16.1: Integrals in One and Two Dimensions

Calculus III

College of the Atlantic. Winter 2016

- 1. Roadkill occurs along a 100 mile stretch of road. The Department of Transportation has determined that the rate of occurrence is given in the table below, in units of roadkill per mile per month.
 - (a) Find upper and lower estimates for the number of animals killed each month on the 100 mile portion of this road.
 - (b) Suppose you knew that the roadkill density was described by the function $\rho(x)$. Write an exact expression for the total number of roadkill per month.
- 2. In a certain forest, the density of rabbits has been carefully measured. The rabbit density is higher in some places than others. The densities, in units of rabbits per square kilometer, are given in the table, where x and y and are measured in meters. Find lower and upper estimates of the total number of rabbits in the region $1 \le x \le 1.6$, $1 \le y \le 1.4$.
- 3. Estimate $\int_A x^2 y \, dA$ where A is the rectangle whose corners are: (0,0), (2,0), (0,1), and (2,1). Do not use the fundamental theorem of calculus. Instead, make a table of numbers with $\Delta x = 0.5$ and $\Delta y = 0.5$.
- 4. Suppose we wish to evaluate $\int_R f(x,y) dA$, where R is the rectangular region with corners (0,0), (2,0), (0,3), and (2,3). Write this as an iterated integral two different ways.
- 5. Suppose we wish to evaluate $\int_R f(x,y) dA$, where R is the triangular region with corners (0,1), (0,0), and (0,2). Write this as an iterated integral two different ways.
- 6. Suppose we wish to evaluate $\int_R f(x,y) dA$, where R is the triangular region with corners (0,1), (0,4), and (4,1). Write this as an iterated integral two different ways.

Mile	Roadkill Density
0	8
25	4
50	3
75	3
100	0

X 1 1.6 1.4 1.2 1.6 0.4 0.2 0.0 0.0 0.0 0.0 0.1 0.1 1.1 1.1 1.8 1.6 1.4 1.1 1.6 1.6 1.6 0.0 0.2 0.0		>																				
8 7.24 6.56 5.96 5.44 4.36 4.16 4.04 4.16 4.	×	7	1.8	1.6	1.4		1	8.0	9.0	9.0	0.5	0				-0.8			-1.4	-1.6	-1.8	-5
7.24 6.48 5.8 5.2 3.24 3.28 3.24 3.28 3.24 3.28 3.24 3.28 3.24 3.28 3.24 3.28 3.24 3.58 3.24 3.58 3.24 3.58 3.24 3.56 3.2 3.25 3.29 3.25 3.25 3.29 3.25 3.26 3.29 3.24 3.59 3.4 3.59 3.4 3.59 3.4 3.59 3.4 3.59 3.59 3.4 3.55 3.59 3.59 3.4 3.59 3.4 3.59 3.4 3.59 3.4 3.59 3.4 3.59 3.4	2.0			95.9	5.96	5.44		_			4.04					4.64		5.44	_	95.9	7.24	∞
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