Personal Statement

Many people in the mechanical community believe that the field of elastodynamics and finite element analysis for wave propagation phenomena are sufficiently mature and need only insignificant improvements. However, anyone who tries to solve these problems, even in the simplest case of linear elastic materials, will find that there is no reliable numerical technique that can yield an accurate solution because of spurious oscillations. Moreover, the amplitude of spurious oscillations in numerical solutions even increases at mesh refinement and at very small time increments. As a consequence, the problem cannot be solved and its numerical result cannot be trusted. According to Robert Cook, author of the popular textbook, *Concepts and Applications of Finite Element Analysis*, “In no way can today’s computer programs for wave propagation and impacted [problems] be treated as ‘black boxes.’ A minimum 6 months to 2 years of experience is needed to be able to use such programs successfully.” It may be unusal to a person without experience in solving these problems to believe that existing methods fail to produce accurate results because of spurious oscillations. For transient acoustic and elastodynamics problems, the existing numerical approaches in space-discretization methods, such as the finite element method, spectral element method, isogeometric analysis, among others, do not include techniques that separate spurious and actual oscillations in the numerical results. My research focuses on the development and implementation of new and more accurate numerical techniques for transient linear acoustic and elastodynamics problems. The application of space-discretization methods to these problems leads to a system of ordinary differential equations in time

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{K}\boldsymbol{U} = \boldsymbol{F}, \tag{1}$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices, respectively; $\boldsymbol{U}(t)$ is the vector of the nodal field variable and $\boldsymbol{F}(t)$ is the vector of the nodal load.
For these problems, the integration of Eq. 1 leads to the appearance of spurious high-frequency oscillations. Both the space-discretization used for the derivation and the time-integration of Eq. 1 affect the spurious oscillations and accuracy of a numerical solution. In order to suppress spurious oscillations, existing time-integration methods (e.g., the Houbolt, the Newmark, the Park, the HHT-α, the TDG and others methods) treat this issue by introducing numerical dissipation or artificial viscosity from the first time increment. However, there is no understanding of how to select a time increment for the time-integration methods with numerical dissipation. If the time increments are very small for these methods, then numerical solutions converge to the solution of a semi-discrete problem such as Eq. 1, and contain large spurious oscillations. If the time increments are large, then numerical solutions can be very inaccurate due to the error accumulation related to numerical dissipation. Moreover, numerical dissipation also affect the accuracy of low modes of a numerical solution.

In contrast to the existing technique, a new two-stage time-integration technique has been recently suggested by Dr. Idesman of Texas Tech to obtain non-oscillatory, accurate, and reliable solutions for acoustic and elastodynamics problems. This technique involves implicit or explicit time-integration methods with zero numerical dissipation for the integration of Eq. 1 as a stage for basic computations and an implicit method to filter spurious oscillations as the filtering stage.

In order to exclude the error in time during the basic computations, time increments are selected at which the numerical solutions are practically independent of the size of time increments and close to the exact solution of Eq. (1). However, the error in space still exists due to the space-discretization of Eq. (1). As mentioned earlier, one of the main issues is the presence of spurious high-frequency oscillations in numerical solutions for transient and acoustic elastodynamics problems, which makes the comparison of accuracy of different numerical results difficult. We will resolve this issue by the application of the two-stage time-integration approach. Because the errors in high frequencies do not affect the accuracy of low frequencies, there is no need to filter spurious oscillations at each time increment. In addition, because the filtering stage is independent of the boundary and initial conditions and can be applied to any loading, it can be used as a pre- or post-processor.

One of the disadvantages of the use of implicit time-integration methods is the need to solve a system of algebraic equations that could require large computational resources for a large number of degrees of freedom (dof). Explicit time-integration methods yield an additional reduction in the computation time compared with implicit time-integration methods. In order to increase accuracy and decrease the computation time, the finite element techniques with reduced dispersion (RD) are suggested for linear acoustic and elastodynamics wave propagation problems using the explicit time-integration methods. The linear elements with RD are based on the modified integration rule approach for the mass and stiffness matrices and on the averaged mass matrix approach. Numerical results show that for elastodynamics and acoustics problems, compared to standard linear elements with explicit time-integration methods at the same accuracy, the linear elements with RD significantly reduce the number of dof by factors of $2-3$ or more, $4-9$ or more, and $8-27$ or more in the 1-D, 2-D, and 3-D cases, respectively. Furthermore, I also developed and implemented numerical algorithms for linear elastodynamics problems on parallel computers. The new capability on parallel computer has showed to greatly reduce computation time. We should also mention that the formulations with RD and the explicit time-integration methods can be directly and effectively used (almost without modifications) on parallel computers because the numerical algorithm includes the matrix and vector multiplications and can be performed at the element level without the solution of a system of algebraic equations.

In summary, the existing numerical methods and all those used in commercial software are incapable of accurately solving acoustic and elastodynamics problems due to the appearance of spurious oscillations. These oscillations are the result of the space-discretization methods. Therefore, a new and reliable two-stage time-integration technique was introduced to separate the spurious and actual oscillations for these problems. This technique is very general and can be applied to any loading as well as for any space-discretization technique and any explicit or implicit time-integration method. Furthermore, the finite element techniques with RD can be easily implemented into existing finite element codes and lead to a significant reduction in computation time at the same accuracy compared to standard finite element with explicit time-integration methods.