

A decorative network diagram in the top-left corner, featuring a complex web of interconnected nodes and edges. Some nodes are highlighted with blue circles or dots, while others are grey. The diagram is composed of thin grey lines and small circular nodes.

# Inverse Matrix

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A decorative network diagram in the bottom-right corner, similar to the one in the top-left. It shows a network of nodes and edges, with several nodes highlighted by blue circles or dots. The overall style is minimalist and technical.



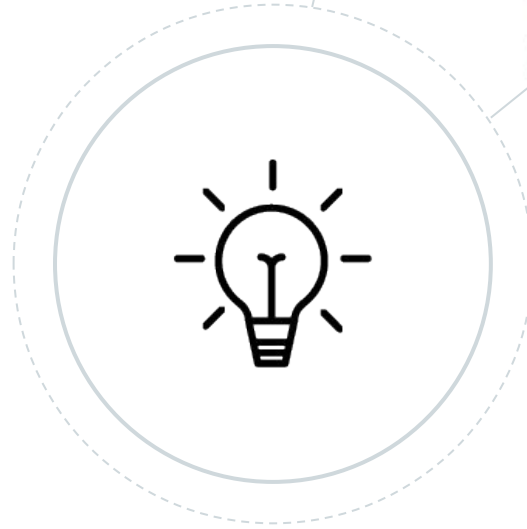
# The Inverse

**A matrix's inverse is the same idea as the reciprocal of a number:**


The reciprocal of **9** is  **$1/9$** ,  
which is also expressed as  
 **$9^{-1}$**

# Key Difference

The reciprocal of a matrix  $\mathbf{M} \neq \mathbf{1}/\mathbf{M}$  because we do not divide by matrices.



\*instead we use an inverse matrix:  $\mathbf{M}^{-1}$  to multiply with other matrices



Finding the inverse of a matrix is  
necessary to solve matrix  
quotients

An analogy can be seen in  
dividing numbers:

$$n / 2 \equiv n * 1/2 \equiv n * 2^{-1}$$

$$4 / 4 \equiv 4 * 1/4 \equiv 4 * 4^{-1} \\ = 1$$

$$M * M^{-1} \\ = I$$


# Identity Matrix

Given a matrix **M**, the inverse **M**<sup>-1</sup> can be multiplied on either side of **M** to get the **identity (I)**

$$M * M^{-1} = I \quad M^{-1} * M = I$$

The 2 x 2 and 3 x 3 identities:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

\*identity is square (has same number of rows as columns)

\*has ones on the diagonals and zeros everywhere else

Solve **A** in the matrix equation:

$$AB = C$$

$$AB B^{-1} = C B^{-1}$$

$$AI = C B^{-1}$$

$$A = C B^{-1}$$



1.

# **Inversion Methods**

how to find an inverse



# Gaussian Elimination

Write down the entries of your matrix into a double-wide matrix on the left side. And on one side, write the entries of the identity matrix.

Use **matrix row operations** to convert the left side of the double-wide into an identity matrix.

\*transform  $[A | I]$  into  $[I | A^{-1}]$

$$\begin{array}{c}
 \left[ \begin{array}{ccc|ccc}
 1 & 3 & 3 & 1 & 0 & 0 \\
 1 & 4 & 3 & 0 & 1 & 0 \\
 1 & 3 & 4 & 0 & 0 & 1
 \end{array} \right]
 \xrightarrow{\substack{-R_1+R_2 \\ -R_1+R_3}}
 \left[ \begin{array}{ccc|ccc}
 1 & 3 & 3 & 1 & 0 & 0 \\
 0 & 1 & 0 & -1 & 1 & 0 \\
 0 & 0 & 1 & -1 & 0 & 1
 \end{array} \right]
 \xrightarrow{-3R_2+R_1}
 \left[ \begin{array}{ccc|ccc}
 1 & 0 & 3 & 4 & -3 & 0 \\
 0 & 1 & 0 & -1 & 1 & 0 \\
 0 & 0 & 1 & -1 & 0 & 1
 \end{array} \right]
 \xrightarrow{-3R_3+R_1}
 \left[ \begin{array}{ccc|ccc}
 1 & 0 & 0 & 7 & -3 & -3 \\
 0 & 1 & 0 & -1 & 1 & 0 \\
 0 & 0 & 1 & -1 & 0 & 1
 \end{array} \right]
 \end{array}$$

## Matrix Row Operations

1. Swap rows
2. Multiply or divide each element in a row by a constant
3. Add or subtract multiples of one row to another row

\*matrix may turn out to be singular

# Gauss-Jordan Elimination







# Thanks!

## Any questions?

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