

Stats 110 Strategic Practice and Homework 2

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July 4, 2015

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1 Strategic Practice

1.1 Inclusion-Exclusion

1. For 7 people, what is the probability that all 4 season occur at least once among their birthdays assuming all seasons are equally likely. A_i is the probability that there are no birthdays in the i^{th} season. Probability that all seasons occur at least once is $1 - P(A_1 \cup A_2 \cup A_3 \cup A_4)$. $A_1 \cap A_2 \cap A_3 \cap A_4 = \emptyset$

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = \sum_i^4 P(A_i) - \sum_{i=1}^3 \sum_{j>i} P(A_i \cap A_j) + \sum_{i=1}^3 \sum_{j>i} \sum_{k>j} P(A_i \cap A_j \cap A_k)$$

$$= 4P(A_1) - 6P(A_1 \cap A_2) + 4P(A_1 \cap A_2 \cap A_3)$$

$$P(A_1) = (3/4)^7, P(A_1 \cap A_2) = \frac{1}{2^7}, P(A_1 \cap A_2 \cap A_3) = \frac{1}{4^7}$$

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 4\left(\frac{3}{4^7}\right) - 6\left(\frac{1}{2^7}\right) + 4\left(\frac{1}{4^7}\right)$$

$$\rightarrow 1 - \left[4\left(\frac{3}{4^7}\right) - 6\left(\frac{1}{2^7}\right) + 4\left(\frac{1}{4^7}\right)\right]$$

The probability that there are no birthdays in all the season is the \emptyset . The probability that there are no birthdays in a given season means you have reduced the number of season by 1, there are now 3 seasons to choose from. And so on for 2 seasons not occurring, and so on for 3 seasons not occurring. Each time you are making the event less probable by shrinking the number of options.

2. Picking Classes Randomly

- (a) Naive: There are 7 classes to take total, there are 30 total classes, 6 each day of the week. 7 classes can be taken either with 2 days with 2 classes and 3 days with 1 or 1 day with 3 classes and 4 days with 1: (2, 2, 1, 1, 1) or (3, 1, 1, 1, 1). First select the days of the week that will have more than one course, then for those days select the required number out of the 6 possible classes, then for the remaining days, select 1 of the possible 6 classes.

$$\frac{\binom{5}{2} \binom{6}{2} \binom{6}{2} \binom{6}{1} \binom{6}{1} \binom{6}{1} + \binom{5}{1} \binom{6}{3} \binom{6}{1} \binom{6}{1} \binom{6}{1} \binom{6}{1}}{\binom{30}{7}}$$

- (b) Inclusion-Exclusion:

2 Homework