Inverse Matrix

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The Inverse

A matrix's inverse is the same idea as the reciprocal of a number:

The reciprocal of **9** is **1/9**,
which is also expressed as
$\mathbf{9^{-1}}$

Key Difference

The reciprocal of a matrix $\mathbf{M} \neq \mathbf{1/M}$ because we do not divide by matrices.

*instead we use an inverse matrix: $\mathbf{M^{-1}}$ to multiply with other matrices

Finding the inverse of a matrix is necessary to solve matrix quotients

An analogy can be seen in dividing numbers:

$$n/\,\mathbf{2} \;\equiv\; n * \mathbf{1/2} \;\equiv\; n * \mathbf{2^{-1}}$$

$$4/\,\mathbf{4} \;\equiv\; 4 * \mathbf{1/4} \;\equiv\; 4 * \mathbf{4^{-1}}$$

$$= \mathbf{1}$$

$$M * \mathbf{M^{-1}}$$

$$= \mathbf{I}$$

Identity Matrix

Given a matrix **M**, the inverse $\mathbf{M}^{-1}$ can be multiplied on either side of **M** to get the **identity (I)**

$$M * M^{-1} = I \qquad M^{-1} * M = I$$

The 2 x 2 and 3 x 3 identities:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

*identity is square (has same number of rows as columns)

*has ones on the diagonals and zeros everywhere else

Solve **A** in the matrix equation:

$$\boldsymbol{A}B = C$$

$$\boldsymbol{A}B\,B^{-1} = C\,B^{-1}$$

$$\boldsymbol{A}I = C\,B^{-1}$$

$$\boldsymbol{A} = C\,B^{-1}$$

1.

Inversion Methods

how to find an inverse

Gaussian Elimination

Write down the entries of your matrix into a double-wide matrix on the left side. And on one side, write the entries of the identity matrix.

Use **matrix row operations** to convert the left side of the double-wide into an identity matrix.

*transform [A | I] into [I | A^{-1}]

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \quad \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \xrightarrow{-R_1+R_2} \\ \xrightarrow{-R_1+R_3} \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-3R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 4 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-3R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \quad \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Matrix Row Operations

1. Swap rows
2. Multiply or divide each element in a row by a constant
3. Add or subtract multiples of one row to another row

*matrix may turn out to be singular

Gauss-Jordan Elimination

Double-wide

Matrix Row Operations

Multiply Matrices to check

Thanks!

Any questions?

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