Dimensional Analysis

Quantitative Skill: Dimensional Analysis

In physics (or any quantitative science) most numbers carry units. For example, the momentum of a cart is not 5, but 5 kg m/s. (See the discussion on *Unit Consistency* on the bottom of p. 23.) For an equation to make sense, the units must be the same on each side. The equation

$$4\text{kg} = 4\text{m} \tag{1}$$

makes no sense. The number 4 equals the number 4, but saying that four kilograms is the same as four meters is clearly wrong. Said another way, we are not free to add any two physical quantities together; it is only meaningful to add (or subtract) quantities that have the same units.

When dealing with algebraic, as opposed to numeric expressions, what matters is the quantity's *dimension*. The term dimension here is used in the following way. If a variable x represents a an area, then we say it dimensions of length squared. Speed has dimensions of length/time.

Examining the dimensions of an algebraic equation give us a useful way to check the results of an algebraic calculation. As an example of this, consider Eq. (C2.3).

Practice: In the following, let x, y, z, r, and R all stand for lengths. Which of the following formula represent a length? Area? Volume? Which are nonsense?

1. $4\pi r^2$.
2. $\frac{4}{3}\pi R^3$
3. xy
4. $x(y+z)$
5. $x(1+y)$
6. $x+y+z$
7. $x(1+\frac{y}{z})$
8. x^x
9. $R^{\frac{x}{y}}$

More practice:

1. Steve Katona weighs 180 lbs. How many kilograms is this?
2. How many meters are in a mile?
3. How many square meters in a square mile?
4. Convert 95 mi/hr to m/s.
5. How many cubic inches are in one cubic meter?