the simplest kind lead inevitably to chaotic phenomena should be taught both early (prior to 'calculus') and widely (in politics, economics, and so on). This book is one attempt to flesh that out, and it is a very largely successful one. The book is split into seven main parts: an introduction to discrete dynamical systems viewed as iteration of a map, some basic tools for graphing iterates, some early models, and a short detour into the history of debates about determinism and the origins of our belief in mathematical models for the physical universe; 'chaos' as observed initially via the logistic equation, Lyapunov exponents, sensitive dependence on initial conditions, bifurcation diagrams, and some statistical language; fractals arising in this context and some fractal geometry, box counting dimension, and more probabilistic language; some complex dynamics and an introduction to the Julia and Fatou set and the idea of parameter spaces, the Mandelbrot set in particular; a quick overview of higher-dimensional and other kinds of systems, notably differential equations and cellular automata; a short narrative overview trying to draw conclusions about the pervasive presence of chaotic phenomena and fractal geometry; and finally three appendices on algebra, statistics, and further reading.

There are many ways to approach material like this, all having advantages and disadvantages. At one extreme the course might demand background in calculus, analysis, measure theory, functional analysis, complex variables, etc. and be able to study dynamical systems with a heavy-laden mathematical toolbox. This would dramatically narrow the number of students exposed to these beautiful and important ideas, but enormously increase the scope of what could be covered. At the other extreme one might abandon all hope and present some of these topics essentially as observed mysterious phenomena, generating beautiful and multi-coloured pictures – a deeply flawed pedagogical approach which is peculiarly tempting in complex dynamics and cellular automata.

This book chooses a more difficult but ultimately more fruitful path. As far as possible (there are flagged exercises requiring a little more) the minimal prerequisites are not exceeded – though getting through all the material would require real perseverance and some innate mathematical ability. Where mathematical concepts are being skipped over (and, of course, a great many concepts and proofs need to be) this is said so clearly,

and some intuition about what is meant by more sophisticated notions like ergodicity is provided. The fact that this material has been honed by real classes comes across clearly – the examples and explanations are invariably carefully thought out and clear. There are also some unconventional asides of great potential value. The third appendix, for instance, is not just a list of additional sources. It also explains what the peer-reviewed literature is, why it matters, and some of the shortcomings and possible pitfalls in the literature. There is also a practical guide to actually accessing some of the literature – if you are not located inside the mathematics department in a research-intense university, how do you lay hands on a research article?
For the right audience and instructor, this is a wonderful book. With considerable effort on both sides it can take a wide audience with modest mathematics to a reasonable understanding of what is behind much of the complex phenomena seen in modern mathematical models of the physical universe.

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Keywords : chaos; fractal geometry; nonlinear dynamics; complex dynamics; cellular automata
Classification :
∗37-01 Instructional exposition (Dynamical systems and ergodic theory)
37D45 Strange attractors, chaotic dynamics
28A80 Fractals
37F50 Small divisors, rotation domains and linearization
00A05 General mathematics