Math 586: Lecture 1

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and Computer Science
University of Illinois at Chicago

Math 586: Lecture 1

Course Introduction

- Math 586: Computational Finance
- Lecture: Monday, Wednesday, and Friday at 10am in Taft Hall, Room 300 (**for now . . .**).
- Instructor: Professor David Nicholls
  - Office: Science and Engineering Offices (SEO) 1219,
  - E-mail: `davidn@uic.edu`,
  - Phone: 312-413-1641.
- Office Hours: Wednesdays 2pm-3pm.
- Text: Wilmott, Howison, & Dewynne, *The Mathematics of Financial Derivatives: A Student Introduction*.

Course Grading

- There are two components to your grade:
  - Four homeworks (60 %)
  - Final project (40 %).
- Homeworks: Homeworks are due **by 2pm**:
  - Friday, February 7,
  - Friday, February 28,
  - Friday, March 21,
  - Friday, April 18.

  HOMEWORKS WILL BE PENALIZED 10 % PER DAY LATE.
- Students with Disabilities: If you are registered with the Office of Disability Services (ODS) please see me ASAP to make arrangements.

My Background

- B.S. (Math) from University of Illinois at Urbana–Champaign.
- Sc.M. (Applied Math) from Brown University.
- Ph.D. (Applied Math) from Brown University.
- Dunham Jackson Assistant Professor at University of Minnesota.
- Assistant Professor at University of Notre Dame.
- Assistant, Associate, and now Full Professor at University of Illinois at Chicago.
- Director of Undergraduate Studies at UIC (2011–2013).

Finance Books

There are many excellent books on mathematical finance and I will borrow from several. Here is a partial list:

1. Higham, *An Introduction to Financial Option Valuation*
2. Wang, *Monte Carlo Simulation with Applications to Finance*
3. Baxter & Rennie, *Financial Calculus : An Introduction to Derivative Pricing*
4. Hull, *Options, Futures, and Other Derivatives*
5. Brandimarte, *Numerical Methods in Finance and Economics: A MATLAB–Based Introduction*
6. Seydel, *Tools for Computational Finance*
7. Kwok, *Mathematical Models of Financial Derivatives*

Chapter 1 (Higham): Options

The term **asset** is used to describe any financial object whose value is known at present but will likely change in the future, e.g.,

- Shares in a company
- Commodities such as gold, oil, or electricity
- Currencies, for example, the value of US$ 100 in euros

Definition: A **European call option** gives its **holder** the right (but not the obligation) to purchase from the **writer** a prescribed asset for a prescribed price at a prescribed time in the future.

- The prescribed purchase price is the **exercise** or **strike price**.
- The prescribed time in the future is known as the **expiry date**.

Example

Example: Suppose that I (the writer) **write** a European call option that gives you (the holder) the right to buy 10 shares of Google stock (On January 7 the price was $S(0) = 734.75$) for \$ 7350 three months from now. After three months there are two actions to take:

1. If the actual value of 10 Google shares turns out to be more than \$ 7350 (say \$ 8000) you would **exercise** your right to buy the shares because you could immediately sell them for a profit (say \$ 8000-\$ 7350=\$ 650).
2. If the actual value of 10 Google shares turns out to be less than \$ 7350 you would **not exercise** as the deal would not be worthwhile.

Since you are not **obliged** to purchase the shares, you **cannot** lose money. Similarly, because there is no coercion, I **cannot** make money! Therefore, this option has a value.

Question: What is the value of this European Call Option?

Put Options and the Study of Options

Definition: A **European put option** gives its **holder** the right (but not the obligation) to sell to the **writer** a prescribed asset for a prescribed price at a prescribed time in the future.

Question? Why study options?

Answer: There are two reasons at least:

1. Options are extremely attractive to investors, both for **speculation** and for **hedging**.
2. There is a systematic way to determine how much they are worth, and hence they can be bought and sold with some confidence.

Hedging

The point of this book is (2.) and we comment on (1.) briefly.
For the **hedging**, consider buying a stock and a "put" on that stock with expiry three months from now:

1. If the stock has gone **up** then you sell the stock at a profit and ignore the put: You make money!
2. If the stock has gone **down** then you sell the stock for a loss and exercise the put: If you have enough puts then you make money!

Another way options can be used to hedge is, e.g., if a company is obliged to make a purchase in a foreign currency sometime in the future. You can effectively fix the exchange rate at today's value so that there is no risk about the rate at time of purchase.

Speculating with Assets and Options

To illuminate the method for speculating with options ("gearing") consider stock with an initial value of $S(0)$, and calls struck at K with value C at time of expiry T. Suppose the asset has value S at $t = T$.

- Speculating with assets only: Buy 1 share and at expiry have:
  - Profit: $S - S(0)$.
  - Relative Profit: $(S - S(0))/S(0) = (1/S(0))S - 1$.
- Speculating with options only: Buy $N = S(0)/C$ calls and at expiry have, if $S < K$:
  - Profit: $0 - S(0)$.
  - Relative Profit: $(0 - S(0))/S(0) = -1$.

  while, if $S > K$,
  - Profit: $N(S - K) - S(0) = (S(0)/C)\{S - K - C\}$.
  - Relative Profit: $(1/C)S - K/C - 1$.

Speculating with Assets and Options, cont.

- It can be shown that $C < S$ so the **slope** in the option–scenario is much **larger**: For S large enough much **larger profit**!
- Clearly, if $S < K$ then one would prefer to have speculated with stocks. However, even if $S > K$ it can be the case that stocks are better.
- The “break–even” point occurs for S^* satisfying

$$S^* - S(0) = (S(0)/C)\{S^* - K - C\},$$

which, upon recalling that $N = S(0)/C$, simplifies to

$$S^* = \frac{NK}{N-1}.$$

- Thus, for $S > S^*$ one would prefer options.

Example: Google

To be more concrete, consider stock in Google (data from GoogleFinance):

- On January 7, 2013 the price of Google stock was $S(0) = 734.75$.
- Consider **calls** with date of expiry **March 16, 2013**.
- With strike $K = 735.00$ the price is $C(0) = 30.90$.
- With 734.75 dollars one could:
  1. Buy 1 share of Google.
  2. Buy (roughly) 23.75 calls.
- If the stock goes up to 800 dollars then:
  1. You make 65.25 dollars, or a 9 % profit.
  2. You make $(23.75 \times 65.00) - 734.75 = 809.00$ dollars, or a 110 % profit!
- If the stock goes down to 672.50 dollars then:
  1. You lose 65.25 dollars, or a 9 % loss.
  2. You make zero dollars, or a 100 % loss.

Payoff Diagrams

- Another attraction of options is that by combining them an investor can take highly specific (and extremely complicated) positions in the market. To see this we introduce the concept of a **payoff diagram**.
- Let K denote the strike price, and $S(T)$ be the asset price at expiry (time T). Note that $S(T)$ will not be known when the contact is written and a central concern of ours will be the modeling of this quantity.
- At expiry, if $S(T) > K$ then the holder of a European call option will buy for K and sell at $S(T)$ to yield a profit of $S(T) - K$.
- However, if $S(T) \leq K$ then the holder gains nothing.
- Hence, the **value** of the European call at expiry denoted by C is

$$C = \max\{S(T) - K, 0\}.$$

Higham Figure 1.1: Call Option

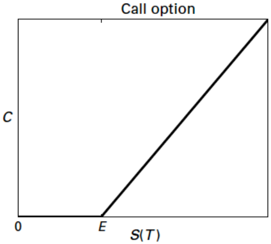


Fig. 1.1. Payoff diagram for a European call. Formula is $C = \max(S(T) - E, 0)$.

Payoff Diagram: European Put

- Consider the same circumstances for a European put.
- At expiry, if $S(T) < K$ then the holder of a European put option will buy at $S(T)$ and sell for K to yield a profit of $K - S(T)$.
- However, if $S(T) \geq K$ then the holder gains nothing.
- Hence, the **value** of the European put at expiry denoted by P is

$$P = \max\{K - S(T), 0\}.$$

- From the plots of these diagrams it is clear why these are called "hockey–stick payoffs."

Higham Figure 1.2: Put Option

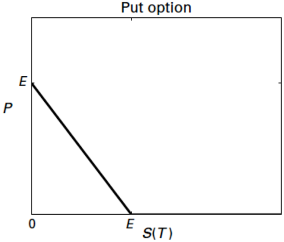
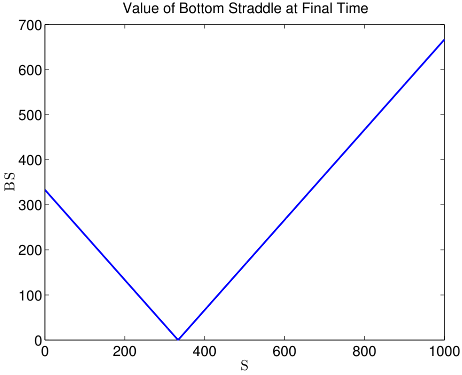


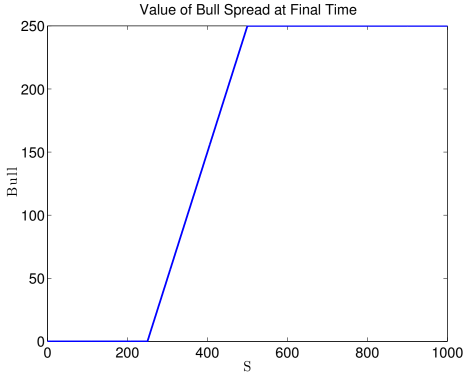
Fig. 1.2. Payoff diagram for a European put. Formula is $P = \max(E - S(T), 0)$.

Value of Bottom Straddle at Final Time



- Hold **both** a call and put struck at the same price $K = 1000/3$.
- The value of this portfolio is $\max\{S - K, 0\}$ plus $\max\{K - S, 0\}$ which equals $|S - K|$.
- This is known as a "Bottom Straddle."

Value of Bull Spread at Final Time



- Hold a call struck at the price $K_1 = 1000/4$.
- Write a call struck at the price $K_2 = 1000/2$.
- The value of this portfolio is $\max\{S - K_1, 0\}$ minus $\max\{S - K_2, 0\}$.
- This is known as a "Bull Spread."

General thoughts . . .

- Please see Hull's book for the details of how options are bought, sold, and used in finance.
- The European calls and puts we've just described are so commonplace that they are called "vanilla calls/puts."
- The options we have described are also called **derivatives** as their values are "derived" from an underlying asset, *not* because of any particular connection to mathematical derivatives.
- There are three broad classes of pricing methods and we will discuss all three:
  1. PDE methods [first]
  2. Binomial methods [second]
  3. Monte Carlo methods [third]