Machine Learning
Prof. Dr. Dominik Heider
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Heinrich-Heine-Universität Düsseldorf

Abgabe von
Taha El Amine Kassabi (takas100)
Emre Gökcek (emgoe104)
Lutfi Orabi (luora100)
Ibrahim Shinahov (ibshi100)

Sheet 2

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Task 3

Show: $Var(X) = E(X^2) - E(X)^2$

$$Var(X) = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2$$

$$E(X) = \frac{1}{n} = \overline{X}, \qquad x_i \in X$$

$$\Rightarrow E(X^2) = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \, , E(X^2) = \overline{X}^2$$

$$E(X^2) - E(X)^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{X}^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\overline{X}^2 + \overline{X}^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\overline{X}\frac{1}{n}\sum_{i=1}^{n} x_i + \overline{X}^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\sum_{i=1}^{n} 2x_i\overline{X} + \frac{1}{n}\sum_{i=1}^{n} \overline{X}^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2x_i\overline{X} + \overline{X}^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2$$