Data Structures and Algorithms

Greatest Common Divisor (GCD) & Prime Factorization

Lab Sheet 07

<u>Answer all questions.</u>

1.Briefly Explain Greatest Common Devisor GCD.

The greatest positive integer that divides all the given integers without leaving a residual is known as the Greatest Common Divisor (GCD) of two or more integers. In other terms, it is the biggest number that can divide the supplied integers in two equal parts.

When two integers "a" and "b" are used, the GCD is typically denoted as " GCD (a, b)". It is a fundamental idea in number theory and has many uses in math and computer science.

For example:

Let's find the GCD of 18 and 24 using the Euclidean algorithm:

GCD (18, 24) = GCD (18, 24 - 18) = GCD (18, 6)

GCD (18, 6) = GCD (18 - 6, 6) = GCD (12, 6)

GCD (12, 6) = GCD (12 - 6, 6) = GCD (6, 6)

GCD (6, 6) = 6

2. Explain the steps of the Euclidean Algorithm.

The Euclidean Algorithm is a method for finding the greatest common divisor (GCD) of two integers. The GCD of two numbers is the largest positive integer that divides both of them without leaving a remainder. It is named after the ancient Greek mathematician Euclid, who first described this algorithm in his work "Elements." The steps of the Euclidean Algorithm are as follows:

Step 1: Start with two positive integers, let's call them "a" and "b," where a > b. If a < b, swap the values of a and b.

Step 2: Divide a by b, and let the remainder be "r." Perform the division: a = q * b + r, where q is the quotient and r is the remainder.

Step 3: If the remainder r is equal to zero (r = 0), then stop. The current value of "b" is the greatest common divisor (GCD) of the original numbers a and b.

Step 4: If the remainder r is not zero (r ≠ 0), update the values as follows: Set a = b, and set b = r.

Step 5: Repeat Steps 2 to 4 with the new values of a and b.

Step 6: Continue repeating Steps 2 to 4 until you reach a point where the remainder r becomes zero.

Step 7: The GCD of the original numbers a and b is the value of b at the point where the remainder becomes zero.

Mathematically, we can represent the Euclidean Algorithm using recursive notation as follows:

GCD (a, b) = {

b if b divides a evenly (i.e., a % b = 0),

GCD (b, a % b) otherwise.

}

3. Write a function using pseudo or source code to find out the GCD using recursive.

```
def GCD recursive (a, b):
    if b == 0:
        return a
    else:
        return GCD recursive (b, a % b)
```

Explain: -

- The function GCD recursive takes two integers, "a" and "b," as inputs. In the base case, if the value of "b" is zero, it means we have found the GCD (it is equal to "a"). So, we return "a" as the results.
- If the value of "b" is not zero, we recursively call the function with the argument's "b" and "a % b." The expression "a % b" calculates the remainder when "a" is divided by "b."
- The function will keep calling itself with updated values of "a" and "b" until the base case is reached (i.e., when "b" becomes zero).
- Once the base case is reached, the function starts returning the GCD step by step through the recursive calls.

4. Try to use the iteration to get the same results.

```
def GCD iterative (a, b):
    while b != 0:
        a, b = b, a % b
    return a
```

- The function GCD iterative takes two integers, "a" and "b," as inputs.

- The while loop continues until "b" becomes zero.
- Inside the loop, we update the values of "a" and "b" as follows:
    - Set "a" to the current value of "b."
    - Set "b" to the remainder when "a" is divided by "b" (i.e., "a % b").
- The loop will continue to execute until "b" becomes zero. At that point, the value of "a" will be the GCD.
- The function returns the GCD.

.

5. What is defined by prime factorization.

The expression of a positive integer as the product of its prime factors is known as prime factorization, and it is a fundamental idea in number theory. In other terms, it multiplies a number's prime numbers to represent it.
Every positive number higher than one can be expressed in a single way using the sum of its prime components. A prime number that divides the provided number without leaving a residual is said to be a prime factor. Prime numbers are those bigger than 1 that can only be divided positively by themselves and by the number 1.

6. Graphically represent how to identify the prime factorization

Step 1:

Start with the number you want to factorize. In this case, let's use the number 24.

Step 2:

Find the smallest prime factor of the number. In this case, the smallest prime factor of 24 is 2

Step 3:

Divide the number by the prime factor you found in step 2. In this case, we divide 24 by 2 to get 12.

Step 4:

Repeat steps 2 and 3 until the number you are dividing is no longer divisible by any prime factor other than 1. In this case, we divide 12 by 2 to get 6, and then divide 6 by 2 to get 3.

Step 5:

The prime factors of the number are the prime factors that you used to divide the number in steps 2-4. In this case, the prime factors of 24 are 2, 2, and 3.

Graphical representation:

```
  2
 / \
2   3
```

The number 2 is at the top of the diagram because it is the largest prime factor of 24. The numbers 2 and 3 are below 2 because they are the prime factors of 12, which is the result of dividing 24 by 2.