Practice Problems for the Midterm
Computational Physics
Kigali Institute of Science and Technology
Semester One: 2011–12

These problems are similar to those that will be on the midterm exam on January 5, 2012.

1. Consider the function $g(x) = x^3 - 2x$
   (a) Sketch the function $g(x)$, and use algebra to find the largest root of $g(x)$.
   (b) By hand, carry out the first three steps in the bisection method to find the largest root of $g(x)$. Start with the interval $[0, 2]$.
   (c) By hand, carry out the first two steps in Newton's method to find the largest root of $g(x)$. Start with $x_0 = 1$.
2. What matlab commands would you need to enter so that it found a numerical solution to the equation $f(z) = \cos(2z) - 1$?
3. What would you need to type to have matlab evaluate the following expressions?
   (a) $3e^{-2}$
   (b) $3\sin(4)$
   (c) $\sqrt{100}$
4. What would you need to type to make matlab produce a smooth plot of the function $f(x) = \cos(t)$ from $t = 1$ to $t = 20$?
5. Use Euler's method to find an approximate solution to the following differential equation
   $$p'(t) \;=\; p^2 - t\;, \quad p(0) = 100\;. \tag{1}$$
   Come up with an estimate for $p(6)$. Use $\Delta t = 2$. If $\Delta t = 1$, would you expect your answer to be more accurate or less accurate? Why?
6. The bisection method for finding the root of a function converges linearly.
   (a) Explain briefly what this means.
   (b) Suppose you know that the root of a function is somewhere between 0 and 10. You need to determine the root to within 0.001. About how many steps of the bifurcation method would you need to apply to accomplish this? Briefly explain your reasoning.
7. Today, the number of mangoes on a mango tree is 100. Mangoes are currently falling off the tree at a rate of 15 per day.
   (a) What is your best estimate for the number of mangoes on tree in three days.
   (b) Why is the above answer an approximation and not an exact value?
   (c) What is your best estimate for the day on which there are no mangoes left on the tree?
8. Consider a "semi-infinite" square well as shown in Fig. 1. In region I the potential is finite. In region III the potential is finite.
   (a) In region I the general solution is:
   $$\psi_I(x) \;=\; Ce^{\beta x} + De^{-\beta x}\;, \quad \text{where} \quad \beta \;=\; \sqrt{2m(V_o - E)/\hbar^2}\;. \tag{2}$$
   Briefly explain why D must equal zero in the above equation.

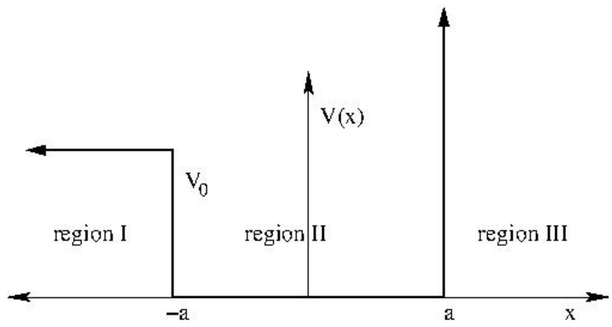

Figure 1: The potential for the infinite square well.

(b) Write down the general solution to Schrödinger's equation in Region II.
(c) Write down the general solution to Schrödinger's equation in Region III.
(d) Write down the boundary conditions that apply to the wavefunctions at $x = -a$. Briefly explain the physical or geometric means of these boundary conditions.

9. For the infinite square well, we found the following energies for bound states with even wave functions:

$$E_n = \frac{(n+1/2)^2\pi^2\hbar^2}{2ma^2} . \tag{3}$$

(a) For a certain system, suppose the ground state ($n = 0$) energy is 2 eV. If the width of the well is doubled, how would the ground state energy change?
(b) If the mass of the particle is doubled, how would the ground state energy change?
(c) If the particle is an electron and the width of the well is 1.24 nm, what is the ground state energy?

10. For the finite square well we found that the allowed energies for even wave functions were given by the following:

$$\alpha \tan(\alpha a) = \beta , \tag{4}$$

where

$$\beta = \sqrt{2m(V_o - E)/\hbar^2} , \tag{5}$$

and

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}}. \tag{6}$$

(a) Show that, in the $V_0 \to \infty$ limit, the solutions to Eq. (4) are equivalent to those of the infinite well, Eq. (3).
(b) Explain how this limiting case can be used to check that a computational solution is giving the correct results.
(c) Why is it a good idea to re-write Eq. (4) using sin and cos (and not tan) before entering it into matlab?