

# Determinants

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## 1 Determinant Computations

1. Compute the determinant of the following matrix

$$A = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$$

- 8
- 17
- 22
- 35

Answer:  $\det(A) = 22$  because  $(3)(4) - (5)(-2) = 12 + 10 = 22$

2. Without evaluating, give the reason why the determinant of the following matrix is equal to zero

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 4 & 3 & 0 \\ 1 & 2 & 3 & 8 \end{pmatrix}$$

- A has two identical rows (columns)
- one row(column) is a multiple of another
- rows (column) of zeros

Answer: it consist a row of zeros so the answer will be a multiple of zeros

3. Compute the determinant of the following matrix

$$A = \begin{pmatrix} 5 & 7 & 2 & -1 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 8 \end{pmatrix}$$

- 300
- 120

- 72
- 240

Answer:  $\det(A) = 240$  since  $A$  is a triangular matrix, it  $\det(A)$  will be the product of the entries on the main diagonal

4. Compute the minor of the following matrix

$$A = \begin{pmatrix} \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix} \end{pmatrix} \text{ What is } M_{13}?$$

- 0
- 1
- 2
- 7

Answer: the answer is 0 since  $M_{13} = \begin{pmatrix} \begin{bmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{bmatrix} \end{pmatrix}$  which equals to  $(0 + 0 + 12) - (12 + 0 + 0) = 0$

5. Find all values of  $x$  for which  $\det(A) = 0$

$$A = \begin{pmatrix} \begin{bmatrix} x-1 & 0 \\ 2 & x+1 \end{bmatrix} \end{pmatrix}$$

- +1
- -1
- A and B
- None of the above

Answer: Both +1 and - works. Since  $\det(A) = (x+1)(x-1)$ , so  $\det(A) = 0$  if and only if  $x = 1$  or  $x = -1$

6. What is the maximum number of zeroes that a 3 by 3 matrix can have without having a zero determinant?

- 3
- 4
- 5
- 6

Answer: the matrix can have at most 6 zeros without determinant being 0. we can show by letting the matrix be a diagonal matrix with nonzero diagonal entries.