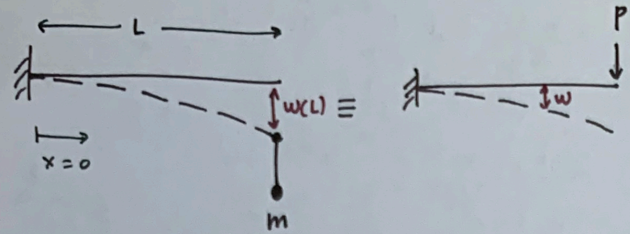
PART I



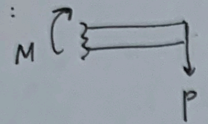
Load @ $x = L$.

w: deflection

, $P = mg$

beam eq., valid for small slopes.

FBD:



check: $M @ x=L : 0$

$M @ x=0 : PL$

$$M = P(L-x) \qquad (1)$$

$$EI\, w''(x) = M \qquad (2)$$

(1,2):

$$\Rightarrow EI\, w''(x) = PL - Px$$

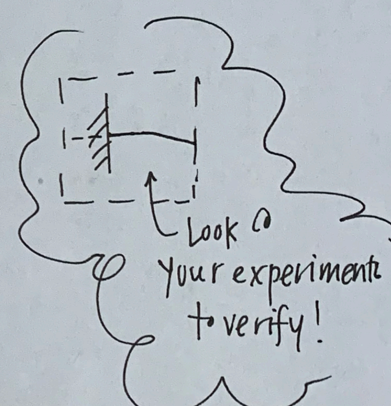
(integrating) $$\Rightarrow EI\, w'(x) = PLx - \frac{Px^2}{2} + c_1 \qquad (3)$$

(integrating) $$\Rightarrow EI\, w(x) = \frac{PLx^2}{2} - \frac{Px^3}{6} + c_1 x + c_2 \qquad (4)$$

Need 2 BCs (2nd order ODE):

No deflection allowed @ clamp: $w(0) = 0$ (5)

No slope " " ": $w'(0) = 0$ (6)



(6) in (3): $c_1 = 0$ (7)

(5,7) in (4): $c_2 = 0$ (8)

(7,8) in (4):

$$\boxed{w(x) = -\frac{Px^2}{6EI}(3L - x)} \qquad (9)$$

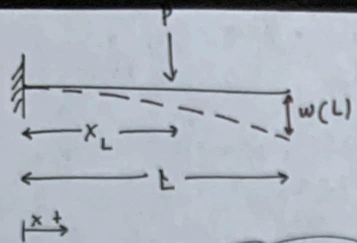
The deflection as a function of x for a force P applied at L on a cantilever clamped @ $x=0$.

Deflection at $x = L$?

(9) $\Rightarrow$ $$w(x=L) = -\frac{PL^2}{6EI}(3L - L) = -\frac{PL^3}{3EI}$$

$$\Rightarrow \boxed{w(L) = -\frac{PL^3}{3EI}} \qquad (10)$$

PART 2



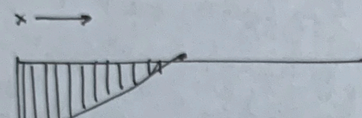
Load at $x = x_L$.

FBD → shear & moment diagram: to get expressions for each along x.

	$x \le x_L$		$x \ge x_L$	
shear: V	$V = P$	(1)	$V = 0$	(2)
moment: M	$M = -P(x_L - x)$	(3)	$M = 0$	(4)

shear: V

moment: M



Split analysis to 2 sections (piece-wise):

$x \le x_L$

$$EI\,w''(x) = M$$

$$\Rightarrow EI\,w''(x) \overset{(3)}{=} -P(x_L - x)$$

$$\Rightarrow EI\,w''(x) = -Px_L + Px$$

$$\Rightarrow EI\,w'(x) = -Px_L x + \frac{Px^2}{2} + c_1 \quad (5)$$

$$\Rightarrow EI\,w(x) = -\frac{Px_L x^2}{2} + \frac{Px^3}{6} + c_1 x + c_2 \quad (6)$$

BCs: $w(0) = 0$ (no deflection @ clamp) (7)

$w'(0) = 0$ (no slope @ clamp) (8)

(7) in (6):

$0 = c_2$

(8) in (5):

$0 = c_1$.

$\overset{(5,6)}{\Rightarrow}$ $w'(x) = -\frac{Px}{2EI}(2x_L - x)$ (9)

$w(x) = -\frac{Px^2}{6EI}(3x_L - x)$ (10)

$x \ge x_L$

$$EI\,w''(x) = M \overset{(4)}{=} 0$$

$$\Rightarrow EI\,w'(x) = c_1 \quad (11)$$

$$\Rightarrow EI\,w(x) = c_1 x + c_2 \quad (12)$$

BCs: continuous beam ⇒ deflection & slope need to match from L & R: use (9) & (10).

(9): $w'(x_L) = -\frac{Px_L^2}{2EI}$ (13)

(10): $w(x_L) = -\frac{Px_L^3}{3EI}$ (14)

(13) in (11):

$$-\frac{\cancel{EI}\,Px_L^2}{2\cancel{EI}} = c_1 \Rightarrow c_1 = -\frac{Px_L^2}{2} \quad (15)$$

(14) in (12):

$$-\frac{Px_L^3}{3} = c_1 x_L + c_2$$

$$\overset{(15)}{\Rightarrow} -\frac{Px_L^3}{3} = -\frac{Px_L^3}{2} + c_2$$

$$\Rightarrow c_2 = \frac{Px_L^3}{6} \quad (16)$$

(11, 12) :

(15,16) in ~~(13,14)~~

$$EIw'(x) = -\frac{Px_L^2}{2}$$

$$\Rightarrow w'(x) = -\frac{Px_L^2}{2EI} \quad (17)$$

$$\Rightarrow EI\,w(x) = -\frac{Px_L^2}{2}x + \frac{Px_L^3}{6}$$

$$\Rightarrow w(x) = \frac{-Px_L^2}{6EI}\left(3x - x_L\right) \quad (18)$$

Summarizing :

$$w(x) = \begin{cases} -\dfrac{Px^2}{6EI}(3x_L - x), & 0 \le x \le x_L \\[2ex] \dfrac{-Px_L^2}{6EI}(3x - x_L), & x_L \le x \le L \end{cases} \quad (19)$$

Max displacement @ $x = L \Longrightarrow$

$$\boxed{w(x=L) = -\frac{Px_L^2}{6EI}(3L - x_L)}$$

Note $\dfrac{dM}{dx} = V(x)$ where V is shear, M is moment

$\dfrac{dV}{dx} = \dfrac{d^2M}{dx^2} = q(x)$ where q is distributed load.

Boundary conditions on w, y-deflection :

		Free	Clamped	Simply-supported
$w = 0$:	no deflection	✗	✓	✓
$w' = 0$:	no slope	✗	✓	✗
$w'' = 0$:	no bending moment (free to rotate)	✓	✗	✓
$w''' = 0$:	no shearing force	✓	✗	✗