

**Moroccan birds, coots, risk unending snow storms for this talk?**  
**(13,2,10)**

**Daniel Heath**

`personalpages.manchester.ac.uk/staff/daniel.heath-2/crosswords.htm`

Pure Postgrad  
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With thanks to tikzpingus

# Cryptic Clues



- **Every** cryptic clue has a definition somewhere.
- 95% of the time it's at the **start** or the **end** of the clue.
- 5% of the time it's the **whole** clue or a *double definition*.



- The rest of the clue is **wordplay**.
- It will try to mislead you with some other strange context!
- Read everything carefully and look out for **homographs**.



# Wordplay

There are **loads** of types of wordplay... but most clues have one or more of these:



Moroccan birds, coots, risk unending snow storms for this talk? (13,2,10)

Moroccan birds, coots, risk unending snow storms for this talk? (13,2,10)



Moroccan

talk

Moroccan birds

this talk

Moroccan birds, coots, risk unending snow storms for this talk? (13,2,10)



Moroccan birds, coots, risk **unending** snow storms for **this talk?** (13,2,10)



Total number of **blue** letters: **26**

Either we anagram:

Moroccan birds coots risk sno

Moroccan birds coots ris snow

Moroccan birds, coots, risk unending snow storms for this talk? (13,2,10)



Moroccan birds coots ris snow anagrams to  
**Combinatorics on Crosswords**



# Crossword Grids

## Definition

An  $n \times n$  array of black and white squares is a *crossword grid* if:

- 1  $n$  is odd.
- 2 There is exactly 1 contiguous region of white squares.
- 3 The grid is  $180^\circ$  rotationally symmetric.
- 4 No run of white squares is exactly 2 squares long.
- 5 Every row and column contains at least 1 white square.

## Definition

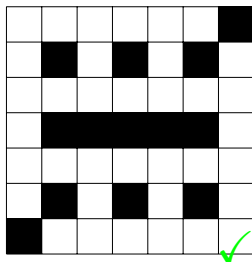
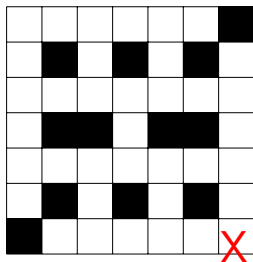
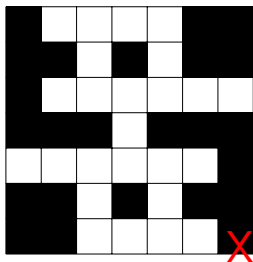
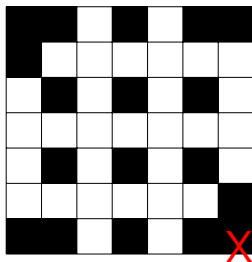
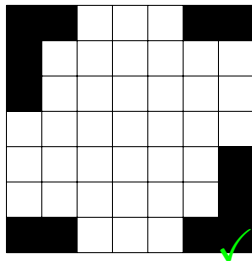
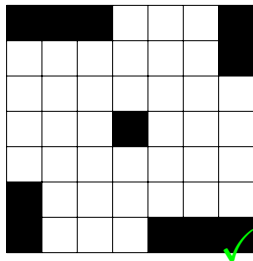
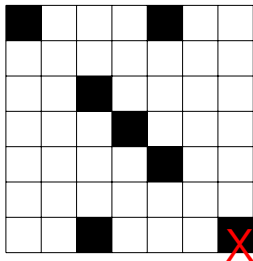
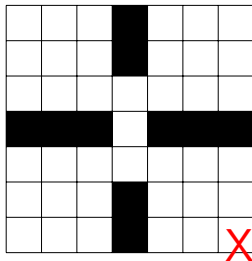
A crossword grid is *American* if **every** white square is *checked* (in an across and down clue).

## Definition

A crossword grid is *British* if:

- 1 Every answer has exactly half (rounded up) of its letters checked.
- 2 There is no run of 3 unchecked squares.
- 3 No run of 2 unchecked squares occurs at the start or end of an answer.

# Plenty of Examples inside (5)



# Combinatorial Questions

- What is the maximum number of clues that can fit in an  $n \times n$  grid?
- What is the minimum number of white squares appearing in an  $n \times n$  grid?
- How many  $n \times n$  grids exist?



## Theorem (Ferland, 2014/2020)

*The maximum number of clues in a  $15 \times 15$  American grid is 96.*

## Theorem (Rabosky, Willmott, 2023)

*For  $n \geq 5$ , the minimum number of white squares in an  $n \times n$  American grid is  $3n + 2$ .*

## Theorem (Ferry, Kleber, 2019)

*The number  $A$  of  $n \times n$  American grids is given by:*

$n$	5	7	9	11	13	15
$A$	12	312	31,187	17,438,702	40,575,832,476	404,139,015,237,875

$n = 17$  has  $A = 17,253,201,646,564,115,439$

$n = 19$  has  $A = 3,185,118,492,387,669,085,270,268$

$n = 21$  has  $A = 2,540,927,098,290,913,247,876,202,007,100$

# Combinatorial Questions 2

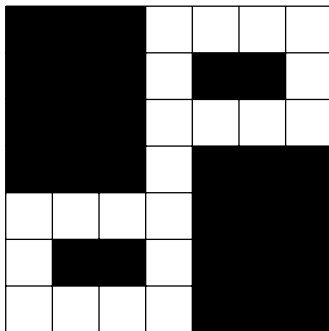
- What is the maximum number of clues that can fit in an  $n \times n$  grid?
- What is the minimum number of white squares appearing in an  $n \times n$  grid?
- How many  $n \times n$  grids exist?



Theorem (Ho, 2021 / H., 2024+)

- *There are 17 5x5 British grids.*
- *There are 556 7x7 British grids.*

This includes rather 'uninteresting' grids...



# Combinatorial Questions 3

## Definition

We call a British grid *interesting* if every row and column contains at least 3 white squares (at least 2 white squares for  $5 \times 5$ ).

## Theorem (H., 2024+)

*The maximum number of clues in an interesting grid is 6 for  $5 \times 5$  and 12 for  $7 \times 7$ .*

## Theorem (H., 2024+)

*The minimum number of white squares in an interesting grid is 15 for  $5 \times 5$ , 28 for  $7 \times 7$  and 37 for  $9 \times 9$ .*

## Theorem (Ho 2021 / H., 2024+)

- *There are 9  $5 \times 5$  interesting British grids.*
- *There are 65  $7 \times 7$  interesting British grids.*
- *There are 13,240  $9 \times 9$  interesting British grids.*

# Toy Story actor gives gratitude? (6)

Thanks!

- Nuns, I stress, needing to reform (7)
- In favour of Republican entering Post Office (3)
- Deity's pet comes back (3)
- Turncoats abandoning HMS inhabited this island, primarily? (6)
- Game in disorder – it's difficult to solve (6)
- Following Harris's lead, Biden gets confused (6)
- Grand! I have beer – and dog returning to offer support (4,1,3,2)
- He had, for example, upset greedy people and prickly sorts (9)
- 'Time to get up, Mr. Warne: would you like some dried fruit?' (4,3,5)