Momentum and Mean-Reversion
in Strategic Asset Allocation[*]

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This version: January 2009

Abstract

We study a dynamic asset allocation problem in which stock returns exhibit short-run momentum and long-run mean reversion. We develop a tractable continuous-time model that captures these two predictability features and derive the optimal investment strategy in closed-form. The model predicts negative hedging demands for medium-term investors, and an allocation to stocks that is non-monotonic in the investor's horizon. Momentum substantially increases the economic value of hedging time-variation in investment opportunities. These utility gains are preserved when we impose realistic borrowing and short-sales constraints and allow the investor to trade on a monthly frequency.

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1 Introduction

Equity returns tend to continue over short horizons (momentum) and revert over longer horizons (mean-reversion).[1] If stock returns are predictable, an investor modifies her optimal strategy in two important ways. First, the investment strategy reflects the current conditioning information. Second, investors with different horizons hold different optimal portfolios as a result of hedging demands.

Recent studies show that mean-reversion in equity returns induces positive hedging demands and economically significant market timing opportunities.[2] However, these studies ignore that returns tend to continue over short horizons. Return continuation adds to the market timing opportunities, and it may also affect hedging demands by increasing the risk of a position in equities. This paper is the first to study a dynamic, long-term investment problem in a financial market that features both momentum and mean-reversion. We show that momentum at the index level does impact hedging demands and market timing opportunities, and that this result is strong at horizons up to five years. In addition, changes in the allocation to stocks induced by momentum yield remarkable gains in investor's welfare. Momentum is most important at short horizons, while mean reversion in equity returns is the dominant force for longer investment horizons. These gains are preserved in the cases of discrete-time trading and standard borrowing and short-sales constraints.

As a starting point, we develop a novel continuous-time financial market model that features both momentum and mean-reversion in equity returns. The state variable that captures momentum in stocks' performance is a weighted average of recent stock returns. Expected returns relate positively to the performance variable, the key feature of momentum. The state variable that captures mean-reversion in equity returns is the dividend yield.[3] Our model is sufficiently tractable to allow the derivation of the investor's optimal strategy in closed form.

Expected returns relate positively to both the performance variable and the mean-reversion variable. However, innovations to expected returns relate positively to innovations in the

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[1]See Lo and Mackinlay (1988), Fama and French (1988), and Poterba and Summers (1988) for seminal contributions.

[2]See for instance Brandt (1999), Campbell and Viceira (1999), and Wachter (2002).

[3]See Cochrane (2007), Boudoukh, Michaely, Roberts, Richardson (2007), Campbell and Thompson (2007), Lettau and Van Nieuwerburgh (2007), Binsbergen and Koijen (2008) for recent studies for evidence on the forecasting power of the dividend yield for aggregate stock market returns.

performance variable, but negatively to innovations in the dividend yield. The difference in the correlation structure of the innovations determines the sign of the hedging demands induced by momentum and mean-reversion. The investor hedges the performance variable by optimally reducing the allocation to stocks, whereas the mean-reversion variable is hedged by optimally increasing the allocation to stocks. Since we find that the performance variable is less persistent than the mean-reversion variable, hedging demands induced by momentum are dominant at short horizons. At longer horizons, mean reversion is most prevalent. Our model therefore implies that the total allocation to stocks does not increase monotonically with the investor's horizon.[4] Consequently, there is a strictly positive investment horizon at which the investor behaves as if she were myopic.

The economic importance of hedging demands is an empirical question. To quantify the economic significance of including momentum as an input for strategic allocation,[5] we calibrate the model on the basis of the CRSP value-weighted and equally-weighted return series of stocks present in the AMEX/NASDAQ/NYSE. The sample period we consider is 1946.1 - 2005.12. The value- and equally-weighted indices exhibit a monthly first-order autocorrelation of about 6% and 20%, respectively. By means of these two indices we explore the predictions of our model in two well-differentiated environments: one, represented by the value-weighted index,in which continuation is rather weak, and the other, represented by the equally-weighted index, in which continuation is more prominently present. It is worth noticing that these numbers are in the ballpark of autocorrelation in other indices of developed economies. For example, the MSCI Europe index has a monthly autocorrelation of 10.32% for the period 1970-2005.

We find that the hedging demands are negative for the first year for the value-weighted index, and for the first five years for the equally weighted index. For longer investment horizons, the hedging demands are positive as mean reversion kicks in. In addition to the change in the optimal strategic allocation, momentum also strongly affects the short-term myopic allocation to stocks via

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[4]Xia (2000) shows that, under incomplete information, the allocation to stocks can have any shape. Note, however, that our result is obtained under complete information. In this sense, it is directly comparable to other results in the extant literature.

[5]In line with the asset allocation literature, we call the optimal long-term allocation the strategic allocation. It is not related to an alternative interpretation of strategic behavior in which you take into account the effect of your actions on the decisions of others, which in turn affect you.

tactical market timing: we find that timing the performance variable is more important than timing the slow-moving mean-reversion variable. This finding is consistent with the abundant evidence on return-chasing behavior of professional fund managers. We further analyze the practical value of our results and solve a discrete-time portfolio choice problem with standard borrowing and short-sales constraints. The benefits of momentum trading decrease with the trading frequency but are still substantial at a monthly frequency. At lower trading frequencies, there is little value of timing past stock market performance.[6] The resulting utility gains are large and unlikely to be wiped out by any realistic amount of transactions costs.

Our finding that hedging demands are negligible and even negative for investment horizons up to five years is consistent with the findings in Brandt (1999) and Ait-Sahalia and Brandt (2001). These studies bypass modeling the return-generating process and estimate the portfolio weights from the Euler conditions directly. They find small or negative hedging demands at short-to intermediate investment horizons. Also, Balvers and Mitchell (1997) show that the optimal allocation to stocks is decreasing in the investment horizon when stock returns are positively correlated. All these papers are crafted in discrete time, and Balvers and Mitchell (1997) accommodate either positive or negative correlation, but not both. We provide a tractable continuous-time model that allows studying the simultaneous impact of momentum and mean-reversion on optimal dynamic asset allocation within a unified framework.

A well-known prediction of standard strategic asset allocation studies is that the fraction of wealth that an investor allocates to stocks is directly related to her investment horizon. This result typically refers to a problem that usually takes a 15- to 20-year perspective and focuses on mean-reversion. We show that even the modest amount of momentum present in the indices we study can substantially weaken, and even reverse, this result at shorter horizons. We would argue that the intermediate investment horizons in which our theory delivers different predictions is relevant for the biggest actors in the financial scene, such as portfolio managers of mutual funds. There is substantial evidence that the investment horizons of many institutional investors have shortened in recent years (see Baker (1998)), a phenomenon that may be due to agency problems that arise in

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[6]This is consistent with Campbell, Chan, and Viceira (2003).

delegated portfolio management. As John Bogle, a veteran of the mutual fund industry puts it:[7]

> "(. . .) a dramatic decline in investment horizons has (. . .) changed the industry.
> . . . Portfolio managers turn over at a rapid rate. The average manager lasts just six
> years, and then a new broom sweeps the portfolio clean. . . . Fund shares - once held by
> long-term investors for an average of 12.5 years - are today held for an average of just
> over two years."

The emphasis on return continuation also relates our paper to the momentum literature pioneered by Jegadeesh and Titman (1993). In these strategies, momentum profits can be generated along three lines: differences in (unconditional) expected returns (see Conrad and Kaul (1998)), positive autocorrelation in winning and losing portfolio returns (see Moskowitz and Grinblatt (1999)), or cross-serial correlations (see Lewellen (2002)). Given the fact that in our economy there is only a single stock index, we can only accommodate the second source of momentum profits. An interesting direction for future research is our model's extension to the optimal-allocation use of positive autocorrelation in momentum portfolios, while taking at the same time mean-reversion into account.[8]

In summary, our paper provides two main contributions to the dynamic asset allocation literature. First, we introduce an intuitive and parsimonious continuous time model that accommodates both momentum and mean-reversion. In this model expected index returns are governed by two state variables, a weighted average of past returns and the dividend yield. The model manages to incorporate the history of stock returns as a state variable while keeping the tractability of a Markovian framework. The model nests the well-known models of Campbell and Viceira (1999) and Wachter (2002), which therefore constitute a natural benchmark for our work. We consider these restricted versions to rigorously study the incremental effect of momentum on strategic asset allocation. Second, we explicitly focus on the case of an investor with an

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[7]How Mutual Funds Lost Their Way, remarks by John C. Bogle, Founder and Former Chairman, The Vanguard Group, The Wall Street Journal, June 20, 2000. Bogle has confirmed these views in The Mutual Fund Industry in 2003: Back to the Future, Before the Harvard Club of Boston, the Harvard Business School Association of Boston, and the Boston Security Analysts Society Boston, Massachusetts, January 14, 2003.

[8]The model developed in this paper could be extended to multiple indices. Such a model can be used to analyze optimal momentum strategies in the presence of mean-reversion as opposed to naively constructed winner and loser portfolios.

intermediate investment horizon, contributing in this way to bridge the gap between the long-term asset allocation literature that emphasizes the economic value of return reversal, and the tactical return continuation or momentum literature.

This paper proceeds as follows. In Section 2, we introduce a new model of stock returns, in which the equity risk premium is driven by both a weighted average of past returns and a mean-reversion variable. In Section 3, the optimal portfolio for a CRRA investor who derives utility from terminal wealth is derived analytically. Section 4 calibrates the model of Section 2, and a restricted version, to the data. In Section 5, we determine the optimal investment strategies and the utility losses from suboptimal strategies. Section 6 finally concludes.

2 Financial market

In this section we introduce an intuitive and parsimonious model of stock returns that accommodates both return continuation over short horizons and return reversals over longer horizons. The model nests the works of Campbell and Viceira (1999) and Wachter (2002), which only accommodate mean-reversion in stock returns.

Momentum over short horizons implies that the recent development of the stock price has predictive power for future returns. To capture this idea, we introduce a state variable that summarizes the past performance of the stock. Mean-reversion in stock returns is usually modeled via persistent financial ratios as the dividend yield or the price earnings ratio predicting future stock returns (see Campbell and Viceira (1999) and Campbell et al. (2003)). In order to allow for both momentum and mean-reversion, we assume that the investor predicts the next return using a weighted average of performance information and a variable capturing mean-reversion.

Define S_t as the level of the index at time t in which the dividends are reinvested. We then characterize the dynamics of stock returns as:

$$\frac{\mathrm{d}S_t}{S_t} = (\phi M_t + (1 - \phi)\mu_t)\,\mathrm{d}t + \sigma_S'\mathrm{d}Z_t,\, 0 \leq \phi < 1, \tag{1}$$

with σ_S a two-dimensional volatility vector and Z_t a two-dimensional vector of independent

Brownian motions. All correlations between the various processes will be captured by the volatility vectors. Note that the instantaneous expected return in (1) depends on two state variables, μ_t and M_t, which we discuss now in detail.

We construct the variable M_t in such a way that it reflects the past performance of the index and term it the *performance variable*. More precisely, we define M_t as a weighted sum of past returns:

$$M_t = \int_0^t e^{-\omega(t-u)} \frac{\mathrm{d}S_u}{S_u}, \tag{2}$$

where $\omega > 0$ and $e^{-\omega(t-u)}$ is the weighting scheme. Because S_t is the value of the index in which dividends are reinvested, M_t accumulates total returns and not only the capital gain of the index.

To simplify the model, we normalize $\omega = 1$. This may seem an ad-hoc restriction at first and it is indeed possible to extend our model by relaxing it. We find, however, that if we select ω to maximize the R-squared of a predictive regression in which both the performance variable and the dividend yield are used to predict future returns, the estimates of ω are close to one. The resulting optimal strategies and the utility costs of suboptimal strategies are also very similar. For tractability, we therefore develop our model under the assumption $\omega = 1$.

The dynamics of the performance variable follows from the dynamics of stock returns. To see this, totally differentiate (2) with respect to time to obtain:

$$\mathrm{d}M_t = \frac{\mathrm{d}S_t}{S_t} - M_t \mathrm{d}t, \tag{3}$$

and replace equation (1) in (3). This results in the following dynamics for the performance variable:

$$\mathrm{d}M_t = (1 - \phi)(\mu_t - M_t)\,\mathrm{d}t + \sigma_S'\mathrm{d}Z_t. \tag{4}$$

It follows from (4) that the performance variable fluctuates around a stochastic mean, μ_t. The variable μ_t captures the long-run mean-reversion in stock returns. We indicate the variable μ_t therefore as the *mean-reversion variable*. In our empirical work, we use the dividend yield to model mean-reversion in stock returns. We assume that μ_t is a stationary variable, following an

Ornstein-Uhlenbeck process:

$$d\mu_t = \alpha \left(\mu_0 - \mu_t\right) dt + \sigma'_\mu dZ_t, \, \alpha > 0, \tag{5}$$

where μ_0 is the (constant) long-run expected rate of return, α is the rate at which μ_t converges to μ_0, and σ_μ a two-dimensional vector of instantaneous volatilities. From equation (1) the (instantaneous) expected return can be recast as:

$$\mathbb{E}_t \left(\frac{dS_t}{S_t}\right) = \left(\mu_t + \phi \left(M_t - \mu_t\right)\right) dt. \tag{6}$$

This representation admits the following interpretation. If $\phi > 0$ and the performance of the stock has in the recent past been above (below) the current stochastic mean conditional on the mean-reversion variable, the investor predicts a return that is above (below) μ_t. In contrast, $\phi = 0$ means that past performance has no predictive power for future stock returns. In this case we recover the classical continuous time models of drift-based predictability in which the stochastic mean μ_t contains only the mean-reversion variable to forecast returns. In the empirical section, we use an affine function of the dividend yield to represent the mean-reversion variable.[9] Along these lines, the model admits both momentum and mean-reversion and nests the return models that have become the workhorse in the strategic asset allocation literature.

Apart from the stock (index), the investor's asset menu also contains a riskless cash account whose price at time t is indicated by B_t. The dynamics of the cash account is given by:

$$\frac{dB_t}{B_t} = rdt, \tag{7}$$

with r the constant riskless interest rate.

Finally, note that although M_t depends on the whole history of the stock, the triplet (S_t, M_t, μ_t) forms a three-dimensional Markov process.

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[9]See for instance Campbell and Viceira (1999) and Wachter (2002).

3   Optimal strategic asset allocation

In this section we solve the strategic asset allocation faced by a long-term investor who derives utility from terminal wealth.[10] We assume that the preferences of the investor can be represented by a CRRA utility index with a constant coefficient of relative risk aversion equal to γ. The investor dynamically allocates the capital available, W_t, to stocks and the cash account. Hence, the investment problem for an investor with an investment horizon equal to $T - t$ can be formalized as:

$$J(W, M, \mu, t, T) = \max_{(\pi_s)_{s \in [t,T]}} \mathbb{E}_t \left(\frac{1}{1 - \gamma} W_T^{1-\gamma} \right), \tag{8}$$

subject to the dynamic budget constraint

$$\frac{\mathrm{d}W_t}{W_t} = \left(\pi_t \left(\mu_t + \phi \left(M_t - \mu_t \right) - r \right) + r \right) \mathrm{d}t + \pi_t \sigma_S' \mathrm{d}Z_t, \tag{9}$$

in which π_t denotes the fraction of wealth allocated to stocks at time t and $J(W, M, \mu, t, T)$ the value function corresponding to the optimal investment strategy. The problem is solved by means of dynamic programming and we refer to Appendix A for details on the derivation. The next proposition provides the main result for the value function and the optimal dynamic strategic allocation.

Proposition 1 *For an investor with an investment horizon $\tau = T - t$ and constant coefficient of relative risk aversion γ, the value function is of the form:*

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\begin{aligned}
J(W, M, \mu, t, T) &= \frac{1}{1 - \gamma} W^{1-\gamma} H(Y, \tau), \\
H(Y, \tau) &= \exp \left(\frac{1}{2} Y' A(\tau) Y + B(\tau)' Y + C(\tau) \right),
\end{aligned}
$$

with $Y = (M, \mu)'$, $A \in \mathbb{R}^{2 \times 2}$, $B \in \mathbb{R}^{2 \times 1}$, and $C \in \mathbb{R}$. A, B, and C are given in Appendix A as the solution to a set of ordinary differential equations. The optimal strategic allocation to stocks is

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[10]Since our financial market model is (dynamically) incomplete, the investment problem with intermediate consumption is non-trivially harder than the terminal wealth problem, see Wachter (2002) and Liu (2006).

given by:

$$\pi_t^\star = \frac{1}{\gamma}\frac{\mu_0 - r}{\sigma_S'\sigma_S} + \frac{1}{\gamma}\frac{\mu_t - \mu_0}{\sigma_S'\sigma_S} \quad + \quad \frac{\phi}{\gamma}\frac{(M_t - \mu_t)}{\sigma_S'\sigma_S} + \frac{1}{\gamma\sigma_S'\sigma_S}\sigma_S'\Sigma'\left(\frac{1}{2}\left(A(\tau) + A(\tau)'\right)Y + B(\tau)\right) \quad (10)$$

with $\Sigma = (\sigma_S, \sigma_\mu)' \in \mathbb{R}^{2\times 2}$. The remainder, $1 - \pi_t^\star$, is invested in the cash account.

This proposition states that the optimal fraction invested in stocks contains four components. The first three components are termed the myopic demand for stocks. It is the allocation that an investor optimally holds if the investment horizon shrinks to zero, and therefore does not care about future investment opportunities. The fourth component is the intertemporal hedging demand (see for instance Merton (1971)). The reason the investor forms hedging demands is to hedge adverse changes in investment opportunities. In the context of our model, poor investment opportunities correspond to low expected returns, which can be the result of poor recent index performance (low value of M_t) or a low value of the dividend yield (and hence a low value of μ_t). Because M_t is positively correlated with returns, the hedging demands are negative. If there is a negative innovation to M_t, then there will be a negative innovation to the index, and the negative hedging demand delivers a positive return, thereby stabilizing wealth. By the same token, because the dividend yield and returns are negatively correlated, the hedging demands are positive. A negative shock to the dividend is likely to go together with a positive return, implying that when expected returns for future periods are low, the positive hedging demand results in a positive return. The size of the hedging demands, for a fixed investment horizon, depends on (i) the correlation between the state variable and returns ("the ability to hedge") and (ii) the persistence of the state variable ("the importance of hedging"). If returns are uncorrelated with the state variables that capture the variation in investment opportunities, the investor cannot hedge the variation therein. If the state variables are very persistent, as is for instance the case for the dividend yield, then a negative shock to the dividend yield is likely to affect expected returns for many consecutive periods. If the state variable is less persistent, like the momentum variable, hedging demands will be smaller because the state variable will revert quicker to its mean. We find, as we show below, that momentum is sufficiently strong to generate economically interesting hedging demands.

4 Model calibration

In this section, we assess empirically the impact of the joint presence of momentum and mean-reversion on strategic asset allocation using the model of Section 2. As argued in the introduction, we focus on investors with a realistic intermediate investment horizon up to five years. In order to highlight the role of momentum, we also estimate a restricted version of our model that allows only for mean-reversion, i.e. $\phi = 0$. This restricted version, in which the equity risk premium is only driven by the mean-reversion variable, has been studied extensively in the literature (see e.g. Campbell and Viceira (1999) and Wachter (2002)) and constitutes therefore a natural benchmark. We discuss the estimation procedure, the data used in estimation, and the resulting estimates of our model and of the restricted version.

In order to estimate the financial market model of Section 2, we require proxies for both the mean-reversion variable, μ_t, and the performance variable, M_t. In line with the literature mentioned above, the mean-reversion variable is modeled affine in the (log) dividend yield, i.e.:[11]

$$\mu_t = \mu_0 + \mu_1(D_t - \mu_D) = \mu_0 + \mu_1 X_t, \tag{11}$$

with D_t indicating the (log) dividend yield, $\mathbb{E}(D_t) = \mu_D$, and $X_t = D_t - \mu_D$ denoting the de-meaned dividend yield. The construction of the performance variable, M_t, is slightly more involved. The investor observes the stock's performance and uses this information to construct the weighted average of past returns. Given that stock returns are observed at discrete points in time, we approximate the integral in (2) using monthly stock returns and through a standard Euler discretization:

$$M_t \approx \sum_{i=1}^{t} e^{-i} \left(\frac{S_{t-i+1} - S_{t-i}}{S_{t-i}} \right). \tag{12}$$

To summarize, the financial market model, which accommodates both momentum and mean-

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[11] The use of the dividend yield is not fully uncontroversial, see Ang and Bekaert (2006) and Goyal and Welch (2003, 2006). However, Lettau and van Nieuwerburgh (2006) and Boudoukh et al. (2006) have shown recently that the predictive power of the dividend yield is significantly enhanced once structural breaks are allowed or the payout yield is used, respectively. Binsbergen and Koijen (2008) report strong predictability of stock returns using the economic restrictions implied by a present-value model.

reversion, is given by:

$$\frac{\mathrm{d}S_t}{S_t} = ((\mu_0 + \mu_1 X_t)(1 - \phi) + \phi M_t) \, \mathrm{d}t + \sigma_S' \mathrm{d}Z_t, \tag{13}$$

$$\mathrm{d}M_t = (1 - \phi)(\mu_0 + \mu_1 X_t - M_t) \, \mathrm{d}t + \sigma_S' \mathrm{d}Z_t, \tag{14}$$

$$\mathrm{d}X_t = -\alpha X_t \mathrm{d}t + \sigma_X' \mathrm{d}Z_t, \tag{15}$$

with $\sigma_X = \sigma_\mu / \mu_1$.

We use monthly US data over the period January 1946 up to December 2005 to estimate the model. The data is obtained from the Center for Research in Security Prices (CRSP). We use the return on both the value-weighted and equally-weighted index, including dividends, on the NYSE, NASDAQ, and AMEX markets. Lo and MacKinlay (1988) and Khil and Lee (2002) have shown that index momentum is more prominently present in the equally-weighted index, which renders a natural application of the model. To calculate the dividend yield, we first construct the dividend payout series using the value-weighted return including dividends, and the price index series associated with the value-weighted return excluding dividends. We take the dividend series to be the sum of dividend payments over the past year. The dividend yield is then the log of the ratio between the dividend and the current price index, in line with Campbell et al. (2003). We set the instantaneous short rate to $r = 4\%$ in annual terms.

In our model of the financial market as summarized in (13), (14), and (15), the uncertainty is driven by two independent Brownian motions. The volatility matrix of returns and the dividend yield is parameterized as:

$$\tilde{\Sigma} = \begin{pmatrix} \sigma_S' \\ \sigma_X' \end{pmatrix} = \begin{pmatrix} \sigma_{S(1)} & 0 \\ \sigma_{X(1)} & \sigma_{X(2)} \end{pmatrix}. \tag{16}$$

That is, $\tilde{\Sigma}$ is the Cholesky decomposition of the instantaneous variance matrix. The first element of Z is the return shock. The second element of Z is the shock to the dividend yield that is orthogonal to returns. This setup follows Sangvinatsos and Wachter (2005) and Binsbergen, Brandt, and Koijen (2005). Next, we discretize the continuous time model exactly at a monthly frequency

(see for instance Sangvinatsos and Wachter (2005)), which results in a tri-variate Gaussian VAR-model for log returns, the performance variable, and the dividend yield, which are all observed. We estimate the model using maximum likelihood restricted to ensure that we fit the empirical autocorrelation structure of stock returns. More specifically, we impose that the model-implied autocorrelations fit the first order sample autocorrelation of 1-month and 24-month returns exactly. For the value-weighted index, the first order autocorrelation of 1-month returns equals 6.2% and of 24-month returns (using non-overlapping returns) -6.9%. For the equally-weighted index, on the contrary, these numbers change to 19.9% and -7.7%, respectively. As such, the estimation method employed can be viewed as a GMM procedure in which both the efficient moments and economically motivated moments are combined. Finally, it is well-known that estimates of the unconditional expectation of persistent series, like the dividend yield, are inaccurate. Therefore, we de-mean the dividend yield using its sample average and use the de-meaned predictor variable, X_t, in estimation.

We are mainly interested in the interaction between momentum and mean-reversion in the strategic asset allocation problem of an investor who has a short to medium-term investment horizon. To draw a comparison with a model that only allows the dividend yield to predict the stock returns, we also estimate the model under the condition that $\phi = 0$. Since this model is, by construction, not able to accommodate both positive and negative autocorrelations in returns, we only require the long-term autocorrelation to match its sample counterpart. By re-estimating the model, we consider an investor who, potentially erroneously, disregards the information contained in the performance variable.

The model estimates in monthly terms are portrayed in Table 1. Panel A portrays the estimation results for the model of Section 2 ('Momentum and mean-reversion') and the restricted model in which only the dividend yield affects the equity risk premium ('Mean-reversion only'), i.e. $\phi = 0$. Panel B compares the sample estimates of the 1-month and 24-month returns first order autocorrelation with the model implied estimates. The results are presented for both the value-weighted index ('VW') and the equally-weighted index ('EW'). For the stock price process, two results are worth mentioning. First, the parameter ϕ, which weighs the relative importance of the momentum and mean-reversion variables in predicting equity returns, is almost three times

	Momentum and mean-reversion		Mean-reversion only	
	VW	EW	VW	EW

$$\frac{\mathrm{d}S_t}{S_t} = (\phi M_t + (1 - \phi)\mu_t)\,\mathrm{d}t + \sigma'_S\mathrm{d}Z_t,\ \mu_t = \mu_0 + \mu_1 X_t$$

	VW	EW	VW	EW
ϕ	0.15	0.39	0	0
μ_0	0.92%	1.15%	0.94%	1.20%
μ_1	0.016	0.012	0.015	0.0059
$\sigma_{S(1)}$	5.24%	6.28%	5.37%	6.68%

$$\mathrm{d}X_t = -\alpha X_t\mathrm{d}t + \sigma'_X\mathrm{d}Z_t$$

	VW	EW	VW	EW
α	0.011	0.0094	0.010	0.0076
$\sigma_{X(1)}$	-5.77%	-8.43%	-5.34%	-6.66%
$\sigma_{X(2)}$	1.35%	1.52%	1.35%	1.45%

Panel B: Autocorrelations

	Momentum and mean-reversion		Mean-reversion only	
	VW	EW	VW	EW
1 month				
Sample	6.2%	19.8%	6.2%	19.8%
Model implied	6.2%	19.8%	-0.3%	-0.4%
24 months				
Sample	-6.9%	-7.7%	-6.9%	-7.7%
Model implied	-6.9%	-7.7%	-6.9%	-7.7%

Table 1: Estimates of the financial market model in Section 2.
We use data over the period January 1946 up to December 2005 to estimate the financial market model in Section 2 ('Momentum and mean-reversion'). Panel A provides the estimates and Panel B portrays both the sample and model implied first-order autocorrelations of 1-month and 24-month returns. The column with the heading 'Mean-reversion only' presents the estimates of a restricted version of the model in which only the dividend yield affects the equity premium, i.e. $\phi = 0$. The model is estimated by means of maximum likelihood with constraints on the autocorrelation function. The abbreviation 'VW' refers to the value-weighted index and likewise 'EW' to the equally-weighted index. The main text provides further details.

higher in the equally-weighted than in the value-weighted index. As we already pointed out, we find that while past performance is a key driver of expected returns in the equally-weighted index, the mean-reversion variable plays a more prominent role in the value-weighted index. Second, the instantaneous volatility of stock returns, $\sigma_{S(1)}$, is lower in the case of the full model as opposed to the restricted model. This is a consequence of the enhanced predictive power of incorporating past performance information. Since the expected returns (μ_0) are comparable, this implies that the myopic allocation to stocks will be higher in the model that accommodates both momentum and mean-reversion. The estimates for the dividend yield reveal the well-documented features of financial ratios, namely high persistence and high (negative) correlation with stock return innovations.

Panel B illustrates that the model accommodating both momentum and mean-reversion is able to fit two autocorrelations with different signs excellently. The restricted model, which has only one state variable, can produce autocorrelations that are either negative or positive, but not both. This potential source of misspecification turns out to be particularly serious in the estimation based on the equally-weighted index.

To further illustrate the dynamics of stock returns in our model, we compute the impact of a return innovation on the future path of stock returns in absence of further shocks. To compute the impact of a return innovation, we define $Y_t^{(h)} = (\log S_t - \log S_{t-h}, M_t, X_t)$, in which h is the period over which we discretize the model. $h = 1$ corresponds to one month for our parameter estimates. Using the results in for instance Sangvinatsos and Wachter (2005), the model can be discretized exactly to a trivariate VAR(1) model:

$$Y_{t+h}^{(h)} = \mu_{(h)} + \Gamma_{(h)} Y_t^{(h)} + \Sigma_{(h)} \varepsilon_{t+h},$$

with:

$$\varepsilon_{t+h} \sim N(0, I), i.i.d.$$

$\Sigma_{(h)}$ is the Cholesky decomposition of the conditional covariance matrix, $var_t \left(Y_{t+h}^{(h)} \right) = \Sigma_{(h)} \Sigma'_{(h)}$, and is lower triangular as a result. The first innovation, $\varepsilon_{t+h(1)}$, therefore corresponds to (geometric)

index returns. We normalize $\log S_t = 0$ and compute the response of the index to an unexpected return as:

$$R(\tau) = \sum_{i=1}^{\tau} Y_{t+ih}, \tag{17}$$

with $\varepsilon_{t+h,(1)} = 1$ and all other innovations are set to zero ($\varepsilon_{t+h,(j)} = 0$ for $j > 1$, $\varepsilon_{t+ih} = 0$ for $i > 1$). To construct Figure 1, we set $h = 1$. We show results over a five-year horizon. The solid (equally-weighted index) and dashed (value-weighted index) curves show the behavior of the equally and value weighted indexes, if we give the first Brownian motion a shock of size $\sigma_{S(1)}$.

Both indexes behave qualitatively in a similar manner. Initially, the index levels go up to the monthly volatility of each index, which is slightly higher for the equally-weighted index (6.28% versus 5.24%). Due to the momentum effect, they further increase. At longer horizons, the mean-reversion effect becomes dominant. The index levels start to fall down steadily to their long-run values. Realized returns are positive for both indexes when momentum dominates, and negative thereafter. Quantitatively, however, the momentum effect is stronger and longer-lived in the equally-weighted case, and determines the separation between the two curves in the graph.

5 Empirical results

5.1 Optimal strategic allocation

In this section we study the implications of the estimates reported in Table 1 for strategic asset allocation. We first analyze the optimal investment strategy of an investor who accommodates both momentum and mean-reversion, and compare the results of this analysis to the case of an investor who restricts her attention to mean-reversion only. Next, we determine the economic loss, measured in utility terms, of not incorporating performance information in the investment policy. Finally, we assess the utility loss of behaving myopically with respect to momentum and mean-reversion.

The optimal allocations to stocks are portrayed in Figure 2 for an investor with a coefficient of relative risk aversion of $\gamma = 5$. The left panels provide the results for the value-weighted index and the right panels for the equally-weighted index. To emphasize the case of the medium term
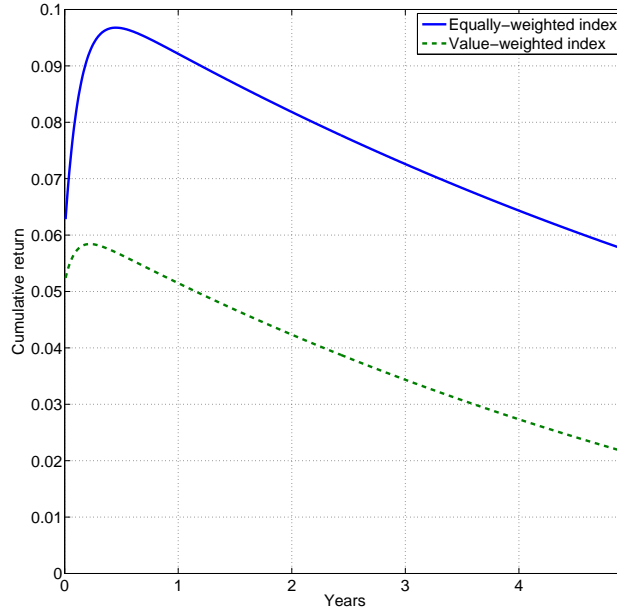
Figure 1: Response of the index to an innovation.
The figure displays the response of the equally- and value-weighted index to a single innovation. The blue solid line corresponds to the equally-weighted index, while the green dashed line corresponds to the value-weighted index. The horizontal axis depicts the time after the shock in years, and the vertical axis displays the cumulative return $(R(\tau))$.

investor, the two top panels focus on investment horizons of up to five years. The two bottom panels extend the results to investment horizons of up to 15 years, i.e. the long-term investment problem that has been studied rigorously in the literature.

Four aspects of Figure 2 are worth highlighting. First, the initial allocation to stocks, i.e. the myopic allocation, is always higher for the investor who uses performance information. This is because the enhanced predictive power sharpens the forecast of future returns, thereby reducing the conditional volatility. Second, investors who acknowledge the strategic nature of the investment problem hold significant hedging demands. The hedging demand of a long-term investor is positive and increases with the investment horizon. This phenomenon is due to long-term mean reversion in stock returns, and it has been widely documented in the literature. As a result of mean-reversion, long-term equity returns are less risky which induces an increase in the optimal allocation. On the other hand, the hedging demand of a short to medium-term investor is greatly affected by return continuation. Once the model is calibrated to the equally-weighted index, the hedging demand is
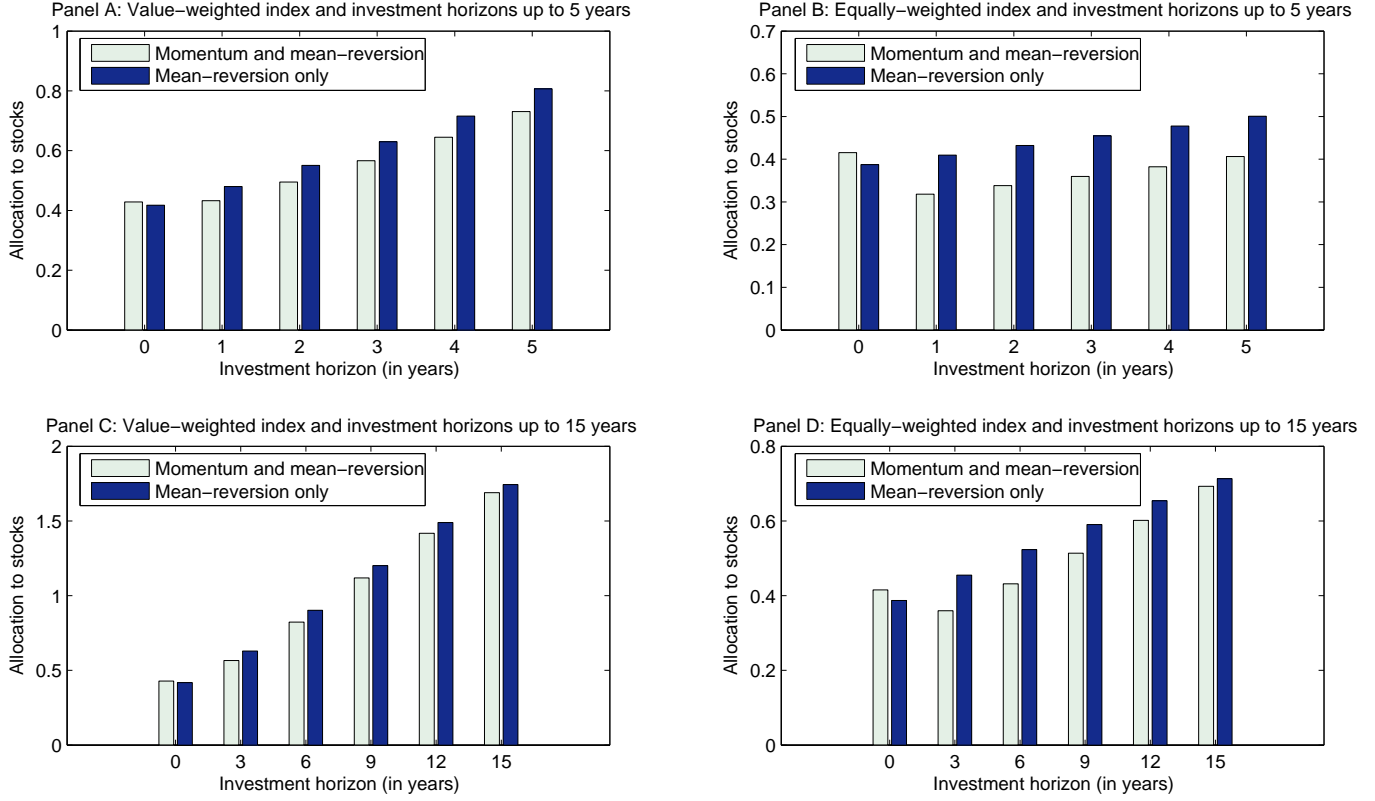
Figure 2: Optimal strategic allocation to stocks for different investment horizons.
Panel A and B consider investment horizons up to 5 years, whereas Panel C and D investment horizons up to 15 years. Panel A and C portray the results for the value-weighted index, whereas Panel B and D are based on the equally-weighted index. In all figures, the light blue bars correspond to the optimal strategy which incorporates momentum and mean-reversion. The dark blue bars, on the contrary, only incorporate mean-reversion. The investor's coefficient of relative risk aversion equals $\gamma = 5$. In all panels, the horizontal axis indicates the investment horizon and the vertical axis the optimal fraction of wealth allocated to stocks.

negative for an investor with an investment horizon of up to five years. Moreover, the hedging demand is also negative for an investor with an investment horizon of up to one year once the model is calibrated to the value-weighted index, even when the evidence of return continuation in this index is rather weak. Momentum leads to an increase of the riskiness of short- to medium-term stock returns. This induces the investor to decrease the optimal allocation to stocks. These results suggest that ignoring return continuation may have nontrivial consequences for the investment policies of medium-term investors. As an example, consider in panel B the investor who has a 5-year investment horizon and who accounts for both momentum and mean-reversion. This investor allocates slightly less than 40% to stocks, while the investor that ignores the presence of return

continuation allocates more than 50% to stocks, a relative increase of 25%.

Third, it turns out that at a long horizon of, say, 15 years, the differences between the two investment strategies are negligible. This, together with the negativity of the hedging demand at medium-term horizons discussed above, implies that the total allocation to stocks is not monotonic in the investment horizon. A striking consequence of the non-monotonicity of the total allocation to stocks is that there exists a positive investment horizon at which the hedging demands to hedge momentum and mean-reversion cancel out exactly, and so the investor behaves as if she were myopic. This investment horizon is about 5 years for the equally-weighted index, and about one year for the value-weighted index. These results are a direct consequence of the interplay between continuation and reversal, and they have not been explored in the literature so far.

Fourth, we find that time-variation in expected returns of the value-weighted index is predominantly driven by changes in the mean-reversion variable, in our case the dividend yield. In contrast, for the equally-weighted index it is mainly performance information what matters for investors with investment horizons up to five years.

To highlight the different impact of momentum and mean-reversion on the strategic allocation, we decompose the total hedging demands into two components for the model that allows for predictability by the performance variable and the dividend yield. The total hedging demands are given by:

$$\frac{1}{\gamma} \left(\sigma'_S \sigma_S \right)^{-1} \sigma'_S \Sigma' J_{WY}, \tag{18}$$

with J_{WY} the partial derivative with respect to wealth W, and the state variables $Y = (M, \mu)$. The hedging demand induced by momentum is therefore given by:

$$\frac{1}{\gamma} \left(\sigma'_S \sigma_S \right)^{-1} \sigma'_S \sigma_S J_{WM} = \frac{1}{\gamma} J_{WM}, \tag{19}$$

and likewise for mean-reversion by:

$$\frac{1}{\gamma} \left(\sigma'_S \sigma_S \right)^{-1} \sigma'_S \sigma_\mu J_{W\mu}. \tag{20}$$

Figure 3 portrays this decomposition of the total hedging demand for the same horizons and indices as in Figure 2. Note that, by definition, both components sum exactly to the total hedging demand. In all cases, the hedging demand of a myopic investor equals by default zero. For longer investment horizons, investors optimally hold significant hedging demands. Once calibrated to the value-weighted index, the hedging demand to hedge mean-reversion dominates the hedging demand induced by momentum after an investment horizon of one year. This is caused by the high persistence of the dividend yield, and also by the relatively minor importance of the performance variable. In case of calibration to the equally-weighted index, we find that the hedging demand induced by momentum exceeds the one induced by mean-reversion, and the total demand is therefore negative. At an investment horizon of 5 years, the two hedging demands approximately cancel and the investor behaves myopically. For longer investment horizons, hedging time-variation in investment opportunities caused by the dividend yield becomes more important, and the total hedging demand is positive.

5.2 Economic costs of sub-optimal strategic allocations

In this section we assess the economic importance of i) taking into account momentum for strategic asset allocation and ii) acting strategically as opposed to myopically. We construct various sub-optimal strategies that shed light on the aspects of the optimal strategic allocation that matter to investors.

We first address the question of how important momentum is, especially for investors with investment horizons up to five years. We therefore evaluate the strategy resulting from the model that ignores momentum, i.e. $\phi = 0$, in the unrestricted model. Recall that the restricted model and the unrestricted model are estimated separately, although the same data and estimation procedures are used in both cases. By estimating a separate model under the constraint, we mimic the behavior of an investor who considers momentum to be of minor importance for her strategic allocation. For both the optimal and sub-optimal investment strategies we determine the certainty equivalent return, and report the annualized loss in certainty equivalent wealth from following a sub-optimal strategic allocation. More specifically, if we denote the value function resulting from the optimal
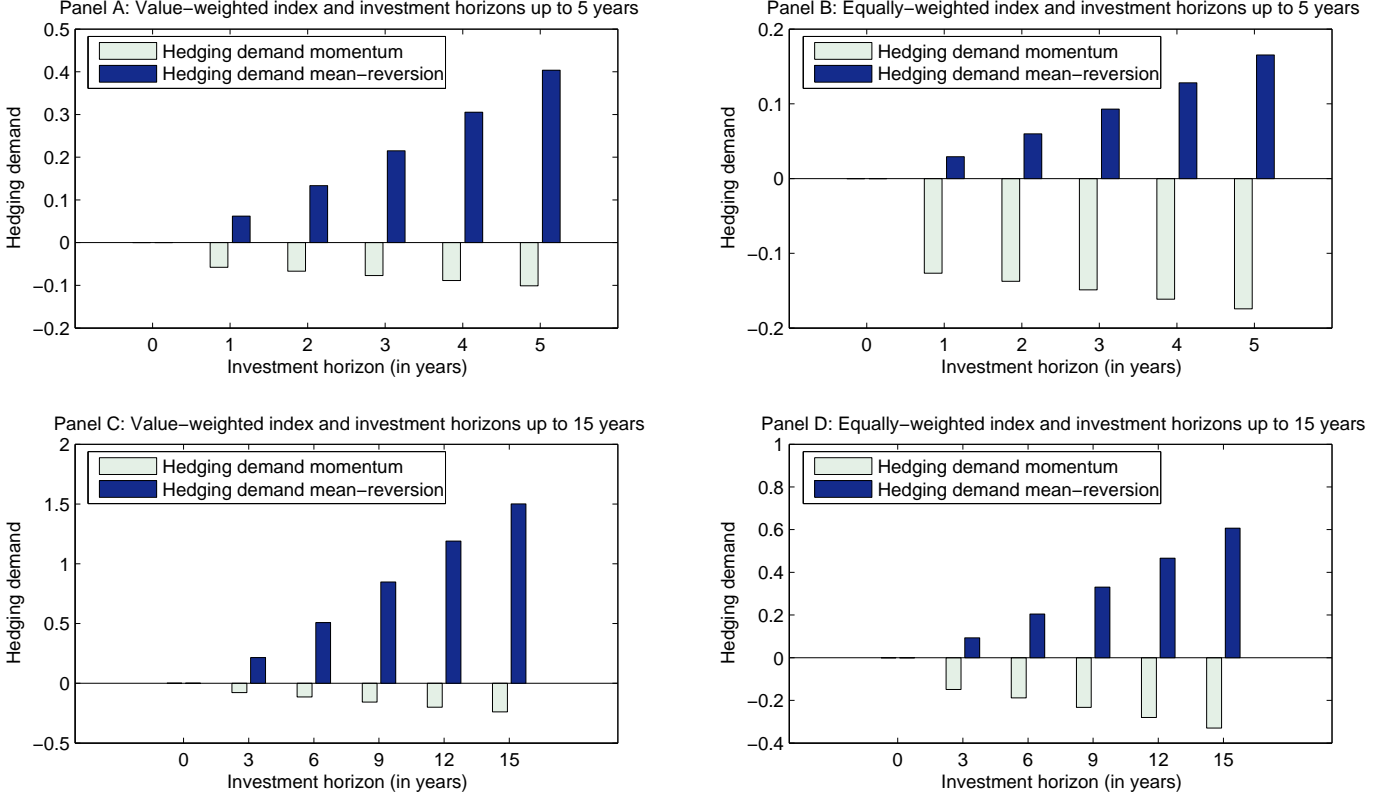
**Figure 3: Decomposition of the total hedging demands.**
Panel A and B consider investment horizons up to 5 years, whereas Panel C and D investment horizons up to 15 years. Panel A and C portray the results for the value-weighted index, whereas Panel B and D are based on the equally-weighted index. In all figures, the light blue bars correspond to the hedging demands induced by momentum. The dark blue bars, on the contrary, correspond to the hedging demands due to mean-reversion. The investor's coefficient of relative risk aversion equals $\gamma = 5$. In all panels, the horizontal axis indicates the investment horizon and the vertical axis the optimal fraction of wealth allocated to stocks.

strategy by J_1 and from the sub-optimal by J_2, the annualized utility costs are given by:

$$C = \left(\frac{J_2}{J_1}\right)^{\frac{1}{T(1-\gamma)}} - 1, \tag{21}$$

with T the investment horizon expressed in years. The utility costs are portrayed in Figure 4.

Panel A portrays the utility costs for the value-weighted index and Panel B for the equally-weighted index. We find that the utility costs of ignoring momentum are substantial for both indices. This is remarkable, given that the evidence of return continuation in the value-weighted index is rather weak. Consider for instance an investor with a coefficient of relative risk aversion
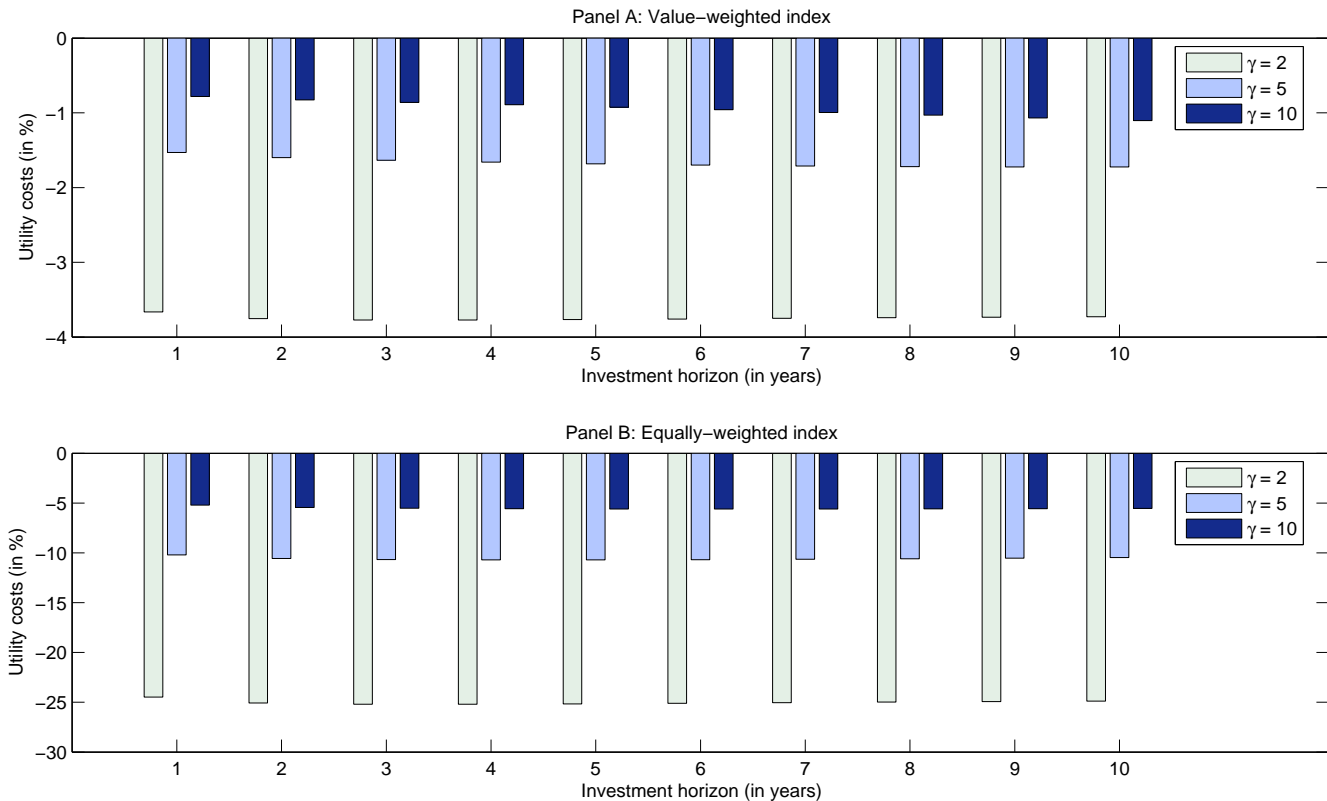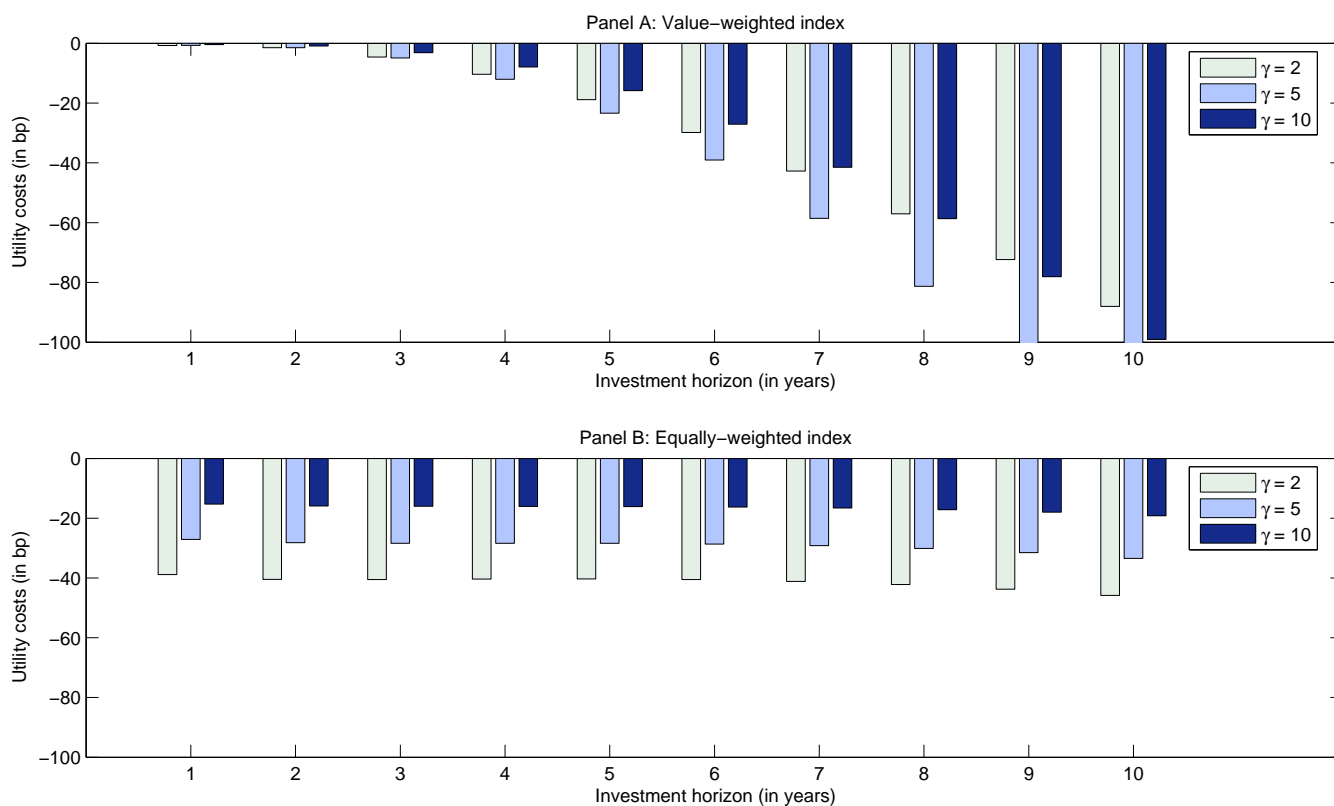
Figure 4: Annualized utility costs of ignoring momentum.
Panel A portrays the utility costs of ignoring momentum for the value-weighted index and Panel B for the equally-weighted index. In order to determine the utility costs, a model which imposes the constraint $\phi = 0$ is estimated. The horizontal axis depicts the investment horizon and the vertical axis the utility costs in percent per year. The results are portrayed for three levels of risk aversion, namely $\gamma = 2, 5, 10$. The main text provides further details.

of $\gamma = 5$. This investor is willing to sacrifice 1.5% per year to be able to implement a momentum strategy on the basis of the value-weighted index and almost 10% in the case of the equally-weighted index, for which past returns have strong predictive power. Note that the annualized utility costs of ignoring momentum are (approximately) invariant to the investment horizon. Despite the strong support for the importance of momentum in strategic problems, these numbers are to be interpreted with some caution. Once the investor incorporates performance information into the investment strategy, the investment policy becomes substantially more volatile. Even small degrees of drift predictability will be maximally exploited in our continuous time model. In practice, however, investors have to trade at discrete intervals and incur in transaction costs once implementing the strategy. Therefore, we will perform the same calculations at the end of this section for an investor

who trades in discrete time and is subject to borrowing and short-sales constraints.

A large part of the utility costs described above arises from the improved tactical opportunities offered by return predictability via past performance. In what follows we also study the strategic aspects of momentum and mean-reversion. Towards this end, we compare a myopic strategy with the optimal strategy, which accounts for time-variation in investment opportunities. An investor implementing the myopic strategy invests the sum of the first three components of equation (10), and therefore does not hold any intertemporal hedging demands. Both strategies are evaluated using the estimated parameters of our general model. The results are depicted in Figure 5.

Figure 5: Annualized utility costs of behaving myopically.
Panel A portrays the utility costs of behaving myopically for the value-weighted index and Panel B for the equally-weighted index. The horizontal axis depicts the investment horizon and the vertical axis the utility costs in basis points per year. The results are portrayed for three levels of risk aversion, namely $\gamma = 2, 5, 10$. The main text provides further details.

Panel A corresponds again to the value-weighted index and Panel B to the equally-weighted index. The results for the two indices are quite different, which reveals the fundamental difference

between momentum and mean-reversion from a strategic perspective. The utility costs of behaving myopically depends crucially on the persistence of predictor variables. If the predictor variable is more persistent, the utility costs of behaving myopically increase more strongly in the investor's horizon. In the context of our model, we find that the dividend yield is more persistent than the performance variable. Also, the mean-reversion variable is more important for the value-weighted index, while the performance variable is more important for the equally-weighted index. This implies that the utility costs of behaving myopically are more dependent on the investor's horizon in case of the value-weighted index. However, it does imply that behaving strategically is also important for a stock index whose expected return is influenced mainly by past performance. Due to the low persistence of return information, the value of hedging is high even for short horizons, but remains constant afterwards. In contrast, for stock indices whose returns are predicted by the dividend yield, the value of hedging for the horizons we are interested in is rather modest. This is due to the high persistence of the dividend yield over time.

We find that the costs of behaving myopically can be higher for less risk-averse investors. This is a consequence of the fact that the utility costs of behaving myopically are generally hump-shaped in risk aversion. For a log-utility investor ($\gamma = 1$), the myopic portfolio is optimal and the utility costs are by definition zero. For an investor who is infinitely risk averse ($\gamma \to \infty$), it is optimal to allocate all wealth to the risk-free cash account. The utility costs of behaving myopically are therefore also equal to zero. For intermediate values of the coefficient of relative risk aversion, that is, $\gamma \in (1, \infty)$, the costs of behaving myopically are strictly positive. Depending on the exact level of risk aversion for which the utility costs of behaving myopically are maximized, the costs can be higher for an investor who is less risk averse.

Staggering as they are, even for the model calibrated to the value-weighted index, the costs of ignoring momentum shown in Figure 4 have been derived under the assumptions that the investor can trade continuously and without any restrictions. Under these stylized assumptions the investor will fully exploit the information in the performance variable. In Table 2 we measure the costs of ignoring momentum for an investor who has been restricted to trade at higher frequencies and on whom borrowing and short-sales constraints have been imposed. The optimal investment strategy and value function have been determined using the simulation-based approach developed in Brandt,

Panel A: Value-weighted index					
γ	2	4	6	8	10
Monthly	-0.3%	-0.4%	-0.4%	-0.3%	-0.3%
Quarterly	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%
Annually	0.0%	0.0%	0.0%	0.0%	0.0%

Panel B: Equally-weighted index					
γ	2	4	6	8	10
Monthly	-4.3%	-3.8%	-2.8%	-2.1%	-1.7%
Quarterly	-2.0%	-1.6%	-1.0%	-0.7%	-0.4%
Annually	-0.2%	-0.1%	0.0%	0.0%	0.0%

Table 2: Annualized utility costs of ignoring momentum for a constrained investor.
Panel A portrays the utility costs of ignoring momentum for the value-weighted index and Panel B for the equally-weighted index. The investor is subject to borrowing and short-sales constraints and trades in discrete time. The trading frequency is either monthly, quarterly, or annually. The utility costs are presented in annualized terms. The results are portrayed for risk aversion levels $\gamma = 2, 4, 6, 8$, and 10. The main text provides further details.

Goyal, Santa-Clara, and Stroud (2005). We refer to Appendix B for further details on the numerical procedure. While less pronounced, the utility gains are remarkable at the monthly trading frequency for the equally-weighted index, and still noticeable for the value-weighted index. An investor with a risk aversion coefficient equal to 2, and trading at this frequency on the equally-weighted (value-weighted) index, is willing to give up 4.5% (30bp) per year of her initial wealth to exploit the information contained in the performance variable. As expected, the gains drop rapidly as the trading frequency decreases. The gains at the quarterly frequency are 2.1% (equally-weighted) and 10bp (value-weighted), and they vanish at the yearly frequency. Thus, while confirming the findings of Campbell, Chan, and Viceira (2003) at quarterly and annual levels, we ascribe them to the trading frequency used. On a monthly basis, we find that investors can benefit from exploiting the information contained in the performance variable.

6 Conclusions

We develop a new model that captures two key predictability features of stock returns: short-term momentum and long-term mean reversion. We derive the optimal strategic asset allocation in closed form. We calibrate our model to the CRSP value- and equally-weighted indices and draw three main conclusions. First, the optimal allocation to stocks is no longer monotone in the investment horizon: it first decreases, because momentum makes stocks riskier in the short run, and subsequently increases, because mean-reversion reduces risk in the long run. Empirically, we find that it takes up to five years (one year) for the equally-weighted (value-weighted) index before the strategic allocation exceeds the myopic allocation. These results challenge the known advice that the allocation to equities increases monotonically with the investor's horizon.

Second, we estimate the utility costs of ignoring momentum. We find that the annualized costs are substantial and relatively insensitive to the investor's horizon. Third, we estimate the costs of behaving myopically as opposed to strategically and find that, when the model is calibrated to the value-weighted index, the costs of acting myopically are gradually increasing in the investor's horizon, although they are relatively small for horizons of up to 5 years. For the equally-weighted index, the value of hedging is substantial at short investment horizons, but remains (approximately) constant up to 5 to 6 years. At this point, mean-reversion is more important, and the value of hedging gradually increases. We show that the utility gains from conditioning the investment strategy on the performance variable persist if the investor is subject to borrowing and short-sales constraints and trades at a monthly frequency. At an annual trading frequency, our estimates indicate that investors cannot benefit anymore from the information contained in the performance variable.

This paper can be extended in various directions. One natural extension of our model is to assess the impact of transaction costs, and parameter uncertainty. Second, we treat momentum as a time-series phenomenon, although there is evidence that cross-serial dependencies may render another source of momentum profits (see Lewellen (2002)). Our framework can be extended to allow for multiple assets. Third, there is abundant empirical evidence indicating that volatilities and correlations are time-varying. Simple rolling estimates suggest that autocorrelations also shift

considerably over time, which we consider to be yet another interesting avenue to explore.

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A Optimal and affine portfolio strategies

In this appendix, we derive the optimal strategic allocation to stocks and the cash account, and the corresponding value function. We first solve for the induced utility function for investment strategies which are affine in the state variables. Next, we show that the optimal policy constitutes a special case of this class of strategies. Using the more general result on strategies which are affine in the state variables, we can determine the utility costs of sub-optimal strategies in Section 5.2. Similar problems have been solved in Sangvinatsos and Wachter (2005) and Liu (2006).

We introduce the following reduced form notation for the dynamics of the financial market model presented in Section 2:

$$\frac{\mathrm{d}S_t}{S_t} = \left(\zeta'Y_t\right)\mathrm{d}t + \sigma_S'\mathrm{d}Z_t, \tag{A.1}$$

$$\mathrm{d}Y_t = \left(\kappa_0 + KY_t\right)\mathrm{d}t + \Sigma\mathrm{d}Z_t, \tag{A.2}$$

with:

$$\zeta = \begin{pmatrix} \phi \\ 1-\phi \end{pmatrix}, \; \kappa_0 = \begin{pmatrix} 0 \\ \alpha\mu_0 \end{pmatrix}, \; K = \begin{pmatrix} -(1-\phi) & (1-\phi) \\ 0 & -\alpha \end{pmatrix}, \tag{A.3}$$

and $\Sigma = (\sigma_S, \sigma_\mu)'$ and $Y_t = (M_t, \mu_t)$.

We next determine the induced utility function of an investment strategy which is affine in the state variables:

$$\pi(\tau, Y) = \theta_0(\tau) + \theta_1(\tau)'Y, \tag{A.4}$$

with $\tau = T - t$ the remaining investment horizon. Both the optimal investment strategy (as we show later formally) and the sub-optimal policies considered in Section 5.2 are of the form of (A.4). The corresponding wealth dynamics is given by:

$$\frac{\mathrm{d}W_t}{W_t} = \left(\pi(\tau, Y_t)\left(\mu_t + \phi\left(M_t - \mu_t\right) - r\right) + r\right)\mathrm{d}t + \pi(\tau, Y_t)\sigma_S'\mathrm{d}Z_t, \tag{A.5}$$

and the utility derived from such a policy is defined by:

$$J\left(W_t, Y_t, \tau\right) = \mathbb{E}_t\left(\frac{W_T^{1-\gamma}}{1-\gamma}\right) \tag{A.6}$$

The induced utility function is a conditional expectation and consequently a martingale. This implies that the drift of the value function satisfies the partial differential equation (PDE):

$$\mathcal{L}J + J_t = 0, \tag{A.7}$$

with $\mathcal{L}$ indicating the infinitesimal generator and subscripts denoting partial derivatives, in this case with respect to time.

It is well-known that the utility function induced by affine policies in an affine financial market model is exponentially quadratic in the state vector Y_t, i.e.:

$$J(W_t, Y_t, \tau) = \frac{W_t^{1-\gamma}}{1-\gamma} H(Y_t, \tau), \tag{A.8}$$

$$H(Y_t, \tau) \equiv \exp\left(\frac{1}{2}Y_t'(\tau)Y_t + B(\tau)'Y_t + C(\tau)\right), \tag{A.9}$$

$$H(Y_t, 0) = 1, \tag{A.10}$$

with $A \in \mathbb{R}^{2\times2}$, $B \in \mathbb{R}^{2\times1}$, $C \in \mathbb{R}$. Substitution of the induced utility function in (A.8) in the PDE in (A.7) and exploiting the affine structure of the investment strategy in (A.4), we obtain a system of ordinary differential equations (ODEs) for the value function coefficients:[12]

$$\begin{aligned}
\dot{A}(\tau) =\ & 2(1-\gamma)\theta_1\zeta' - \gamma(1-\gamma)\sigma_S'\sigma_S\theta_1\theta_1' + K'\left(A(\tau) + A(\tau)'\right) \\
& + (1-\gamma)\theta_1\sigma_S'\Sigma'\left(A(\tau) + A(\tau)'\right) \\
& + \frac{1}{4}\left(A(\tau) + A(\tau)'\right)\Sigma\Sigma'\left(A(\tau) + A(\tau)'\right),
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
\dot{B}(\tau)' =\ & (1-\gamma)\left(\theta_0\zeta' - r\theta_1'\right) - \gamma(1-\gamma)\sigma_S'\sigma_S\theta_0\theta_1' \\
& + \frac{1}{2}\kappa_0'\left(A(\tau) + A(\tau)'\right) + B(\tau)'K \\
& + \frac{1}{2}(1-\gamma)\theta_0\sigma_S'\Sigma'\left(A(\tau) + A(\tau)'\right) \\
& + (1-\gamma)B(\tau)'\Sigma\sigma_S\theta_1' + \frac{1}{2}B(\tau)'\Sigma\Sigma'\left(A(\tau) + A(\tau)'\right),
\end{aligned} \tag{A.12}$$

$$\begin{aligned}
\dot{C}(\tau) =\ & (1-\gamma)(1-\theta_0)r - \frac{1}{2}\gamma(1-\gamma)\theta_0^2\sigma_S'\sigma_S + \kappa_0'B(\tau) + \frac{1}{2}B(\tau)'\Sigma\Sigma'B(\tau) \\
& + \frac{1}{4}\text{trace}\left(\left(A(\tau) + A(\tau)'\right)\Sigma\Sigma'\right) + (1-\gamma)\theta_0\sigma_S'\Sigma'B(\tau),
\end{aligned} \tag{A.13}$$

together with the boundary conditions:

$$A(0) = 0_{2\times2},\ B(0) = 0_{2\times1},\ C(0) = 0. \tag{A.14}$$

Solving for A, B, and C restitutes the utility function J induced by the affine policy (A.4).

Next, we derive the optimal strategic allocation and show that it is affine in the state variables. The Bellman equation corresponding to the optimization problem specified in (8) subject to (9) is

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[12]We drop the indices of the policy coefficients θ_0 and θ_1 for notational convenience.

31

given by:

$$\max_{\pi}(\mathcal{L}J + J_t) = 0, \tag{A.15}$$

which leads to the following first order condition:

$$J_W W_t(\zeta' Y_t - r) + J_{WW} W_t^2 \pi^\star(\tau, Y_t)\sigma'_S\sigma_S + W_t\sigma'_S\Sigma' J_{WY} = 0. \tag{A.16}$$

We conjecture the value function to be of the exponentially quadratic form as in (A.8). This results immediately in the optimal strategic allocation given in Proposition 1 with $\theta_0^\star$ and $\theta_1^\star$ given by:

$$\theta_0^\star(\tau) \;\;=\;\; \frac{-r}{\gamma\sigma'_S\sigma_S} + \frac{\sigma'_S\Sigma'B(\tau)}{\gamma\sigma'_S\sigma_S}, \tag{A.17}$$

$$\theta_1^\star(\tau) \;\;=\;\; \frac{1}{\gamma\sigma'_S\sigma_S}\begin{pmatrix} \phi \\ 1-\phi \end{pmatrix} + \frac{1}{2\gamma\sigma'_S\sigma_S}\left(A(\tau) + A(\tau)'\right)\Sigma\sigma_S. \tag{A.18}$$

These coefficients of the optimal strategic allocation can be used in turn to determine the value function coefficients.

B Optimal portfolio choice in discrete time

We use the simulation-based approach to portfolio choice as it has been introduced by Brandt et al. (2005). We consider an investor which optimizes its portfolio subject to short-sale and borrowing constraints. The discrete time problem is then given by

$$\max_{(x_s)_{s=t}^{T-1}\in\mathcal{K}} \mathbb{E}_t\left(\frac{1}{1-\gamma}W_T^{1-\gamma}\right), \tag{B.1}$$

with W_t indicating wealth, subject to the dynamic budget constraint

$$W_{t+1} = W_t\left(x_t\frac{S_{t+1}}{S_t} + (1 - x_t)\exp(r)\right), \tag{B.2}$$

with

$$\mathcal{K} = \{x \mid x \geq 0, x^\top\iota \leq 1\}. \tag{B.3}$$

The principle of dynamic programming is used to determine the optimal portfolio strategy. Starting at time $T-1$, we first solve

$$\max_{x\in\mathcal{K}} \mathbb{E}_{T-1}\left(\frac{1}{1-\gamma}\left(x_{T-1}\frac{S_T}{S_{T-1}} + (1 - x_{T-1})\exp(r)\right)^{1-\gamma}\right), \tag{B.4}$$

where the homogeneity of the power utility index is exploited. The main complication is that this

conditional expectation cannot be calculated analytically. In line with Brandt et al. (2005) and Longstaff and Schwartz (2001), we approximate the conditional expectation via a projection on a set of basis functions in the state variables, i.e.

$$\mathbb{E}_{T-1}\left(\frac{1}{1-\gamma}\left(x_{T-1}\frac{S_T}{S_{T-1}}+(1-x_{T-1})\exp(r)\right)^{1-\gamma}\right) \simeq \zeta^\top f\left(Y_{T-1}\right), \tag{B.5}$$

where ζ denote the projection coefficients and $Y = (M, D)^\top$. In order to estimate the projection coefficients, ζ, we simulate M paths of both state variables and stock returns on the basis of the discrete time model. We indicate the paths with $(\omega_1, ..., \omega_M)$. Next, the projection coefficients are estimated via a cross-sectional regression across all paths, which results in the following estimator

$$\begin{aligned}
\hat{\zeta} &= \left(\sum_{i=1}^M f\left(Y_{T-1}(\omega_i)\right) f\left(Y_{T-1}(\omega_i)\right)^\top\right)^{-1} \\
&\times \left(\frac{1}{1-\gamma}\sum_{i=1}^M f\left(Y_{T-1}(\omega_i)\right)\left(x_{T-1}\frac{S_T}{S_{T-1}}(\omega_i)+(1-x_{T-1})\exp(r)\right)^{1-\gamma}\right),
\end{aligned} \tag{B.6}$$

where $f\left(X_{T-1}(\omega_i)\right)$ is a column vector containing the values of the basis functions, evaluated at the state variables in branch ω_i. Next, we consider a grid of possible portfolios at every branch at every point in time and solve for the optimal portfolio, with a grid space of one percent. This avoids numerical problems possibly arising when we apply the iterative approach used in Brandt et al. (2005). Along these lines, we can proceed backwards and solve for the optimal investment strategy at each point in time. This procedure results in the optimal initial strategic allocation and an estimate of the value function. This method is in particular well suited to deal with restricted information sets, since we can easily reduce the state vector. This allows us to estimate the utility gain from conditioning the investment strategy on both the mean-reversion and the performance variable as opposed to only the mean-reversion variable.