Institute for Logic, Language and Computation

University of Amsterdam

MULTI-LAYER BELIEF MODEL

Compiègne - November 2022

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supervised by

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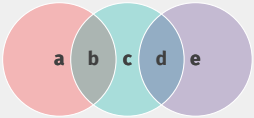
in collaboration with

Aybüke Özgün

Input:

$\mathcal{E} = \Big\{$ {a,b}, {b,c,d}, {d,e} $\Big\}$



$\xrightarrow{\odot}$

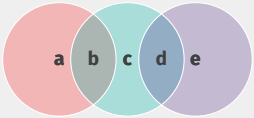
Do I believe in P$= \{a, b, e\}$**?**

YES NO

Input:

$$\mathcal{E} = \left\{ \underset{0.4}{\{a,b\}}, \underset{0.7}{\{b,c,d\}}, \underset{0.5}{\{d,e\}} \right\}$$

DST



Do I believe in P$= \{a, b, e\}$**?**

YES	NO
$Bel(P) \geq 0$	$Bel(P) = 0$

Input:

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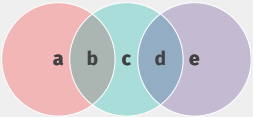


Do I believe in P$= \{a, b, e\}$**?**

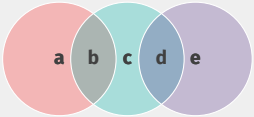
YES	NO
$Bel(P) \geq 0$	$Bel(P) = 0$

✓ **Uncertain Evidence**

✗/? **Conflictive Evidence**

Input:

$\mathcal{E} = \Big\{\{a,b\}, \{b,c,d\}, \{d,e\}\Big\}$



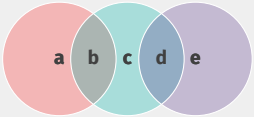
Topology of Evidence

Do I believe in P$= \{a, b, e\}$**?**

YES NO

Input:

$\mathcal{E} = \Big\{\{a,b\}, \{b,c,d\}, \{d,e\}\Big\}$



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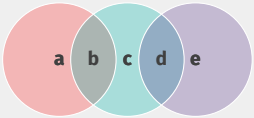

DST
Topology
of
Evidence

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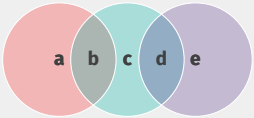

DST
Topology
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Do I believe in P$= \{a, b, e\}$?

YES NO

✓ **Uncertain Evidence** ✓ **Conflictive Evidence**

Input:

S $\mathcal{E}^Q$

Multi-Layer Belief Model (1/4)

Input:

S $\mathcal{E}^Q$

Domain: $2^S \times [0, 1]$

Example: $\{a, b, c, d, e\}$

Input:

S $\mathcal{E}^Q$ $\mathcal{E}$

Domain: $2^S \times [0, 1]$ 2^S

Example: $\{a, b, c, d, e\}$

0.4 0.7 0.5

$S = \{a, b, c, d, e\}$

$\mathcal{E} = \Big\{\{a, b\}, \{b, c, d\}, \{d, e\}\Big\}$

$\longrightarrow$

$\mathcal{J} = \Big\{\{b, d\}, \{b, c, d\}\Big\}$


: High level of skepticism

Introduction to the Topology of Evidence

(Baltag, A., Bezhanishvili, N., Özgün, A., Smets, S. (2016). Justified Belief and the Topology of Evidence.)

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$$\tau_{\mathcal{E}} = \begin{array}{l} \emptyset, \\ \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\}, ..., \\ S \end{array}$$

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$S = \{a, b, c, d, e\}$

$\mathcal{E} = \Big\{\{a, b\}, \{b, c, d\}, \{d, e\}\Big\}$

$T = \{b, d\}$ is **dense** in $\tau_{\mathcal{E}}$

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$$\mathcal{A} = \begin{array}{l} \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\}, ..., \\ S \end{array}$$

$$\mathcal{J} = \begin{array}{l} \{b, c, d\}, \\ \{b, d\}, ..., \\ S \end{array}$$

$S = \{a, b, c, d, e\}$

$\mathcal{E} = \Big\{\{a, b\}, \{b, c, d\}, \{d, e\}\Big\}$

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Low skepticism

$$\mathcal{J}^{DS} = \begin{array}{l} \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\} \\ S \end{array}$$

$S = \{a, b, c, d, e\}$

$\mathcal{E} = \Big\{\{a, b\}, \{b, c, d\}, \{d, e\}\Big\}$

$$\tau_{\mathcal{E}} = \begin{array}{l} \emptyset, \\ \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\}, ..., \\ S \end{array}$$

Low skepticism

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High skepticism

$$\mathcal{J}^{TE} = \begin{array}{l} \{b, c, d\}, \\ \{b, d\} \\ S \end{array}$$

Input:

S $\mathcal{E}^Q$

Domain: $2^S \times [0, 1]$

Example: $\{a, b, c, d, e\}$

Input:

S $\mathcal{E}^Q$

Frame of Justification:

$\mathcal{J}^\#$

Domain: $2^S \times [0,1]$ 2^S

Example: $\{a,b,c,d,e\}$

$\Big\{\{b,d\},\{b,c,d\},S\Big\}$

$S = \{a, b, c, d, e\}$

$\mathcal{E} = \left\{ \underset{0.4}{\{a,b\}}, \underset{0.7}{\{b,c,d\}}, \underset{0.5}{\{d,e\}} \right\}$

$$\delta(\boldsymbol{E}) = \prod_{E_i \in \boldsymbol{E}} p_i \prod_{E_j \notin \boldsymbol{E}} 1 - p_j$$

$$\delta : 2^{\mathcal{E}} \rightarrow [0, 1]$$

$S = \{a, b, c, d, e\}$

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$$\delta(\boldsymbol{E}) = \prod_{E_i \in \boldsymbol{E}} p_i \prod_{E_j \notin \boldsymbol{E}} 1 - p_j$$

$\Big\{\{\emptyset\}\Big\} \mapsto 0.09$

$\Big\{\{a, b\}\Big\} \mapsto 0.06$

$\Big\{\{b, c, d\}\Big\} \mapsto 0.21$

$\Big\{\{d, e\}\Big\} \mapsto 0.09$

$\Big\{\{a, b\}, \{b, c, d\}\Big\} \mapsto 0.14$

$\Big\{\{b, c, d\}, \{d, e\}\Big\} \mapsto 0.21$

$\Big\{\{d, e\}, \{a, b\}\Big\} \mapsto 0.06$

$\Big\{\{a, b\}, \{b, c, d\}, \{d, e\}\Big\} \mapsto 0.14$

Input:

S $\mathcal{E}^Q$

Frame of Justification:

$\mathcal{J}^\#$

Domain: $2^S \times [0,1]$ 2^S

Example: $\{a,b,c,d,e\}$

$\Big\{\{b,d\},\{b,c,d\},S\Big\}$

Input:

S $\mathcal{E}^Q$

Frame of Justification:

$\mathcal{J}^{\#}$

Mass Function:

$$\delta(\boldsymbol{E}) = \prod_{E_i \in \boldsymbol{E}} p_i \prod_{E_j \notin \boldsymbol{E}} 1 - p_j$$

Domain: $2^S \times [0, 1]$ 2^S $2^{\mathcal{E}} \to [0, 1]$

Example: $\{a, b, c, d, e\}$

$\Big\{\{b, d\}, \{b, c, d\}, S\Big\}$

Evidence allocation functions

$$f : 2^{\mathcal{E}} \to \tau_{\mathcal{E}}$$

$$\left\{\emptyset\right\} \mapsto S$$

$$i(\boldsymbol{E}) = \bigcap \boldsymbol{E}$$

$$u(\boldsymbol{E}) = \bigcup \boldsymbol{E}$$

Evidence allocation functions

$$f : 2^{\mathcal{E}} \to \tau_{\mathcal{E}} \qquad \left\{\emptyset\right\} \mapsto S \qquad i(\boldsymbol{E}) = \bigcap \boldsymbol{E} \qquad u(\boldsymbol{E}) = \bigcup \boldsymbol{E}$$

$$\boldsymbol{E} = \{\{d, e\}, \{a, b\}\} \qquad i(\boldsymbol{E}) = \emptyset \qquad u(\boldsymbol{E}) = \{a, b, d, e\}$$

$$\delta_\tau(\boldsymbol{T}) = \sum_{\boldsymbol{E}:f(\boldsymbol{E})=\boldsymbol{T}} \delta(\boldsymbol{E})$$

$$\delta_\tau(T) = \sum_{\boldsymbol{E}:f(\boldsymbol{E})=T} \delta(\boldsymbol{E}) \qquad \delta_\#(A) = \frac{\delta_\tau(A)}{\sum_{T \in \mathcal{J}^\#} \delta_\tau(T)}$$

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Input:

$S \qquad \mathcal{E}^Q$

Frame of Justification:

$\mathcal{J}^{\#}$

Evidence Allocation Function:

$f : 2^{\mathcal{E}} \to \tau_{\mathcal{E}}$

Mass Function:

$\delta : 2^{\mathcal{E}} \to [0, 1]$

Input: $S \quad \mathcal{E}^Q$

Frame of Justification: $\mathcal{J}^{\#}$

Evidence Allocation Function: $f : 2^{\mathcal{E}} \to \tau_{\mathcal{E}}$

Mass Function: $\delta : 2^{\mathcal{E}} \to [0, 1]$

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$$\tau_{\mathcal{E}} = \begin{array}{l} \emptyset, \\ \{a, b\}, \{b, c, d\}, \{d, e\}, \\ \{b\}, \{d\} \\ \{b, d\}, \ldots, \\ S \end{array}$$

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Do I believe in P$= \{a, b, e\}$?

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$$\mathcal{J}^{TE} = \begin{array}{l} \{b,c,d\}, \\ \{b,d\} \\ S \end{array}$$

 : Yes, with degree 0.30.

 : No, I don't.

NEXT STEPS

An Epistemic Logic for the model

Degree of Belief

✓ **Uncertain Evidence**
✗/? **Conflictive Evidence**

Epistemic Logic

✗ **Uncertain Evidence**
✓ **Conflictive Evidence**

Degree of Belief

✓ **Uncertain Evidence**
✓ **Conflictive Evidence**

Degree of Belief

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✗/? **Conflictive Evidence**

Epistemic Logic

✗ **Uncertain Evidence**
✓ **Conflictive Evidence**

Degree of Belief

✓ **Uncertain Evidence**
✓ **Conflictive Evidence**

Epistemic Logic

✓ **Uncertain Evidence**
✓ **Conflictive Evidence**

- Theoretical results
- Computational results
- Experimental results

1. Are there any special cases to check with this model?

2. Do you have other theoretical results to prove in mind?

3. How to test the model experimentally?