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Algebraic Nonlinear Identification and Output Tracking Control of Synchronous Generator using Differential Flatness*

Debarshi Patanjali Ghoshal[1], Shaunak Sinha[2], Hannah Michalska[3]

***Abstract*—A kernel-based approach is explored to enhance robustness of flatness-based nonlinear tracking control design for a synchronous generator machine. The design involves full system identification and nonlinear filtering of the system state, to permit effective implementation of a nonlinear controller based on differential flatness of the model. The difficulty associated with robust implementations of flatness-based controllers resides in the necessity of fast and accurate estimation of higher order derivatives of the noisy, observed flat output. The recently developed forward-backward kernel estimation methods [1], lend themselves powerfully for this task. Two LTI surrogate models are used with the nonlinear model of the machine to serve identification and filtering of the state, and are switched seamlessly to generate persistent excitation for the purpose of a complex nonlinear identification of all the system parameters. The approach does not require a separate start-up phase for identification purposes. The need for on-line adaptive identification and associated re-tuning of the controller is detected and implemented during full operation of the machine. Neither the identification nor the state estimation procedures need any re-initialization while rendering improved accuracy of derivative estimates due to the forward-backward smoothing feature of the kernels involved.**

Keywords: Nonlinear identification; Output tracking control; Synchronous generator

I. INTRODUCTION

The general purpose of this note is to demonstrate how the recently proposed forward-backward kernel-based identification and state estimation approaches, initially introduced in [2], [3], [4], and [5], can be employed to benefit practical control problems. In particular, since the forward-backward kernels have been primarily introduced for linear systems, the goal is to show that their application extends to nonlinear systems as well.

The biggest hurdles to overcome in modern control and estimation theory are perceived to be associated with system nonlinearity and general complexity, as compounded with the difficulties of model construction and uncertainty of observation. Hence follows a rapid development in machine learning methods and the success of on-line approaches such as predictive control, moving window estimators, or non-parametric identification. There is a continued and particular demand for the construction of accurate signal differentiators and, more generally, state and parameter estimators, as the latter are instrumental in the implementation of countless control strategies, notably those based on the notion of differential flatness [6], [7]. In such applications the speed of derivative estimation is usually critical as dictated by higher priorities such as system stability. A large selection of on-line differentiating methods exists, see e.g. [8], [9], [10], [11], [12]. Estimation methods based on finite time algebraic differentiators have been exhaustively discussed and employed in many examples in [13]. While one must be weary of the term "exact" there are more and less accurate differentiation algorithms, but most employ some a priori function approximation, be it splines, or related kernels functions. Fast and accurate differentiation is hence inadvertently associated with general, finite time (also called finite-interval) estimation. In this regard, kernel estimation methods have a certain advantage as they are often finite-interval. Kernel methods are not new; see e.g. [14], [15], [16], [17], [18], and a recent survey of kernel methods for system identification [19]. Sensitivity to noise in the observed signals is undoubtedly the biggest vulnerability of on-line differentiators. There is little room for improvement unless additional information is available to alleviate uncertainty. What makes the forward-backward kernels in [1] differ from others is that such additional information is sought in system differential invariance that is deterministically dependable.

The present contribution shows several ways in which the kernels of [1] can be harnessed to design an adaptive tracking controller for a synchronous generator machine whose heavily nonlinear model has known structure, but needs full multivariate parametric identification as well as state estimation:

- Given a general nonlinear model structure of a generator with all parameters unknown, a parametric LTI 4-th order system is employed as a local surrogate model to be identified and thus estimate the related generator parameters and states in a moving-window minimum-energy estimator that uses the kernel system representation of [1] together with a persistently exciting system input which is a combination of a flatness-based control signal with arbitrary parameters and a fast oscillating exogenous signal.
- As the model parameter estimates start to converge, the surrogate 4-th order model is gradually transitioned to a simpler, 3-rd order surrogate model which is sufficient for the purpose of nonlinear estimation of the state and matches a different differential invariant. The latter is employed in closed loop with the flatness-based controller now employing the estimated parameter values. The transition from the 4-th order model to the 3-rd order model is dictated by the need for renewed system identification while monitoring the tracking control error to remain within given safety zone.
- The adaptive multi-model tracking control approach proves flexible and reliable while maintaining the generator machine in operation for all time.

This application of the basic kernel approach of [1] indicates that it can be successfully used for multiple purposes in linear and also nonlinear systems. It should be noted that in the moving-window regime the system invariance exploited by the kernels is *local* and corresponds to local "linearization" of the system. Continuity of estimation is maintained by the dynamic component of the minimum-energy estimator of which the local kernel estimation is a part.

II. TRACKING CONTROL PROBLEM FOR THE SYNCHRONOUS GENERATOR

The model for the synchronous generator machine is the following nonlinear 3-rd order system ; see [7], [20]

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -b_1 x_3 \sin x_1 - b_2 x_2 + P\\ \dot{x}_3 &= b_3 \cos x_1 - b_4 x_3 + E - u; \quad F = x_1\end{aligned} \tag{1}$$

with input u and a "flat" output F which has the physical meaning of a load angle. The values of the six system parameters b_1, b_2, b_3, b_4, P, and E, are unknown and thus

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Authors are with the Department of Electrical, Computer & Software Engineering, McGill University, 3480 University Street, Montreal H3A 2A7, Quebec, Canada.

[1] debarshi.ghoshal@mail.mcgill.ca
[2] shaunak.sinha@mail.mcgill.ca
[3] Corresponding author hannah.michalska@mcgill.ca

must be identified to implement a control strategy.

The control problem is stated as that of output trajectory tracking for the load angle F. The desired trajectory to be tracked is defined as the one that transfers the load angle $x_1 = F$ from an initial equilibrium point $x_1(t_0) = x_{E1}$ at $t_0 = 0$ to another equilibrium point $x_1(T) = x_{E2}$ at a given time $T > t_0$. A specific trajectory linking the desired equilibria is proposed in [13] as the following Bézier interpolating polynomial function:

$$F^*(t) = x_1(t_0) + (x_1(T) - x_1(t_0))\phi(t, t_0, T) \quad (2)$$

where $\phi(t, t_0, T)$ is a polynomial satisfying the boundary conditions

$$\phi(t_0, t_0, T) = 0, \qquad \phi(T, t_0, T) = 1$$

Here, the polynomial ϕ is defined so that

$$\phi(t, t_0, T) = \Delta^5(t)[r_1 - r_2\Delta(t) + \cdots - r_6\Delta^5(t)] \quad (3)$$

with $\Delta = [t - t_0]/(T - t_0)$ and

$$r_1 = 252, \quad r_2 = 1050, \quad r_3 = 1800,$$
$$r_4 = 1575, \quad r_5 = 700, \quad r_6 = 126$$

If the system parameters are known then the load angle tracking problem is solvable by invoking differential flatness; see [7], [13]. To this end it is easy to verify that the system states and its input can all be expressed in terms of the output variable F and its time derivatives $F^{(i)}, i = 1, 2, 3$; specifically

$$\begin{aligned}
x_1 &= F \\
x_2 &= \dot{F} \\
x_3 &= \frac{P - b_2\dot{F} - \ddot{F}}{b_1 \sin F} \\
u &= E - b_4\left(\frac{P - b_2\dot{F} - \ddot{F}}{b_1 \sin F}\right) + b_3 \cos F \\
&\quad - \frac{(-b_2\ddot{F} - F^{(3)})\sin F - (P - b_2\dot{F} - \ddot{F})\dot{F}\cos F}{b_1 \sin^2 F}
\end{aligned} \quad (4)$$

which implies that system (1) is differentially flat, with flat output $F = x_1$. It is then immediate to see that the reference trajectory $F^*(t), t \geq 0$, can be reached by the feedback control

$$\begin{aligned}
u &= E - b_4\left(\frac{P - b_2\dot{F} - \ddot{F}}{b_1 \sin F}\right) + b_3 \cos F \\
&\quad - \frac{(-b_2\ddot{F} - v)\sin F - (P - b_2\dot{F} - \ddot{F})\dot{F}\cos F}{b_1 \sin^2 F}
\end{aligned} \quad (5)$$

with the new input v satisfying

$$\begin{aligned}
v =& [F^*(t)]^{(3)} - \lambda_2[\ddot{F} - \ddot{F}^*(t)] \\
&- \lambda_1[\dot{F} - \dot{F}^*(t)] - \lambda_0[F - F^*(t)]
\end{aligned} \quad (6)$$

Equations (4) indicate that if the system (1) is controlled by (5) - (6) then the closed loop system dynamics follows the linear equation

$$\begin{aligned}
F^{(3)}(t) =& [F^*(t)]^{(3)} - \lambda_2[\ddot{F}(t) - \ddot{F}^*(t)] \\
&- \lambda_1[\dot{F}(t) - \dot{F}^*(t)] - \lambda_0[F(t) - F^*(t)]
\end{aligned} \quad (7)$$

Hence, the positive controller gains $\{\lambda_0, \lambda_1, \lambda_2\}$ must be chosen to provide an adequate tracking stability margin.

The nonlinear estimation challenge

To implement the feedback controller, the system output F, and its first two time derivatives, $\dot{F}$ and $\ddot{F}$, need to be known or else be reconstructed from observed output data. Clearly, the knowledge of the system parameters is also mandatory or else the linearizing property of the flatness-based control will be lost as the system will fail to follow the dynamics of (7). The estimation challenge is hence two-fold:

1) The six system parameters b_1, b_2, b_3, b_4, P, and E, must be estimated during machine operation. With F as the only observed system variable, this can be done using equation (5), in which the input and output u and F are known or directly observed, respectively. However, for the parametric identification to be successful, the output derivatives $F(i), i = 1, 2, 3$, in (5) must be reconstructed by way of a parallel nonlinear filtering algorithm. The approach to the resolution of the derivative estimation problem was conceived by combining a local double-kernel representation of the nonlinear system (4) with the concept of the moving-window, minimum-energy filtering as presented in [21], [22]. Since the highest order of the estimated derivative is three, a 4-th order LTI surrogate local kernel model is employed for the task and fitted to the observed output in a moving-window regime.
2) If the parametric system identification is successful, then the flatness-based nonlinear control will take hold while the closed-loop dynamics will become that of (7) which is in fact a homogeneous LTI system. During this phase the state estimation needed to implement the control (5) - (6) can thus be carried out using a simpler LTI kernel representation based on a 3-rd order system invariance.

The estimation algorithms of 1) and 2) are explained next followed by an outline of a combined identification-control strategy involving gradual transition between phases 1) and 2).

III. A KERNEL-BASED MOVING-WINDOW MINIMUM ENERGY NONLINEAR STATE ESTIMATOR

In the absence of the linearizing action of the flat feedback control which can be constructed only with full knowledge of the system parameters, the estimation of the "state vector" $[F, F^{(1)}, F^{(2)}, F^{(3)}]$ is really a nonlinear filtering problem. Here, it is solved by drawing on the idea of the minimum energy filters as first suggested by Fleming in [22] and Krener in [21] with a full convergence analysis presented in [23]. However, the original minimum energy filters require the knowledge of the model and, adaptive versions of the latter have not been found. Moreover, the minimum energy filters employ explicit system linearizations. Still, as demonstrated in [21] their performance is superior to the extended and unscented Kalman filters. The kernel-based moving-horizon version of the minimum energy filter was briefly introduced in [5] in the context of linear parameter-varying systems and favourably compared with a kernel adaptation of the Bayesian dynamic regression of [24] in the thesis [25]. It is summarized here for completeness of exposition in the particular context of the system at hand.

Consider a 4-th order LTI model

$$\begin{aligned}
y^{(4)}(t) + a_3y^{(3)}(t) + a_2y^{(2)}(t) + a_1y^{(1)}(t)& \\
+ a_0y(t) = u(t)&
\end{aligned} \quad (8)$$

where $F(t) := y(t), t \in [t_k, t_{k+1}]$ is the observed output trajectory and $u(t), t \in [t_k, t_{k+1}]$ is a known input function. The model can be thought of as a surrogate *local* model for the nonlinear system to be adaptively fitted to the measured output over a time window $[t_k, t_{k+1}]$ for the purpose of estimating both the parameter vector (9)

$$w_k := [a_0 \quad a_1 \quad a_2 \quad a_3]^T \quad (9)$$

and the state $y^{(i)}(t_k^*) \approx F^{(i)}(t_k^*)$ for $i = 0, 1, \cdots, 4$ at any $t_k^* \in (t_k, t_{k+1})$. Such an estimation problem lends itself directly to the application of the kernel-based method of [1] which employs special kernel representations for systems such as (8) as cited below for LTI systems of order n.

Theorem 1: Let $[a, b]$ be any interval on $\mathbb{R}$. There exist kernel functions K_y, K_u, defined on $[a, b] \times [a, b]$ that yield compact integral operators acting on $L^2[a, b]$, such that the differential system (8) has an equivalent integral representation for $t \in [a, b]$

$$y(t) = \int_a^b K_y(t,\tau) y(\tau)\ d\tau + \int_a^b K_u(t,\tau) u(\tau)\ d\tau \qquad (10)$$

where, additionally, K_y is a *linear function of the system parameters* $a_0, \cdots, a_{n-1}$. The regularity properties of the kernels K_y, K_u, permit differentiation of (10) up to order $n-1$ yielding corresponding recursive expressions for the derivatives of the output $y^{(i)}$ for $i = 1, \cdots, n-1$

$$y^{(i)}(t) = \sum_{m=0}^{i-1} f_y^{i,m}(t) y^{(m)}(t) + \sum_{m=0}^{i-1} f_u^{i,m}(t) u^{(m)}(t) + \int_a^b K_y^i(t,\tau) y(\tau)\ d\tau + \int_a^b K_u^i(t,\tau) u(\tau)\ d\tau \qquad (11)$$

where f_y^i, f_u^i, $i = 0, \cdots, n-2$, are rational functions on $[a, b]$ and the kernels K_y^i and K_u^i are obtained by direct differentiation of K_y, K_u with respect to t.

□

The proof of this theorem for a general order n can be found in [1].

Since the kernel K_y is linear in the system parameters, the integral representation (10) can be written as

$$y(t) - g_n(t, y) - h(t, u) = \sum_{i=0}^{n-1} a_i g_i(t, y) \qquad (12)$$

$$g_i(t, y) := \int_a^b K_{y,i}(t,\tau) y(\tau) d\tau \quad i = 0, \ldots, n \qquad (13)$$

$$h(t, u) := \int_a^b K_u(t,\tau) u(\tau)\ d\tau$$

for some "component kernels" of $K_{y,i}$. Taking $a := t_k$, $b := t_{k+1}$ let $s^k := \{t_1^k, \cdots, t_N^k\} \subset (t_k, t_{k+1}]$, be a given discrete set of distinct time instants. The N copies of equation (12) for all members of s^k can then be stacked in the form of a matrix equation, where the index k indicates the dependence on the estimation window $[t_k, t_{k+1}]$

$$Q_k(y) = P_k(y)\bar{a}\ ; \quad y : [t_k, t_{k+1}] \to \mathbb{R}$$

$$Q_k(y) \stackrel{def}{=} \begin{bmatrix} q(t_1^k) \\ \vdots \\ q(t_N^k) \end{bmatrix} \quad ; \quad \bar{a} \stackrel{def}{=} \begin{bmatrix} a_0 \\ \vdots \\ a_{n-1} \end{bmatrix} \quad ; \qquad (14)$$

$$P_k(y) \stackrel{def}{=} \begin{bmatrix} p_1(t_1^k) \cdots p_n(t_1^k) \\ \ddots \\ p_1(t_N^k) \cdots p_n(t_N^k) \end{bmatrix}$$

$$q(t_i^k) = y(t_i^k) - g_n(t_i^k, y) - h(t_i^k, u);\ i = 1, \cdots, N$$

$$p_j(t_i^k) = g_{j-1}(t_i^k, y);\ j = 1, \cdots, n \qquad (15)$$

Identification and state estimation for system (8) in window k, requires substituting: $n = 4$, $\bar{a} := w_k$, $y(t) = y_M(t); t \in [t_k, t_{k+1}]$ in the estimation equation (14), where y_M denotes the measured system output F. Under practical identifiability condition ; see [1], the knots s^k are assumed to be such that $rank\ P_k(y_M) = n$ which would yield an estimate $\hat{w}_k := P_k(y_M)^\dagger Q_k(y_M)$, *in a single isolated window* k, where the pseudo-inverse $P_k^\dagger$ is the left inverse of $P_k(y_m)$. The corresponding estimates $y_E, y_E^{(i)}, i = 1, 2, 3$, of the output and its derivatives in (8) over the window k, would then be calculated from the equations of Theorem 1; specifically

$$y_E(t) = \int_a^b K_y(t,\tau) y_M(\tau)\ d\tau + \int_a^b K_u(t,\tau) u(\tau)\ d\tau \quad (16)$$

$$y_E^{(i)}(t) = \sum_{m=0}^{i-1} f_y^{i,m}(t) y_E^{(m)}(t) + \sum_{m=0}^{i-1} f_u^{i,m}(t) u^{(m)}(t) + \int_a^b K_y^i(t,\tau) y_M(\tau)\ d\tau + \int_a^b K_u^i(t,\tau) u(\tau)\ d\tau \qquad (17)$$

for $i = 1, 2, 3$, where all the functions and kernels depend on the value of the estimated parameter $\hat{w}_k$ and the input function $u(t), t \in [t_k, t_{k+1}]$ is known.

If the local model (8) is to be useful for adaptive filtering of the nonlinear system then the "observation window" must be traveling forward in time while the local model is re-fitted, allowing its parameters to vary with time. The local estimates over any given window k must effectively provide local linearization of the true nonlinear system output. Adaptation of the surrogate model is achieved as the estimation window is moved forward ; each window giving rise to new estimates of the surrogate model parameters $\hat{w}_i$. If the underlying system dynamics is smooth, it is reasonable to expect that the estimated parameters exhibit a similar degree of smoothness. For all practical purposes, the dynamics of the parameters is adequately captured by a local auto-regressive model which leads to a dynamic regression formulation, essentially equivalent to a single step of a minimum energy filter, as explained in [25]. The associated minimum energy estimation problem is stated in window k as the following convex programming problem

$$\text{MEE:} \quad \underset{w_k \in \mathbb{R}^4}{\text{minimize}} \{ \gamma ||w_k - \hat{w}_{k-1}||_2^2 + \lambda ||w_k||_2^2 + ||P_k(y_M) w_k - Q_k(y_M)||_2^2 \} \qquad (18)$$

where $\hat{w}_{k-1}$ is the optimal parameter estimate computed over the previous window $(k-1)$, γ and λ are penalty coefficients, here chosen to be 0.25 and 0.5 respectively, and y_M is the measured output over window k. Adaptation of the local model to the underlying nonlinear system is achieved through the repeated solution of the above MEE for $k = 1, 2, \cdots$, as the observation window is shifted forward by discrete quanta. The shift parameter $\Delta > 0$ is chosen to be a fraction of the window size.

The estimates of the nonlinear system output and its derivatives are carried out directly from equations (16) using the consecutive optimal parameter estimates $\hat{w}_k$. For best results, the window size is kept small with the state estimates $y_E(t_k^*), y_E^{(i)}(t_k^*), i = 1, 2, 3$ evaluated at the centers of the windows, i.e. $t_k^* = 0.5(t_k + t_{k+1})$.

While the kernel based MEE estimator delivers estimates of the "flat" system states

$$[F_E, F_E^{(1)}, F_E^{(2)}, F_E^{(3)}](t_k^*) = [y_E, y_E^{(1)}, y_E^{(2)}, y_E^{(3)}](t_k^*) \quad (19)$$

for $k = 1, 2, \cdots$, in the nonlinear equations (4), the parameter estimates $\hat{w}_k, k = 1, 2, \cdots$ of the surrogate models in consecutive windows are only needed to calculate the correct expressions for the estimating kernels in (16). The estimates of the six unknown parameters of the synchronous generator are then obtained from the estimates of the flat system states (19), as described next.

IV. Optimization-Based Parameter Identification for the Synchronous Generator

The estimates of the nonlinear system output and its derivatives (19) are used for the estimation of the generator parameters by first re-writing the input expression (5) in terms of (19) in a simpler re-arranged form:

$$\begin{aligned}
&G(p;t) := \\
&[Eb_1 \sin^2 y_E - b_4 P \sin y_E + b_2 b_4 y_E^{(1)} \sin y_E \\
&+ b_4 y_E^{(2)} \sin y_E + b_1 b_3 \sin^2 y_E \cos y_E \\
&+ b_2 \big(y_E^{(2)} \sin y_E - [y_E^{(1)}]^2 \cos y_E\big) + y_E^{(3)} \sin y_E \\
&+ P y_E^{(1)} \cos y_E - y_E^{(1)} y_E^{(2)} \cos y_E \\
&- b_1 u \sin^2 y_E](t) = 0 \ ; \quad t \geq 0
\end{aligned} \tag{20}$$

that should hold for all time if the flat estimates coincide with their true counterparts, the input function u is known exactly, and the parameter values are correct. The argument p in the function G defined in (20) denotes the aggregated parameter vector

$$p := [b_1 \quad b_2 \quad b_3 \quad b_4 \quad P \quad E]^T \tag{21}$$

The parameter identification procedure imposes itself as the following.

Least squares optimization for nonlinear parametric identification

Conceptually, an estimate of the parameter p is sought by on-line minimization of the cumulative cost function for some convexifying term with weight $\epsilon > 0$ and forgetting factor $\alpha > 1$:

$$J_k(p) := \epsilon ||p||_2^2 + \sum_{i=1}^{k} \alpha^{-(k-i)} G(p; t_i^*)^2; \quad k = 1, 2, \cdots \tag{22}$$

with t_k^* denoting the centers of the consecutive windows in the nonlinear state estimation. Despite the quadratic convexifying term with weight ϵ, the cost function J is not convex thus, to limit the size of the essentially global optimization problem, the last is best solved by adding physically meaningful compact constraints on the components $p_i, i = 1, \cdots 6$, of the vector p; in this case box constraints are applied:

$$p \in C := \{p \in \mathbb{R}^6 \mid 0 \leq p_i \leq c_p, \ i = 1, \cdots, 6\} \tag{23}$$

where the bound $c_p > 0$ is chosen to be suitably large. Repetitive minimization of J_k over consecutive windows of the MEE yields a sequence of estimates

$$\hat{p}_k \in \arg\min\{J_k(p) \mid p \in C\} \quad k \in \mathbb{N} := \{1, 2, \cdots\} \tag{24}$$

Any accumulation point p^* of sequence (24), $\{\hat{p}_{k_i}\}_{i\in\mathbb{N}} \to p*$ is considered an estimate of the system parameter p provided that it is not a boundary point of the constraint set C; the existence of interior accumulation points is encouraged by choosing a large value of c_p. The on-line optimization is not overly demanding as at each iteration k, the optimal value at iteration $k-1$ can be used to improve upon.

As is well known, for the identification process to be successful, the input function u has to be persistently exciting, i.e. has to be sufficiently rich to fully excite the dynamics of the system over a broad frequency range, ([26], [27]). In the identification experiment presented here a very simple sinusoidal signal with modulated frequency was found sufficient to render identifiability, [28].

V. Combined State and Parameter Estimation in Flatness-Based Tracking

The discussion in section II points to the fact that the flatness based tracking control law (5) - (6) has a linearizing effect on the closed loop system only if: (i) it employs the correct system parameters, and (ii) it has access to the correct full state information i.e. the values of the flat output F and its two derivatives $\dot{F}$ and $\ddot{F}$. Moreover, for the purpose of tracking, such access must be instantaneous as $F(t), \dot{F}(t), \ddot{F}(t)$ appear in the control (5). In summary, the following process control design factors interplay:

- The lack of knowledge of the correct system parameters used in the control (5) - (6) makes the closed loop system follow a nonlinear third order dynamics which is unlikely to achieve tracking. A 4-th order nonlinear state observer/filter is necessary to estimate the states $F, F^{(i)}, i = 1, 2, 3$. Such an observer should be robust with respect to measurement noise and be as fast as is achievable to shorten the time needed for system identification.
- System identification requires the use of a persistently exciting input and good estimates of the flat state $F, F^{(i)}, i = 1, 2, 3$.
- When the system parameters are identified correctly, the closed loop dynamics changes order to three; see (7). The closed loop dynamics under the action of the flat control becomes a homogeneous LTI system

  $$z^{(3)} + \lambda_2 z^{(2)} + \lambda_1 z^{(1)} + \lambda_0 z = 0 \tag{25}$$

  under the simple change of variables :

  $$z^{(i)} := F^{(i)} - [F^*]^{(i)}; \quad i = 0, 1, 2, 3; \tag{26}$$

  $$z(0) := z = F - F^* \tag{27}$$

  The reference trajectory and its derivatives are known functions. Hence, using this substitution in the closed loop system describing the tracking error, the following output is obtained $\bar{F} := F - F^*$.
- The error dynamics (25) remains time invariant as long as the system parameters do not fluctuate and the states $F, \dot{F}, \ddot{F}$ are estimated correctly. The closed loop error dynamics lends itself ideally to finite-time state estimation as asymptotic observers will change the dynamics, calling for a rigorous assessment of observer-based asymptotic tracking stability.

It follows that the nonlinear MEE estimator of section III is ideally suited to deliver the four state estimates $F, F^{(i)}, i = 1, 2, 3$, for the purpose of the least squares nonlinear identification algorithm of section IV. A different state estimator is called for when the system has already been identified, see below.

A 3-rd order kernel state estimator for the closed loop tracking error control

On the other hand, the state estimation of the LTI tracking error system (25), i.e. the estimation of $z^{(i)}, i = 0, 1, 2$, from the observed z_M does not require a 4-th order MEE estimator because (25) is a third order system whose parameters $\lambda_2, \lambda_1, \lambda_0$ are known by design. A finite time state estimator is still a priority, but is readily available in terms of equations (16) with the substitution of $n = 3$, and the values of $\lambda_2, \lambda_1, \lambda_0$ in place of the kernel parameters as explicitly given by the closed loop invariance (25).

Simultaneous system identification and tracking control

Finally, the identification and tracking control error need not be carried out in separate stages. In fact, as noted in extensive simulations, best overall results are obtained when the actions of the two controllers: the flatness-based tracking controller u,

(5) - (6), and an open loop persistently exciting controller, u_p, are *blended* in the form of a linear combination:

$$u_b(t) := u(t) + \beta(t)u_p(t); \quad t \geq 0, \tag{28}$$

$$\beta : [0, \infty) \to [0, 1] \tag{29}$$

with $\beta(0) = 1$ corresponding to the prevalence of the persistent excitation signal for the purpose of more efficient system identification, and with $\beta(t) \approx 0$ for large times at which the identification process is considered completed. Such control design secures continuous operation of the synchronous generator machine and, more importantly, avoids abrupt switching between control regimes, that was seen to cause large oscillations. The exact blending function β is subject to design whose formula is best determined by trial and error.

The 4-th and 3-rd order state estimators could also be blended in a similar manner. In the case of state estimation, however, it is more practical to switch between estimators when their respective estimates are close.

VI. Simulation Results

System identification and tracking control of the synchronous generator machine were implemented as described in sections II - V. The persistent excitation was chosen in the form of a frequency modulated sinusoid :

$$u_p(t) = 8sin(2\pi f t)$$

where the frequency f is varied linearly from 1 Hz to 8 Hz over the simulation time of $20\,\mathrm{sec}$. The nonlinear and persistently exciting controls were blended according to (28) with $\beta(t) = 1$ through the initial phase of the identification. Once the estimated parameters start converging the value of β is lowered in steps of 0.02 to effectuate a semi-smooth transition between the excitation and tracking control laws. When the magnitude of estimated parameter changes satisfy a threshold, the state estimator is switched from the 4-th MEE to a finite-time 3-rd order one.

The true and estimated parameter values achieved during this control and identification policy are displayed in table I below.

Parameter	$\|\mathbf{p}_{\mathbf{true}}\|_2$	$\|\mathbf{p}_{\mathbf{estimated}}\|_2$
b_1	34.29	34.2564
b_2	0	0
b_3	0.1490	0.1491
b_4	0.3341	0.3406
P	28.220	27.7363
E	0.2405	0.2418

TABLE I
TRUE AND ESTIMATED PARAMETER VALUES OF THE SYNCHRONOUS GENERATOR WITH CONTROL BLENDING

For comparison, table II shows estimates obtained using the persistent excitation alone. The blended control input is

Parameter	$\|\mathbf{p}_{\mathbf{true}}\|_2$	$\|\mathbf{p}_{\mathbf{estimated}}\|_2$
b_1	34.29	34.2110
b_2	0	0.0213
b_3	0.1490	0.1484
b_4	0.3341	0.3242
P	28.220	28.0193
E	0.2405	0.2306

TABLE II
TRUE AND ESTIMATED PARAMETER VALUES OF THE SYNCHRONOUS GENERATOR USING PERSISTENT EXCITATION ALONE.

shown in Figure 1 while the corresponding system output is shown in Figure 2 against the reference trajectory. It is seen that the magnitude of the control input is confined to the interval of $[+15, -15]$. The green vertical line marks the time instant when the parameter estimation is deemed complete triggering the switch between the two state estimators: the 4-th order MEE and the 3-rd order kernel-based one.

Images of the estimates of $y(t)$ and its time derivatives are seen in Figures 3, 4, 5 and 6, respectively. Each figure contains 2 subplots. The upper subplot shows true and estimated signals (while super-imposed on one-another) over the entire simulation period. The estimates match very closely, *the estimated curves are virtually indistinguishable*. Therefore the lower subplot in every figure presents a highly magnified view of the upper subplot over a small region so that the two curves can be seen clearly.

A. Discussion of Results

The simulation results show that blending of the persistently exciting signal with the nonlinear control does not interfere with the nonlinear identification process. As expected, nonlinear identification, is very sensitive to the type of persistent excitation signal employed. Experimentation with noisy output measurement would provide additional insight into the quality of the approach and will be carried out within a forthcoming publication. The finite-time kernel state estimation of [1] was already shown to be very robust with respect to noise.

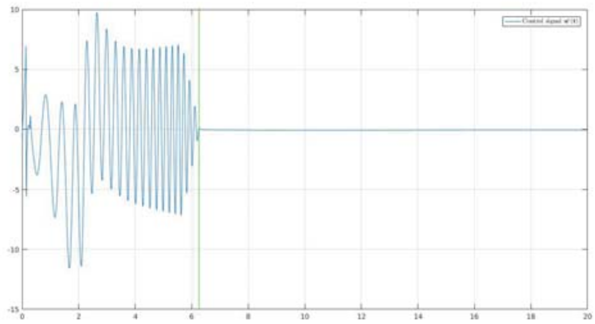

Fig. 1. System input signal over the entire simulation time

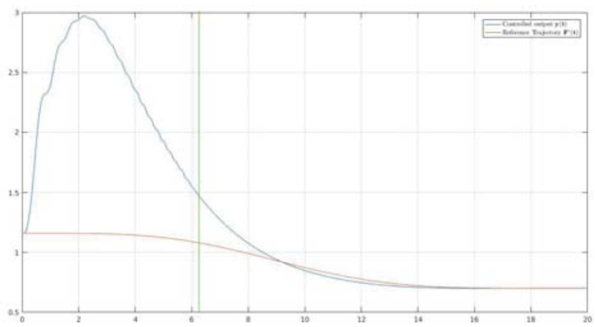

Fig. 2. Tracked output trajectory

VII. Conclusions

Although flatness-based nonlinear control is certainly not new, see [13], it has been employed here to show how the forward-backward kernel system representation of [1] can be harnessed to accomplish full nonlinear system identification and finite time state estimation. The local kernel representation of nonlinear systems employed in this example is especially enticing as it effectively leads to an adaptive version of the minimum energy nonlinear state estimator. Explicit system linearization of a known model in the classical MEE is replaced

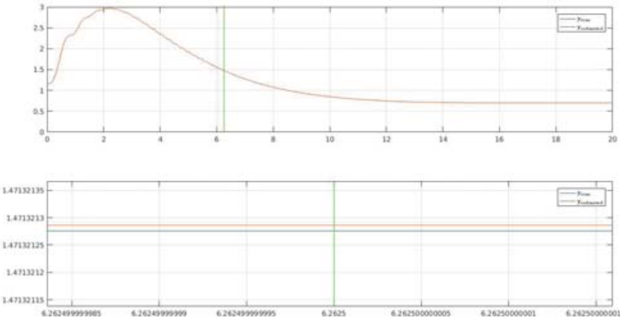

Fig. 3. True and estimated $y(t)$ signals

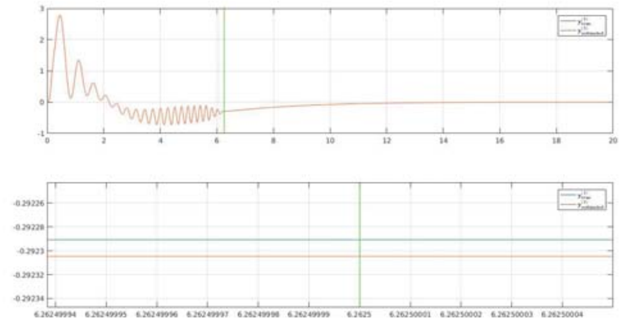

Fig. 4. True and estimated $y^{(1)}(t)$ signals

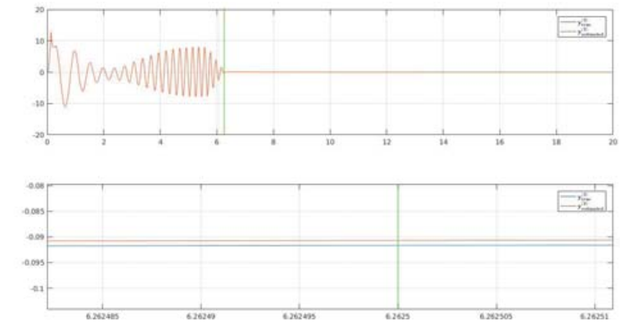

Fig. 5. True and estimated $y^{(2)}(t)$ signals

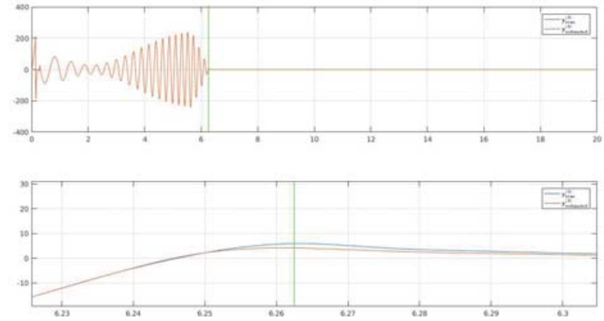

Fig. 6. True and estimated $y^{(3)}(t)$ signals

here by fitting of a local differential invariant. Although we have no experimental results, the simulation model is detailed enough (includes all the important equations) to merit the current exposition.

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