Discrete Math Set Theory

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1 Introduction

2 Questions

1. Given the sets $A = \{2, 20, 22, 5, 13\}$ and $B = \{6, 9, 20, 12, 22\}$, what is the sum of all the elements in $A \cup B$?

   - 89
   - 131
   - 47
   - 98

   Answer: 89

   $A \cup B$ is the set of elements in A or B. So we have $A \cup B = A + B - A \cap B$, where $A \cap B = \{20, 22\}$. Thus $A \cup B = \{2, 5, 6, 9, 12, 13, 20, 22\}$. Hence the sum is $2 + 5 + 6 + 9 + 12 + 13 + 20 + 22 = 89$.

2. Symmetric Difference

   $$A = \{x \mid -8 < x < 4, x \text{ is an integer}\},$$

   $$B = \{x \mid -2 < x \leq 11, x \text{ is an integer}\}.$$

   What is $|A \triangle B|$, the size of the symmetric difference of A and B?

   - 13
   - 11
   - 14
   - 9

   Answer: 14

   Observe that $A \cup B$ is the set of integers x satisfying $-8 < x \leq 11$ and $A \cap B$ is the set of integers x satisfying $-2 < x < 4$. This implies

   $$\begin{aligned} A \triangle B &= (A \cup B) - (A \cap B) \\ &= \{x \mid -8 < x \leq -2, \text{ or } 4 \leq x \leq 11, x \text{ is an integer}\}. \end{aligned}$$

Thus, the number of elements in the set $A \triangle B$ is

$$|A \triangle B| = (-2 - (-8)) + (11 - 4 + 1) = 14.$$

3. In counting the numbers from 1 to 99 (inclusive), what is the total number of times that the digit 3 is used?

   Details and assumptions The number 12 uses the digit 1 once and the digit 2 once. The number 111 uses the digit 1 three times.

   - 3
   - 9
   - 10
   - 20

   Answer: 20

4. Ian picked 68 distinct integers out of the first 100 positive integers. What is the minimum number of odd integers that Ian must have picked?

   - 50
   - 18
   - 34
   - 51

   Answer: 18

   Since there are 50 even integers out of the first 100 positive integers, at most 50 of the distinct integers in are even. Hence, at least 68 - 50 = 18 integers are odd. It is clear how to create such a set - pick all 50 even integers, and then any 18 odd integers.

5. 100 students take an exam with two questions a and b. If 66 students solved question a, 54 students solved question b, and 18 students solved neither of the two questions, how many students solved only a?

   - 28
   - 36
   - 38
   - 34

   Answer: 28

   Let A and B be the sets of students who solved a and b, respectively. Then the number of students who solved at least one question is $|A \cup B| = 100 - 18 = 82$ because 18 students out of 100 solved neither of the two questions. Hence, the number of students who solved both a and b can be calculated as follows:

$$\begin{aligned}|A \cap B| &= |A| + |B| - |A \cup B| \\ &= 66 + 54 - 82 \\ &= 38.\end{aligned}$$

Thus, the number of students who solved only a, or equivalently, who solved a but did not solve b is

$$\begin{aligned}|A \setminus B| &= |A| - |A \cap B| \\ &= 66 - 38 \\ &= 28.\end{aligned}$$

6. Person "A" says the truth 60% of the time, and person "B" does so 90% of the time. In what percentage of cases are they likely to contradict each other in stating the same fact?

   - 42%
   - 54%
   - 60%
   - 36%

   Answer: 42%

   when they both say true the probability is 60*90/100=54;

   When the both say false at same time is 40*10/100=4; So the say always the same fact is 54+4=58%; So the percentage of cases that they contradict each other is 100-58=42%