

# fairadapt: Causal Reasoning for Fair Data Pre-processing

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## Abstract

Machine learning algorithms are useful for various predictions tasks, but they can also learn how to discriminate, based on gender, race or other sensitive attributes. This realization gave rise to the field of fair machine learning, which aims to measure and mitigate such algorithmic bias. This manuscript describes the R-package **fairadapt**, which implements a causal inference pre-processing method. By making use of the causal graphical model and the observed data, the method can be used to address hypothetical questions of the form “What would my salary have been, had I been of a different gender/race?”. Such individual level counterfactual reasoning can help eliminate discrimination and help justify fair decisions. We also discuss appropriate relaxations which assume certain causal pathways from the sensitive attribute to the outcome are not discriminatory.

*Keywords:* algorithmic fairness, causal inference, machine learning.

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## 1. Introduction

Machine learning algorithms have become prevalent tools for decision-making in socially sensitive situations, such as determining credit-score ratings or predicting recidivism during parole. It has been recognized that algorithms are capable of learning societal biases, for example with respect to race (Larson, Mattu, Kirchner, and Angwin 2016) or gender (Lambrecht and Tucker 2019; Blau and Kahn 2003), and this realization seeded an important debate in the machine learning community about fairness of algorithms and their impact on decision-making.

In order to define and measure discrimination, existing intuitive notions have been statistically formalized, thereby providing fairness metrics. For example, *demographic parity* (Darlington 1971) requires the protected attribute  $A$  (gender/race/religion etc.) to be independent of a constructed classifier or regressor  $\hat{Y}$ , written as  $\hat{Y} \perp\!\!\!\perp A$ . Another notion, termed *equality of odds* (Hardt, Price, Srebro *et al.* 2016), requires equal false positive and false negative rates of classifier  $\hat{Y}$  between different groups (females and males for example), written as  $\hat{Y} \perp\!\!\!\perp A \mid Y$ . To this day, various different notions of fairness exist, which are sometimes incompatible (Corbett-Davies and Goel 2018), meaning not of all of them can be achieved for a predictor  $\hat{Y}$  simultaneously. There is still no consensus on which notion of fairness is the correct one.

The discussion on algorithmic fairness is, however, not restricted to the machine learning domain. There are many legal and philosophical aspects that have arisen. For example, the legal distinction between disparate impact and disparate treatment (McGinley 2011) is important for assessing fairness from a judicial point of view. This in turn emphasizes the

importance of the interpretation behind the decision-making process, which is often not the case with black-box machine learning algorithms. For this reason, research in fairness through a causal inference lens has gained attention.

A possible approach to fairness is the use of counterfactual reasoning (Galles and Pearl 1998), which allows for arguing what might have happened under different circumstances that never actually materialized, thereby providing a tool for understanding and quantifying discrimination. For example, one might ask how a change in sex would affect the probability of a specific candidate being accepted for a given job opening. This approach has motivated another notion of fairness, termed *counterfactual fairness* (Kusner, Loftus, Russell, and Silva 2017), which states that the decision made, should remain fixed, even if, hypothetically, some parameters such as race or gender were to be changed (this can be written succinctly as  $\hat{Y}(a) = \hat{Y}(a')$  in the potential outcomes notation). Causal inference can also be used for decomposition of the parity gap measure (Zhang and Bareinboim 2018),  $\mathbb{P}(\hat{Y} = 1 \mid A = a) - \mathbb{P}(\hat{Y} = 1 \mid A = a')$ , into the direct, indirect and spurious components (yielding further insights into the demographic parity as a criterion), as well as the introduction of so-called resolving variables Kilbertus, Carulla, Parascandolo, Hardt, Janzing, and Schölkopf (2017), in order to relax the possibly prohibitively strong notion of demographic parity.

The following sections describe an implementation of the fair data adaptation method outlined in Plecko and Meinshausen (2020), which combines the notions of counterfactual fairness and resolving variables, and explicitly computes counterfactual values for individuals. The implementation is available as R-package **fairadapt** from CRAN. Currently there are only few packages distributed via CRAN that relate to fair machine learning. These include **fairml** (Scutari 2021), which implements the non-convex method of Komiyama, Takeda, Honda, and Shimao (2018), as well as packages **fairness** (Kozodoi and V. Varga 2021) and **fairmodels** (Wiśniewski and Biecek 2021), which serve as diagnostic tools for measuring algorithmic bias and provide several pre- and post-processing methods for bias mitigation. The only causal method, however, is presented by **fairadapt**. Even though theory in fair machine learning is being expanded at an accelerating pace, good quality implementations of the developed methods are often not available.

The rest of the manuscript is organized as follows: In Section 2 we describe the methodology behind **fairadapt**, together with quickly reviewing some important concepts of causal inference. In Section 3 we discuss implementation details and provide some general user guidance, followed by Section 4, which illustrates the usage of **fairadapt** through a large, real-world dataset and a hypothetical fairness application. Finally, in Section 5 we elaborate on some extensions, such as Semi-Markovian models and resolving variables.

## 2. Methodology

First, the intuition behind **fairadapt** is described using an example, followed by a more rigorous mathematical formulation, using Markovian structural causal models (SCMs). Some relevant extensions, such as the Semi-Markovian case and the introduction of so called *resolving variables*, are discussed in Section 5.

### 2.1. University Admission Example

Consider the example of university admission based on previous educational achievement and

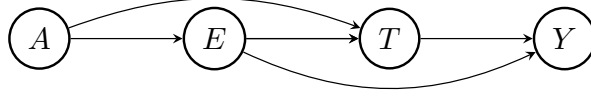


Figure 1: University admission based on previous educational achievement  $E$  combined with an admissions test score  $T$ . The protected attribute  $A$ , encoding gender, has an unwanted causal effect on  $E$ ,  $T$ , as well as  $Y$ , which represents the final score used for the admission decision.

an admissions test. Variable  $A$  is the protected attribute, describing candidate gender, with  $A = a$  corresponding to females and  $A = a'$  to males. Furthermore, let  $E$  be educational achievement (measured for example by grades achieved in school) and  $T$  the result of an admissions test for further education. Finally, let  $Y$  be the outcome of interest (final score) upon which admission to further education is decided. Edges in the graph in Figure 1 indicate how variables affect one another.

Attribute  $A$ , gender, has a causal effect on variables  $E$ ,  $T$ , as well as  $Y$ , and we wish to eliminate this effect. For each individual with observed values  $(a, e, t, y)$  we want to find a mapping

$$(a, e, t, y) \longrightarrow (a^{(fp)}, e^{(fp)}, t^{(fp)}, y^{(fp)}),$$

which represents the value the person would have obtained in an alternative world where everyone was female. Explicitly, to a male person with education value  $e$ , we assign the transformed value  $e^{(fp)}$  chosen such that

$$\mathbb{P}(E \geq e \mid A = a') = \mathbb{P}(E \geq e^{(fp)} \mid A = a).$$

The key idea is that the *relative educational achievement within the subgroup* remains constant if the protected attribute gender is modified. If, for example, a male has a higher educational achievement value than 70% of males in the dataset, we assume that he would also be better than 70% of females had he been female<sup>1</sup>. After computing transformed educational achievement values corresponding to the *female* world ( $E^{(fp)}$ ), the transformed test score values  $T^{(fp)}$  can be calculated in a similar fashion, but conditioned on educational achievement. That is, a male with values  $(E, T) = (e, t)$  is assigned a test score  $t^{(fp)}$  such that

$$\mathbb{P}(T \geq t \mid E = e, A = a') = \mathbb{P}(T \geq t^{(fp)} \mid E = e^{(fp)}, A = a),$$

where the value  $e^{(fp)}$  was obtained in the previous step. This step can be visualized as shown in Figure 2.

As a final step, the outcome variable  $Y$  remains to be adjusted. The adaptation is based on the same principle as above, using transformed values of both education and the test score. The resulting value  $y^{(fp)}$  of  $Y = y$  satisfies

$$\mathbb{P}(Y \geq y \mid E = e, T = t, A = a') = \mathbb{P}(Y \geq y^{(fp)} \mid E = e^{(fp)}, T = t^{(fp)}, A = a). \quad (1)$$

<sup>1</sup>This assumption of course is not empirically testable, as it is impossible to observe both a female and a male version of the same individual.

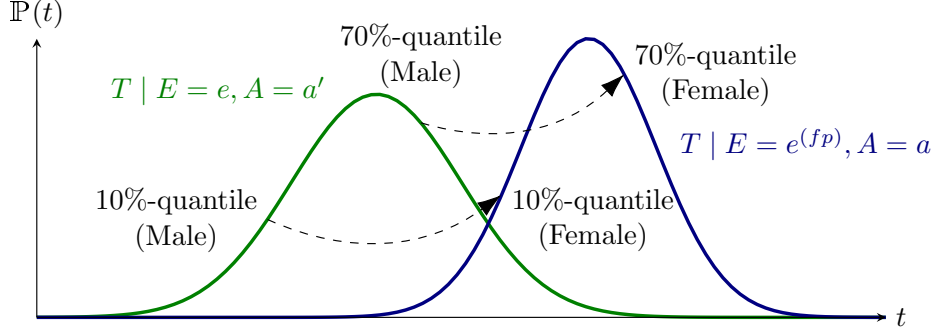


Figure 2: A graphical visualization of the quantile matching procedure. Given a male with a test score corresponding to the 70% quantile, we would hypothesize, that if the gender was changed, the individual would have achieved a test score corresponding to the 70% quantile of the female distribution.

This form of counterfactual correction is known as *recursive substitution* (Pearl 2009, Chapter 7) and is described more formally in the following sections. The reader who is satisfied with the intuitive notion provided by the above example is encouraged to go straight to Section 3.

## 2.2. Structural Causal Models

In order to describe the causal mechanisms of a system, a *structural causal model* (SCM) can be hypothesized, which fully encodes the assumed data-generating process. An SCM is represented by a 4-tuple  $\langle V, U, \mathcal{F}, \mathbb{P}(u) \rangle$ , where

- $V = \{V_1, \dots, V_n\}$  is the set of observed (endogeneous) variables.
- $U = \{U_1, \dots, U_n\}$  are latent (exogeneous) variables.
- $\mathcal{F} = \{f_1, \dots, f_n\}$  is the set of functions determining  $V$ ,  $v_i \leftarrow f_i(\text{pa}(v_i), u_i)$ , where  $\text{pa}(V_i) \subset V, U_i \subset U$  are the functional arguments of  $f_i$  and  $\text{pa}(V_i)$  denotes the parent vertices of  $V_i$ .
- $\mathbb{P}(u)$  is a distribution over the exogeneous variables  $U$ .

Any particular SCM is accompanied by a graphical model  $\mathcal{G}$  (a directed acyclic graph), which summarizes which functional arguments are necessary for computing the values of each  $V_i$  and therefore, how variables affect one another. We assume throughout, without loss of generality, that

- (i)  $f_i(\text{pa}(v_i), u_i)$  is increasing in  $u_i$  for every fixed  $\text{pa}(v_i)$ .
- (ii) Exogeneous variables  $U_i$  are uniformly distributed on  $[0, 1]$ .

In the following section, we discuss the Markovian case in which all exogeneous variables  $U_i$  are mutually independent. The Semi-Markovian case, where variables  $U_i$  are allowed to have a mutual dependency structure, alongside the extension introducing *resolving variables*, are discussed in Section 5.

## 2.3. Markovian SCM Formulation

Let  $Y$  take values in  $\mathbb{R}$  and represent an outcome of interest and  $A$  be the protected attribute taking two values  $a, a'$ . The goal is to describe a pre-processing method which transforms the entire data  $V$  into its fair version  $V^{(fp)}$ . This can be achieved by computing the counterfactual values  $V(A = a)$ , which would have been observed if the protected attribute was fixed to a baseline value  $A = a$  for the entire sample.

More formally, going back to the *university admission* example above, we want to align the distributions

$$V_i \mid \text{pa}(V_i), A = a \text{ and } V_i \mid \text{pa}(V_i), A = a',$$

meaning that the distribution of  $V_i$  should be indistinguishable for both female and male applicants, for every variable  $V_i$ . Since each function  $f_i$  of the original SCM is reparametrized so that  $f_i(\text{pa}(v_i), u_i)$  is increasing in  $u_i$  for every fixed  $\text{pa}(v_i)$ , and also due to variables  $U_i$  being uniformly distributed on  $[0, 1]$ , variables  $U_i$  can be seen as the latent *quantiles*.

The algorithm proposed for data adaption proceeds by fixing  $A = a$ , followed by iterating over descendants of the protected attribute  $A$ , sorted in topological order. For each  $V_i$ , the assignment function  $f_i$  and the corresponding quantiles  $U_i$  are inferred. Finally, transformed values  $V_i^{(fp)}$  are obtained by evaluating  $f_i$ , using quantiles  $U_i$  and the transformed parents  $\text{pa}(V_i)^{(fp)}$  (see Algorithm 1).

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**Algorithm 1:** Fair Data Adaptation

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**Input:**  $V$ , causal graph  $\mathcal{G}$   
 set  $A \leftarrow a$  for everyone  
**for**  $V_i \in \text{de}(A)$  *in topological order* **do**  
     learn function  $V_i \leftarrow f_i(\text{pa}(V_i), U_i)$   
     infer quantiles  $U_i$  associated with the variable  $V_i$   
     transform values as  $V_i^{(fp)} \leftarrow f_i(\text{pa}(V_i)^{(fp)}, U_i)$   
**end**  
**return**  $V^{(fp)}$

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The assignment functions  $f_i$  of the SCM are of course unknown, but are learned non-parametrically at each step. Algorithm 1 obtains the counterfactual values  $V(A = a)$  under the  $do(A = a)$  intervention for each individual, while keeping the latent quantiles  $U$  fixed. In the case of continuous variables, the latent quantiles  $U$  can be determined exactly, while for the discrete case, this is more subtle and described in Plecko and Meinshausen (2020, Section 5).

### 3. Implementation

In order to perform fair data adaption using the **fairadapt** package, the function **fairadapt()** is exported, which returns an object of class **fairadapt**. Implementations of the base R S3 generics **print()**, **plot()** and **predict()**, as well as the generic **autoplot()**, exported from **ggplot2** (Wickham 2016), are provided for **fairadapt** objects, alongside **fairadapt**-specific implementations of S3 generics **visualizeGraph()**, **adaptedData()** and **fairTwins()**. Finally, an extension mechanism is available via the S3 generic function **computeQuants()**, which is used for performing the quantile learning step.

The following sections show how the listed methods relate to one another alongside their intended use, beginning with constructing a call to `fairadapt()`. The most important arguments of `fairadapt()` include:

- **formula**: Argument of type `formula`, specifying the dependent and explanatory variables.
- **adj.mat**: Argument of type `matrix`, encoding the adjacency matrix.
- **train.data** and **test.data**: Both of type `data.frame`, representing the respective datasets.
- **prot.attr**: Scalar-valued argument of type `character` identifying the protected attribute.

As a quick demonstration of fair data adaption using, we load the `uni_admission` dataset provided by **fairadapt**, consisting of synthetic university admission data of students. We subset this data, using the first `n_samp` rows as training data (`uni_trn`) and the following `n_samp` rows as testing data (`uni_tst`). Furthermore, we construct an adjacency matrix `uni_adj` with edges `gender`  $\rightarrow$  `edu`, `gender`  $\rightarrow$  `test`, `edu`  $\rightarrow$  `test`, `edu`  $\rightarrow$  `score`, and `test`  $\rightarrow$  `score`. As protected attribute, we choose `gender`.

```
R> n_samp <- 200
R>
R> uni_dat <- data("uni_admission", package = "fairadapt")
R> uni_dat <- uni_admission[seq_len(2 * n_samp), ]
R>
R> head(uni_dat)
```

	gender	edu	test	score
1	1	1.3499572	1.617739679	1.9501728
2	0	-1.9779234	-3.121796235	-2.3502495
3	1	0.6263626	0.530034686	0.6285619
4	1	0.8142112	0.004573003	0.7064857
5	1	1.8415242	1.193677123	0.3678313
6	1	-0.3252752	-2.004123561	-1.5993848

```
R> uni_trn <- head(uni_dat, n = n_samp)
R> uni_tst <- tail(uni_dat, n = n_samp)
R>
R> uni_dim <- c("gender", "edu", "test", "score")
R> uni_adj <- matrix(c(
+           0,      0,      0,      0,
+           1,      0,      0,      0,
+           1,      1,      0,      0,
+           0,      1,      1,      0),
+                    ncol = length(uni_dim),
+                    dimnames = rep(list(uni_dim), 2))
R>
R> basic <- fairadapt(score ~ ., train.data = uni_trn,
```

```

+               test.data = uni_tst, adj.mat = uni_adj,
+               prot.attr = "gender")
R>
R> basic

```

Fairadapt result

Formula:

```
score ~ .
```

```

Protected attribute:          gender
Protected attribute levels:   0, 1
Number of training samples:   200
Number of test samples:      200
Number of independent variables: 3
Total variation (before adaptation): -0.6757414
Total variation (after adaptation):  -0.1551275

```

The implicitly called `print()` method in the previous code block displays some information about how `fairadapt()` was called, such as number of variables, the protected attribute and also the total variation before and after adaptation, defined as

$$\mathbb{E}[Y \mid A = a] - \mathbb{E}[Y \mid A = a'] \text{ and } \mathbb{E}[Y^{(fp)} \mid A = a] - \mathbb{E}[Y^{(fp)} \mid A = a'],$$

respectively, where  $Y$  denotes the outcome variable. Total variation in the case of a binary outcome  $Y$ , corresponds to the parity gap.

### 3.1. Specifying the Graphical Model

As the algorithm used for fair data adaption in `fairadapt()` requires access to the underlying graphical causal model  $\mathcal{G}$  (see Algorithm 1), a corresponding adjacency matrix can be passed as `adj.mat` argument. The convenience function `graphModel()` turns a graph specified as an adjacency matrix into an annotated graph using the **igraph** package (Csardi and Nepusz 2006). While exported for the user to invoke manually, this function is called as part of the `fairadapt()` routine and the resulting **igraph** object can be visualized by calling the S3 generic `visualizeGraph()`, exported from `fairadapt` on an object of class `fairadapt`.

```
R> uni_graph <- graphModel(uni_adj)
```

A visualization of the **igraph** object returned by `graphModel()` is available from Figure 3. The graph shown is equivalent to that of Figure 1 as they both represent the same causal model.

### 3.2. Quantile Learning Step

The training step in `fairadapt()` can be carried out in two slightly distinct ways: Either by specifying training and testing data simultaneously, or by only passing training data (and

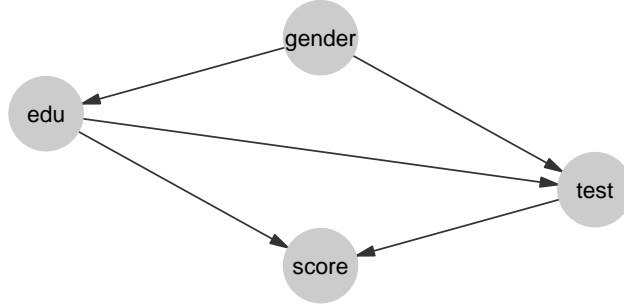


Figure 3: The underlying graphical model corresponding to the university admission example (also shown in Figure 1).

at a later stage calling `predict()` on the returned `fairadapt` object in order to perform data adaption on new test data). The advantage of the former option is that the quantile regression step is performed on a larger sample size, which can lead to more precise inference in practice.

The two dataframes passed as `train.data` and `test.data` are required to have column names which also appear in the row and column names of the adjacency matrix, alongside the protected attribute  $A$ , passed as scalar-valued character vector `prot.attr`.

The quantile learning step of Algorithm 1 can in principle be carried out by several methods, three of which are implemented in `fairadapt`:

- Quantile Regression Forests (Meinshausen 2006).
- Non-crossing quantile neural networks (Cannon 2018).
- Linear Quantile Regression (Koenker and Hallock 2001).

Using linear quantile regression is the most efficient option in terms of runtime, while for non-parametric models and mixed data, the random forest approach is well-suited, at the expense of a slight increase in runtime. The neural network approach is, substantially slower when compared to linear and random forest estimators and consequently does not scale well to large sample sizes. As default, the random forest based approach is implemented, due to its non-parametric nature and computational speed. However, for smaller sample sizes, the neural network approach might in fact be a superior choice. A quick summary outlining some differences between the three natively supported methods is available from Table 1.

The above set of methods is not exhaustive. Further options are conceivable and therefore `fairadapt` provides an extension mechanism to account for this. The `fairadapt()` argument `quant.method` expects a function to be passed, a call to which will be constructed with three unnamed arguments:

1. A `data.frame` containing data to be used for quantile regression. This will either be the `data.frame` passed as `train.data`, or depending on whether `test.data` was specified, a row-bound version of train and test datasets.
2. A logical flag, indicating whether the protected attribute is the root node of the causal graph. If the attribute  $A$  is a root node, we know that  $\mathbb{P}(X \mid \text{do}(A = a)) = \mathbb{P}(X \mid A =$



	Random Forests	Neural Networks	Linear Regression
R-package	<b>ranger</b>	<b>mcqrnn</b>	<b>quantreg</b>
<code>quant.method</code>	<code>rangerQuants</code>	<code>mcqrnnQuants</code>	<code>linearQuants</code>
complexity	$O(np \log n)$	$O(np n_{\text{epochs}})$	$O(p^2 n)$
default parameters	$ntrees = 500$ $mtry = \sqrt{p}$	2-layer fully connected feed-forward network	"br" method of Barrodale and Roberts used for fitting
$T(200, 4)$	0.4 sec	89 sec	0.3 sec
$T(500, 4)$	0.9 sec	202 sec	0.5 sec

Table 1: Summary table of different quantile regression methods.  $n$  is the number of samples,  $p$  number of covariates,  $n_{\text{epochs}}$  number of training epochs for the neural network.  $T(n, 4)$  denotes the runtime of different methods on the university admission dataset, with  $n$  training and testing samples.

- a*). Therefore, the interventional and conditional distributions are in this case the same, and we can leverage this knowledge in the quantile learning procedure, by splitting the data into  $A = 0$  and  $A = 1$  groups.
3. A logical vector of length `nrow(data)`, indicating which rows in the `data.frame` passed as `data` correspond to samples with baseline values of the protected attribute.

Arguments passed as `...` to `fairadapt()` will be forwarded to the function specified as `quant.method` and passed after the first three fixed arguments listed above. The return value of the function passed as `quant.method` is expected to be an S3-classed object. This object should represent the conditional distribution  $V_i \mid \text{pa}(V_i)$  (see function `rangerQuants()` for an example). Additionally, the object should have an implementation of the S3 generic function `computeQuants()` available. For each row  $(v_i, \text{pa}(v_i))$  of the `data` argument, the `computeQuants()` function uses the S3 object to (i) infer the quantile of  $v_i \mid \text{pa}(v_i)$ ; (ii) compute the counterfactual value  $v_i^{(fp)}$  under the change of protected attribute, using the counterfactual values of parents  $\text{pa}(v_i^{(fp)})$  computed in previous steps (values  $\text{pa}(v_i^{(fp)})$  are contained in the `newdata` argument). For an example, see `computeQuants.ranger()` method.

### 3.3. Fair-Twin Inspection

The university admission example presented in Section 2 demonstrates how to compute counterfactual values for an individual while preserving their relative educational achievement. Setting candidate gender as protected attribute and gender level *female* as baseline value, for a *male* student with values  $(a, e, t, y)$ , his *fair-twin* values  $(a^{(fp)}, e^{(fp)}, t^{(fp)}, y^{(fp)})$ , i.e., the values the student would have obtained, had he been *female*, are computed. These values can be retrieved from a `fairadapt` object by calling the S3-generic function `fairTwins()` as:

```
R> ft_basic <- fairTwins(basic, train.id = seq_len(n_samp))
R> head(ft_basic, n = 3)
```

	gender	score	score_adapted	edu	edu_adapted	test
1	1	1.9501728	0.96725357	1.3499572	0.67290529	1.6177397
2	0	-2.3502495	-2.35024955	-1.9779234	-1.97792341	-3.1217962
3	1	0.6285619	0.03824946	0.6263626	0.02188639	0.5300347

	test_adapted
1	0.9070710
2	-3.1217962
3	-0.2313427

In this example, we compute the values in a *female* world. Therefore, for *female* applicants, the values remain fixed, while for *male* applicants the values are adapted, as can be seen from the output.

## 4. Illustration

As a possible real-world use of **fairadapt**, suppose that after a legislative change the US government has decided to adjust the salary of all of its female employees in order to remove both disparate treatment and disparate impact effects. To this end, the government wants to compute the counterfactual salary values of all female employees, that is the salaries that female employees would obtain, had they been male.

To do this, the government is using data from the 2018 American Community Survey by the US Census Bureau, available in pre-processed form as a package dataset from **fairadapt**. Columns are grouped into demographic (**dem**), familial (**fam**), educational (**edu**) and occupational (**occ**) categories and finally, salary is selected as response (**res**) and sex as the protected attribute (**prt**):

```
R> gov_dat <- data("gov_census", package = "fairadapt")
R> gov_dat <- get(gov_dat)
R>
R> head(gov_dat)
```

	sex	age	race	hispanic_origin	citizenship	nativity	marital
1:	male	64	black	no	1	native	married
2:	female	54	white	no	1	native	married
3:	male	38	black	no	1	native	married
4:	female	41	asian	no	1	native	married
5:	female	40	white	no	1	native	married
6:	female	46	white	no	1	native	divorced

	family_size	children	education_level	english_level	salary
1:	2	0	20	0	43000
2:	3	1	20	0	45000
3:	3	1	24	0	99000
4:	3	1	24	0	63000
5:	4	2	21	0	45200
6:	3	1	18	0	28000

	hours_worked	weeks_worked	occupation	industry	economic_region
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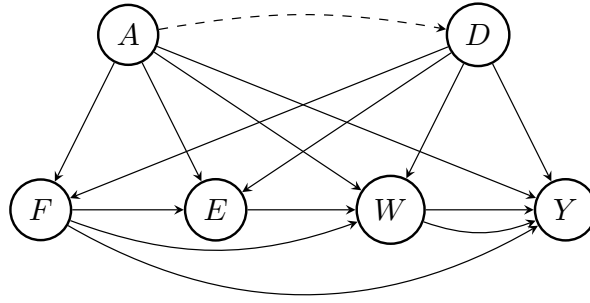


Figure 4: The causal graph for the government-census dataset.  $D$  are demographic features,  $A$  is gender,  $F$  represents marital and family information,  $E$  education,  $W$  work-related information and  $Y$  the salary, which is also the outcome of interest.

1:	56	49	13-1081	928P	Southeast
2:	42	49	29-2061	6231	Southeast
3:	50	49	25-1000	611M1	Southeast
4:	50	49	25-1000	611M1	Southeast
5:	40	49	27-1010	611M1	Southeast
6:	40	49	43-6014	6111	Southeast

```
R> dem <- c("age", "race", "hispanic_origin", "citizenship",
+          "nativity", "economic_region")
R> fam <- c("marital", "family_size", "children")
R> edu <- c("education_level", "english_level")
R> occ <- c("hours_worked", "weeks_worked", "occupation",
+          "industry")
R>
R> prt <- "sex"
R> res <- "salary"
```

The hypothesized causal graph for the dataset is given in Figure 4. According to this, the causal graph can be specified as an adjacency matrix `gov_adj` and as confounding matrix `gov_cfd`:

```
R> cols <- c(dem, fam, edu, occ, prt, res)
R>
R> gov_adj <- matrix(0, nrow = length(cols), ncol = length(cols),
+                  dimnames = rep(list(cols), 2))
R> gov_cfd <- gov_adj
R>
R> gov_adj[dem, c(fam, edu, occ, res)] <- 1
R> gov_adj[fam, c(edu, occ, res)] <- 1
R> gov_adj[edu, c(occ, res)] <- 1
R> gov_adj[occ, res] <- 1
R>
R> gov_adj[prt, c(fam, edu, occ, res)] <- 1
```

```
R>
R> gov_cfd[prt, dem] <- 1
R> gov_cfd[dem, prt] <- 1
R>
R> gov_grph <- graphModel(gov_adj, gov_cfd)
```

Before applying `fairadapt()`, we first log-transform the salaries and look at respective densities by sex group. We subset the data by using the first `n_samp` samples for training, the subsequent `n_pred` samples for predicting and plot the data before performing the adaption.

```
R> gov_dat$salary <- log(gov_dat$salary)
R>
R> n_samp <- 2000
R> n_pred <- 5
R>
R> gov_trn <- head(gov_dat, n = n_samp)
R> gov_prd <- tail(gov_dat, n = n_pred)
```

There is a clear shift between the two sexes, indicating that **male** employees are currently better compensated when compared to **female** employees. However, this differences in **salary** could, in principle, be attributed to factors apart from gender inequality, such as the economic region in which an employee works. This needs to be accounted for as well, i.e., we do not wish to remove differences in salary between economic regions.

```
R> gov_ada <- fairadapt(salary ~ ., train.data = gov_trn,
+                      adj.mat = gov_adj, prot.attr = prt)
```

After performing the adaptation, we can investigate whether the salary-gap has shrunk. The densities after adaptation can be visualized using the **ggplot2**-exported S3 generic function `autoplot()`:

```
R> autoplot(gov_ada, when = "after") +
+   theme_bw() +
+   ggtitle("Adapted salary density by gender")
```

If we are provided with additional testing data, and wish to adapt this as well, we can use the base R S3 generic function `predict()`:

```
R> predict(gov_ada, newdata = gov_prd)
```

	sex	age	race	hispanic_origin	citizenship	nativity
1:	female	19	white	no	1	native
2:	female	46	white	no	1	native
3:	female	24	AIAN	no	1	native
4:	female	23	AIAN	no	1	native
5:	female	50	white	no	1	native

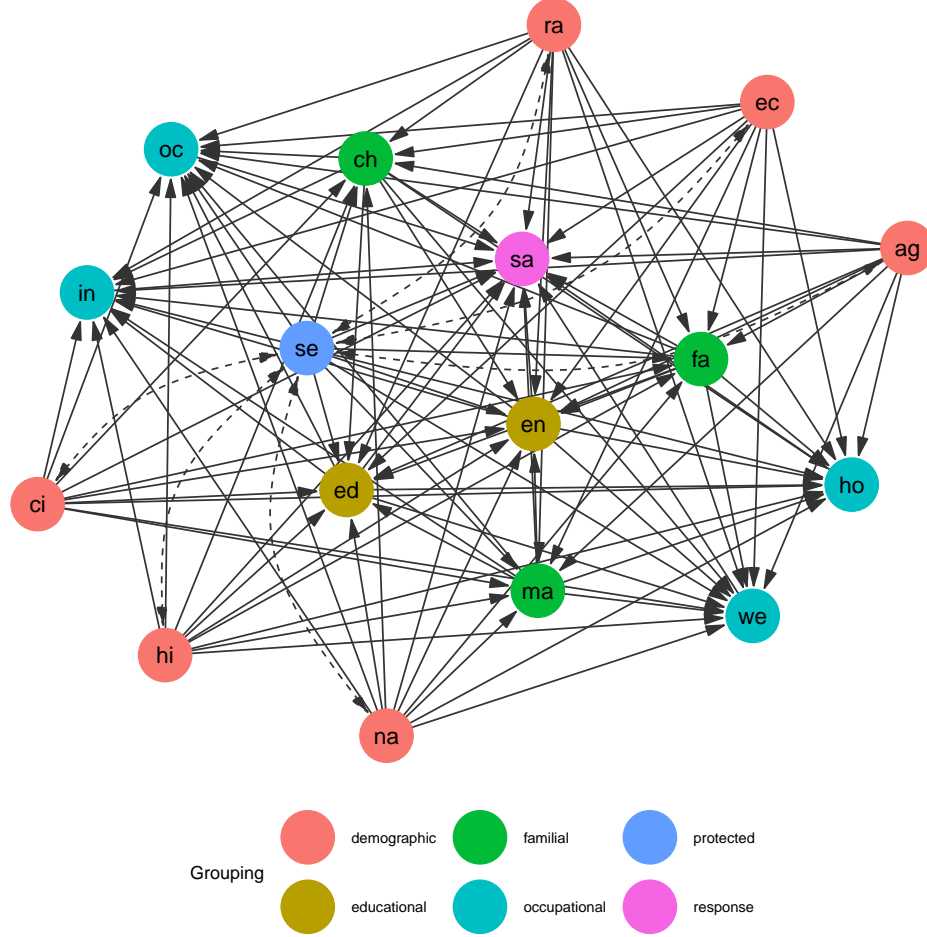


Figure 5: Full causal graph for the government-census dataset, expanding the grouped view presented in Figure 4. *Demographic* features include age (**ag**), race (**ra**), whether an employee is of Hispanic origin (**hi**), is US citizen (**ci**), whether the citizenship is native (**na**), alongside the corresponding economic region (**ec**). *Familial* features are marital status (**ma**), family size (**fa**) and number of children (**ch**), *educational* features include education (**ed**) and English language levels (**en**), and *occupational* features, weekly working hours (**ho**), yearly working weeks (**we**), job (**oc**) and industry identifiers (**in**). Finally, the yearly salary (**sa**) is used as *response* variable and employee sex (**se**) as *protected* attribute.



Figure 6: Visualization of salary densities grouped by employee sex, indicating a shift to higher values for male employees. This uses the US government-census dataset and shows the situation before applying fair data adaption, while Figure 7 presents transformed salary data.

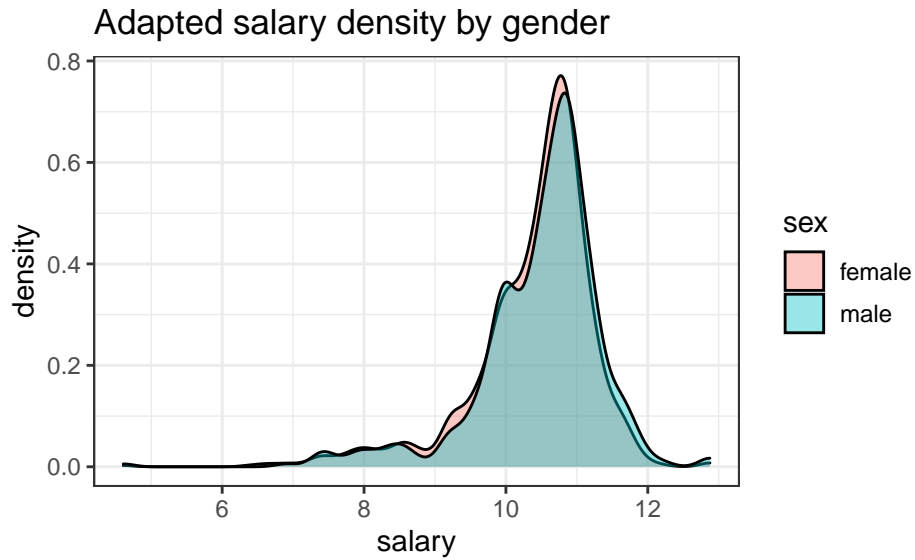


Figure 7: The salary-gap between male and female employees of the US government according to the government-census dataset is clearly reduced when comparing raw data (see Figure 6) to transformed salary data as yielded by applying fair data adaption using employee sex as protected attribute and assuming a causal graph as shown in Figure 5.

	marital	family_size	children	education_level	english_level
1:	never married	5	2	16	0
2:	married	2	0	18	0
3:	never married	4	2	19	0
4:	never married	3	2	19	1
5:	married	2	0	19	0

	salary	hours_worked	weeks_worked	occupation	industry
1:	7.003065	25	49	37-3011	23
2:	9.667765	40	0	43-6014	52M1
3:	10.126631	40	49	33-3012	623M
4:	9.903488	40	26	43-4171	9211MP
5:	11.472103	40	49	43-5031	92MP

	economic_region
1:	Rocky Mountain
2:	Rocky Mountain
3:	Rocky Mountain
4:	Rocky Mountain
5:	Rocky Mountain

Finally, we can do fair-twin inspection using the `fairTwins()` function of **fairadapt**, to retrieve counterfactual feature values:

```
R> fairTwins(gov_ada, train.id = 1:5,
+           cols = c("sex", "age", "education_level", "salary"))
```

	sex	age	age_adapted	education_level	education_level_adapted
1	male	64	64	20	20
2	female	54	54	20	20
3	male	38	38	24	24
4	female	41	41	24	24
5	female	40	40	21	21

	salary	salary_adapted
1	10.66896	9.998798
2	10.71442	10.714418
3	11.50288	11.608236
4	11.05089	11.050890
5	10.71885	10.718852

Values are unchanged for female individuals (as *female* is used as baseline level), as is the case for **age**, which is not a descendant of the protected attribute **sex** (see Figure 5). However, variables **education\_level** and **salary** do change for males, as they are descendants of the protected attribute **sex**.

The variable **hours\_worked** is also a descendant of *A* and one might argue that this variable should *not* be adapted in the procedure, i.e., that it remain the same, irrespective of employee sex. This is the idea behind *resolving variables*, which we discuss next, in Section 5.1.

## 5. Extensions

Several extensions to the basic Markovian SCM formulation introduced in Section 2.3 exist, some of which are available for use in `fairadapt()` and are outlined in the following sections.

### 5.1. Adding Resolving Variables

Kilbertus *et al.* (2017) discuss that in some situations the protected attribute  $A$  can affect variables in a non-discriminatory way. For instance, in the Berkeley admissions dataset (Bickel, Hammel, and O’Connell 1975) we observe that females often apply for departments with lower admission rates and consequently have a lower admission probability. However, we perhaps would not wish to account for this difference in the adaptation procedure, if we were to argue that applying to a certain department is a choice everybody is free to make. Such examples motivated the idea of *resolving variables* by Kilbertus *et al.* (2017). A variable  $R$  is called resolving if

- (i)  $R \in \text{de}(A)$ , where  $\text{de}(A)$  are the descendants of  $A$  in the causal graph  $\mathcal{G}$ .
- (ii) The causal effect of  $A$  on  $R$  is considered to be non-discriminatory.

In presence of resolving variables, computation of the counterfactual is carried out under the more involved intervention  $\text{do}(A = a, R = R(a'))$ . The potential outcome value  $V(A = a, R = R(a'))$  is obtained by setting  $A = a$  and computing the counterfactual while keeping the values of resolving variables to those they *attained naturally*. This is a nested counterfactual and the difference in Algorithm 1 is simply that resolving variables  $R$  are skipped in the for-loop. In order to perform fair data adaptation with the variable `test` being resolving in the `uni_admission` dataset used in Section 3, the string `"test"` can be passed as `res.vars` to `fairadapt()`.

```
R> fairadapt(score ~ ., train.data = uni_trn, test.data = uni_tst,
+           adj.mat = uni_adj, prot.attr = "gender", res.vars = "test")
```

Fairadapt result

Formula:

```
score ~ .
```

Protected attribute:	gender
Protected attribute levels:	0, 1
Resolving variables:	test
Number of training samples:	200
Number of test samples:	200
Number of independent variables:	3
Total variation (before adaptation):	-0.6757414
Total variation (after adaptation):	-0.4160513

As can be seen from the respective model summary outputs, the total variation after adaptation, in this case, is larger than in the `basic` example from Section 3, with no resolving



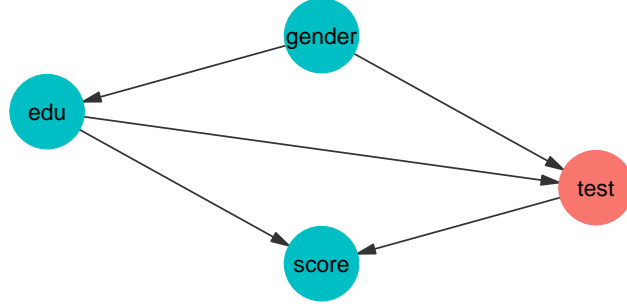


Figure 8: Visualization of the causal graph corresponding to the university admissions example introduced in Section 1 with the variable `test` chosen as a *resolving variable* and therefore highlighted in red.

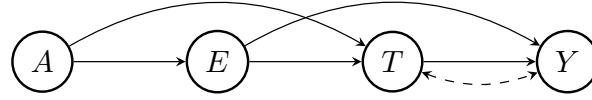


Figure 9: Causal graphical model corresponding to a Semi-Markovian variant of the university admissions example, introduced in Section 3. and visualized in its basic form in Figures 1 and 3. Here, we allow for the possibility of a mutual dependency between the latent variables corresponding to test and final scores.

variables. The intuitive reasoning here is that resolving variables allow for some discrimination, so we expect to see a larger total variation between the groups.

```
R> uni_res <- graphModel(uni_adj, res.vars = "test")
```

A visualization of the corresponding graph is available from Figure 8, which highlights the resolving variable `test` in red. Apart from color, the graphical model remains unchanged from what is shown in Figure 3.

## 5.2. Semi-Markovian and Topological Ordering Variant

Section 2 focuses on the Markovian case, which assumes that all exogeneous variables  $U_i$  are mutually independent. However, in practice this requirement is often not satisfied. If a mutual dependency structure between variables  $U_i$  exists, this is called a Semi-Markovian model. In the university admission example, we could, for example, have  $U_{\text{test}} \not\perp U_{\text{score}}$ , i.e., latent variables corresponding to variables test and final score being correlated. Such dependencies between latent variables can be represented by dashed, bidirected arrows in the causal diagram, as shown in Figures 9 and 10.

There is an important difference in the adaptation procedure for Semi-Markovian case: when inferring the latent quantiles  $U_i$  of variable  $V_i$ , in the Markovian case, only the direct parents  $\text{pa}(V_i)$  are needed. In the Semi-Markovian case, due to correlation of latent variables, using only the  $\text{pa}(V_i)$  can lead to biased estimates of the  $U_i$ . Instead, the set of direct parents needs to be extended, as described in more detail by Tian and Pearl (2002). A brief sketch of the

argument goes as follows: Let the *C-components* be a partition of the set  $V$ , such that each *C-component* contains a set of variables which are mutually connected by bidirectional edges. Let  $C(V_i)$  denote the entire *C-component* of variable  $V_i$ . We then define the set of extended parents as

$$\text{Pa}(V_i) := (C(V_i) \cup \text{pa}(C(V_i))) \cap \text{an}(V_i),$$

where  $\text{an}(V_i)$  is the set of ancestors of  $V_i$ . The adaptation procedure in the Semi-Markovian case in principle remains the same as outlined in Algorithm 1, with the difference that the set of direct parents  $\text{pa}(V_i)$  is replaced by  $\text{Pa}(V_i)$  at each step.

To include the bidirectional confounding edges in the adaptation, we can pass a `matrix` as `cfd.mat` argument to `fairadapt()` such that:

- `cfd.mat` has the same dimension, column and row names as `adj.mat`.
- `cfd.mat` is symmetric.
- As is the case with the adjacency matrix passed as `adj.mat`, an entry `cfd.mat[i, j] == 1` indicates that there is a bidirectional edge between variables `i` and `j`.

The following code performs fair data adaptation of the Semi-Markovian university admission variant with a mutual dependency between the variables representing test and final scores. For this, we create a matrix `uni_cfd` with the same attributes as the adjacency matrix `uni_adj` and set the entries representing the bidirected edge between vertices `test` and `score` to 1. Finally, we can pass this confounding matrix as `cfd.mat` to `fairadapt()`. A visualization of the resulting causal graph is available from Figure 10.

```
R> uni_cfd <- matrix(0, nrow = nrow(uni_adj), ncol = ncol(uni_adj),
+                   dimnames = dimnames(uni_adj))
R>
R> uni_cfd["test", "score"] <- 1
R> uni_cfd["score", "test"] <- 1
R>
R> semi <- fairadapt(score ~ ., train.data = uni_trn,
+                   test.data = uni_tst, adj.mat = uni_adj,
+                   cfd.mat = uni_cfd, prot.attr = "gender")
```

Alternatively, instead of using the extended parent set  $\text{Pa}(V_i)$ , we could also use the entire set of ancestors  $\text{an}(V_i)$ . This approach is implemented as well, and available by specifying a topological ordering. This is achieved by passing a `character` vector, containing the correct ordering of the names appearing in `names(train.data)` as `top.ord` argument to `fairadapt()`. The benefit of using this option is that the specific edges of the causal model  $\mathcal{G}$  need not be specified. However, in the linear case, specifying the edges of the graph, so that the quantiles are inferred using only the set of parents, will in principle have better performance.

### 5.3. Questions of Identifiability

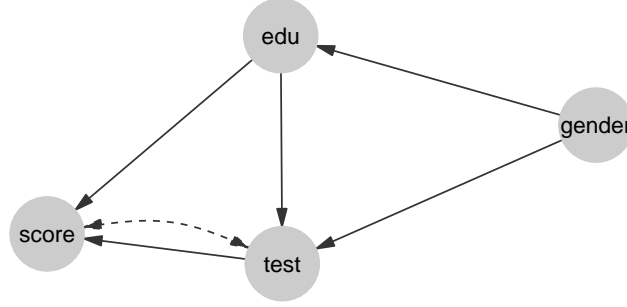


Figure 10: Visualization of the causal graphical model also shown in Figure 9, obtained when passing a confounding matrix indicating a bidirectional edge between vertices `test` and `score` to `fairadapt()`. The resulting Semi-Markovian setting is also handled by `fairadapt()`, extending the basic Markovian formulation introduced in Section 2.3.

So far we did not discuss whether it is always possible to carry out the counterfactual inference described in Section 2. In the causal literature, an intervention is termed *identifiable* if it can be computed uniquely using the data and the assumptions encoded in the graphical model  $\mathcal{G}$ . An important result by Tian and Pearl (2002) states that an intervention  $\text{do}(X = x)$  on a singleton variable  $X$  is identifiable if there is no bidirected path between  $X$  and  $\text{ch}(X)$ . Therefore, our intervention of interest is identifiable if one of the two following conditions are met:

- The model is Markovian.
- The model is Semi-Markovian and,
  - (i) there is no bidirected path between  $A$  and  $\text{ch}(A)$  and,
  - (ii) there is no bidirected path between  $R_i$  and  $\text{ch}(R_i)$  for any resolving variable  $R_i$ .

Based on this, the `fairadapt()` function may return an error, if the specified intervention is not possible to compute. An additional limitation is that `fairadapt` currently does not support *front-door identification* (Pearl 2009, Chapter 3), meaning that certain special cases which are in principle identifiable are not currently handled. We hope to include this case in a future version.

## References

- Bickel PJ, Hammel EA, O’Connell JW (1975). “Sex Bias in Graduate Admissions: Data from Berkeley.” *Science*, **187**(4175), 398–404.
- Blau FD, Kahn LM (2003). “Understanding International Differences in the Gender Pay Gap.” *Journal of Labor Economics*, **21**(1), 106–144.
- Cannon AJ (2018). “Non-Crossing Nonlinear Regression Quantiles by Monotone Composite Quantile Regression Neural Network, With Application to Rainfall Extremes.” *Stochastic Environmental Research and Risk Assessment*, **32**(11), 3207–3225.

- Corbett-Davies S, Goel S (2018). “The Measure and Mismeasure of Fairness: A Critical Review of Fair Machine Learning.” *arXiv preprint arXiv:1808.00023*.
- Csardi G, Nepusz T (2006). “The **igraph** Software Package for Complex Network Research.” *InterJournal, Complex Systems*, 1695. URL <https://igraph.org>.
- Darlington RB (1971). “Another Look at Cultural Fairness.” *Journal of Educational Measurement*, **8**(2), 71–82.
- Galles D, Pearl J (1998). “An Axiomatic Characterization of Causal Counterfactuals.” *Foundations of Science*, **3**(1), 151–182.
- Hardt M, Price E, Srebro N, *et al.* (2016). “Equality of Opportunity in Supervised Learning.” In *Advances in neural information processing systems*, pp. 3315–3323.
- Kilbertus N, Carulla MR, Parascandolo G, Hardt M, Janzing D, Schölkopf B (2017). “Avoiding Discrimination Through Causal Reasoning.” In *Advances in Neural Information Processing Systems*, pp. 656–666.
- Koenker R, Hallock KF (2001). “Quantile Regression.” *Journal of Economic Perspectives*, **15**(4), 143–156.
- Komiyama J, Takeda A, Honda J, Shimao H (2018). “Nonconvex Optimization for Regression with Fairness Constraints.” In *International Conference on Machine Learning*, pp. 2737–2746. PMLR.
- Kozodoi N, V Varga T (2021). *fairness: Algorithmic Fairness Metrics*. R package version 1.2.2, URL <https://CRAN.R-project.org/package=fairness>.
- Kusner MJ, Loftus J, Russell C, Silva R (2017). “Counterfactual Fairness.” In *Advances in Neural Information Processing Systems*, pp. 4066–4076.
- Lambrecht A, Tucker C (2019). “Algorithmic Bias? An Empirical Study of Apparent Gender-Based Discrimination in the Display of STEM Career Ads.” *Management Science*, **65**(7), 2966–2981.
- Larson J, Mattu S, Kirchner L, Angwin J (2016). “How We Analyzed the COMPAS Recidivism Algorithm.” *ProPublica* (5 2016), **9**.
- McGinley AC (2011). “Ricci v. DeStefano: Diluting Disparate Impact and Redefining Disparate Treatment.” *Scholarly Works*, **646**.
- Meinshausen N (2006). “Quantile Regression Forests.” *Journal of Machine Learning Research*, **7**(Jun), 983–999.
- Pearl J (2009). *Causality*. Cambridge University Press.
- Plecko D, Meinshausen N (2020). “Fair Data Adaptation with Quantile Preservation.” *Journal of Machine Learning Research*, **21**, 1–44.
- Scutari M (2021). *fairml: Fair Models in Machine Learning*. R package version 0.6, URL <https://CRAN.R-project.org/package=fairml>.

- Tian J, Pearl J (2002). “A General Identification Condition for Causal Effects.” In *Eighteenth National Conference on Artificial Intelligence*, pp. 567–573. American Association for Artificial Intelligence, USA.
- Wickham H (2016). *ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag New York. URL <https://ggplot2.tidyverse.org>.
- Wiśniewski J, Biecek P (2021). “**fairmodels**: A Flexible Tool For Bias Detection.” *arxiv*. URL <https://arxiv.org/abs/2104.00507>.
- Zhang J, Bareinboim E (2018). “Fairness in Decision-Making: The Causal Explanation Formula.” In *Thirty-Second National Conference on Artificial Intelligence*. American Association for Artificial Intelligence, USA.

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