Stats 110 Strategic Practice and Homework 2

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1 Strategic Practice

1.1 Inclusion-Exclusion

1. For 7 people, what is the probability that all 4 season occur at least once among their birthdays assuming all seasons are equally likely. A_i is the probability that there are no birthdays in the i^{th} season. Probability that all seasons occur at least once is $1 - P(A_1 \cup A_2 \cup A_3 \cup A_4)$. $A_1 \cap A_2 \cap A_3 \cap A_4 = \emptyset$

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = \sum_{i}^{4} P(A_i) - \sum_{i=1}^{3} \sum_{j>i} P(A_i \cap A_j) + \sum_{i=1}^{3} \sum_{j>i} \sum_{k>j} P(A_i \cap A_j \cap A_k)$$

$$= 4P(A_1) - 6P(A_1 \cap A_2) + 4P(A_1 \cap A_2 \cap A_3)$$

$P(A_1) = (3/4)^7$, $P(A_1 \cap A_2) = \frac{1}{2^7}$, $P(A_1 \cap A_2 \cap A_3) = \frac{1}{4^7}$

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 4(\frac{3}{4^7}) - 6(\frac{1}{2^7}) + 4(\frac{1}{4^7})$$

$$\rightarrow 1 - [4(\frac{3}{4^7}) - 6(\frac{1}{2^7}) + 4(\frac{1}{4^7})]$$

The probability that there are no birthdays in all the season is the $\emptyset$. The probability that there are no birthdays in a given season means you have reduced the number of season by 1, there are now 3 seasons to choose from. And so on for 2 seasons not occurring, and so on for 3 seasons not occurring. Each time you are making the event less probable by shrinking the number of options.

2. Picking Classes Randomly

(a) Naive Method: There are 7 classes to take total, there are 30 total classes, 6 each day of the week. 7 classes can be taken either with 2 days with 2 classes and 3 days with 1 or 1 day with 3 classes and 4 days with 1: (2, 2, 1, 1, 1) or (3, 1, 1, 1, 1). First select the days of the week that will have more than one course, then for those days select the required number out of the 6 possible classes, then for the remaining days, select 1 of the possible 6 classes.

$$\frac{\binom{5}{2}\binom{6}{2}\binom{6}{2}\binom{6}{1}\binom{6}{1}\binom{6}{1}+\binom{5}{1}\binom{6}{3}\binom{6}{1}\binom{6}{1}\binom{6}{1}\binom{6}{1}}{\binom{30}{7}}$$

(b) Inclusion-Exclusion Method: To find the probability of classes on each day of the week, consider the complement, that is the probability that at least one day does not have classes. To take 7 classes total, there must be at least 2 days with classes because of the constraint that each day has 6 classes. Let $B_i = A_i^c$, where A_i is the probability of class on that day.

$$P(atLeastOneDayWithNoClass) =$$
$$\sum_i P(NoClassOnDayA_i)$$
$$-\sum_{i<j} P(NoClassOnDayA_iandA_j)$$
$$+\sum_{i<j<k} P(NoClassOnDayA_iandA_jandA_k)$$

There must be two days with classes, therefore there is no need to consider no classes on 4 days and no classes on 5 days.

$$P(B_1 \cup B_2 \cup \cdots \cup B_5) = \sum_i P(B_i) - \sum_{i<j} P(B_i \cap B_j) + \sum_{i<j<k} P(B_i \cap B_j \cap Back_k)$$

Now, to fill in the probabilities:

$$P(B_1) = \frac{\binom{24}{7}}{\binom{30}{7}}, P(B_1 \cap B_2) = \frac{\binom{18}{7}}{\binom{30}{7}}, P(B_1 \cap B_2 \cap B_3) = \frac{\binom{12}{7}}{\binom{30}{7}}$$

Other intersections are similar.

$$P(B_1 \cup B_2 \cup \cdots \cup B_5) = 5\frac{\binom{24}{7}}{\binom{30}{7}} - \binom{5}{2}\frac{\binom{18}{7}}{\binom{30}{7}} + \binom{5}{3}\frac{\binom{12}{7}}{\binom{30}{7}}$$

Therefore:

$$P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) = 1 - P(B_1 \cup B_2 \cup \cdots \cup B_5) = 5\frac{\binom{24}{7}}{\binom{30}{7}} - \binom{5}{2}\frac{\binom{18}{7}}{\binom{30}{7}} + \binom{5}{3}\frac{\binom{12}{7}}{\binom{30}{7}}$$

1.2 Independence

2 Homework