Stats 110 Notes

Daniel Flannery

June 20, 2015

1 Lecture 1: Probability and Counting

1.1 Notes

- A **Sample Space** is the set of all possible outcomes of an experiment
- An **event** is a subset of the sample space.
- **Naive Definition of Probability**:

$$P(A) = \frac{\#\ of\ favorable\ poutcomes}{\#\ possible\ outcomes}$$

- **The Multiplication Rule** - if you have experiment with n possible outcomes, and for each outcome of the first experiment there are n outcomes for the second experiments, ..., for each r^{th} experiment there are n_r outcomes, then there are $n_1 * n_2 * ... * n_r$ overall possible outcomes. Assumes all outcomes are equally likely within a finite sample space
- **Binomial Coefficient**:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

  0 if $k > n$

  How do you get there? Number of subsets of size k of group n. Choosing k from n in order: $n * (n-1)(n-2) * ... * (n-k+1)$ Now if order does not matter: $\frac{n*(n-1)(n-2)*...*(n-k+1)}{k!}$ Now through canceling we reach the formula for the binomial coefficient.

1.2 Example: Full House, 5 Card Hand

There are 13 suits. We must choose one suit then 3 of the 4 possible cards. Then we must choose another suit of the remaining 12, then select 2 of the 4 possible cards. Then divide by the sample space which is all possible 5 card hands from a deck of 52 cards.

$$\frac{13\binom{4}{3} * 12\binom{4}{2}}{\binom{52}{5}}$$

1.3 Sampling Table

Sampling Table		
	Order Matters	**Order Doesn't Matter**
Replace	n^k	$\binom{n+k-1}{k}$
Don't Replace	$n(n-1)(n-2)...(n-k+1)$	$\binom{n}{k}$

2 Lecture 2: Story Proofs, Axioms of Probability

2.1 Notes

- Classic and Fundamental Relationship:

$$\binom{n}{k} = \binom{n}{n-k}$$

- Exploring picking k times from set of n objects with replacement where order doesn't matter. Let's try out a few cases:

  Exploration

  - Case 1: $k = 0 \rightarrow \binom{n-1}{0} = 1$
  - Case 2: $k = 1 \rightarrow \binom{n}{1} = n$
  - Case 3 (Simplest non-trivial example): $n = 2 \rightarrow \binom{k+1}{k} = \binom{k+1}{1} = k + 1$
  - This is equivalent to the number of ways to put k indistinguishable particles into n distinguishable boxes, which can be visualized as such:

    $$\cdots || \cdot\cdot | \cdot$$

    The $|$ represent partitions for the 'boxes' and the $\cdot$ is a particle. There are then k $\cdot$'s and $n-1$ $|$'s. This can be simplified again to be thought of just arranging symbols, and you are choosing to place the symbols. There are $k+n-1 = n+k-1$ symbols, so choose where to place the separators or the particles. Choose where the $\cdot$'s are, and the $|$'s are then positioned, and vice versa.

    $$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

- **Story Proof**: proof by interpretation.

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    $$n\binom{n-1}{k-1} = k\binom{n}{k}$$

    Pick k people out of n with designated as the president, so you can first pick the president out of n, then select who can be in the club. Or you can pick the club k out of n then choose the president from the k people.

- **Vandermonde Identity**:

$$\binom{m+n}{k} = \sum_{j=0}^{k} \binom{m}{j}\binom{n}{k-j}$$

  You have two groups with labeled people containing m and n people. You need to select k people total from both groups. The summation refers to the combinations of selecting j from m and $k-j$ from n such that you have $j+k-j = k$ total people from the $m+n$ total people in two groups. Each instance is multiplied together due to the multiplication rule.

- **The Non-Naive Definition of Probability**
  - A probability sample consists of **S** and **P** where **S** is a sample space and **P**, a function which takes an event $A \subseteq S$ as input and returns $P(A) \in [0, 1]$ as output such that:
    - (1) $P(\emptyset) = 0$, $P(S) = 1$
    - (2)

$$P(\cup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} P(A_n)$$

    if $A_1, A_2, ..., A_n$ are disjoint.