```
clear variables;
```

Wireless Communications Project: 28 GHz patch antenna design and beamforming

Patch antenna design

A patch antenna at 28 GHz is designed. To this end, the specifications of the antenna are set according to the standard Rogers4350B substrate in accordance to the material library in High Frequency Structure Simulator (HFSS).

The height of the substrate (0.15748 mm) is based on manufacturer datasheet for Rogers4350B and is the thickest substrate thickness available with the thinnest at 0.1 mm. Thereafter, using the substrate properties, the approximate equations in [1] are applied to obatin patch dimensions in patchDim.m.

[1] Balanis, Constantine A. *Antenna theory: analysis and design*. John wiley & sons, 2015.

First, the width of the patch is obatined as:

$$W = \frac{1}{2f_r\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2}{\epsilon_r+1}} = \frac{\upsilon_0}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}}$$

then the effective permittivity is given by:

$$\underline{W/h > 1}$$

$$\epsilon_{\text{reff}} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-1/2}$$

The length of the patch antenna is found by:

$$L = \frac{1}{2f_r\sqrt{\epsilon_{\text{reff}}}\sqrt{\mu_0\epsilon_0}} - 2\Delta L$$

where \delta{L} is

$$\frac{\Delta L}{h} = 0.412\frac{(\epsilon_{\text{reff}}+0.3)\left(\frac{W}{h}+0.264\right)}{(\epsilon_{\text{reff}}-0.258)\left(\frac{W}{h}+0.8\right)}$$

The width of the feed line to the patch is an important parameter to keep the characteristic impedance of the feed line at 50 Ohms. The approximate width is obtained from [2].

[2] M. Graham and H. Johnson, High-speed digital design: A handbook of black magic. Prentice Hall, 1993.

The following equation provides the necessary width of the feedline (w).

$$Z_0[\Omega] = \frac{87}{\sqrt{\epsilon_r+1.41}}ln\left(\frac{5.98h}{0.8w+t}\right) \text{ for } 0.1 < \frac{w}{h} < 2; 1 < \epsilon_r < 15,$$

The width requirement is a constant value and thus introduces difficulty in design as frequencies are increased and antenna dimensions become smaller. Thus antennas at higher frequencies may be manufactured on silicon wafer substrates used for IC fabrication.

```
fc=28e9; % frequency of operation

%% Substrate properties
hs=0.15748; %mm-- height of substrate
t=18e-3; %thickness of top-layer metal in mm
er=3.66; %permittivity of substrate
Lt=0.004; %loss tangent
Lg=10; %mm-- ground plane width
substrate = dielectric('Name','Rogers4350B','EpsilonR',er,'LossTangent',Lt,...
    'Thickness',hs*1e-3);
[Lp,Wp,w0]=patchDim(fc,hs,er,t);
fprintf(1,"Length of the patch: %7.4f mm",Lp);
```

```
Length of the patch:  2.7551 mm
```

```
fprintf(1,"Width of the patch: %7.4f mm",Wp);
```

```
Width of the patch:  3.5096 mm
```

```
fprintf(1,"Width of microstrip feed: %7.4f mm",w0);
```

```
Width of microstrip feed:  0.3002 mm
```

The patch antenna design chosen is chosen with an inset feed.

The edge of the patch antenna presents a higher input resistance due to the width of the patch. However, the input resistance changes as the feed line is pushed more into the antenna. The distance of the inset is calculated using equations provided in [1] impleted in patchInset.m.

The input resistance at the edge of the patch antenna is calculated as:

$$R_{in} = \frac{1}{2(G_1 \pm G_{12})}$$

$$G_1 = \begin{cases} \frac{1}{90}\left(\frac{W}{\lambda_0}\right)^2 & W \ll \lambda_0 \\ \frac{1}{120}\left(\frac{W}{\lambda_0}\right) & W \gg \lambda_0 \end{cases}$$

Here W is the width of the patch antenna.

$$G_{12} = \frac{1}{120\pi^2}\int_0^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 J_0(k_0 L \sin\theta)\sin^3\theta \, d\theta$$

where J_{0} is the Bessel function of the first kind of order zero. k_{0} is the wave number.

Then, input resistance at an inset distance y_{0} is given by:

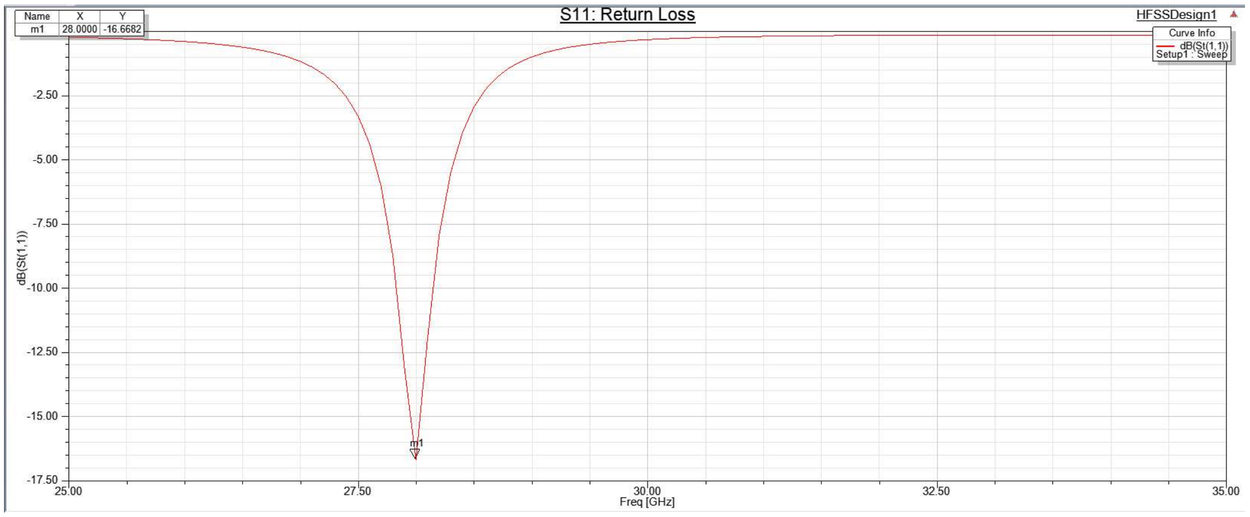
$$\begin{aligned} R_{in}(y = y_0) &= \frac{1}{2(G_1 \pm G_{12})}\cos^2\left(\frac{\pi}{L}y_0\right) \\ &= R_{in}(y = 0)\cos^2\left(\frac{\pi}{L}y_0\right) \end{aligned}$$

The approximate values here obtained are tuned in HFSS, particularly the length of the patch. The revised value is used.

```
%The obtained dimensions are simulated in HFSS. Results obtained are
%slightly modified as:
Lp=2.72;
y0=patchInset(fc,Wp,Lp);
fprintf(1,"Inset on the patch: %7.4f mm",y0);
```

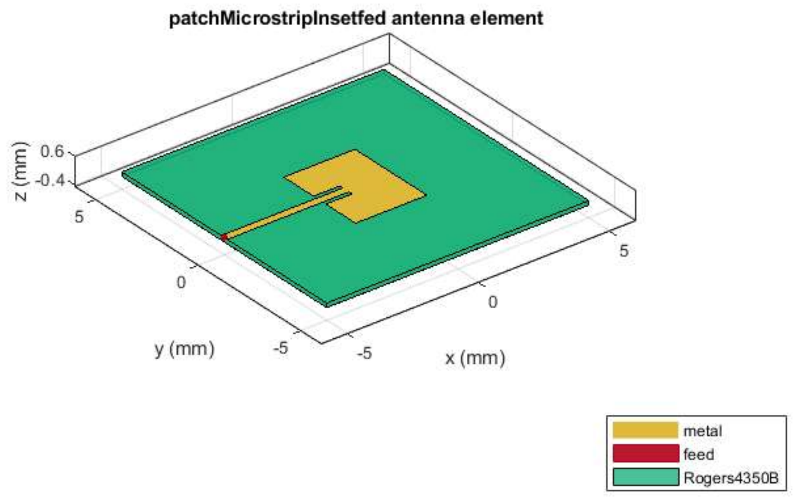
```
Inset on the patch:  0.9894 mm
```

From HFSS, the s-parameters for the designed antenna are obtained as shown below with a return loss of 16 dB at 28 GHz.

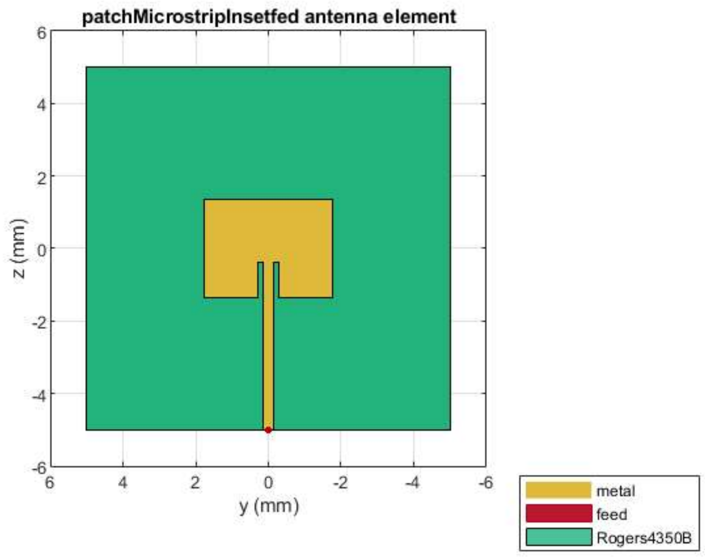


The patch antenna is hereafter implemented in Matlab for simulations:

```
%matlab function takes all dimensions in meters. The results are in mm
insetpatch = patchMicrostripInsetfed("Width",Wp*1e-3,"Length",Lp*1e-3,...
    "Height",hs*1e-3,"Substrate",substrate,"NotchLength",y0*1e-3,...
    "NotchWidth",2*w0*1e-3,"StripLineWidth",w0*1e-3,"GroundPlaneLength",Lg*1e-3,...
    "GroundPlaneWidth",Lg*1e-3,"FeedOffset",[-Lg/2*1e-3 0]);

insetpatch.show();
```



```
insetpatch.Tilt=-90;
insetpatch.TiltAxis=[0,1,0];
insetpatch.show();
view(-90,0);
```



```
%The electric firsl pattern is generated from the anteena simulations in
%HFSS.
eFieldTot=csvread("E_fields28.csv",1,0);
az=eFieldTot(:,1);
theta=eFieldTot(:,2);
Etotal=eFieldTot(:,3).*1e-3;
Etotal=flipud(Etotal); %custom antenna element takes elevation values from -90 to 90
                        % but fields are stored 90 to -90
el=theta-90;% changing theta to elevation from -90 to 90 to match the flip

%antenna directivity may be calculated from the electric field values.
Epow=abs(Etotal).^2;
elrad=deg2rad(el);
scale=mean(cos(elrad))/mean(Epow.*cos(elrad));
dir=scale*Epow;
dir_db=pow2db(dir);

rad_eff=0.82211; %radiation efficiency from HFSS
fprintf(1,"Radiation efficiency evaluated in HFSS: %7.4f",rad_eff);
```

```
Radiation efficiency evaluated in HFSS:  0.8221
```

```
A_gain=rad_eff.*dir;
gain_db=pow2db(A_gain);

fprintf(1,"Max directivity is: %7.4f dBi",max(dir_db));
```

```
Max directivity is:  7.4657 dBi
```

```
fprintf(1,"Max gain is: %7.4f dBi",max(gain_db));
```

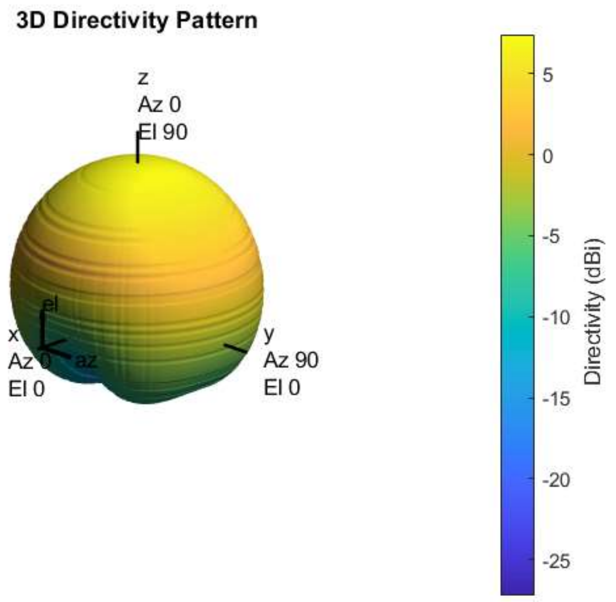
```
Max gain is:  6.6150 dBi
```

```
check=mean(dir.*cos(elrad))/mean(cos(elrad));
fprintf(1,"Average value of directivity*cos(elevationAngle) over integration sphere: %7.4f",check);
```

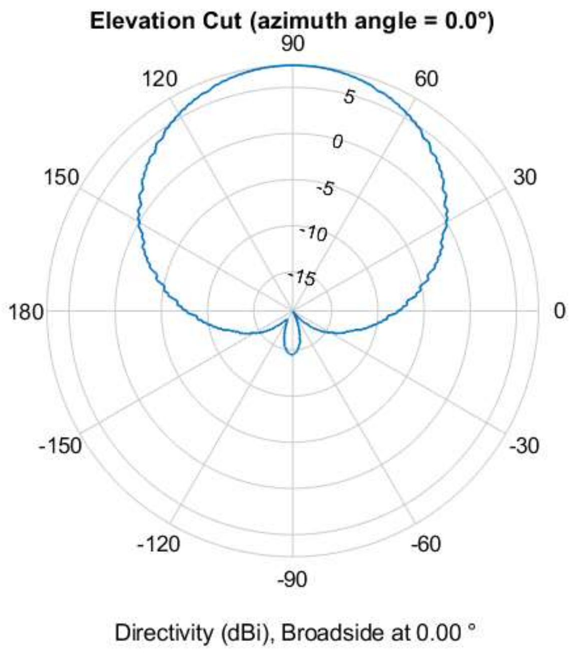
```
Average value of directivity*cos(elevationAngle) over integration sphere:  1.0000
```

```
elvals=(-90:2:90)';
azvals=(-180:2:180)';
```

```
nel = length(elvals);
naz = length(azvals);

dirMat=reshape(gain_db,naz,nel);
dirMat=dirMat';

im=find(azvals==0);
dirMat=circshift(dirMat,[0,im-1]);

phasePattern=zeros(nel,naz);
ant=phased.CustomAntennaElement('AzimuthAngles',azvals,'ElevationAngles',...
    elvals,'MagnitudePattern',dirMat,'PhasePattern',phasePattern);

ant.pattern(fc);
```



```
azcut=0;
ant.patternElevation(fc,azcut);
```



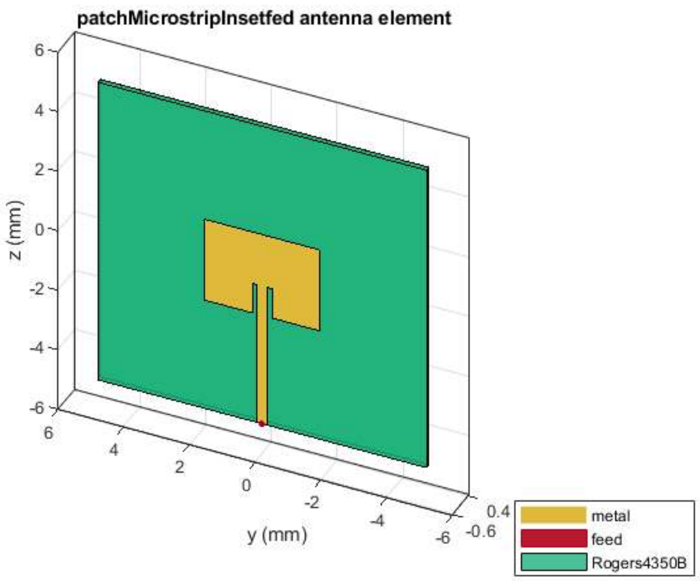
```
vslice0=ant.patternElevation(fc,azcut);
el_slice=-180:180;
dirmax=max(vslice0);
I=(vslice0>=dirmax-3);
angles=el_slice(I);
angles=sort(angles,'descend');
HPBW=angles(1)-angles(end);
fprintf(1,"Half power beamwidth: %7.4f degrees",HPBW);
```

```
Half power beamwidth: 72.0000 degrees
```

```
I=(el>=angles(end))&(el<=angles(1));
Ptot=mean(Epow.*cos(elrad));
Phpbw=mean(Epow.*cos(elrad).*I);
fprintf(1,"Power fraction in HPBW: %7.4f",Phpbw/Ptot);
```

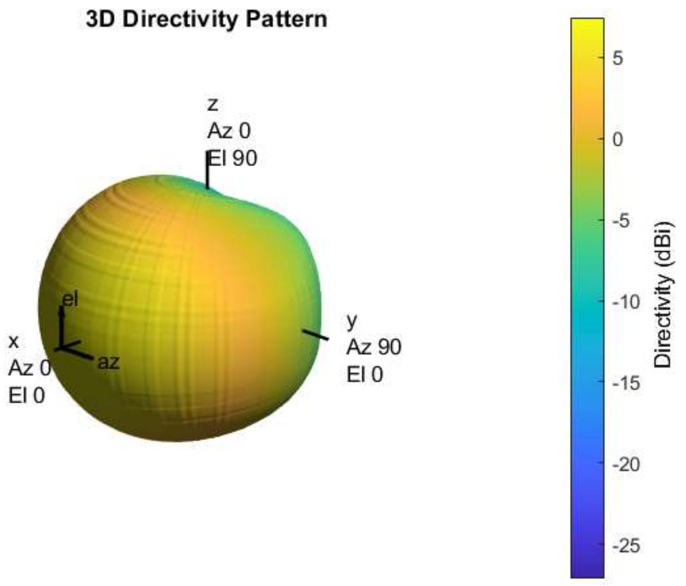
```
Power fraction in HPBW:  0.4147
```

```

insetpatch.show();
view(-65,35);
```



The antenna element is tilied for alignment with X-axis and implementation into Matlab phased array toolbox.

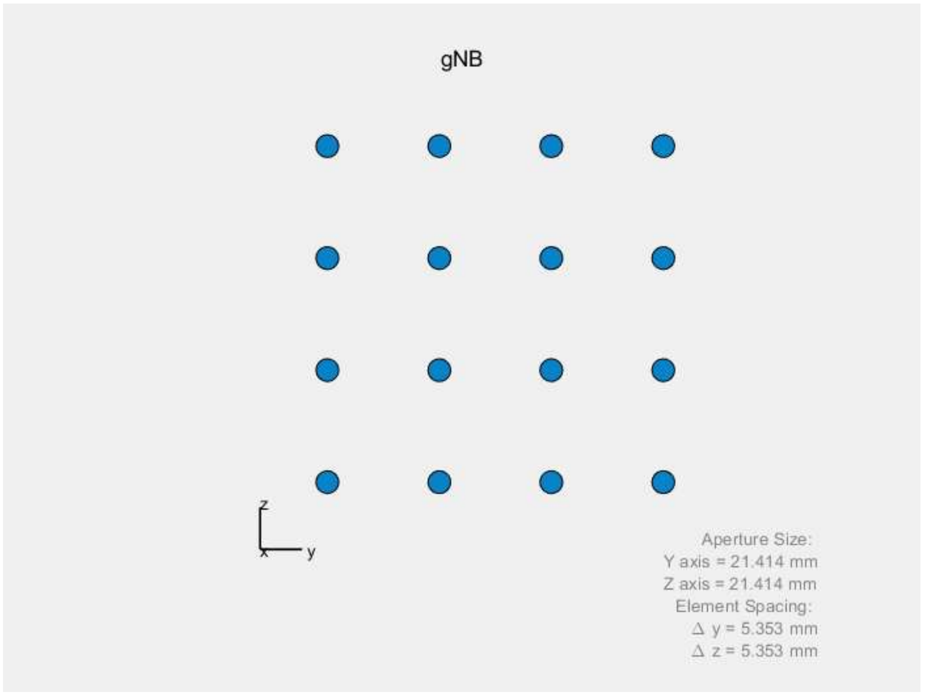
```
newax = roty(90);
dirMatX=rotpat(dirMat,azvals,elvals,newax);
antX=phased.CustomAntennaElement('AzimuthAngles',azvals,'ElevationAngles',...
    elvals,'MagnitudePattern',dirMatX,'PhasePattern',phasePattern);

antX.pattern(fc);
```



Beamforming

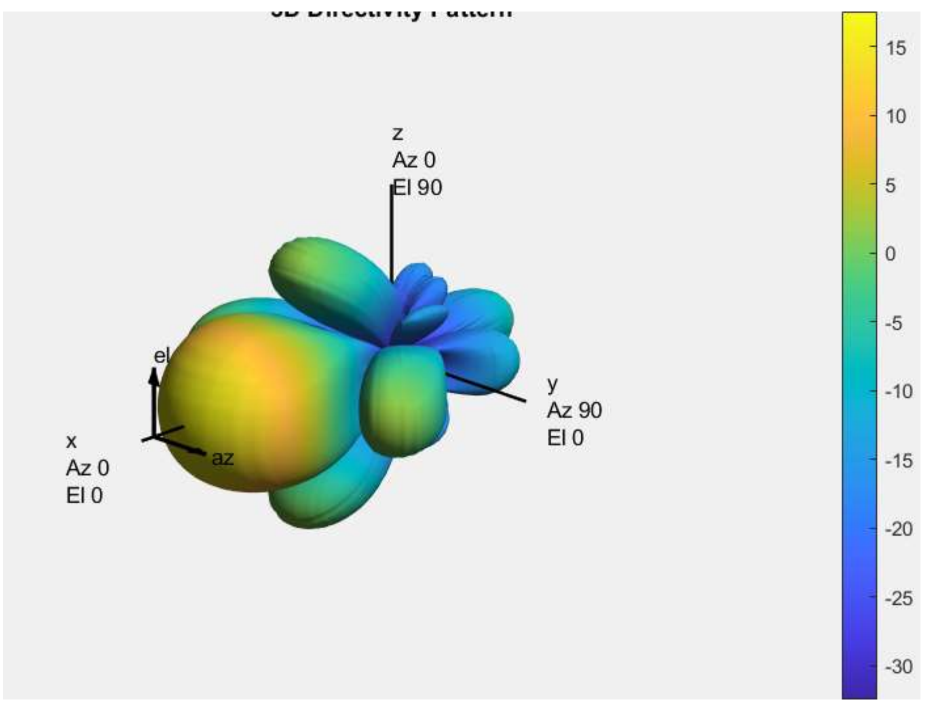
The designed patch array is implemented into phased arrays for simulating beamforming. The ArrayPlatform class has been used to obtain normalized steering vectors.

```
% Number of antennas
nantgNB = [4,4];
nantUE = 8;

lambda = physconst('Lightspeed') / fc;
dsep = 0.5*lambda;
arrgNB = phased.URA(nantgNB,dsep,'Element',antX,'ArrayNormal','x');
viewArray(arrgNB,'Title',"gNB");
```

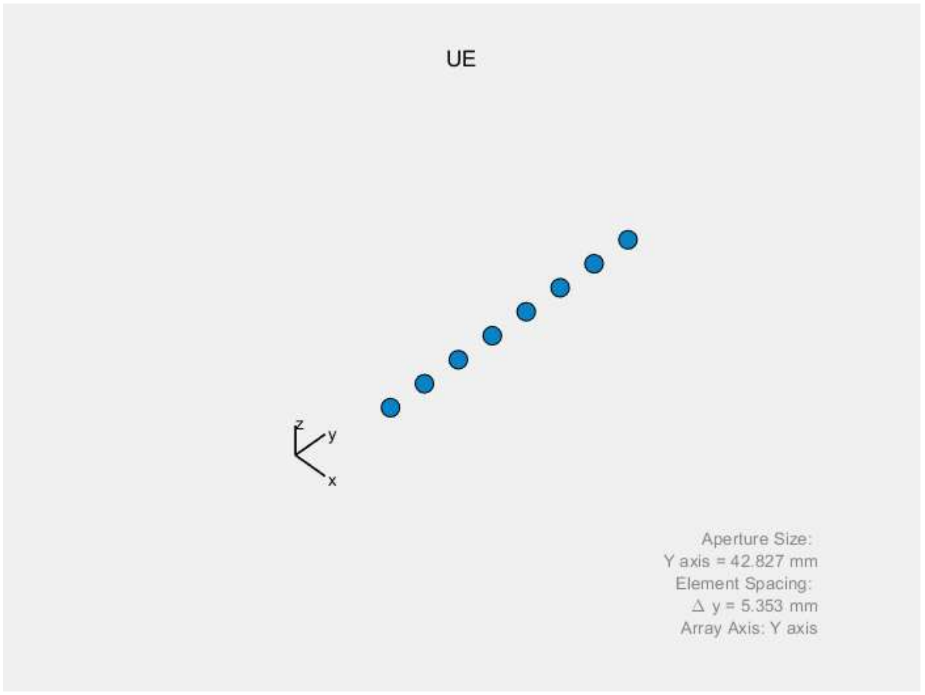


```
sv = phased.SteeringVector('SensorArray',arrgNB);
ang0 = [0; 0];
sv.release();
```

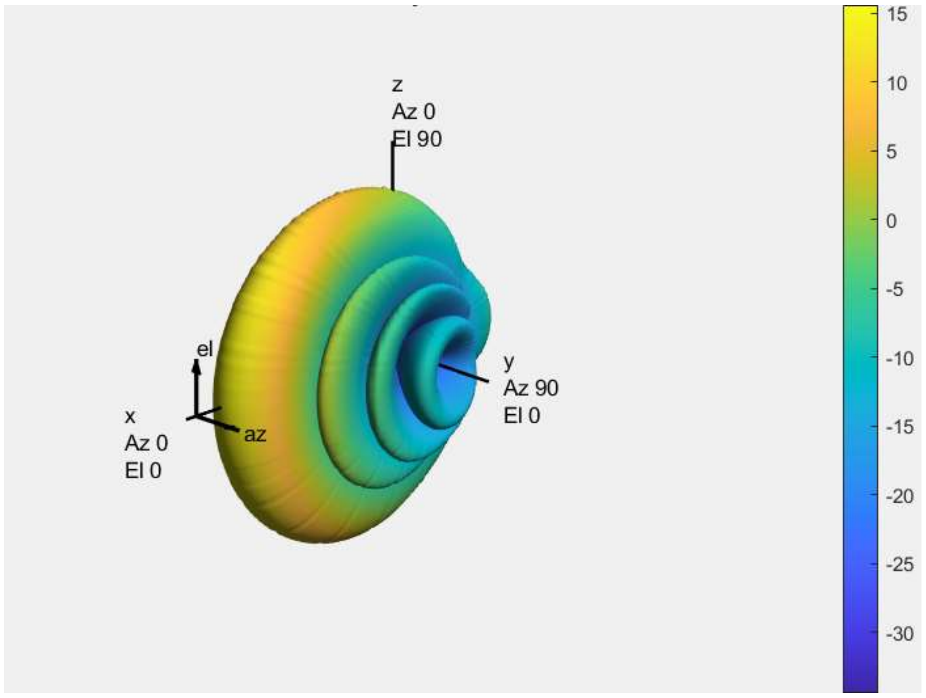
```
u0 = sv(fc, ang0);
u0 = u0 / norm(u0);
arrgNB.pattern(fc,'Weights', u0);
```



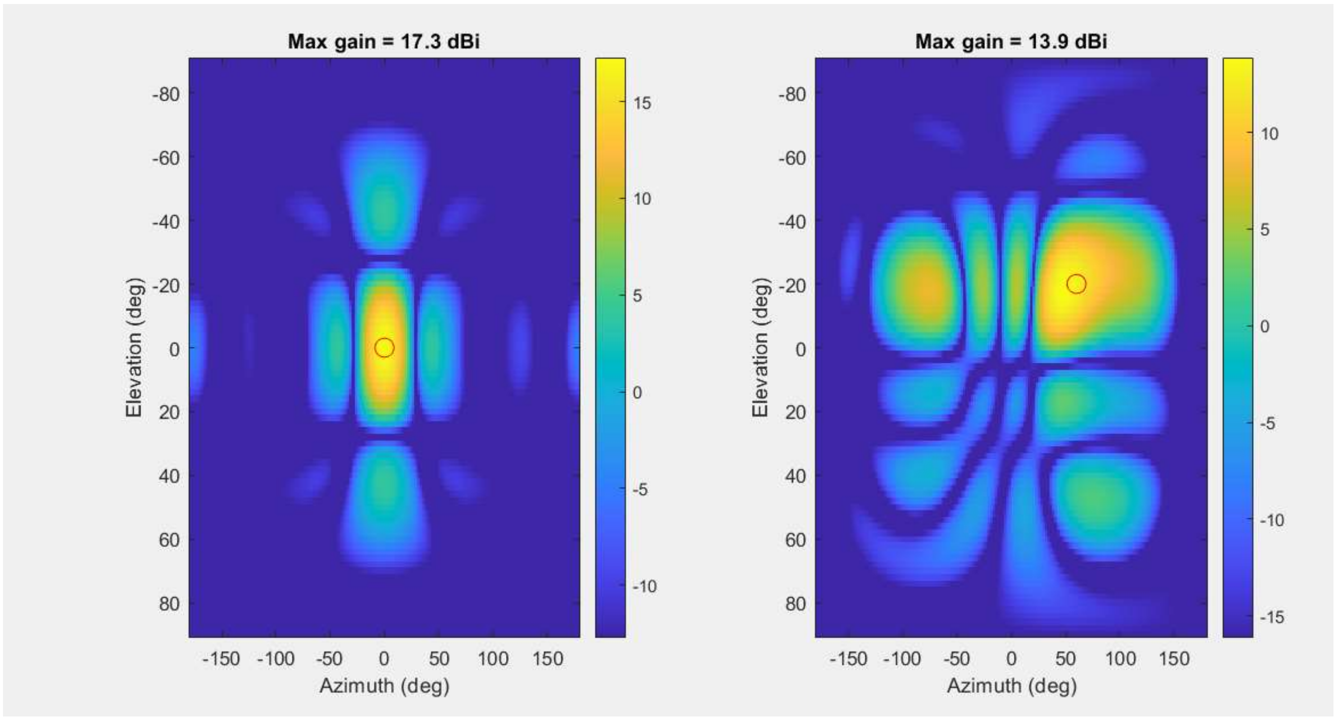
```
arrUE = phased.ULA(nantUE,dsep,'Element',antX);
viewArray(arrUE,'Title',"UE");
```



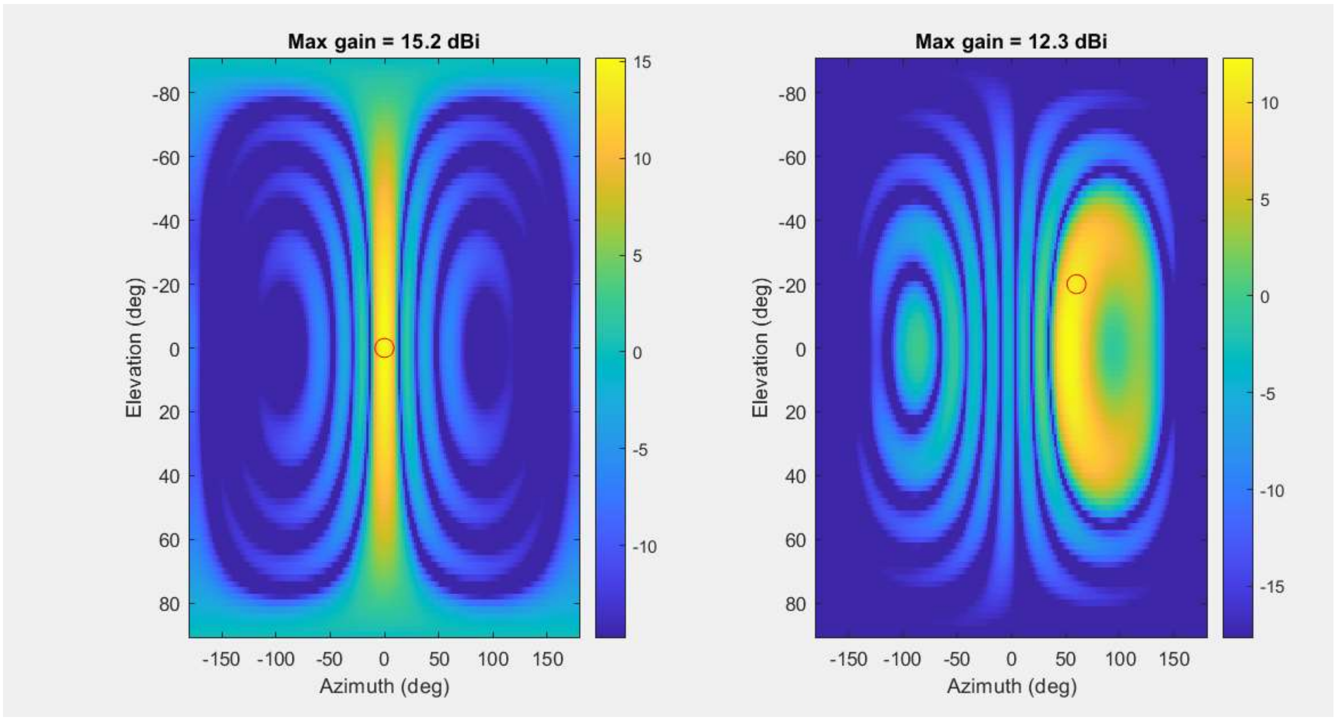
```
sv = phased.SteeringVector('SensorArray',arrUE);
ang0 = [0; 0];
sv.release();
u0 = sv(fc, ang0);
u0 = u0 / norm(u0);
arrUE.pattern(fc,'Weights', u0);
```



```
arrPlatformgNB = ArrayPlatform('elem', antX, 'arr', arrgNB, 'fc', fc);
arrPlatformgNB.computeNormMatrix();

arrPlatformUE =  ArrayPlatform('elem', antX, 'arr', arrUE, 'fc', fc);
arrPlatformUE.computeNormMatrix();

% Angles to get the beamforming vectors. Values chosen as an example
az0 = [0, 60]';
el0 = [0, -20]';
nw = length(az0);

%the steering vectors
svgNB = arrPlatformgNB.step(az0,el0,true);
svUE = arrPlatformUE.step(az0,el0,true);

% Get grid of values
[azMat, elMat] = meshgrid(azvals, elvals);
azVal = azMat(:);
elVal = elMat(:);

% The BF vectors
w = conj(svgNB);
wnorm = sqrt(sum(abs(w).^2));
w = w ./ wnorm;
w = w/norm(w);

% Normalized steering vectors at each of the plot angles
Sv = arrPlatformgNB.step(azVal,elVal);

clf;
posOld = get(gcf, 'Position');
set(gcf,'Position', [0,0,1000,500]);

% Looping over the beamforming vectors
for i = 1:nw
    gain = mag2db(abs(w(:,i).'*Sv));%the array gain
    gainMax = max(gain);
    gain = reshape(gain, nel, naz);

    subplot(1,nw,i);
    imagesc(azvals,elvals,gain,[gainMax-30, gainMax]);
    colorbar();
    hold on;
    plot(az0(i),el0(i), 'ro', 'Markersize', 10);
    hold off;
    xlabel('Azimuth (deg)');
    ylabel('Elevation (deg)');
    title(sprintf('Max gain = %3.1f dBi', gainMax));
end
```



```
% The BF vectors
wUE = conj(svUE);
w1norm = sqrt(sum(abs(wUE).^2));
wUE = wUE ./ w1norm;
wUE = wUE/norm(wUE);

% Normalized steering vectors at each of the plot angles
SvUE = arrPlatformUE.step(azVal,elVal);

clf;
posOld = get(gcf, 'Position');
set(gcf,'Position', [0,0,1000,500]);

% Looping over the beamforming vectors
for i = 1:nw
    gainUE = mag2db(abs(wUE(:,i).'*SvUE));%the array gain
    gainMaxUE = max(gainUE);
    gainUE = reshape(gainUE, nel, naz);

    subplot(1,nw,i);
    imagesc(azvals,elvals,gainUE,[gainMaxUE-30, gainMaxUE]);
    colorbar();
    hold on;
    plot(az0(i),el0(i), 'ro', 'Markersize', 10);
    hold off;
    xlabel('Azimuth (deg)');
    ylabel('Elevation (deg)');
    title(sprintf('Max gain = %3.1f dBi', gainMaxUE));
end
```

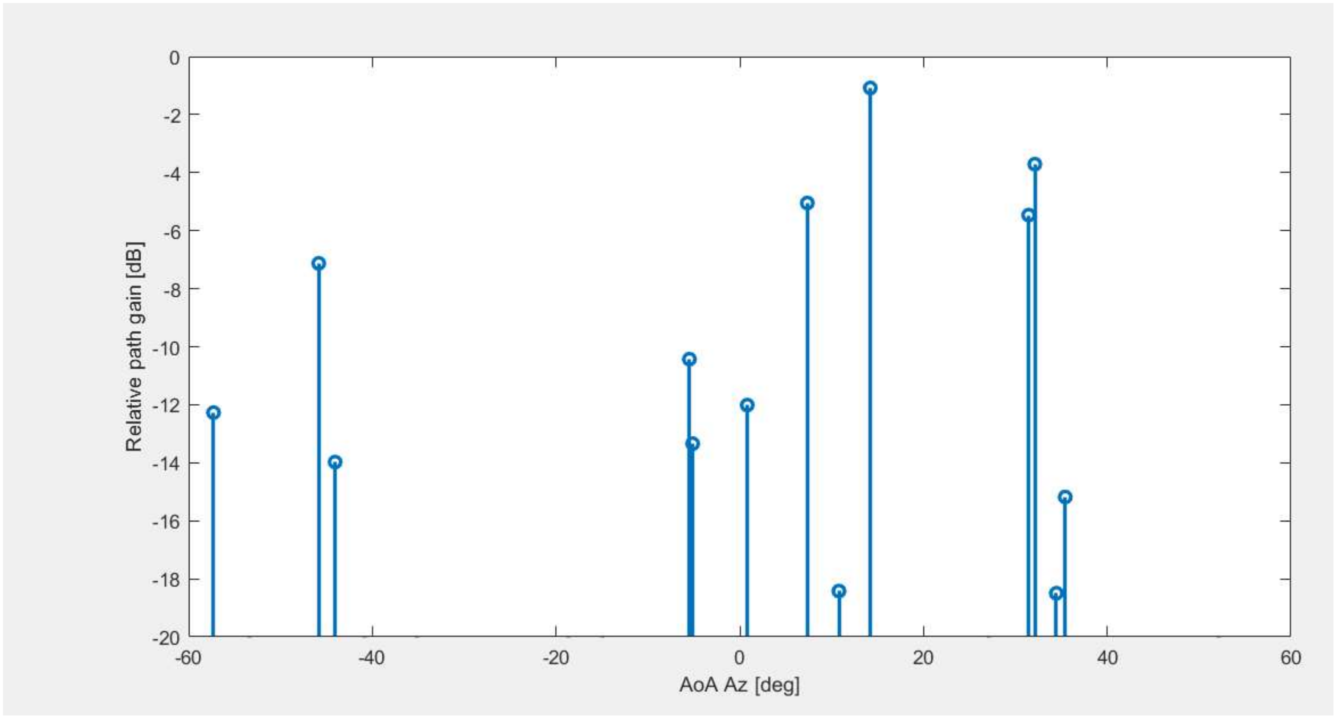


Beamforming results at both UE and gNB shows maximum power at the intended direction of transmission.

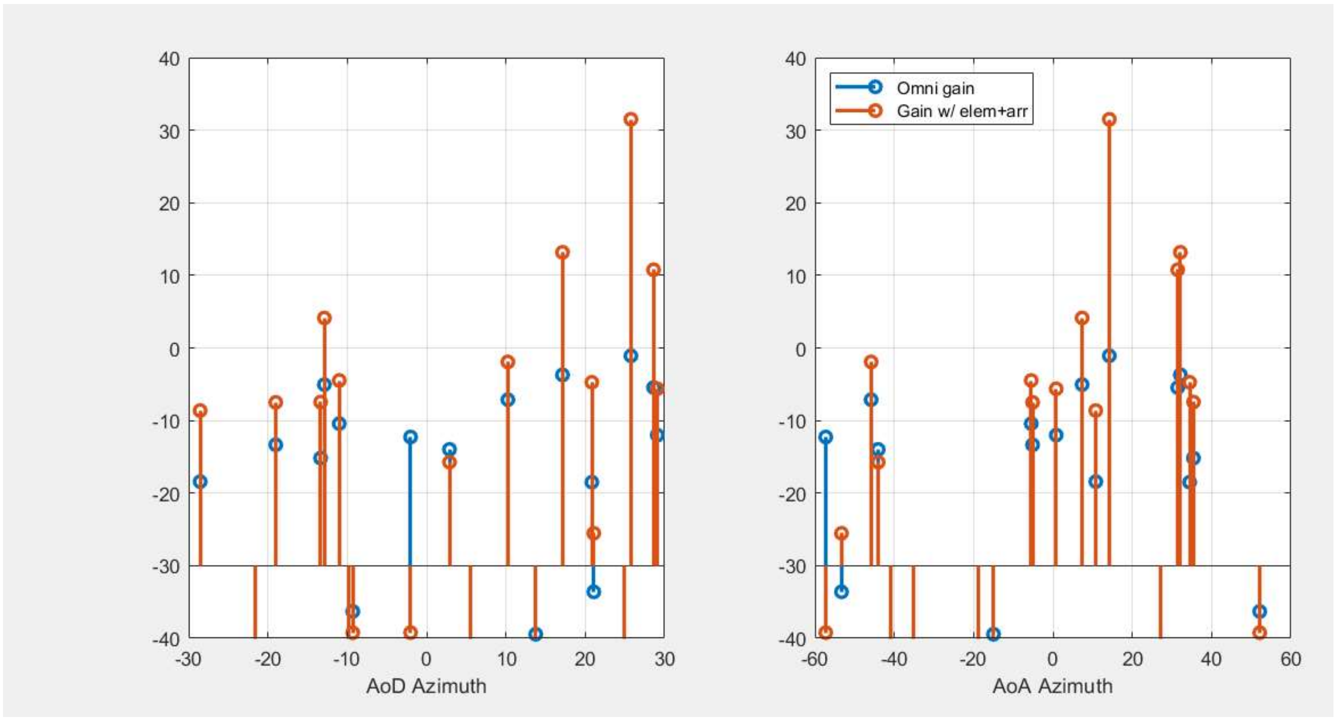
Implementing the beamforming for a narrowband MIMO channel

```
%Considering a high   High-dimensional arrays
%with 4x4 URA at the gNB and 8x1 ULA at the UE.
% The paths are angularly clustered for a lower rank channel.
% Path clusters
npaths = 20;
dlysprd = 100e-9;  % delay spread

gain = -exprnd(20,npaths,1);
[~,maxgainIdx] = max(gain);

% Random phases
phase = unifrnd(-pi,pi,npaths,1);

dly= unifrnd(-dlysprd,dlysprd,npaths,1);

% Generate random paths
    % Use spatially clustered paths
aodAz = unifrnd(-30,30,npaths,1);
aoaAz = unifrnd(-60,60,npaths,1);
aoaEl = unifrnd(-10,10,npaths,1);
aodEl = unifrnd(-10,10,npaths,1);

% Plot the paths vs. AoA
clf;
set(gcf,'Position', [0,0,500,400]);
stem(aoaAz, gain, 'Linewidth', 2, 'BaseValue', -20);
ylim([-20,0]);
xlabel('AoA Az [deg]');
ylabel('Relative path gain [dB]');

%Finding the direction of the maximum power multipath component
maxAz=aoaAz(maxgainIdx);
maxEl=aoaEl(maxgainIdx);
arrPlatformUE.alignAxes(maxAz, maxEl);

%obtaining the steering vectors for each of the path directions
[svTx, elemGainTx] = arrPlatformgNB.step(aodAz, aodEl);
```

```
[svRx, elemGainRx] = arrPlatformUE.step(aoaAz, aoaEl);
gainElem = gain + elemGainTx + elemGainRx;

% Get maximum gain
[~,im] = max(gainElem);

% Find TX BF vector
wtx = conj(svTx(:,im));
wtx = wtx / norm(wtx);

% Find RX BF vector
wrx = conj(svRx(:,im));
wrx = wrx / norm(wrx);

%implementing the gain form the arrays to the directions
arrGainTx = mag2db(abs(wtx.'*svTx))';
arrGainRx = mag2db(abs(wrx.'*svRx))';
gainArr = gain + arrGainTx + arrGainRx;


set(gcf,'Position', [0,0,1000,500]);
```



```
clf;
subplot(1,2,1);
stem(aodAz, [gain gainArr], 'BaseValue',-30, 'LineWidth', 2);
xlabel('AoD Azimuth');
grid on;
ylim([-40,40]);
subplot(1,2,2);
stem(aoaAz, [gain gainArr], 'BaseValue',-30, 'LineWidth', 2);
xlabel('AoA Azimuth');
legend('Omni gain', 'Gain w/ elem+arr', 'Location', 'northwest');
grid on;
ylim([-40,40]);
```

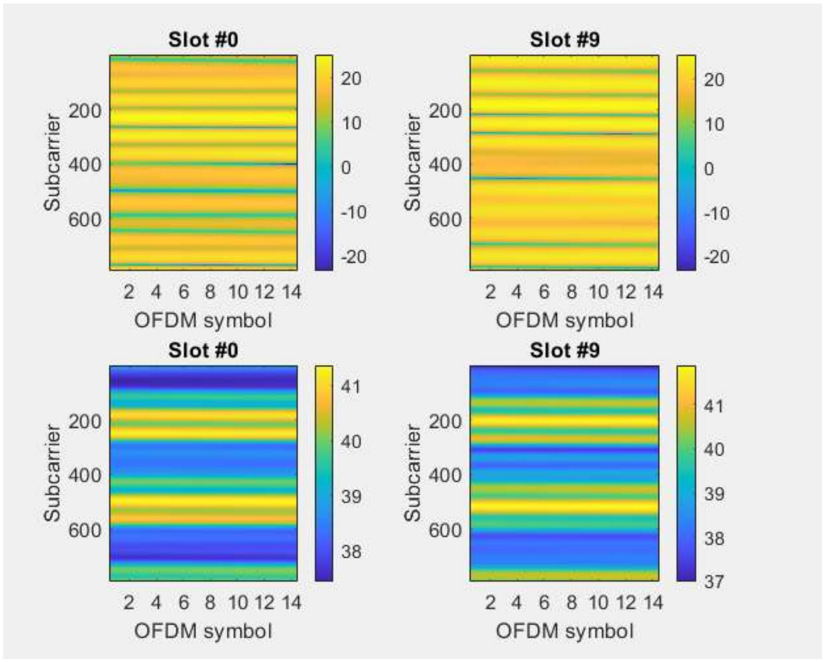


Beamforming in the maximum directions results in one dominant path.

OFDM channel implementation

Observations are made over two slots (0 and 9) in frame 0 for variability. FDMIMOChan class used to obatin the channel grid.Next, the SNR using instantaneous BF is computed. For each point `chanGrid(:,:,n,t), the SVD yields` the maximum gain with BF.

```
SubcarrierSpacing = 120;  % SCS in kHZ
NRB = 66;  % number of resource blocks
nscPerRB = 12;  % number of sub-carriers per RB

carrierConfig = nrCarrierConfig(...
    'NSizeGrid', NRB, 'SubcarrierSpacing', SubcarrierSpacing);
waveformConfig = nrOFDMInfo(carrierConfig);

Enoise = -5;  % Energy per noise sample in dB
fdchan = FDMIMOChan(carrierConfig, 'txArrPlatform', arrPlatformgNB, 'rxArrPlatform', arrPlatformUE, ...
    'aoaAz', aoaAz, 'aodAz', aodAz, 'aoaEl', aoaEl, 'aodEl', aodEl,  ...
    'gain', gain, 'dly', dly, 'fc', fc, 'Enoise', Enoise);
gainComplex = fdchan.gainComplex; % the complex gains are required for finding the spatial covariance matrices
gainComplex = gainComplex / norm(gainComplex);

nrx = arrUE.getNumElements();
ntx = arrgNB.getNumElements();
frameNum = 0;
slotNum = [0,9];
nslot=length(slotNum);
clf;
for i=1:nslot
    [chanGrid, noiseVar] = fdchan.step(frameNum, slotNum(i)); % chanGrid of size nrx x ntx x nsc x nt
                                            % representing the MIMO channel matrix components over
                                            % frequency and time
    set(gcf,'Position', [0,0,500,400]);
    chanGainSing = squeeze( abs(chanGrid(3,4,:,:)).^2 ); %removes dimensions of length 1 from array
    snrSing = 10*log10(chanGainSing / noiseVar);
    subplot(2,nslot,i);
    imagesc(snrSing);
    colorbar();
    xlabel('OFDM symbol');
    ylabel('Subcarrier');
    title(sprintf('Slot #%d', slotNum(i)));
    nsc = size(chanGrid,3);
    nsym = size(chanGrid,4);
    snrInst = zeros(nsc,nsym);
    for n = 1:nsc
        for t = 1:nsym
            chani = squeeze(chanGrid(:,:,n,t));
            [wrxi,si,wtxi] = svds(chani, 1);

            snrInst(n, t) = pow2db(si^2/noiseVar);
        end
    end
    subplot(2,nslot,nslot+i);
    imagesc(snrInst);
    colorbar();
    xlabel('OFDM symbol');
    ylabel('Subcarrier');
    title(sprintf('Slot #%d', slotNum(i)));
end
```



The figure is plotted with the first row showing the OFDM channel grid without beamforming.

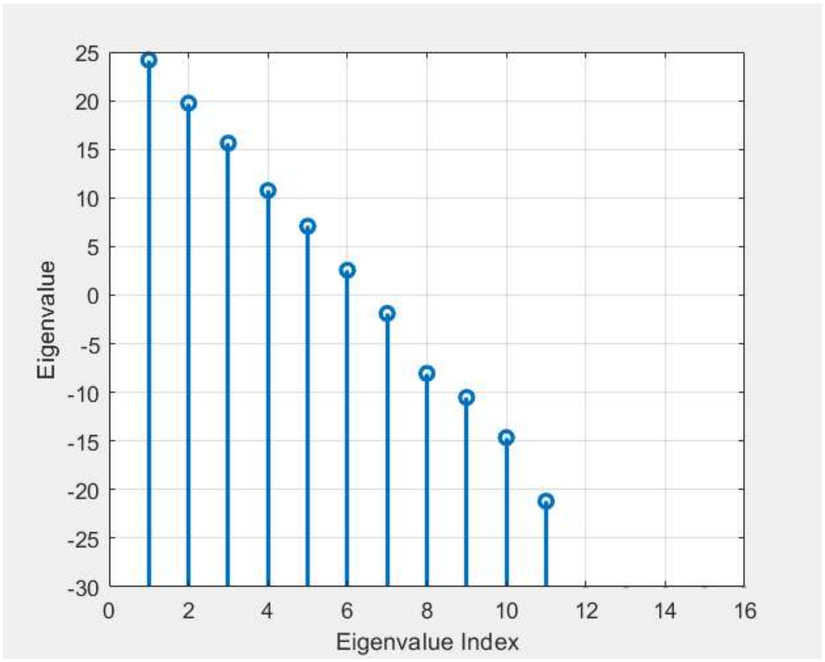
The second row has beamforming implemented on the channel. The variation across time and frequency is significantly lower and the maximum gain on the dominant path is much higher corresponding to the subcarriers with the maximum gain before beamforming.

Statistical Precoding in MIMO

A statistical pre-coding helps obtain CSI-T partially using the TX spatial covariance matrix. This matrix is an average of `H'*H` over many fading realizations that depends on the large scale channel parameters.

```
Qtx = reshape(svTx,ntx,1,npaths).*reshape(conj(svTx),1,ntx,npaths) ...
    .*reshape(abs(gainComplex).^2,1,1,npaths);
Qtx = nrx*sum(Qtx, 3);

% The eigenvectors in the TX directions can be obtained from the TX spatial covariance matrix

% Get the eigenvalues
[V,lam] = eig(Qtx);
lam = real(diag(lam));

% Sort the eignvalues
[lam,I] = sort(lam, 'descend');
```

```
Vstat = V(:,I);

% Plot the eigenvalues
clf;
stem(pow2db(lam), 'LineWidth', 2, 'Basevalue', -30);
xlabel('Eigenvalue Index');
ylabel('Eigenvalue');

ylim([-30, 25]);
grid on;
```

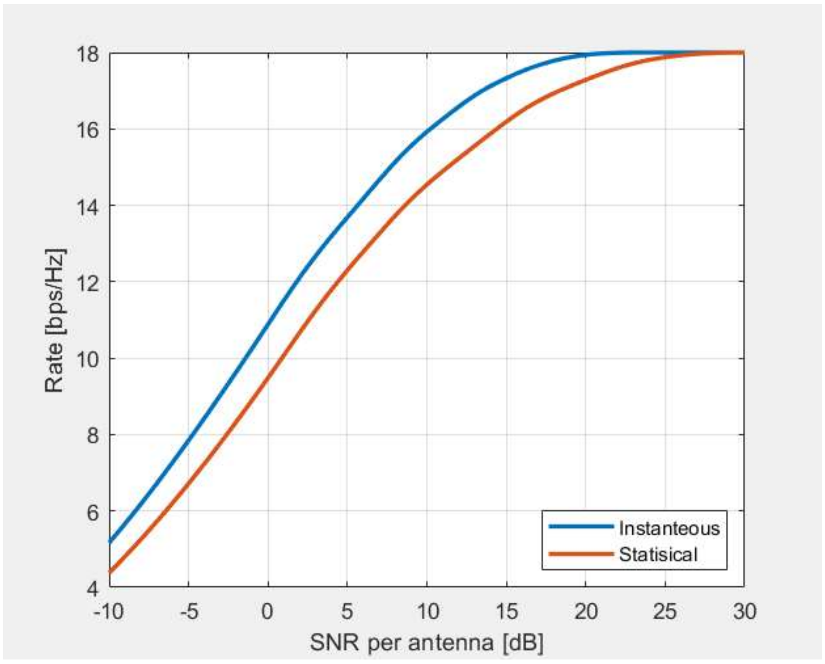


Eigenvalues show tthat the gains from using multiple antennas at input and output are on the order of ntx x nrx; ntx and nrx represnt number of gNB and UE antennas respectively.

Next, the rate with instantaneous and statistical pre-coding is estimated over multiple independent channel realizations

```
nsamp = 100;  % Number of random samples
r = 4;        % TX rank

% Rate model parameters
bwScale = 0.5;
maxSE = 4.5;

% Random phases on each path
phase = unifrnd(0,2*pi,nsamp,npaths);

% Eigenvalues for the pre-coded system with
% instanteous and statistical pre-coding
LamInst = zeros(nsamp,r);
LamStat = zeros(nsamp,r);

for i = 1:nsamp

    % Generate complex gains in this sample
    g = gainComplex.*exp(1i*phase(i,:).');

    % Get the instantaneous H
    H = reshape(svRx,nrx,1,npaths).*reshape(svTx,1,ntx,npaths).*reshape(g,1,1,npaths);
    H = sum(H, 3);

    % Compute the instaneous eigenvalues
    s = svd(H, 'econ');
    LamInst(i,:) = s(1:r).^2;
    % Compute the statistically pre-coded system
    G = H*Vstat(:,1:r);

    % Compute the eigenvalues with statistical pre-coding
    s = svd(G, 'econ');
    LamStat(i,:) = s(1:r).^2;

end
```

```
snrTest = linspace(-10,30,100)';
nsnr = length(snrTest);

rateInst = zeros(nsnr,1);
rateStat = zeros(nsnr,1);
for i = 1:nsnr

    snr = snrTest(i);

    % Get the rate with instantaneous pre-coding
    rate = sum(min(bwScale*log2(1 + db2pow(snr)*LamInst/r), maxSE), 2);
    rateInst(i) = mean(rate);

    % Get the rate with statistical pre-coding
    rate = sum(min(bwScale*log2(1 + db2pow(snr)*LamStat/r), maxSE), 2);
    rateStat(i) = mean(rate);
end

plot(snrTest, [rateInst rateStat], 'Linewidth', 2);
grid on;
legend('Instanteous', 'Statisical', 'location', 'southeast');
xlabel('SNR per antenna [dB]');
ylabel('Rate [bps/Hz]');
```



The curve shows close performace between rates with instantaneous and statistical precoding with TX rank of 4. The difference is reduced as higher transmission rank is used.