An evolutionary analysis of the stag hunt game

Eine evolutionäre Analyse des Hirschjagdspiels

Bachelor Thesis

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06.06.2017

Frankfurt am Main

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Notation Index

Δ_i	Mixed strategy space of player i
$\Gamma(\cdot,\cdot,\cdot)$	A game in normal form
$\mathbb{S}$	Set of pure strategies
ρ_{ij}	Conditional switch rate for switching from strategy i to strategy j
I	Set of players
S_i	Individual set of pure strategies for player i
$x_i(t)$	Share of population choosing strategy i at time t
A,B	Payoff matrices of the row and the column player,
$\beta_i(y)$	Best-reply for player i against strategy y
$\bar{F}(\mathbf{x}(t))$	Average payoff of the population at time t
$\hat{F}(\cdot,\cdot)$	Mixed strategy payoff function
$\hat{F}^i$	Expected payoff of an individual with strategy i against the population state $\mathbf{x}(t)$ at time t
F	Pure strategy payoff function
$F_i(\cdot)$	Pure payoff function for player i
$\mathbb{N}$	Natural numbers
$\mathbb{R}$	Real numbers
$\dot{x}$	$\frac{dx}{dt}$, Derivative with respect to time
$\varphi(x)$	The polynomial of the replicator dynamic
$\mathbf{x}(t)$	Population state vector at time t
x_0	Initial condition for a dynamical system

1 Introduction

The beginnings of game theory date back to Johann von Neumann in 1928, who developed a mathematical framework for modelling situations with strategic interactions of rational agents (Von Neumann, 1928). In a situation of strategic interaction, the outcome for an agent does not only depend on his choice, but also on the choice of the other agents. In economics, such situations often present themselves as a social dilemma, a situation where a group of agents is prevented to achieve the best outcome for the group, because of strategic considerations of the individual agent.

Asking a social scientist about *the* example of a social dilemma, one usually gets to hear the story about two prisoners, arrested and separately interrogated by an authority. Lacking the amount of evidence to sue the criminals for serious crimes, the authority offers them a deal simultaneously. A confession would grant a prisoner amnesty and hence, he could escape the punishment for the serious crimes. However, if both chose to confess, they receive the full sharpness of the law. Not admitting the crimes would leave the authority with too little evidence, resulting only in a sentence for minor crimes. This story, developed by Albert Tucker in 1950 and suitably named prisoner's dilemma (PD) is usually represented in a table, as shown in figure 1, with the options for each prisoners denoted by C, remaining silent, and D, admitting their crimes. The total amount of

		Player 2	
		C	D
Player 1	C	-1, -1	-4, 0
	D	0, -4	-3, -3

Figure 1: A parametrized representation of the prisoner's dilemma

years for the group is lowest if both do not confess, but each individual prisoner has the incentive to take the deal of the authority as this leaves him, independently of what his fellow prisoner does, with a shorter time in prison. The dilemma clearly consists of the conflict between individual interest and the benefit of the group (Skyrms, 2004).

Skyrms argues that there is "another story that became a game" which describes a different but underrepresented social dilemma (Skyrms, 2004, p. 1). In *A Discourse on Inequality*, Rousseau outlines the situation of hunters, heading out to hunt stag. However, if a hare runs past a hunter, he might consider to shoot the hare, scaring of all stags in the forest. The total reward for the group is surely lower, but it is rational for the single hunter if he expects the others to act the same, given the opportunity. In contrast to PD, it is not the hunters individual interest that is competing with the benefit of the group, but the uncertainty about what the other hunters choose to do. In fact, none of the hunters would mind his choice not to shoot a hare when returning with a stag, as his personal benefit is greater with a stag as bounty. Figure 2 represents this dilemma for two hunters

		Player 2	
		S	H
Player 1	S	5, 5	0, 4
	H	4, 0	3, 3

(a) Numerical example

		Player 2	
		S	H
Player 1	S	a, a	b, c
	H	c, b	d, d

(b) Parametrized: $a > c \geq d > b$

Figure 2: The stag hunt game: Numerical example and parametrized

with a numerical example and parametrized. Considering the numerical example, when both coordinate on stag hunting, denoted by S, they receive 5. If one of them decides to shoot the hare, he gets 4, whereas the other hunter is left with nothing. Coordination on hare hunting yields both with 3. Clearly, hunting hare is safer as it yields a hunter at least 3, whereas stag hunting may leave a hunter with nothing. In the representation with parameters in figure 2b, these are usually assumed to satisfy the condition $a > c \geq d > b$. The relevance of this restriction will be discussed throughout the text. In game theory, this game is known as the stag hunt (SH) game and is considered as a coordination game.

In both outcomes, collective stag hunting and collective hare hunting, each individual hunter has no incentive to change his action. Hence, both outcomes are what game theory calls a Nash equilibrium, originally formulated by John Nash in 1950 (Nash, 1950). In a Nash equilibrium all players of a game choose a best-reply against the strategies of the other players, so that none of them has an incentive to deviate unilateral from his decision. Although this is the mainly used solution concept, it does not give a definite answer in the stag hunt game.

One might argue that both hunters can speak to each other beforehand and assure each other that hare hunting is more attractive to both them. However, Camerer (2003) argues that empirical observation and theory suggest that the problem is not solved that easily. Furthermore, in a wide range of economic applications interaction of agents happen repeatedly. Analyzing repeated prisoner's dilemmas, it turns out that the resulting game can actually be interpreted as a stag-hunt game with two Nash equilibria (Skyrms, 2004).

The connection between these two social dilemmas advocates that the selection of equilibria should be investigated more deeply. An interesting approach to the equilibrium selection problem comes from biology. Evolutionary game theory was pioneered by Maynard J. Smith and George R. Price with their work on the conflict of animals in populations (Smith & Price, 1973). Started as a refinement of the Nash equilibrium, the concept of an evolutionary stable strategy, various other authors, for example Taylor and Jonker (1978), Hofbauer, Schuster, and Sigmund (1979) and Zeeman (1981), developed a "time dependent dynamical extension of game theory" (Hanauske, 2011, p. 55). In contrast to traditional game theory, the evolutionary approach models agents in a large population, interacting repeatedly in a strategic environment. Furthermore, the agents are not assumed to be rational, but follow a specific rule, updating their strategy. As outlined in the rest of this thesis, the evolutionary approach offers an answer to the coordination problem in the stag hunt game.

The thesis is organized as follows. Section 2 introduces the framework of traditional game theory for the stag hunt game and discusses the traditional approach to the equilibrium selection problem. Section 3 introduces evolutionary game theory and presents the replicator dynamic. Later on, it will be discussed how the evolutionary approach selects between multiple equilibria. Section 4 demonstrates the effect of a network externality in the underlying framework. Looking for evidence on how real people play the stag hunt game, section 5 considers experimental literature on the topic. Section 6 finally concludes the presented analysis.

2 The Stag Hunt Game

As already explained, the stag hunt game is played by two players who choose their strategies simultaneously. Both have information about the strategies of the other player and the payoffs they and their opponents receive. In game theory such a game is called a *normal form* game with complete information. Typically, such a game is formalized as a Triplet $\Gamma = (I, \mathbb{S}, F)$, with the set of players $I = \{1, 2, ..., n\}$, where $n \in \mathbb{N}$ is the total number of players. Expressing all possible states of the game, the pure strategy space of the game $\mathbb{S}$ is defined as the cartesian product of the individual pure strategy spaces S_i for a player $i \in I$, $\mathbb{S} = \times_i S_i$. The individual pure strategy space is defined as $S_i = \{1, 2, ..., m_i\}$, where $m_i \in \mathbb{N}$ denotes the total number of strategies available to player i. The payoff function of the game is denoted by $F : S \to \mathbb{R}^n$. As this text focuses upon the stag hunt game, these definitions reduce to the following. By setting $n = 2$, we define the set of two hunters playing SH as $I = \{1, 2\}$. In the SH game, each hunter can choose one out of two pure strategies, namely, hunting stag (strategy 1 or S) or hunting hare (strategy 2 or H). Therefore, the pure strategy space is defined by $S_i = \{1, 2\}$, which results by setting $m_i = 2$. The strategies are called pure to distinguish them from the later introduced mixed strategies. In the SH game both players choose from the same set of strategies such that $S_1 = S_2 = S$. Combining the strategy spaces of the two players with the cartesian product yields the definition of the pure strategy space of the game $\mathbb{S} = S \times S = S^2$. Games played by two players with two strategies each, are often said to be 2x2 games.

Considering the story Rousseau told, the hunters do not just hunt for their pleasure, but to sell their loot for a reward or to feed their families. This is captured by the definition of an individual payoff function for player i, $F_i : S \to \mathbb{R}$, which maps a payoff $F_i(s) \in \mathbb{R}$ to every state of the game $s = (s_1, s_2) \in S^2$, where s_i is a pure strategy of player i. A state of the game s is sometimes simply called strategy profile. The payoffs are usually interpreted as *von-Neumann* utility when played by individuals or households and may be interpreted as earnings regarding firms (Fudenberg & Levine, 1998).

In the special case of two players, one can define a payoff matrix for player 1, $A \in \mathbb{R}^{2\times 2}$ and for player 2, $B \in \mathbb{R}^{2\times 2}$. The elements of the matrices A_{kl} and B_{kl} state the payoffs to the players for a given strategy profile. Hence, the elements are defined as $A_{kl} = F_1(k, l)$ and $B_{kl} = F_2(k, l)$ with the pure strategies $k, l \in S$. Using the representation in figure 2b, one can identify the payoff matrices for the players as

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad B = \begin{pmatrix} a & c \\ b & d \end{pmatrix}. \tag{1}$$

For the SH game studied in this thesis, the following definition is important:

Definition 1. *A two player game $\Gamma = (I, S_1 \times S_2, A, B)$ is called symmetric if the players of the game have the same strategy space $S_1 = S_2 = S$ and for the payoff matrices the condition $B = A^T$ holds. Therefore, the game is well-defined as $\Gamma = (I, S^2, A)$.*

Clearly, the SH game satisfies this condition. Both hunters have the same strategies available and the payoff matrices defined in equation (1) are the transpose of the other. Hence, it is irrelevant which player is labeled as player 1 or 2, because they are identical with respect to their strategies and payoffs.

Usually a game is extended by the possibility of the players to play *mixed strategies*. Intuitively, the hunters in the SH game could choose whether to shoot the stag or the hare by using a randomization tool such as flipping a coin. However, randomization is not really satisfying in most applications (Radner & Rosenthal, 1982). An alternative interpretation, outlined by Rubinstein, is that mixed strategies are actually deterministic, but seem to be random because they depend on not modelled private information of the individuals

(Rubinstein, 1991, p. 914). In the evolutionary context, discussed in section 3, mixed strategies have a different, unproblematic interpretation. Apart from the interpretation problem, mixed strategies are appealing because they ensure the existence of a Nash equilibrium, which is defined later in section 2.1. Formally, every player of the game assigns a probability distribution over his pure strategies space. A strategy $x_i \in \Delta_i$ of player $i \in I$ is called a *mixed strategy*, where Δ_i is the mixed strategy space

$$\Delta_i = \{x_i = (x_{i1}, x_{i2})^T \in \mathbb{R}^2 : \sum_{k=1}^{2} x_{ik} = 1, x_{ik} \geq 0 \quad \forall k \in S\}.$$

In a symmetric game, the mixed strategy spaces of the players are equal, as both players are not restricted in assigning probabilities to their pure strategies. For the SH game this means $\Delta_1 = \Delta_2 = \Delta$. With this notation, a pure strategy can be interpreted as a mixed strategy, that assigns probability one to the pure strategy chosen and zero to all other strategies. This is represented by the unit vectors $e_k \in \Delta$, where $k \in \{1, 2\}$. Hence, $e_1 = (1, 0)^T$ is the pure strategy hunting stag and $e_2 = (0, 1)^T$ denotes the pure strategy hunting hare. The combined mixed strategy space of the game is $\Delta^2 = \Delta \times \Delta$. From now on, except otherwise indicated, $x \in \Delta$ denotes a (mixed) strategy chosen by player 1 and $y \in \Delta$ a (mixed) strategy of player 2. Again, similar to the pure strategy case, the mixed strategy payoff $\hat{F}_i : \Delta^2 \to \mathbb{R}$ maps any state in the mixed strategy space $(x, y) \in \Delta^2$ to an expected payoff $\hat{F}_i(x, y) \in \mathbb{R}$. With the matrix notation this is defined as:

$$\hat{F}_1(x, y) = x^T A y$$
$$\hat{F}_2(x, y) = x^T A^T y$$

Summarizing the notation, the SH game can be defined as a symmetric two-player normal form game $\Gamma = (I = \{1, 2\}, \Delta^2, \hat{F})$.

2.1 Solution Concepts

In describing the behavior of agents, game theory developed a wide range of tools to solve these strategic interactions. First of all, for each individual a best-reply is defined by simply describing the strategies available to the agent which yield the highest expected payoff against a given strategy of the other player. Formally, the *best-reply* $\beta_i(y)$ for player $i \in I$ to a strategy $y \in \Delta$ played by $j \neq i$ is defined as:

$$\beta_i(y) = \{x \in \Delta : \hat{F}(x, y) \geq \hat{F}(x', y), \quad \forall x' \in \Delta\} \tag{2}$$

Based on agents choosing a best-reply, the most frequently used solution concept of a normal form game is the *Nash equilibrium.* The Nash equilibrium was named by its proposer John Nash in 1950 and due to its prominence it is sometimes simply referred to as equilibrium or NE. In the following, the definition of a Nash equilibrium in a two player normal form game is stated.

Definition 2. *A state of* $(x^*, y^*) \in \Delta^2$ *is called a Nash equilibrium if it holds that*

$$(x^*)^T A y^* \geq x^T A y^*, \quad \forall x \in \Delta$$
$$(x^*)^T A^T y^* \geq (x^*)^T A^T y \quad \forall y \in \Delta$$

It is called symmetric if $x^* = y^*$*. A Nash equilibrium is called a strict Nash equilibrium if the inequality is strict.*

Actually, only one inequality is needed in a symmetric game as payoffs and strategy spaces are identical and so every state $(y^*, x^*) \in \Delta^2$ is also a Nash equilibrium. Equivalently to the definition, one can say that a strategy profile is a Nash equilibrium if all players choose a best-reply in that state of the game and hence, no player has the incentive to deviate from his choice given the choice of the other players. Nash (1950) proved the existence of such an equilibrium in any normal form game with mixed strategies. Imposing the Nash equilibrium as a solution concept implies that agents are *perfectly rational* and have *common knowledge* of the payoff matrix and the strategies available to all other players. In the stag hunt game, this means that players are able to calculate every outcome and additionally know that other players are rational and consequently also know that other players know that they are rational and so forth (Fudenberg & Levine, 1998). Furthermore, it is assumed that they strive to maximize their own payoff.

Analyzing the SH game, one finds three symmetric Nash equilibria. The first one consists of both players choosing strategy 1, hunting stag, $(e_1, e_1) \in \Delta^2$, where both players receive the payoff (a). None of them has an incentive to change his decision as for both playing the pure strategy 1 is a best-reply. A unilateral deviation would lead to the payoff c $(< a)$. The second equilibrium is the state of both players choosing strategy 2, hunting hare, $(e_2, e_2) \in \Delta^2$. Again, by definition, both play a best-reply and a unilateral deviation of a player results in a lower payoff $b < d$. Additionally to these Nash equilibria in pure strategies, there is a symmetric mixed strategy equilbrium where both players assign a probability $q^* = \left(\frac{d-b}{a-c+d-b}\right)$ to strategy 1. To see this, a player is indifferent between choosing one of his pure strategies against y, $\hat{F}(e_1, y^*) = \hat{F}(e_2, y^*)$, exactly when $y^* = (q^*, 1-q^*)^T$. The best-reply for a player against y^* can be obtained by equating the partial derivative of his payoff function with 0 at $y = y^*$, $\left.\frac{\partial \hat{F}(x,y)}{\partial y}\right|_{y=y^*} = 0$. One finds that the best-reply is also $x = y^*$.

For further analysis it is convenient to use the fact that Nash equilibria are only defined by comparison of payoff differences. Hence, any affine transformation of the payoff matrix does not change the best-replies of the players and thus, does not affect the set of Nash equilibria of the game (Weibull, 1997, pp. 17-19). Furthermore, adding a constant to every entry in a column of the payoff matrix, called a *local shift*, does not affect the payoff differences a player considers when valuating which strategy is better against a given strategy of the other player. Formally, let A_{kl} denote the elements of the player's payoff matrix. A local shift to column l^* transforms this payoff matrix into the payoff matrix A^*:

$$A^*_{kl} = \begin{cases} A_{kl} + v & \text{, for } l = l^*, v \in \mathbb{R} \\ A_{kl} & \text{, else} \end{cases}$$

Applying this to the stag hunt game, one can use two local shifts to turn the payoff matrix into a diagonal matrix with elements $A_{11} = \alpha_1$ and $A_{22} = \alpha_2$, where $\alpha_1 = (a - c)$ and $\alpha_2 = (d - b)$. Weibull (1997, p. 28) groups all symmetric 2x2 games into four categories with different equilibrium properties. The stag hunt game is considered a coordination game, with $\alpha_1, \alpha_2 > 0$ and in contrast to the class of games with dominated strategies, for example the PD, the solution of the game is not that obvious as there are multiple Nash equilibria. This will be further discussed in section 2.3.

2.2 Evolutionary Solution Concept

The stag hunt game has two Nash equilibria. When a game exhibits multiple equilibria it seems to be a reasonable question which equilibrium will be played. To answer this, game theorists tried to construct refinements of the Nash equilibrium in cases where some equilibria were unconvincing. Motivated by the context of evolution in animal populations Smith and Price (1973) constructed the refinement of an *evolutionary stable strategy* (ESS). In the biological context of Smith and Price (1973), a game is played by animals that are of a specific phenotype. A phenotype is interpreted as a strategy in the underlying game. The animals of the population are randomly matched in pairs, playing the normal form game. Payoffs are interpreted as *Darwinian fitness*, the number of expected offspring to a phenotype. Hence, the payoffs determine the survivability of the phenotype in the population. The ESS concept answers the question which compositions of the population are stable against an invasion by other animals from outside.

Definition 3. *A strategy $x \in \Delta$ is called a evolutionary stable strategy (ESS) if it holds that*

$$\hat{F}(x,x) \geq \hat{F}(y,x) \quad \forall y \tag{3}$$

$$\hat{F}(y,x) = \hat{F}(x,x) \Rightarrow \hat{F}(x,y) > \hat{F}(y,y) \quad \forall y \neq x \tag{4}$$

Equation (3) requires an ESS x to be a better reply to itself than any other strategy y. If there exists a strategy y with the same payoff, equation (4) demands that choosing x against y is better than choosing y against itself. Indeed, an ESS is used in a Nash equilibrium as it needs to be a best-reply to satisfy the condition in equation (3). In addition, a strategy used in a strict Nash equilibrium is always an ESS, since it strictly satisfies (3). In the stag hunt game, not all strategies are evolutionary stable. The pure strategies e_1 and e_2 are evolutionary stable, since both are used in a strict symmetric Nash equilibria and thereby satisfy condition (3) strictly. Now let y denote the strategy in the mixed NE $y = (q^*, 1-q^*)^T$, with $q = \left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)$. To prove that a strategy is not an ESS it suffices to find one strategy for which the inequalities do not hold. For example consider strategy e_1. The payoff against strategy y is, independently of the other strategy, equal to $\left(\frac{\alpha_2\alpha_1}{\alpha_2+\alpha_1}\right)$, i.e. (3) is an equality. Proceeding with (4), one finds $\hat{F}(y,e_1) = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} < \alpha_1 = \hat{F}(e_1,e_1)$, implying that y is not an ESS. Hence, the concept of an ESS does not help to select one unique solution in the stag hunt game. However, its importance for evolutionary game theory will be put into sharper relief when considering evolutionary dynamics in section 3.

2.3 Equilibrium Selection

Clearly, game theory has the aspiration to provide an unique solution for every strategic interaction. As seen in the stag hunt game, the most frequently used solution concept, the Nash equilibrium, and refinements such as the ESS are not sufficient to select a unique equilibrium, even in the case of a simple 2x2 SH game. This does not satisfy game theorists, since it is not clear which equilibrium is eventually played by the agents. Or how Weibull puts it, this kind of coordination games "caused[1] game theorists and users of noncooperative game theory a fair amount of frustration" (Weibull, 1997, p. 30).

A closer look at the two pure Nash equilibria in the Stag hunt game shows their difference. Considering the normal form representation in figure 2, the Nash equilibrium in which both players choose strategy one has the highest payoff for both players. (e_1, e_1) is then said to *payoff-dominate* or to *Pareto-dominate* (e_2, e_2). Equivalently, the equilibria

[1] And still causes.

are said to be *Pareto-ranked.* In the prisoner's dilemma, briefly described in the introduction, the payoff-dominant outcome is not a Nash equilibrium, due to the fact that every agent has the incentive to deviate and accept the deal of the authority. On the contrary, in the SH game every agent plays a best-reply in the Pareto efficient outcome, but since there is another Nash equilibrium it is not clear which one is played based on equilibrium play alone. Following Schelling (1960, p. 57), one may argue that Pareto-dominance characterizes (e_1, e_1) as a focal point. A focal point is an outcome of a game that is psychologically prominent and hence, might attract players.

However, the equilibrium point (e_2, e_2) exhibits a property called *risk-dominance.* Consider the numerical example in figure 2a. Strategy H ensures a player with a payoff of at least 3, whereas the payoff for strategy S varies between 5 and 0. Undoubtedly, strategy H is less risky than strategy S if a player is unsure about the strategy choice of the other player. Harsanyi and Selten (1988) defined this selection criterion based on the risk for the players associated with an equilibrium point using a tracing procedure. Basically, in this tracing procedure agents use common knowledge to form new beliefs about the choice of their opponent, considering that the opponent also has a less risky strategy. It is noteworthy that "risk-dominance is theoretically completely unrelated to risk aversion" as on one hand, the payoffs are in terms of von-Neumman utility and on the other hand, proceeding with the updating of their beliefs using common knowledge finally leaves the agents with certainty that the less risky strategy will be played by their opponent (Straub, 1995, p. 341). A detailed discussion of the tracing procedure is beyond the scope of this text. Nevertheless, risk-dominance is easily defined for the SH game.

Definition 4. *The Nash equilibrium* (e_2, e_2) *risk-dominates* (e_1, e_1)*, if* $(d-b)^2 > (a-c)^2$.

The squares on the left and the right side of the inequality are named *Nash products.* As mentioned in Weibull (1997), a Nash equilibrium risk-dominates if it is Pareto-dominant in the normalized payoff version of the game, associated with the diagonal payoff matrix, as definition (4) corresponds to $\alpha_1 < \alpha_2$.

Therefore, both pure Nash equilibria have a certain appeal for game theorists to be favored as an outcome of the game. The question is whether people rather play safe and coordinate on the hare hunting equilibrium, terminating in a social dilemma as there exists an outcome in which everyone of them is better off. Or whether they are able to recognize the payoff-dominance of stag hunting, trusting each other not to react to a hare passing by. Indeed, there is no consensus which equilibrium will be played. Although Harsanyi and Selten (1988) introduced the risk-dominance criterion, their general theory of equilibrium selection favors payoff-dominance in the SH game. They argue that "if each player knows the other to be fully rational [...] they should trust each other to play" strategy S (Harsanyi & Selten, 1988, p. 89). Furthermore, Harsanyi and Selten expect players to coordinate on the Pareto-dominant equilibrium if they are allowed to communicate beforehand, sometimes referred to as *cheap talk* when it does not directly affect the payoffs (Farrell & Rabin, 1996, p. 104), because "an agreement to do so is self-stabilizing". In contrast to that, Aumann (1990) points out that a message from player 1 to player 2, saying that he intends to coordinate on the payoff-dominant equilibrium, does not in general contain useful information for player 2. Indeed, in the numerical example of the SH game in figure 2a a player wants his opponent to choose strategy S independently of his choice, because hare hunting alone is better (payoff: 4) than hare hunting together (payoff: 3). Despite this argument, Farrell and Rabin (1996, p. 114) expect that "cheap talk will do a good deal to bring Artemis and Calliope[2] to the stag hunt". The experimental evidence will give a hint on how cheap talk affects real players. "Convinced by Aumann's argument", Harsanyi formulated a new theory of equilibrium selection in 1995, solely focussing on risk-dominance as selection criterion, hence favoring the hare equilibrium in the SH game (Harsanyi, 1995, pp. 92,94,96).

In the social sciences, the infinitely repeated version of the prisoner's dilemma has attracted wide interest for studying which factors affect cooperation. The famous book "The evolution of cooperation" by Robert Axelrod studies the success of different strategies using computer simulations of the repeated prisoner's dilemma. The typical theoretical framework usually only recognizes the Pareto-dominance of some equilibria in the repeated game. Intriguingly, Blonski and Spagnolo (2015) show that the underlying structure actually is similiar to the stag hunt game, as risk-dominance and Pareto-dominance select different equilibria. This connection to one of the most frequently used frameworks of a social dilemma stresses the importance of the equilibrium selection problem in the stag hunt game.

The evolutionary analysis outlined in the next section gives an answer to the equilibrium selection problem. Moreover, the answer is directly connected to the risk-dominance criterion.

3 The Evolutionary Stag Hunt Game

Evolutionary games consist of a population of agents that are randomly matched over time to play a stage game. In an evolutionary interpretation of the stag hunt game, it is repeatedly played as the stage game. Usually, the number of agents is assumed to be

[2]The names Farrell and Rabin (1996) used for player 1 and player 2.

(infinitely) large, such that on one hand, the effect of a single individual on the population is small, and on the other hand, the interaction happens anonymously. Furthermore, agents are described as myopically, so that they do not discount later payoffs. Populations can either be *monomorphic* or *polymorphic*. Monomorphic populations consist of agents that are only able to play pure strategies, whereas in polymorphic populations mixed strategies are allowed. In the following, the focus will be on monomorphic populations as they have a straightforward interpretation and the main implications do not differ. Formally, let $p_j(t) \geq 0$ denote the number of individuals in the population playing strategy $j \in \{1, 2\}$ at time $t \in \mathbb{R}$ and let $P = p(t) = p_1(t) + p_2(t)$ describe the total number of all individuals in the population. Note, $P = p(t)$ $\forall t$ meaning that the population is not growing. The vector $\mathbf{x}(t) = (x_1(t), x_2(t))^T = \left(\frac{p_1(t)}{p(t)}, \frac{p_2(t)}{p(t)}\right)^T$ is called the state vector of the population at time $t \in \mathbb{R}$. Each component of the population state vector represents the share of agents choosing a specific strategy. As the individuals can only choose between strategy 1 and 2 in the SH game, it holds that $x_2(t) = 1 - x_1(t)$. Mentioned earlier, a stage game played by a population enables a new interpretation of mixed strategies, because $\mathbf{x}(t)$ is an element of the mixed strategy space Δ for all t. Hence, mixed strategies in monomorphic games are simply the state of the population, which is the division of players on the two pure strategies in the SH game. This has, however, little relevance for the interpretation in the traditional game (Rubinstein, 1991, pp. 914-915).

Out of convenience, let the expected payoff to a strategy $j \in \{1, 2\}$ of an individual randomly matched against another individual in the current population state $\mathbf{x}(t)$ at time t be denoted by $F^j(\mathbf{x}) = \hat{F}(e_j, \mathbf{x}(t))$.

3.1 Revision Protocols

This section motivates the use of revision protocols to model the behavior of agents. In contrast to the traditional approach, agents in evolutionary game theory are usually neither able to calculate best-replies, nor are they observing the information relevant to perform the necessary calculations. Revision protocols formalize the idea that agents are following a certain rule by which they change their strategy. The following derivation is by Sandholm (2010). Typically it is assumed that agents have an inner alarm clock which rings at a rate R following an exponential distribution. Of course, this does not translate in economic applications to individuals having literal clocks around their wrist, but illustrates the idea of a randomly occurring chance to reconsider their strategy choice. The agents' clocks are assumed to be independent of each other, such that the individuals chance to revise does not dependent on others. Whenever the clock of an agent rings, he receives a revision opportunity which means that he changes his strategy with the probability $\frac{\rho_{ij}}{R}$, where $\rho_{ij}(\hat{F}, \mathbf{x}(t))$ is called the conditional switch rate, representing the rule by which agents change from strategy i to strategy j. The conditional switch rate usually depends on the current state of the population $\mathbf{x}(t)$ at time t and the mixed strategy payoff function of the underlying game $\hat{F}$. In a two-strategy game there are two conditional switch rates, ρ_{12} describing the rate of switching from strategy 1 to 2 and ρ_{21}, the rate for switching from strategy 2 to 1. For ρ_{ii} no actual switch happens. Switch rates result in different individual behavior rules, and furthermore in different dynamics for the whole population. An evolutionary dynamic describes the change of the population vector $\mathbf{x}(t)$ in time. The derivation of the dynamic can be done for a general conditional switch rate. Every agent receives Rdt revision opportunities, i.e. the clock rings, in a time interval $[0, dt]$ as it follows a Poisson distribution with mean Rdt (Sandholm, 2010, p. 123). If the population in the stag hunt game is at state $\mathbf{x}(t)$, the number of agents currently playing strategy i receiving a revision opportunity during the time interval of length dt is Px_iRdt. As Sandholm (2010) argues, this is an approximation because the state of the population $\mathbf{x}(t)$ may change during the time interval dt. With the probability for an agent playing strategy i switching to strategy j, one gets $Px_i\rho_{ij}dt$ for the expected number of switches for strategy i during $[0, dt]$ across the population. In a game with two strategies, the change in the number of agents in the population playing strategy 1 is determined by agents switching to strategy 2 and agents with strategy 2 switching to 1. Hence, this leads to

$$Pdx_1 = \underbrace{-Px_1\rho_{12}dt}_{\text{Switches from 1 to 2}} + \underbrace{Px_2\rho_{21}dt.}_{\text{Switches from 2 to 1}} \tag{5}$$

Dividing by P and dt yields the differential equation for the change in the share of agents playing strategy 1, $\dot{x}_1 = \frac{dx_1}{dt}$. The sum of the time derivatives of population shares must equal zero and so $\dot{x}_2(t) = -\dot{x}_1(t)$. There are different plausible revision protocols studied in the literature. Sandholm (2010) discusses various forms of these revision protocols, such as logit choice, comparison to average payoff and best response protocol. They all contain different assumptions about the information an individual agent can obtain and about the ability of the agents to process this information. Hence, the resulting evolutionary dynamics have different properties.

With respect to the implied dynamic, the *pairwise proportional imitation* protocol will be outlined hereafter. It assumes that whenever an agent receives a revision opportunity, he randomly gets to know another agent's strategy and its payoff received in the current state of the population. The switching probability of an agent choosing strategy i is proportional to the difference between the observed payoff $F^j(\mathbf{x})$ of the other agent with strategy j and his own payoff $F^i(\mathbf{x})$. This can formally be expressed as

$$p_{ij} = \begin{cases} x_j \left[F^j(\mathbf{x}) - F^i(\mathbf{x})\right] & \text{, for } F^j(\mathbf{x}) - F^i(\mathbf{x}) > 0 \\ 0 & \text{, else.} \end{cases} \tag{6}$$

A story of traders on a marketplace can be told to illustrate this revision protocol. Suppose the traders interact on a market selling some good. They implicitly play a game with each other as their choice influences the earnings of the others. The strategies may be the way they organize their market stall or what position on the market they choose. Usually each trader does not observe the strategies other traders use to sell things, because the market place is large and the bustle going on is complicated. Hence, a trader is usually committed to a strategy he got to know a while ago. As he returns to his tavern, he occasionally gets to meet another trader. Being proud of their earnings, they boast about their individual strategy. In this way, they get to know other traders' strategies and as they all dream of a live in pageantry, they might imitate the other traders. However, they are more likely to imitate traders with a higher payoff compared to theirs, because such traders feel superior and thus, are the loudest in the tavern.

Plugging the conditional switch rate of the pairwise imitation protocol (6) into the equation for the change of the population share playing strategy 1, $\dot{x}_1(t)$, leads to the *replicator dynamic*:

$$\begin{aligned} \dot{x}_1 &= 2x_1x_2\left[F^1(\mathbf{x}) - F^2(\mathbf{x})\right] \\ &= 2x_1\left[(1-x_1)F^1(\mathbf{x}) - x_2F^2(\mathbf{x})\right] \\ &= 2x_1\left[F^1(\mathbf{x}) - \bar{F}(\mathbf{x})\right], \end{aligned} \tag{7}$$

with the average payoff across the population $\bar{F}(\mathbf{x}) = x_1F^1(\mathbf{x}) + x_2F^2(\mathbf{x})$. This equation fully describes the evolution of the population in time. Properties and the connection to the stage game are discussed in the next section.

3.2 Replicator Dynamic

The replicator dynamic, one of the most popular dynamics in evolutionary game theory, has a wide range of applications and attractive properties, such as population genetics, ecology, abiogenesis, and of course game theory (Hofbauer & Sigmund, 1998, p. 203). It was mathematically formulated by Taylor and Jonker (1978) and termed after the biological concept of a replicator, which was introduced in 1982 by Dawkins as the "fundamental unit[s] of natural selection, the basic thing[s] that survive or fail to survive" (Dawkins, 2016, p. 254). In a purely game theoretical fashion, strategies are the replicators as they are propagated through the population based on the revision protocol. Using the matrix notation, the replicator dynamic for the strategies $j \in \{1, 2\}$ can be formulated as

$$\dot{x}_j = x_j \left[(A\mathbf{x})_j - \left(\mathbf{x}^T A \mathbf{x} \right) \right]. \tag{8}$$

The first term in the bracket $(A\mathbf{x})_j$ is the expected payoff to strategy j against the current population state at time t. The second term is the average payoff of the whole population $(\mathbf{x}^T A\mathbf{x}) = \bar{F}(\mathbf{x})$. The growth of the share $\dot{x}_j(t)$ at time t depends on the share of the population currently using strategy j and the excess payoff of that strategy, the difference between the expected payoff to strategy j against the current state of the population and the average payoff for the whole population. Intuitively, the share of a strategy in the population increases (decreases) if the strategy yields a higher (lower) than average payoff. An evolutionary dynamic that satisfies this property is called *payoff monotone* (Szabó & Fath, 2007, p. 30). Comparing equation (8) with equation (7), one notices the lack of the factor 2. This is simple due to the derivation of equation (7). In fact, any positive transformation of the payoff matrix results only in a change in speed of the replicator dynamic (Weibull, 1997, p. 73). Another property of the replicator dynamic is the low data requirement. Seen in the previous section, it was not necessary to assume that agents have much knowledge about the game or the population. It was sufficient to assume that an agent gets to know the payoff of one random player. This is interesting for applications where it is reasonable to assume that players cannot obtain much information.

Contrary to the interpretation that agents following a revision protocol are "boundedly rational", Gintis (2000) argues that "this is very misleading, because the real issue is the distribution of the information" (Gintis, 2000, p. 273). He claims, that even if they were not "bounded" they could not calculate a best-reply since they do not have the information about the state of the population. For clarification, consider a representative agent in the population. At time t he plays strategy 1 and hence he has an expected payoff $F^1(\mathbf{x}) = \alpha_1 x_1$. But he actually receives either the payoff α_1 or α_2 when matched against another agent with strategy 1 or 2, respectively. Based on that information and the knowledge of another agents payoff, the agent cannot deduce the composition of the population which is needed to choose a strategy that maximizes the expected payoff. However, one can qualify this argument by assuming that agents have the information but cannot process it. In the trader analogy, traders might be able to see the difference of the other traders' market stalls, but cannot recognize the secret of their success without others telling them.

A further property of the replicator dynamic is the lack of mutation or an error. In other words, a strategy that has not existed or has vanished from the population will not be in a future state of the population. That means, if only a subset $S' \subset S$ of the set of pure strategies is used in a population state, any future population state can also only contain strategies from this subset S'. Hence, if a strategy in the stag hunt game vanishes, the population state is in one of the two pure Nash equilbria. For further use it is practical to express the replicator dynamics for the parametrized SH game using the matrix in equation (1) and define $x_1(t) := x,\ x_2(t) = 1 - x$:

$$\begin{aligned} \dot{x} := \varphi(x) &= x^2(a - c + 2d - 2b) - x^3(a - c + d - b) - x(d - b) \\ &= x^2(\alpha_1 + 2\alpha_2) - x^3(\alpha_1 + \alpha_2) - x(\alpha_2) \end{aligned} \tag{9}$$

The last step follows by using local shifts on the matrix A. This does not change the dynamic at all because it preserves the relative difference of payoffs (Weibull, 1997, p. 73). The polynomial $\varphi(x)$ is of degree 3 and contains all information about the dynamic. Actually, as outlined below, it shows to which state the population converges. In figure 3 the graph of $\varphi(x)$ for the parameters $a = 5, b = 0, c = 4, d = 3$ is shown. The ordinate represents the polynomial and the abscissa represents the population share choosing strategy 1. Hence, it is between 0 and 1. Roots of the polynomial are the values of x for which $\varphi(x)$ crosses the horizontal dashed gray line. The vertical dashed gray line indicates the inner root, the root for an $x \in (0, 1)$.

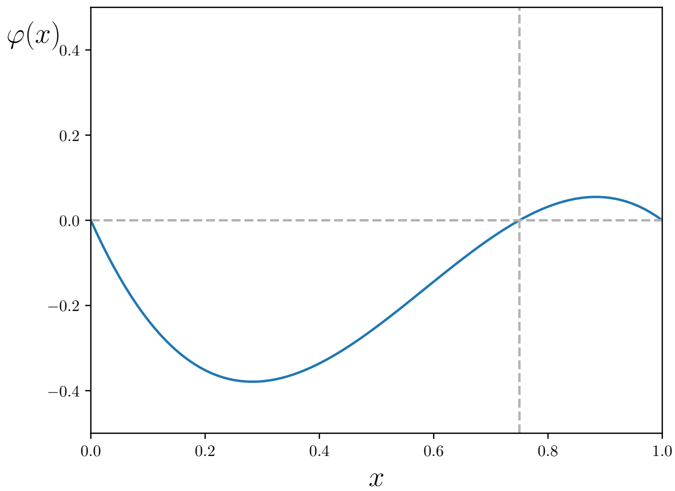


Figure 3: $\varphi(x)$ for the parameter setting $a = 5, b = 0, c = 4, d = 3$

A solution through an initial state x_0 of the dynamical system is called a *trajectory*, whereby it is distinguished between a forward trajectory for $t \to \infty$ and backward trajectory for $t \leftarrow \infty$. Analytic solutions, i.e. solutions one can express with the use of basic functions, can only be derived for simple cases. However, the existence and uniqueness of a solution through any initial state for the replicator dynamic is guaranteed by the theorem of *Picard-Lindelöf*, theorem 6.1 in Weibull (1997, p. 74).

Once a solution is obtained, the question whether there is a connection between the evolutionary dynamic and the equilibria in the stage game arises. To see this, consider the concept of a fixed point. A point $\mathbf{x}^* = (x_1^*, x_2^*)^T \in \mathbb{R}^2$ of a dynamical system, such as the replicator dynamic in equation (8), is called a fixed point if $\dot{\mathbf{x}} = 0$ at $\mathbf{x}^*$, i.e. it stays in the current state for all $t \to +\infty$. Analyzing equation (8), this happens if either a strategy is not present in the population or the excess payoff of a strategy is zero. Consequently, considering equation (9), in the stag hunt game a fixed point can be characterized by an x^* for which $\varphi(x^*) = 0$. The fixed points of the dynamic coincide with the Nash equilibria of the stage game as $\varphi(x^*) = 0$ for $x^* \in \{0, \frac{\alpha_2}{\alpha_1+\alpha_2}, 1\}$. It suffices to calculate the roots of the polynomial $\varphi(x)$, either numerical or, as the Nash equilibria are usually calculated beforehand, reducing the degree by polynomial division.

In general, the replicator dynamic lacks the *Nash stationarity* property and thus there are games with fixed points which are not a Nash equilibrium (Sandholm, 2010). For a trivial example, consider the prisoner's dilemma in figure 1. A population consisting only of cooperating agents stays in that state which is not a Nash equilibrium forever. However, inserting one single defecting agent would lead all other agents to imitate the strategy defect, as it has a higher payoff, so that in the end the whole population defects. Therefore, one is usually interested in the fixed points of a dynamical system that are stable in terms of some disturbance of the system.

A quite strict and useful concept of stability, often used in evolutionary game theory, is *asymptotic stability.* It essentially requires that a system in a specific fixed point returns to the fixed point after a perturbation. Hence, the system tends to move back to an asymptotically stable fixed point once disturbed. Formally, an ϵ-perturbation of $\dot{x} = \varphi(x)$ is a trajectory of the system with the initial condition x_0 in some ball $B_\epsilon(x^*)$ of radius $\epsilon > 0$ and $x_0 \neq x^*$. In one dimension a ball $B_\epsilon(x)$ is simply an interval $[x^*-\epsilon, x^*+\epsilon]$ around the fixed point x^*. The fixed point x^* is then called *asymptotically stable* if there exists an ϵ-perturbation for which $x(t) \to x^*$ as $t \to \infty$. The *basin of attraction* of a fixed point x^* is defined as the set of points $x_0 \in \mathbb{R}$ for which a trajectory through x_0 approaches the fixed point x^*. For the one dimensional case, this is simply the interval of x for which any solution to the differential equation with the initial conditions in this interval converges to the fixed point. With the following theorem independently contributed by Hartman (1960) and Grobman (1959), one can easily find the asymptotically stable fixed points in the stag hunt game:

Definition 5. *If a one dimensional dynamical system* $\dot{x}(t) = \varphi(x)$ *has a hyperbolic fixed point* x^**,* x^* *is asymptotically stable if its linearization* $\dot{x} = \varphi'(x^*)x$ *at that fixed point is asymptotically stable, with* $\frac{\partial \varphi(x)}{\partial x} := \varphi'(x)$*. A fixed point is called hyperbolic in one dimension if* $\varphi'(x^*) \neq 0$*. The linearization is stable at that point if* $\varphi'(x^*) < 0$ *and not stable if* $\varphi'(x^*) > 0$*.*

The solution to the linearization can be analytically derived by a standard tool for differential equations - separation of variables and integration. Accordingly, the linearization around the fixed point x^* is $x(t) = x_0 e^{\varphi'(x^*)t}$, where $x_0 = x(t = 0)$ denotes the initial condition, i.e. the share of agents choosing strategy 1 in the beginning. Applying the theorem to equation (9), one finds that the fixed point $x = 0$, corresponding to the Nash equilibrium of hunting hare, is asymptotically stable, since $\varphi'(0) = -\alpha_2 < 0$. The linearization around the fixed point is $x(t) = x_0 e^{-\alpha_2 t}$, which clearly approaches zero as $t \to \infty$. Similarly, the Nash equilibrium of hunting stag at $x = 1$ is an asymptotically stable fixed point as $\varphi'(1) = -\alpha_1 < 0$ with linearization $x(t) = x_0 e^{-\alpha_1 t}$. At the inner fixed point one gets $\varphi'(\frac{\alpha_2}{\alpha_1+\alpha_2}) = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} > 0$. By the linearization theorem the inner fixed point is not asymptotically stable. In general, more sophisticated mathematical tools such as the theorem of Lyapunov and entropy functions are needed to prove whether a fixed point is asymptotically stable (Friedman, 1998).

Another way to look at the asymptotic stability of the inner fixed point is the following. Suppose the population is currently at the inner fixed point and now some agents come from outside into the population using strategy 1, represented by the share ϵ, $\frac{\alpha_1}{\alpha_1+\alpha_2} > \epsilon > 0$. The new population state is $x_\epsilon = \left(\frac{\alpha_1}{\alpha_1+\alpha_2} + \epsilon\right)$. An asymptotically stable fixed point is now expected to withstand the invasion and make the outsiders imitate the prevailing strategy composition. Hence, the change of the population share $\dot{x}$ should be negative. Plugging x_ϵ into (9) one finds that $\dot{x} > 0$, the population share choosing strategy 1 grows for every $\epsilon > 0$, converging to the fixed point $x = 1$. Conversely, for an invasion by a share ϵ of agents choosing strategy 2, $0 > \epsilon > \frac{\alpha_2}{\alpha_1+\alpha_2}$, agents start switching away from strategy 1 to strategy 2 ($\dot{x} < 0$) and hence, the population share of agents playing strategy 1 decreases, until it reaches the fixed point $x = 0$. Hence, only population state which involve the play of an ESS are asymptotically stable.

The key things to remember are that, a Nash equilibrium is a rest point of the replicator dynamic. Furthermore, not all rest points are Nash equilibria in general. And finally, strict Nash equilibria are asymptotically stable. These results do not just hold for the evolutionary stag hunt game, but are true for the replicator dynamic in general. In the literature these connections between the stage game and the replicator dynamic are summarized in the *Folk Theorem of Evolutionary Game theory* (Szabó & Fath, 2007, p. 25).

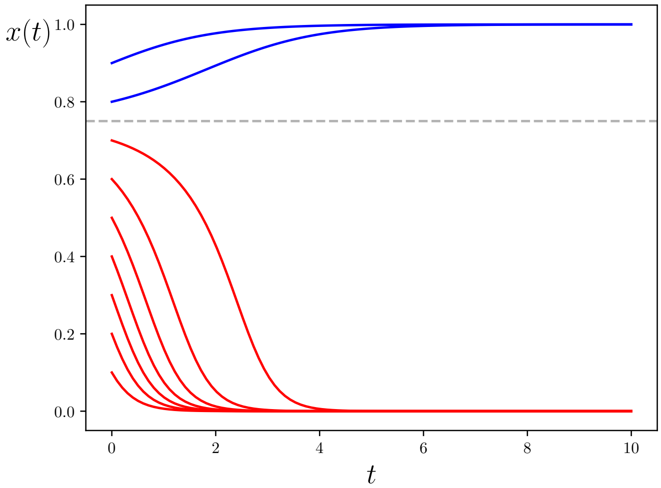


Figure 4: Replicator dynamics in the stag hunt game for $\alpha_1 = 1,\ \alpha_2 = 3$ with different initial conditions

Interestingly, every payoff monotone dynamic satisfies this theorem, as shown in Hofbauer and Sigmund (2003), and hence, many dynamics have the same dynamic properties for the stag hunt game.

In figure 4 the graph of the replicator dynamic in the stag hunt game for $\alpha_1 = 1$ and $\alpha_2 = 3$ are plotted. The horizontal axis represents time t, while the vertical axis shows the share of the population choosing strategy 1. The dashed gray horizontal line represents the inner fixed point of the dynamic at 75%. Red trajectories converge to the population state $x = 0$, the hare hunting equilibrium, whereas blue trajectories converge to the population state in which all agents hunt stag.

Concerning the equilibrium selection, the evolutionary approach with replicator dynamic has a definite answer. The population will be in one of the three Nash equilibria depending on the initial condition after some time. However, an equilibrium that is not asymptotically stable is rather implausible, because it only emerges from one initial condition in which the population is already exactly in that population state. Intuitively, one additional agent playing a pure strategy suffices to get the population state moving to one of the pure equilibria. As the model outlined here is only a deterministic approximation, any stochastic shock would lead to such a disturbance and hence would start convergence to one of the states with evolutionary stable strategies. The population converges to the stag hunting or the hare hunting equilibrium if the initial conditions lie in their basin of attraction. As discussed earlier, the population converges to the state $x = 1$, the stag hunting equilibrium of the stage game, for all initial conditions $x_0 \in (\frac{\alpha_2}{\alpha_1+\alpha_2}, 1]$. Any trajectory of the dynamical system with initial conditions $x_0 \in [0, \frac{\alpha_2}{\alpha_1+\alpha_2})$ leads to the hare hunting equilibrium.

Interestingly, one observes another connection of the dynamic with the stage game. By definition (4), the hare hunting equilibrium risk-dominates if $\alpha_2 > \alpha_1$. Thus, the basin of attraction is larger for an equilibrium that risk-dominates the other. With the parameter setting shown in figure 4, 75% of the possible initial conditions lead to the risk-dominant equilibrium, whereas only 25% of them lead to the payoff-dominant equilibrium. This connection was for example also shown by Kandori, Mailath, and Rob (1993) in a stochastical model. The convergence to the risk-dominant equilibrium in one dimension was independent of the specific adjustment process underlying the dynamic, as long it satisfied payoff monotonicity. However, it is not clear what specifies the initial conditions which ultimately determine the selected equilibrium in the deterministic case. If it is assumed, for example, that agents choose a random strategy in a first round of the game independently from each other, one can say that the equilibrium with a larger basin of attraction, the risk-dominant equilibrium, is reached more likely. Otherwise, a "slice of history", knowledge about how the population has played before, is needed to know in which population state the dynamic starts (Friedman, 1998).

4 Coordination and Network Externalities

This section investigates the effect of a network externality on the evolutionary stag hunt game. The following situation is inspired by an example given in Kandori et al. (1993). Consider students in university being assigned to projects in their classes with other students at university. Nowadays, most of the projects involve some kind of software tool to either present the results, analyze the subject or just manage the process of aggregating knowledge. Usually the choice of the tools can be simplified to the choice of proprietary software or open source software. For reasons of simplification, assume that both software types are capable of producing the same quality of output for the projects and are similiarly easy to handle. The only difference is the high license cost for proprietary software. Common examples of this are Open Office vs. Microsoft Word for text editing, Python vs. Matlab for scientific computing and R vs. Stata for statistical analysis. Assigned to a project, students can either choose proprietary software denoted as strategy 1 or open source software denoted as strategy 2. If both students choose open source software they receive utility (a). Coordinating on proprietary software leaves both with a utility (d) which is smaller than (a) as they have to pay for the licenses. In the case where students fail to coordinate on a software tool, the student with proprietary software gets (c). However, the other student has to transfer his contributions to the standards of the proprietary software as portability is usually only possible in this direction. Hence, this student only gets (b). The parameters fulfill the condition $a > c \geq d > b$. Evidently, the structure of the game is precisely that of a stag hunt game. Coordination on open source software is the payoff-dominant Nash equilibrium, but it is risk-dominated by the equilibrium in which the students coordinate on proprietary software.

For simplification, assume that during their studies, a large body of homogeneous students is randomly paired by their professor into groups of two for a project, implicitly playing the coordination game outlined. Concerning the behavior of an individual student, I assume that they are not perfectly rational and do not adjust their software choice each time they have to do a project. For simplicity, students in this model behave according to the revision protocol of pairwise proportional imitation discussed in section 3.1. This applies as students tend to use software they always used and only change it, if they get to know someone being more successful with another software choice. Therefore, the dynamic in the model can be described by the replicator dynamic derived for the stag hunt game in equation (9). This model would not differ in terms of convergence and stability of Nash equilibria to the evolutionary stag hunt game above.

However, I want to introduce *network externalities* in this model. The interest of the economic literature turned to open source software development, because it is commonly observed that users contribute to programming projects without receiving a monetary compensation for it. In fact users share their source code publicly to the open source community, being motivated by peer recognition and the signaling for career concerns (Lerner & Tirole, 2002, p. 21). The utility for a student increases with the size of the community as more students contributing code enhance the variety and usefulness of the software. To incorporate this in the model, let the utility for a student in the case of coordination on open source software be a function of population size using that strategy $f(x)$. Preserving the general structure of the game, it is useful to focus on the functional specification $f(x) = a + e(x)$, where (a) is the payoff parameter used previously and $e(x)$ denotes the network externality. A positive network externality satisfies $e(x) \geq 0$ and $\frac{de(x)}{dx} > 0$, as an increase of the population share choosing the strategy also increases the utility due to the network. Including the parameter (a) ensures that the utility of this strategy profile is never below the utility of coordination on proprietary software. Otherwise, the structure[3] of the stage game would differ fundamentally with respect to the population share choosing strategy 1.

The payoff matrix, using local shifts, is $A = \begin{pmatrix} \alpha_1 + u(x) & 0 \\ 0 & \alpha_2 \end{pmatrix}$. Substituting into the

[3] Although it might be interesting to relax this assumption. For example, one might study a change of the stage game from prisoner's dilemma to stag hunt game for a sufficiently high population share choosing strategy 1.

replicator dynamic (9) yields:

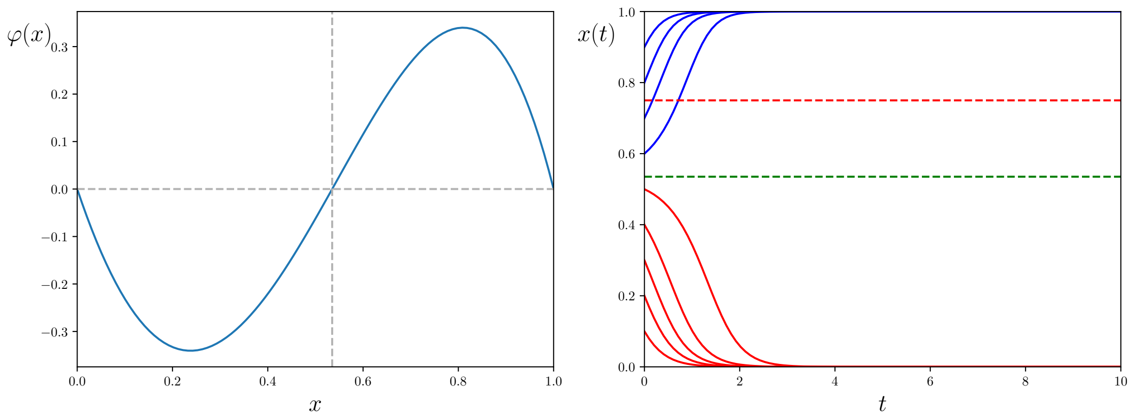
$$\dot{x} = \varphi(x) = x^2(\alpha_1 + e(x) + 2\alpha_2) - x^3(\alpha_1 + e(x) + \alpha_2) - x(\alpha_2) \tag{10}$$

By plugging $x = 0$ and $x = 1$ into equation (10) and by applying the linearization theorem, one can ensure that the fixed points with pure strategies and their property of asymptotic stability did not change ($\varphi'(1) = -\alpha_1 - e(1)$, $\varphi'(0) = -\alpha_2 - e(0)$). The literature on network externalities usually assumes a diminishing marginal effect of the network externality $\frac{d^2e(x)}{dx^2} < 0$ (Lin, 2008, p. 73). However, the analysis presented in this text will focus on the linear form $e(x) = \beta x$ with the externality parameter $\beta \in \mathbb{R}_+$. This restriction does not change the main implication of the model. Despite not changing the stability of the pure states, the externality affected the inner fixed point of the dynamic. The polynomial $\varphi(x)$ is now of degree 4:

$$\dot{x} = \varphi(x) = -\beta x^4 - x^3(\alpha_1 + \alpha_2 - \beta) + x^2(\alpha_1 + 2\alpha_2) - x(\alpha_2) \tag{11}$$

Using the knowledge from the general case, we can find the roots, i.e. fixed points of the dynamic, by reducing the degree with the two roots known. The polynomial of degree 2 can then be solved applying a quadratic formula[4], so that the fixed point with a mixed population state is $x_3 = -\frac{\alpha_1+\alpha_2}{2\beta} + \sqrt{\frac{(\alpha_1+\alpha_2)^2}{4\beta^2} + \frac{\alpha_2}{\beta}}$. The new inner fixed point is always lower than the inner fixed point without externality. It cannot coincide with the stag hunt equilibrium, as it satisfies $0 < x_3 < \frac{\alpha_2}{\alpha_1+\alpha_2}$. Taking the limit $\lim_{\beta \to 0} x_3 = \frac{\alpha_2}{\alpha_2+\alpha_1}$ verifies the intuition that for a diminishing network externality parameter β the model without externality is obtained. The plot of the dynamic and the polynomial is shown in figure 5. Note that the share of students choosing strategy 1 is now lower at the inner fixed point, displayed by the gray dashed lines in subfigure 5a, showing that the externality has a positive effect on the attractiveness of choosing open source software. Using the argument developed above, the inner fixed point is still not asymptotically stable. Any feasible share of outsider ϵ with a pure strategy would lead to convergence to one of the pure population states as $|\varphi(x_3 + \epsilon)| > 0$. Indeed, the basin of attraction of the equilibrium in which all students are using open source software became larger, depicted by the distance to the inner fixed point. This is illustrated by the dashed, red horizontal line, the inner fixed point without externality, and the dashed, green horizontal line, the inner fixed point with externality, in figure 5b. The basins of attraction have equal size for $\beta^* = 2(\alpha_2 - \alpha_1)$, attracting an equal amount of initial conditions. However, the proprietary equilibrium looses its risk-dominance property only for $1 > x > \frac{\alpha_2 - \alpha_1}{\beta}$, using definition

[4] The solution with negative sign was omitted, since it has no economic interpretation.



(a) The polynomial $\varphi(x)$ (b) The dynamic $x(t)$ for different x_0

Figure 5: The linear externality model for parameters $\alpha_1 = 1, \alpha_2 = 3, \beta = 3$

4. The right hand side of the equation must be lower than one to make a change of the risk-dominance property feasible. In this model, the size of the basins of attraction and the risk-dominance property do not coincide in general, since the externality affects the Nash product associated with the pure strategy 1 equilibrium. Consider for example the parameter setting $\alpha_1 = 1, \alpha_2 = 3$, and $\beta = 3$ in figure 5. The basins of attraction are not equal, since $3 < \beta^* = 4$. All trajectories starting with $x_0 \in (\frac{-2+\sqrt{13}}{3}, 1]$ converge to the open source equilibrium and hence the proprietary equilibrium attracts all trajectories with $x_0 \in [0, \frac{-2+\sqrt{13}}{3})$. Clearly, the latter basin of attraction is larger ($\frac{-2+\sqrt{13}}{3} > 0.5$), but the open source equilibrium gets risk-dominant for $x > \frac{2}{3}$. This implies that the network externality not only increases the utility obtained from the choice of open source software, but also, combined with a high enough user base, makes this choice risk-dominant.

The model presented here is a simplification of the real world. First, the students are expected to be able to review which software is appropriate for their projects, hence have more information available than assumed in the model. Secondly, talking to their fellow student seems possible before deciding which software to use for the project. However, as the experimental literature that will be discussed in section 5, and Aumann (1990) suggest, cheap talk does not generally solve the coordination problem. Additionally, the way the payoff of a strategy increases with the population size using it, the network externality, is modeled quite naively. Students at university and social networks in general are not fully connected as students, for example, only do projects with fellow students in the same classes. Furthermore, students usually try to do projects with people they are socially connected with. In fact, the assumption about a fully connected network is “one of the main criticisms of evolutionary game theory” (Hanauske, 2011, p. 246). As a consequence, many researchers recently gave attention to the structure of the network underlying the interaction of agents and evolving of network structure (Szabó & Fath, 2007, p. 46). For example, Ohtsuki, Hauert, Lieberman, and Nowak (2006) found that fewer connectivity leads to more cooperation for a range of network topologies, agreeing with the intuition "The fewer friends I have the more strongly my fate is bound to theirs" (Ohtsuki et al., 2006, p. 1). Additionally, Santos, Pacheco, and Lenaerts (2006) found in a model with a heterogeneous population with respect to the links of each agent that cooperation on the payoff-dominant outcome is easier to achieve than in a fully connected population. Although the model with a positive linear network externality is far too simple to capture the effect of complex, adapting networks, it gives an intuition of how coordination of a small group, in this case two players, can be affected by the choice of the whole population, as it changes the structure of the coordination game they play.

5 Experimental Evidence

A third approach beside the traditional and the evolutionary treatment of coordination problems is what Camerer calls the "fundamentally empirical" approach (Camerer, 2003, p. 336). Hence, this section consults the experimental economic literature regarding laboratory coordination games to provide evidence how actual people choose their strategies in such situations and which factors might influence them. Precisely, do people, if they are able to coordinate on an equilibrium at all, play the risk-dominant or the payoff-dominant equilibrium? And how does their choice change, playing the game repeatedly? Due to the rich experimental literature on coordination in social dilemmas, the focus will be on studies with a framework close to the evolutionary SH game. A comparison will show the shortcomings and what is described well by the evolutionary framework.

Due to the different notations authors use, a translation into the present notation was performed. On these grounds, the strategy used in the payoff-dominant equilibrium of the SH game will be either called *strategy S* or in analogy to the stag hunt story *hunting stag*. Consistently, the strategy used in the risk-dominant equilibrium is called *strategy H* or *hunting hare*.

The papers of Van Huyck, Battalio, and Beil (1990) and Cooper, DeJong, Forsythe, and Ross (1992) are seen as the first experiments investigating coordination games (Devetag & Ortmann, 2007). In Van Huyck et al. (1990), subjects did not play a stag hunt, but an order-statistics game. These games differ in that subjects can choose from a broader set of strategies, ordered from high to low, and hence there are more than two equilibria in the game. Played by a group, the players' payoffs depend on the lowest strategy one of their group members chose, thus the name 'weak-link' order statistics game. Nevertheless, the structure is similar to the stag hunt in the way that the equilibria are Pareto-ranked, with one safe strategy (Devetag & Ortmann, 2007). So evidence from these games is likely to be transferable to the coordination problem in the stag hunt game. In Van Huyck et al. (1990) subjects played the order-statistics game in groups of 14-16 for 10 periods. They only received information about their payoff, but that was sufficient for the players to find out what the lowest choice of one subject in their group was. Astonishingly, in every experiment the groups converged to the equilibrium with the lowest payoff. This result was preserved across all groups in another five periods of this game, although the subjects played an alternative payoff matrix, embracing the payoff-efficient equilibrium, in between. Replications of these results have been performed numerously, with varying group sizes and slightly changed payoff matrices (Devetag & Ortmann, 2007, p. 6).

Cooper et al. (1992) used a two-player stag hunt game[5] and one augmented by a dominated strategy. They found that subjects in randomly matched one-shot games, knowing only about their own received payoffs, also failed to coordinate without any pre-play communication. However, they find that two-way communication, in which both players were able to send a message to their matched partner containing which strategy they intend to choose, solves the coordination problem in the SCG. In their game, both players had an incentive to truthfully tell their opponents what they intend to do as ($c = d$). Therefore, this does not reject Aumanns hypothesis.

Nevertheless, Clark, Kay, and Sefton (2001) ran experiments with a game in which players had an incentive to want their opponents to choose strategy S independently of their choice ($c > d$). In this game, when both players announced S, only 25% coordinated on the Pareto-dominant equilibrium. Furthermore, they introduced a game similar to the typical stag hunt, but with ($c < d$). Here, a player saying he intends to choose "S" is better off if the other player also chooses S, and so "communication may be expected to be most credible in Aumann's sense" (Clark et al., 2001, p. 508). They found that agreements on a strategy helped to coordinate, but players equally agreed to the Pareto-dominated equilibrium. All in all, pre-play communication helps to coordinate, but which equilibrium is reached depends on the payoff matrix.

However, the early approaches by Cooper et al. (1992) and Van Huyck et al. (1990) suggest that without cheap talk coordination failure[6] is common in the laboratory (Devetag & Ortmann, 2007, p. 2).

[5]They call it simple coordination game (SCG).

[6]Here in the meaning of not being able to coordinate on the payoff-dominant equilibrium.

5.1 Variation of the Payoff Matrix

In the evolutionary games discussed above, a group of players is randomly matched against each other to play a stag hunt game. For that reason, the implementation of random matching in a group closely resembles an evolutionary game. Random matching was conducted by Battalio, Samuelson, and Van Huyck (2001). Subjects played three stag hunt games with the typical symmetric pure strategy equilibrium and the symmetric mixed strategy equilibrium with $(0.8, 0.2)^T \in \Delta$. Figure A.1 shows the payoff matrices of the games noted as $2R, R$ and $0.6R$. The game $2R$ does not satisfy the condition for the parameters used throughout this text as $c < d$ $(35 < 40)$. The notation for the games become clear when observing the *optimization premium* of the games. While controlling for the basin of attraction of the equilibria - the mixed equilibrium is equal in all games - they want to explore the effect of an increase in the "premium for playing a best-response" (Battalio et al., 2001, p. 751). In order to investigate this, they define the optimization premium $r_j(y)$ of a game j as the difference in payoffs choosing the pure strategies e_1 and e_2 while expecting the opponent to play a strategy $y = (q, 1-q)^T \in \Delta$:

$$r_j(y) = \hat{F}_j(e_1, y) - \hat{F}_j(e_2, y) = \delta_j(q - q^*), \tag{12}$$

with the *optimization premium parameter* δ_j and the probability used in the mixed strategy equilibrium q^*. In the notation of the parametrized SH game the optimization premium parameter is $\delta_j = a - c + d - b$. The optimization premium is only different from zero if a player expects the other player not to use the mixed Nash equilibrium strategy.

The parameters for the payoff matrices used in this study are reported in figure A.1. First of all, they find support for their hypothesis that subjects coordinate less frequently on the payoff-dominant equilibrium in games with a larger optimization premium. In game $0.6R$ ($\delta_{0.6R} = 50$) two cohorts and in R ($\delta_R = 25$) one cohort converged to the payoff-dominant equilibrium. There was no cohort in the game with the lowest optimization premium parameter, $\delta_{2R} = 15$, that coordinated on stag hunting. The deterministic replicator dynamic does not offer a description of this behavior, since convergence to an equilibrium only depends on its basin of attraction once the dynamic started, which do not differ for the three games. While "all 24 cohorts start in the basin of attraction of the risk-dominant equilibrium" (Battalio et al., 2001, p. 755), all cohorts should converge to the risk-dominant equilibrium according to the dynamic. A crossing of the "best-response separatrix" (Battalio et al., 2001), i.e. mixed-strategy equilibrium, can not happen in the deterministic replicator dynamic. Furthermore, as they conjectured, a larger optimization premium increases the speed of convergence. Hence, coordination on a equilibrium was achieved fastest in the $2R$ game. This is consistent with a postulated replicator dynamic for these games. Starting from (8), one can derive that the dynamics of this games are the same up to a change in speed, proportional to the ratio of optimization premia of the games. Thus, in game R the change in the population share playing strategy S is half the speed of $2R$ and five thirds of $0.6R$, $\dot{x}_R = \frac{1}{2}\dot{x}_{2R} = \frac{5}{3}\dot{x}_R$.

Schmidt, Shupp, Walker, and Ostrom (2003) designed their experiment so that the effect of differing risk levels could be observed. It stands to reason to relabel the strategies in the game, because in their treatment the payoff-dominant equilibria is in the right corner of the normal form game representation, contrary to the notation used throughout this text. Comparing risk levels between games, they used a measure formulated by Selten (1995), defined as:

$$R = ln\left(\frac{\hat{F}(e_2, e_2) - \hat{F}(e_1, e_2)}{\hat{F}(e_1, e_1) - \hat{F}(e_2, e_1)}\right) = ln\left(\frac{d-b}{a-c}\right) \tag{13}$$

The risk measure R is positive if the Nash equilibrium (e_2, e_2) is risk-dominant, and negative if the payoff-dominant equilibrium inhibits both properties. For $R = 0$, the mixed strategy equilibrium is risk-dominant. Risk-dominance of a game compared to another is stronger if it has a higher value of R. In contrast to Battalio et al. (2001), they do not use the proposed optimization premium, but rather measure the payoff-dominance level in relative terms as $P = \frac{a-d}{a}$.[7] The payoff matrices of the games are shown in figure A.2. Comparing game 2 with game 3, and game 1 with game 4, the games only differ in their degree of risk-dominance and hence one can investigate if subjects react to a change in R ceteris paribus. For game 1 and game 3 the basins of attraction of both equilibria are equal as the Mixed-NE is at $(\frac{1}{2}, \frac{1}{2})$ and for game 2 and 3 the Mixed-NE are at $(\frac{3}{4}, \frac{1}{4})$, resulting in the typically larger basin of attraction of the equilibrium (e_2, e_2). Therefore, it is expected from the replicator dynamic, of course depending on the initial condition of the game, that subjects are more likely attracted to the hare hunting equilibrium in game 2 and 3. In their implementation of a random matching protocol, Schmidt et al. (2003) found evidence that an increase in R leads to a higher choice of the strategy in the risk-dominant Nash equilibrium. Players did not react to changes in P, the level of payoff dominance.

According to Rydval and Ortmann (2005), the risk measure R can be interpreted as the risk concerned with a players' own deviation from one strategy to another. Rydval and Ortmann (2005, p. 19) suggest "[R]ather than their own deviation [...], players might worry about their opponent's choices", and hence a "definition of risk other than that embedded

[7] As they mention, the games in Battalio et al. (2001) vary in their level of P accordingly.

in the definition of risk dominance" is necessary. Dubois, Willinger, and Van Nguyen (2012, p. 371) introduced an alternative measure for risk, named the *relative riskiness* of a game, defined as $RR = \frac{|d-c|}{a-b}$. Intuitively, a player committed to strategy S receives less payoff $(a-b)$ when his opponent switches from strategy S to H. Similar thoughts lead to a difference of $(c-d)$ for a player committed to strategy H. Hence, the RR is simply the ratio of those payoff differences caused by an opponents deviation. Alternatively, it can be interpreted as the ratio of the standard deviations of the payoffs (Dubois et al., 2012, p. 372). The relative riskiness measure is close to one if both strategies involve similar risk. Accordingly, a lower relative riskiness measure implies that strategy H has a lower risk compared to strategy S. A game with a lower RR indicates lower relative riskiness of the strategy S to H compared to another game. Observing the values of the relative riskiness by Dubois et al. (2012) and the risk-dominance measure of Schmidt et al. (2003) for the games used in the papers, it is clear that both measures are in a conflict. As by the measure Schmidt et al. (2003) used, risk-dominance is kept constant in the games of Battalio et al. (2001), whereas relative riskiness indicates a variation. On the other hand, relative riskiness cannot distinguish between the games 1,2 and 3 of Schmidt et al. (2003), since $RR = 0$. In an early working version of their paper they explicitly excluded the case $c \neq d$ and stated that "a more general measure of relative riskiness should allow for the case where c=d as compared to $(a-b)$" (Dubois, Willinger, & Van Nguyen, 2008). The distinction between the two measures can be illustrated by two games. Suppose a game A with the parameters $(a, b, c, d) = (9, 0, 8, 7)$ and a game B with $(9, 0.5, 6.5, 6)$. Both games have the same optimization premium $\delta_A = \delta_B = 8$. Game B has a lower risk than A in regard to the relative riskiness measure as $RR_B = \frac{1}{17} < RR_A = \frac{1}{9}$. Conversely, the risk measure R indicates that risk-dominance is stronger in game A because $R_A = ln(7) > R_B = ln(2.2)$.

The experimental design of Dubois et al. (2012) is similar to that of Battalio et al. (2001). Game 1 replicated game $0.6R$ of Battalio et al. (2001) (Compare figure A.3 and A.1). As per their conjecture, they found that a lower relative riskiness in a game decreases the rate at which players choose strategy S, keeping the optimization premium constant. This is congruent with the intuition that the severity of the impact connected to the uncertainty about an opponents choice, expressed in the difference in payoffs a deviation of the opponent would cause, effects a players strategy choice. On the other hand, keeping the relative riskiness constant and varying the optimization premium, they could not find an effect on the frequency of strategy S. In measuring the "coordination success", they deployed the concept of "strong coordination" (Dubois et al., 2012). In other words, they count the occurrence of periods in which a group uniformly adopts a strategy so that every pair of subjects lands in one of the Nash equilibria. Dubois et al. (2012) intend to sort out “fortuitous coordination”, coordination as a consequence of subjects being randomly matched with each other (Dubois et al., 2012, p. 373). Comparing strong coordination between the games they found a non-significant difference between game 1 and game 2 in which only the relative riskiness ceteris paribus was changed. Contrary to that, there was a significant difference in the commonness in game 3 to game 2. Since game 3 and 2 only differ in their optimization premia, Dubois et al. (2012) conclude that a higher optimization premium leads to more frequent (strong) coordination. As they mention, this supports the observation of Battalio et al. (2001) that coordination success depends positively on the optimization premium.

On the basis of the evidence provided in Battalio et al. (2001), Schmidt et al. (2003) and Dubois et al. (2012), it seems fair to suggest that the structure of the payoff matrix has a strong impact on the way people play the stag hunt game. Additionally, the treatment of Dubois et al. (2012) suggests that suspects not only react to a risk measure that is theoretically derived from the risk-dominance criterion. However, the story the evidence tells so far is quite frightening. Failure to coordinate on the payoff-dominant equilibrium is apparently common.

A conflicting result was presented by Rankin, Van Huyck, and Battalio (2000). Following an argument of Kreps (1990) that identical strategic interactions, such as the play of completely identical stag hunt games in the laboratory, “take us very little distance outside the laboratory” (Kreps, 1990, p. 212), they designed an experiment with a randomly perturbed stag hunt game. Essentially, the payoff to strategy S is fixed, whereas the payoff to strategy H varies randomly between the games. This is depicted in the payoff matrix (14).

$$A = \begin{pmatrix} 1 & 0 \\ x & x \end{pmatrix} \tag{14}$$

The parameter x is equally distributed between $(0, 1)$ and, once a sequence was calculated, used for all cohorts participating in the experiment. This variation justifies to describe the games as “similar” but not “identical”, hence the structure stays the same, but the risk-dominance property varies. Indeed, if x is greater than 0.5, risk-dominance and payoff-dominance select different equilibria, (e_2, e_2) and (e_1, e_1), respectively. For a value of x smaller than 0.5 both select the equilibrium in strategy S. One finds that the optimization premium for this randomly perturbed games does not change, since $\delta = 1 - x + x + 0 = 1$. The relative riskiness measure RR cannot discriminate between these games, since $|d - c| = 0 \; \forall x$. R is positive for $x > 0.5$ and negative for $x < 0.5$. The question Rankin et al. (2000) want to investigate is, whether this framework of slightly varied situations may lead the groups to form a "convention" which deductive principle to choose. In contrast to the studies performed with identical stag hunt games, they observe coordination to the payoff dominant equilibrium across all cohorts. Whereas in the first ten periods of play risk dominance seems to have some explanatory power for the choice of strategies, in the last ten rounds 91% chose the payoff dominant action. In the cases of coinciding equilibrium selection of risk-dominance and payoff-dominance ($x < 0.5$), all subjects in group 1-5 and 95% of the subjects in group 6 coordinated on that equilibrium. For the other case a ($x > 0.5$) high coordination on the payoff-dominant strategy was observed as well. The hypothesis of an emerging convention based on the deductive principle payoff-dominance was supported by a frequency of strategy H of 80% to 95%. Contrary to previous experiments, this sends quite a positive image about coordination, as this is "dramatically at odds with claims that coordination failure is common" (Devetag & Ortmann, 2007, p. 9).

A major difference between the evolutionary agents and real people are the cognitive abilities. Certainly, it is to suspect that real players, opposed to the agents following a simple imitation rule, can take into account what strategies were played in the rounds before. Battalio et al. (2001) used a logistic-response model to investigate if the optimization premium affects the reliance of subjects on the history of play. Indeed, they found the highest sensitivity of individuals to previous rounds in game $2R$. Dubois et al. (2012) confirmed this result, also using a logistic-response model. Additionally, they could not reject the null hypothesis that sensitivity to previous play is significantly different for game 2 and 1. Hence, they did not find an effect of a change in the relative riskiness. Interestingly, the logistic-response model can be formulated as a single-population continuous-time logistic-response dynamic (Battalio et al., 2001, p. 752). The derivation is due to Fudenberg and Levine (1998). Its dynamic properties are different than that of the replicator dynamic as it does not satisfy payoff monotonicity. For example in the stag hunt game the dynamic does not have the same inner fixed point as the replicator dynamic. Instead, the literature speaks of a logit equilibrium (Battalio et al., 2001). The discussion of the explicit dynamic is beyond the scope of this thesis. However, it is interesting to note that revision protocols can be motivated to incorporate that agents use the history of play for their future strategy choices.

5.2 Details of the Implementation

As outlined, most studies concerned with the stag hunt game in the laboratory focused on the characteristics of the payoff matrix. However, details of the implementation, such as the matching protocol influence behavior sharply. Not only Van Huyck et al. (1990) found convergence to the payoff-dominant equilibrium in their two-player fixed matching implementation. Clark and Sefton (2001) also found different strategy choices between randomly matched one-shot games and fixed matching protocol games. The latter lead to significantly higher choices of the strategy in the efficient equilibrium and less "disequilibrium play" in general (Clark & Sefton, 2001). Devetag and Ortmann (2007) mention that one-shot games imply a random matching protocol. Yet it is different to random matching on group basis, where subjects can play against the same opponent again and hence there is the possibility that a "convention" emerges like in Rankin et al. (2000).

Even more subtle experiment design differences are conjectured to have an effect. For example Devetag and Ortmann (2007) suggest that the formulation "you will remain grouped with the same seven other participants for the next 75 rounds" in the instruction of the Rankin et al. (2000) experiment may increased trust within the group and hence was a cause for the exceptional coordination success. Comparatively, the instructions in Dubois et al. (2012) read "At the beginning of each period, in each group (composed of 8 participants), the computer system will form 4 pairs of subjects. [...] The experiment involves 75 rounds"(Dubois et al., 2012, p. 378). While essentially containing the same information, it does not emphasize the fact that subjects can encounter the same player again and consequently might have influenced the subjects to play the safer strategy.

An interesting experimental approach was executed by Dufwenberg and Gneezy (2005). They tried to evaluate whether there is a difference in coordination behavior across genders. Playing the weak-link order statistics game of Van Huyck et al. (1990), groups of only females or only males faced the coordination problem with Pareto-ranked equilibria. The motivation of observing gender differences was "never explicitly pointed out to the subjects" (Dufwenberg & Gneezy, 2005). However, they did not find that the groups play differently. Differences in the beginning "disappeared fast and no difference is found in later stages" (Dufwenberg & Gneezy, 2005, p. 235). All groups converged to the choice of the minimum contribution. As they mention, it might be useful to investigate coordination of groups that also have some other characteristics in common. They refer to the differences group identity has for females and males playing a public goods game. Namely, Croson, Marks, and Snyder (2008) found that females in a sorority playing the public goods game perform better than females with no specific affiliation. Contrarily, males in a fraternity performed worse than males without a group identity.

Interestingly, this and other factors concerning the group of subjects playing the coordination game, might be a rationale for the initial conditions in the evolutionary dynamic. If there is no "slice of history" available, the initial conditions could capture the composition of the subjects interacting, reflecting how well trust or cultural conventions are embedded in this group. It might be interesting to observe the coordination of groups where the composition of the group was pointed out to each subject. For example, I suspect that the behavior of people that are permanently confronted with risky decisions and the risky decisions of others, knowing that they are interacting with her kind, may not be as sensitive to risk-dominance as people that do not have this experience. Nevertheless, it seems difficult, at least for the author, to justify the reduction of the inheritance of cultural values or conventions to one[8] dimension such as the initial condition x_0.

6 Conclusion

Answering the question which factors affect social cooperation, one might only be able to give a partial answer. This is because the problem of cooperation can occur due to different reasons. The origin of the dilemma described in this thesis is the uncertainty of individuals about their counterparts choice. While one outcome guarantees everyone a higher payoff, they may fear that the other player deviates and hence, also choose the safer option. In the social sciences, the most frequently used framework for social dilemmas involves a conflict between the individual interest and the interest of the group. However, Camerer, for instance, argues that various situations that "are thought to be a prisoner's dilemma are actually a stag hunt game" if the described situation "is likely enough to be repeated, evokes emotion, has enough synergy or excludability" (Camerer, 2003, pp. 376-377). Hence, the stag hunt game finds a wide range of different applications, for example the coordination in the macroeconomy (Bryant, 1994), publishing decisions in academia (Hanauske, 2011) and in the discussion of the social contract (Skyrms, 2004).

The empirical evidence considered in section 5 suggests that failure to coordinate on the payoff-dominant outcome for the group in pure stag hunt games is common. In general, even preplay communication does not solve this. However, situations in the real world that can be treated as a coordination game may not involve the identical structure all the time. The results of Rankin et al. (2000) suggest that people can learn to coordinate when confronted with variation and hence, we might not have to be that pessimistic about coordination. Evolutionary game theory does not describe all varieties of the behavior in the laboratory. As a model should, it offers a simplification that helps to understand some of the patterns observable when real people interact. Additionally, it emphasizes that for a wide range of behavioral rules coordination on the risk-dominant equilibrium is more likely. While applied in various disciplines such as biology, psychology and political science,

[8]Of course, in games with more than two strategies the initial conditions are represented by a vector.

for example Friedman (1998) argues that the full potential of evolutionary game theory for economic applications has not yet been unlocked. He suggests that "economists must re-adapt evolutionary theory to economics" before it will be widely accepted (Friedman, 1998, p. 18). The underlying concept of behavior and the simplification of an infinite population have to be carefully reflected when applying the framework to economic problems. Furthermore, questioning the structure of the population is important, as the underlying social network may be pivotal for coordination success.

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Appendices

A Games of the Experimental Literature

	S	H
S	$45,45$	$0,35$
H	$35,0$	$40,40$

$\delta_{2R} = 50$

(a) Game $2R$

	S	H
S	$45,45$	$0,40$
H	$40,0$	$20,20$

$\delta_R = 25$

(b) Game R

	S	H
S	$45,45$	$0,42$
H	$42,0$	$12,12$

$\delta_{0.6R} = 15$

(c) Game $0.6R$

Figure A.1: Games used in Battalio, Samuelson, and Van Huyck (2001)

	S	H
S	$100,100$	$20,60$
H	$60,20$	$60,60$

$P = \frac{2}{5}$, $R = 0$

(a) Game 1

	S	H
S	$100,100$	$20,80$
H	$80,20$	$80,80$

$P = \frac{1}{5}$, $R = ln(3)$

(b) Game 2

	S	H
S	$100,100$	$60,80$
H	$80,60$	$80,80$

$P = \frac{1}{5}$, $R = 0$

(c) Game 3

	S	H
S	$100,100$	$0,80$
H	$80,0$	$60,60$

$P = \frac{2}{5}$, $R = ln(3)$

(d) Game 4

Figure A.2: Games used in Schmidt, Shupp, Walker, and Ostrom (2003)

	S	H
S	$45,45$	$0,42$
H	$42,0$	$12,12$

$RR_{Game1} = \frac{2}{3}$

(a) Game 1

	S	H
S	$40,40$	$20,37$
H	$37,20$	$32,32$

$RR_{Game2} = \frac{1}{4}$

(b) Game 2

	S	H
S	$44,44$	$4,38$
H	$38,4$	$28,28$

$RR_{Game3} = \frac{1}{4}$

(c) Game 3

Figure A.3: Games used in Dubois, Willinger, and Van Nguyen (2012)

Ehrenwörtliche Erklärung

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Frankfurt am Main, den   Unterschrift: