

MULTI-LAYER

BELIEF MODEL

COMPIÈGNE - NOVEMBER 2022

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supervised by

RONALD DE HAAN

in collaboration with

AYBÜKE ÖZGÜN

THE PROBLEM

Input:

$$\mathcal{E} = \Big\{ \text{\{a,b\}, \{b,c,d\}, \{d,e\}} \Big\}$$







NO

20

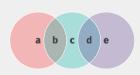


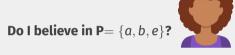
MOTIVATION (1/3)

Input:

$$\mathcal{E} = \left\{ \begin{smallmatrix} \{a,b\} \\ \text{o.4} \end{smallmatrix}, \begin{smallmatrix} \{b,c,d\} \\ \text{o.7} \end{smallmatrix}, \begin{smallmatrix} \{d,e\} \\ \text{o.5} \end{smallmatrix} \right\}$$









YES

$$Bel(P) \ge O$$
 $Bel(P) = O$

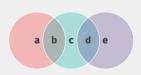
NO

MOTIVATION (1/3)

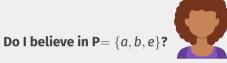
Input:

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YES NO

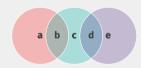
 $Bel(P) \geq 0$ Bel(P) = 0

X/? Conflictive Evidence

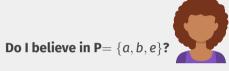
MOTIVATION (2/3)

Input:

$$\mathcal{E} = \Big\{ \{\mathsf{a,b}\}, \{\mathsf{b,c,d}\}, \{\mathsf{d,e}\} \Big\}$$



Topology of Evidence

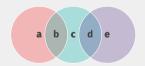


YES

NO

Input:

$$\mathcal{E} = \Big\{ \{\mathsf{a,b}\}, \{\mathsf{b,c,d}\}, \{\mathsf{d,e}\} \Big\}$$



Topology of Evidence

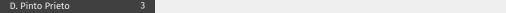


YES

NO

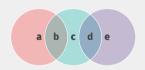
V Uncertain Evidence

✓ Conflictive Evidence

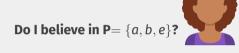


Input:

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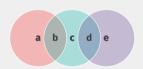


YES NO

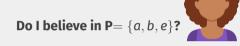


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Topology of Evidence



YES

NO

/ (



D. Pinto Prieto

Uncertain Evidence

MULTI-LAYER BELIEF MODEL (1/4)

Input:

 \mathcal{E}^{Q}

MULTI-LAYER BELIEF MODEL (1/4)

Input:

$$\mathcal{E}^{Q}$$

Domain: $2^S \times [0,1]$

Example: $\{a, b, c, d, e\}$

MULTI-LAYER BELIEF MODEL (1/4)

Input:

$$\mathcal{E}^{Q}$$

 \mathcal{E}

Domain:

$$2^{\mathsf{S}}\times [0,1]$$

Example: $\{a, b, c, d, e\}$



$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \Big\{ \{a, b\}, \{b, c, d\}, \{d, e\} \Big\}$$

$$\mathcal{J} = \left\{ \{b, d\}, \{b, c, d\} \right\}$$



: High level of skepticism

INTRODUCTION TO THE TOPOLOGY OF EVIDENCE

(Baltag, A., Bezhanishvili, N., Özgün, A., Smets, S. (2016). Justified Belief and the Topology of Evidence.)

$$S = \{a, b, c, d, e\}$$

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$$\mathcal{E} = \left\{\{a, b\}, \{b, c, d\}, \{d, e\}\right\}$$

$$au_{\mathcal{E}} = egin{array}{l} \emptyset, \ \{a,b\}, \{b,c,d\}, \{d,e\}, \ \{b\}, \{d\} \ \{b,d\}, ..., \ \mathcal{S} \end{array}$$

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 $T = \{b, d\}$ is **dense** in $\tau_{\mathcal{E}}$

INTRODUCTION TO THE TOPOLOGY OF EVIDENCE

(Baltag, A., Bezhanishvili, N., Özgün, A., Smets, S. (2016). Justified Belief and the Topology of Evidence.)

$$S = \{a,b,c,d,e\}$$

$$\mathcal{E} = \Big\{\{a,b\},\{b,c,d\},\{d,e\}\Big\}$$

$$au_{\mathcal{E}} = \begin{cases} \emptyset, \\ \{a,b\}, \{b,c,d\}, \{d,e\}, \\ \{b\}, \{d\}, \\ \{b,d\}, ..., \end{cases}$$

$$\mathcal{A} = \begin{cases} \{a,b\}, \{b,c,d\}, \{d,e\}, \\ \{b\}, \{d\}, \\ \{b,d\}, ..., \\ S \end{cases}$$

$$T = \{b, d\}$$
 is **dense** in $au_{\mathcal{E}}$

$$\mathcal{J} = \begin{cases} \{b, c, d\}, \\ \{b, d\}, ..., \end{cases}$$

$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{\{a, b\}, \{b, c, d\}, \{d, e\}\right\}$$

$$au_{\mathcal{E}} = egin{array}{l} \emptyset, \ \{a,b\}, \{b,c,d\}, \{d,e\}, \ \{b\}, \{d\} \ \{b,d\}, ..., \ \mathbf{S} \end{array}$$

$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{ \{a, b\}, \{b, c, d\}, \{d, e\} \right\}$$

$$au_{\mathcal{E}} = egin{array}{l} \emptyset, \ \{a,b\}, \{b,c,d\}, \{d,e\}, \ \{b\}, \{d\} \ \{b,d\}, ..., \ \mathbf{S} \end{array}$$



$$\mathcal{J}^{ extsf{DS}} = egin{array}{l} \{a,b\}, \{b,c,d\}, \{d,e\}, \ \{b\}, \{d\} \ \{b,d\} \ S \ \end{array}$$

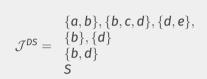
$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{ \{a, b\}, \{b, c, d\}, \{d, e\} \right\}$$

$$au_{\mathcal{E}} = egin{array}{l} \emptyset, \ \{a,b\}, \{b,c,d\}, \{d,e\}, \ \{b\}, \{d\} \ \{b,d\}, ..., \ {\sf S} \end{array}$$



Low skepticism





High skepticism

$$\mathcal{J}^{\mathsf{TE}} = egin{array}{l} \{b,c,d\}, \ \{b,d\} \ \mathsf{S} \end{array}$$

MULTI-LAYER BELIEF MODEL (2/4)

Input:

$$\mathcal{E}^{Q}$$

Domain: $2^S \times [0,1]$

Example: $\{a, b, c, d, e\}$

MULTI-LAYER BELIEF MODEL (2/4)

Input:

 \mathcal{E}^{Q}

Frame of Justification:

 $\mathcal{J}^{\#}$

Domain:

$$2^{\text{S}}\times [0,1]$$

2^S

Example: $\{a, b, c, d, e\}$

0.4 0.7 0.5

$$\Big\{\{b,d\},\{b,c,d\},S\Big\}$$

$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{ \begin{cases} \{a, b\}, \{b, c, d\}, \{d, e\} \\ 0.4, 0.7, 0.5 \end{cases} \right\}$$

$$\delta: \mathbf{2}^{\mathcal{E}} \rightarrow [\mathbf{0}, \mathbf{1}]$$

$$\delta(\mathbf{E}) = \prod_{\mathsf{E}_i \in \mathbf{E}} p_i \prod_{\mathsf{E}_j \notin \mathbf{E}} 1 - p_j$$

QUANTITATIVE LAYER (2/2)

$$\left\{\emptyset\right\}\mapsto 0.09$$

$$\Big\{\{a,b\},\{b,c,d\}\Big\}\mapsto 0.14$$

$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{ \begin{cases} \{a, b\}, \{b, c, d\}, \{d, e\} \\ 0.4, 0.7, 0.5 \end{cases} \right\}$$

$$\{a,b\}$$
 \mapsto 0.06

$$\Big\{\{b,c,d\},\{d,e\}\Big\}\mapsto exttt{0.21}$$

$$\{\{b,c,d\}\}\mapsto 0.21$$

$$\Big\{\{d,e\},\{a,b\}\Big\}\mapsto 0.06$$

$$\delta(\mathbf{E}) = \prod_{E_i \in \mathbf{E}} p_i \prod_{E_i \notin \mathbf{E}} 1 - p_j$$

$$\{d,e\}$$
 \mapsto 0.09

$$\{\{a,b\},\{b,c,d\},\{d,e\}\}\mapsto 0.14$$

MULTI-LAYER BELIEF MODEL (3/4)

Input:

Frame of Justification:

S \mathcal{E}^{Q}

 $\mathcal{J}^{\#}$

Domain:

 $2^{\mathsf{S}}\times [\mathsf{O},\mathsf{1}]$

2^S

Example: $\{a, b, c, d, e\}$

.5

 $\Big\{\{b,d\},\{b,c,d\},S\Big\}$

MULTI-LAYER BELIEF MODEL (3/4)

Input:

S \mathcal{E}^{Q}

Frame of **Justification:**

 $\mathcal{J}^{\#}$

Mass **Function:**

$$\delta(\mathbf{E}) = \prod_{E_i \in \mathbf{E}} p_i \prod_{E_j \notin \mathbf{E}} 1 - p_j$$

Domain:

$$2^{\mathsf{S}}\times [0,1]$$

2^S

$$2^{\mathcal{E}} \to [0,1]$$

Example: $\{a, b, c, d, e\}$



0.4 0.7 0.5

 $\Big\{\{b,d\},\{b,c,d\},S\Big\}$

Evidence allocation functions

$$f: \mathtt{2}^{\mathcal{E}}
ightarrow oldsymbol{ au}_{\mathcal{E}}$$

$$\{\emptyset\}\mapsto S$$
 $i(\mathbf{E})=\bigcap \mathbf{E}$

$$i(\mathbf{E}) = \bigcap \mathbf{I}$$

$$u(\mathbf{E}) = \bigcup \mathbf{E}$$

Evidence allocation functions

$$f: \mathtt{2}^{\mathcal{E}}
ightarrow oldsymbol{ au}_{\mathcal{E}}$$

$$\left\{\emptyset\right\}\mapsto\mathsf{S}\qquad \qquad i(\mathbf{E})=\bigcap\mathbf{E}$$

$$u(\mathbf{E})=\bigcup\mathbf{E}$$

$$egin{aligned} \emph{\emph{i}}(\emph{\emph{E}}) &= \emptyset \ \emph{\emph{E}} &= \{\{d,e\},\{a,b\}\} \ \emph{\emph{u}}(\emph{\emph{\emph{E}}}) &= \{a,b,d,e\} \end{aligned}$$

BRIDGING LAYER (2/2)

$$\delta_{ au}(extbf{T}) = \sum_{ extbf{E}: f(extbf{E}) = extbf{T}} \delta(extbf{E})$$

BRIDGING LAYER (2/2)

$$\delta_{\tau}(T) = \sum_{\mathbf{E}: f(\mathbf{E}) = T} \delta(\mathbf{E})$$
 $\delta_{\#}(A) = \frac{\delta_{\tau}(A)}{\sum\limits_{T \in \mathcal{J}^{\#}} \delta_{\tau}(T)}$

BRIDGING LAYER (2/2)

$$\delta_{\#}(A) = rac{\delta_{ au}(A)}{\sum\limits_{T \in \mathcal{J}^{\#}} \delta_{ au}(T)}$$

$$Bel^{\#}(P) = \sum_{J \subseteq P} \delta_{\#}(J)$$

MULTI-LAYER BELIEF MODEL (4/4)

Input:

 \mathcal{E}^{Q} S

Frame of **Justification:**

 $\mathcal{J}^{\#}$

Evidence Allocation **Function:**

Mass **Function:**

 $f: 2^{\mathcal{E}}
ightarrow au_{\mathcal{E}} \qquad \qquad \delta: 2^{\mathcal{E}}
ightarrow [\mathtt{0},\mathtt{1}]$

MULTI-LAYER BELIEF MODEL (4/4)

Input:

S \mathcal{E}^{Q} Frame of **Justification:**

.7#

Evidence Allocation **Function:**

Mass **Function:**

 $f: 2^{\mathcal{E}} \to \tau_{\mathcal{E}}$ $\delta: 2^{\mathcal{E}} \to [0,1]$

$$\delta_{ au}(T) = \sum_{m{E}: f(m{E}) = T} \delta(m{E})$$

$$\delta_{\#}(A) = \frac{\delta_{\tau}(A)}{\sum\limits_{T \in \mathcal{J}^{\#}} \delta_{\tau}(T)}$$

$$Bel^{\#}(P) = \sum_{J \subseteq P} \delta_{\#}(J)$$

MULTI-LAYER BELIEF MODEL (4/4)

Input:

S

 \mathcal{E}^{Q}

Frame of **Justification:**

.7#

Evidence Allocation **Function:**

Mass **Function:**

 $f: 2^{\mathcal{E}} \to \tau_{\mathcal{E}}$ $\delta: 2^{\mathcal{E}} \to [0,1]$

$$\delta_{ au}(extbf{T}) = \sum_{ extbf{\textit{E}}: f(extbf{\textit{E}}) = extbf{T}} \delta(extbf{\textit{E}})$$

$$\delta_{\#}(A) = rac{\delta_{ au}(A)}{\sum\limits_{T \in \mathcal{J}^{\#}} \delta_{ au}(T)}$$

Bel
$$^{\#}(P) = \sum_{J \subseteq P} \delta_{\#}(J)$$

EXAMPLE

$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{ \begin{cases} \{a, b\}, & \{b, c, d\}, & \{d, e\} \\ 0.4, & 0.7 \end{cases}, & \begin{cases} d, e \} \\ 0.5 \end{cases} \right\}$$

$$au_{\mathcal{E}} = egin{array}{l} \emptyset, \ \{a,b\}, \{b,c,d\}, \{d,e\}, \ \{b\}, \{d\}, \ \{b,d\}, ..., \ & \mathsf{S} \end{array}$$



$$\mathcal{J}^{DS} = \begin{cases} \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\} \end{cases}$$



$$\mathcal{J}^{TE} = \begin{cases} \{b, c, d\}, \\ \{b, d\} \\ S \end{cases}$$

EXAMPLE

$$S = \{a, b, c, d, e\}$$

$$\mathcal{E} = \left\{ \begin{cases} \{a, b\}, \{b, c, d\}, \{d, e\} \\ 0.4 \end{cases} \right\}$$
a b c d e

$$au_{\mathcal{E}} = \begin{cases} \emptyset, \\ \{a,b\}, \{b,c,d\}, \{d,e\}, \\ \{b\}, \{d\}, \\ \{b,d\}, ..., \end{cases}$$

Do I believe in P= $\{a, b, e\}$?



$$\mathcal{J}^{DS} = \begin{cases} \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\} \end{cases}$$



$$\mathcal{J}^{\mathsf{TE}} = \begin{cases} \{b, c, d\}, \\ \{b, d\} \end{cases}$$



: Yes, with degree 0.30.



: No, I don't.

NEXT STEPS

AN EPISTEMIC LOGIC FOR THE MODEL

DEGREE OF BELIEF

Uncertain EvidenceConflictive Evidence

- **EPISTEMIC LOGIC**
- X Uncertain Evidence
- ✓ Conflictive Evidence



DEGREE OF BELIEF

- Uncertain Evidence
- ✓ Conflictive Evidence

AN EPISTEMIC LOGIC FOR THE MODEL

DEGREE OF BELIEF

Uncertain Evidence
X/? Conflictive Evidence

EPISTEMIC LOGIC

Uncertain EvidenceConflictive Evidence



DEGREE OF BELIEF

- ✓ Uncertain Evidence
- ✓ Conflictive Evidence

EPISTEMIC LOGIC

- / Uncertain Evidence
- ✓ Conflictive Evidence

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ASSESSING THE MODEL

■ Theoretical results

■ Computational results

■ Experimental results

MY QUESTIONS

1. Are there any special cases to check with this model?

2. Do you have other theoretical results to prove in mind?

3. How to test the model experimentally?

d.pintoprieto 20 / 20