Rapport de laboratoire 1: modélisation et résolution de problèmes avec IPOPT

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Dawut Esse

```
using Pkg
Pkg.activate("rapport_env")    # activate a virtual environment
Pkg.add(["JuMP", "Ipopt"]);
```

Modélisation d'un problème avec contraintes

Modéliser le problème

$$\min_{x\in\mathbb{R}^2} \; (x_1-2)^2+(x_2-1)^2 \quad \text{s.c. } x_1^2-x_2\le 0,\; x_1+x_2\le 2$$

à l'aide de `ADNLPModels.jl` et le résoudre avec IPOPT à l'aide de `NLPModelsIpopt.jl`. Vous pouvez fournir à IPOPT un point initial de votre choix. Il ne requiert pas un point initial réalisable (c'est-à-dire qui satisfait les contraintes).

Nous avons vu en classe comment modéliser un problème sans contraintes. Dirigez-vous vers https://jso.dev/ADNLPModels.jl/stable/ pour découvrir comment modéliser des contraintes. Effectuez les opérations suivantes :

1. résolvez ce problème avec IPOPT et faites afficher la solution ;

```
# Insérez votre code ici
using JuMP, Ipopt
using LinearAlgebra

m1 = Model(Ipopt.Optimizer)
@variable(m1, x[1:2])
@NLobjective(m1, Min, (x[1] - 2)^2 + (x[2] - 1)^2)
@NLconstraint(m1, x[1]^2 - x[2] <= 0)
@constraint(m1, x[1] + x[2] <= 2)

# Résolution
optimize!(m1)
```

```julia
# 1. Solution trouvée
sol1 = value.(x)
println("Solution x* = ", sol1)
```

```

******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
 Ipopt is released as open source code under the Eclipse Public License (EPL).
         For more information visit https://github.com/coin-or/Ipopt
******************************************************************************

This is Ipopt version 3.14.17, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        4
Number of nonzeros in Lagrangian Hessian.............:        3

Total number of variables............................:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        2
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        2

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  5.0000000e+00 0.00e+00 1.60e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.8768873e+00 7.78e-01 3.74e+00  -1.0 1.07e+00    -  4.56e-01 1.00e+00f  1
   2  2.3931799e+00 0.00e+00 1.88e+00  -1.0 1.01e+00    -  1.00e+00 1.00e+00h  1
   3  1.0488320e+00 0.00e+00 1.03e-01  -1.0 8.48e-01    -  1.00e+00 1.00e+00f  1
   4  1.0296800e+00 0.00e+00 6.38e-04  -1.7 2.73e-01    -  1.00e+00 1.00e+00h  1
   5  1.0011635e+00 0.00e+00 2.10e-04  -3.8 2.68e-02    -  1.00e+00 1.00e+00f  1
   6  1.0000063e+00 0.00e+00 7.94e-08  -5.7 1.54e-03    -  1.00e+00 1.00e+00h  1
   7  9.9999999e-01 0.00e+00 5.79e-12  -8.6 7.04e-06    -  1.00e+00 1.00e+00h  1

Number of Iterations....: 7

                                   (scaled)                 (unscaled)
Objective...............:   9.9999998506912280e-01    9.9999998506912280e-01
Dual infeasibility......:   5.7880367165807911e-12    5.7880367165807911e-12
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   2.5673012579471588e-09    2.5673012579471588e-09
Overall NLP error.......:   2.5673012579471588e-09    2.5673012579471588e-09
```

```
Number of objective function evaluations             = 8
Number of objective gradient evaluations             = 8
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 8
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 8
Number of Lagrangian Hessian evaluations             = 7
Total seconds in IPOPT                               = 14.160

EXIT: Optimal Solution Found.
Solution x* = [1.0000000074654387, 1.000000008683609]
```

2. donnez le statut final d'IPOPT ;

```julia
# Insérez votre code ici
println("Statut : ", termination_status(m1))
```

```
Statut : LOCALLY_SOLVED
```

3. Validez manuellement que la solution vérifie les contraintes ;

```julia
# Insérez votre code ici
println("C1 = x1^2 - x2 = ", sol1[1]^2 - sol1[2])
println("C2 = x1 + x2 =   ", sol1[1] + sol1[2])
```

```
C1 = x1^2 - x2 = 6.247268480308321e-9
C2 = x1 + x2 =   2.0000000161490474
```

4. faites afficher les résidu d'optimalité calculés par IPOPT, contenues dans `stats.primal_feas` et `stats.dual_feas`, respectivement. NB: `primal_feas` donne la satisfaction des contraintes et `dual_feas` est la norme du gradient du lagrangien du problème.

```julia
# Insérez votre code ici

using JuMP, Ipopt, LinearAlgebra

# 1. Création du modèle et des contraintes
m1 = Model(Ipopt.Optimizer)
@variable(m1, x[1:2])
@NLobjective(m1, Min, (x[1] - 2)^2 + (x[2] - 1)^2)
c1 = @NLconstraint(m1, x[1]^2 - x[2] <= 0)
c2 = @constraint(m1, x[1] + x[2] <= 2)

# 2. Résolution
optimize!(m1)
sol1 = value.(x)
```

```
# 3. Affichage de la solution
println("Solution x* = ", sol1)

# 4. Résidus d'optimalité
# --- primal feasibility
viol1 = max(sol1[1]^2 - sol1[2], 0.0)
viol2 = max(sol1[1] + sol1[2] - 2, 0.0)
println("Primal feas (max violation): ", max(viol1, viol2))

# --- dual feasibility
 1 = dual(c1)
 2 = dual(c2)
g1 = 2*(sol1[1] - 2) +  1*2*sol1[1] +  2
g2 = 2*(sol1[2] - 1) -  1 +  2
println("Dual feas (norm grad Lagrangien): ", norm([g1, g2]))
```

```
This is Ipopt version 3.14.17, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        4
Number of nonzeros in Lagrangian Hessian.............:        3

Total number of variables............................:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        2
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        2

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  5.0000000e+00 0.00e+00 1.60e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.8768873e+00 7.78e-01 3.74e+00  -1.0 1.07e+00    -  4.56e-01 1.00e+00f  1
   2  2.3931799e+00 0.00e+00 1.88e+00  -1.0 1.01e+00    -  1.00e+00 1.00e+00h  1
   3  1.0488320e+00 0.00e+00 1.03e-01  -1.0 8.48e-01    -  1.00e+00 1.00e+00f  1
   4  1.0296800e+00 0.00e+00 6.38e-04  -1.7 2.73e-01    -  1.00e+00 1.00e+00h  1
   5  1.0011635e+00 0.00e+00 2.10e-04  -3.8 2.68e-02    -  1.00e+00 1.00e+00f  1
   6  1.0000063e+00 0.00e+00 7.94e-08  -5.7 1.54e-03    -  1.00e+00 1.00e+00h  1
   7  9.9999999e-01 0.00e+00 5.79e-12  -8.6 7.04e-06    -  1.00e+00 1.00e+00h  1

Number of Iterations....: 7

                                   (scaled)                 (unscaled)
Objective...............:   9.9999998506912280e-01    9.9999998506912280e-01
```

```
Dual infeasibility......:   5.7880367165807911e-12    5.7880367165807911e-12
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   2.5673012579471588e-09    2.5673012579471588e-09
Overall NLP error.......:   2.5673012579471588e-09    2.5673012579471588e-09


Number of objective function evaluations             = 8
Number of objective gradient evaluations             = 8
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 8
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 8
Number of Lagrangian Hessian evaluations             = 7
Total seconds in IPOPT                               = 0.009

EXIT: Optimal Solution Found.
Solution x* = [1.0000000074654387, 1.000000008683609]
Primal feas (max violation): 1.6149047432634234e-8
Dual feas (norm grad Lagrangien): 3.9999999701324573
```

Modélisation d'un problème dégénéré

Modéliser le problème

$$\min_{x \in \mathbb{R}} \; x \quad \text{s.c. } x^2 = 0$$

à l'aide de `ADNLPModels.jl` et le résoudre avec IPOPT à l'aide de `NLPModelsIpopt.jl`.

Un solveur comme IPOPT ne requiert pas un point initial réalisable. Utilisez le point initial $x = 1$.

```julia
# Insérez votre code ici
###############################################################################
# 0.  Packages
using ADNLPModels          # description du problème
using NLPModelsIpopt       # appel d'IPOPT

# 1.  Définition du problème
f(x) = x[1]                # objectif
c(x) = [x[1]^2]            # contrainte  g(x) = 0
x0    = [1.0]              # point initial (non réalisable)

nlp = ADNLPModel(
         f, x0;            # objectif + point initial
         c    = c,         # fonction de contraintes
         lcon = [0.0],     # bornes inférieures
         ucon = [0.0]      # bornes supérieures
```

```
    )

# 2.  Appel d'IPOPT
stats = ipopt(nlp; print_level = 5)      # ou print_level = 0 pour silence

# 3.  Résumé rapide
println("\n======= IPOPT =======")
println("Statut    : ", stats.status)
println("Itérations: ", stats.iter)
println("x*        : ", stats.solution)
println("f(x*)     : ", stats.objective)
println("‖g(x*)‖   : ", stats.primal_feas)   # violation contrainte
println("‖ L‖     : ", stats.dual_feas)     # résidu dual
################################################################################
```

```
This is Ipopt version 3.14.17, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:        0

Total number of variables............................:        1
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.0000000e+00 0.00e+00 1.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1 -9.9990000e+03 0.00e+00 1.00e+00  -1.7 1.00e+04  -4.0 1.00e+00 1.00e+00f  1
   2 -3.9999000e+04 0.00e+00 1.00e+00  -1.7 3.00e+04  -4.5 1.00e+00 1.00e+00f  1
   3 -1.2999900e+05 0.00e+00 1.00e+00  -1.7 9.00e+04  -5.0 1.00e+00 1.00e+00f  1
   4 -3.9999900e+05 0.00e+00 1.00e+00  -1.7 2.70e+05  -5.4 1.00e+00 1.00e+00f  1
   5 -1.2099990e+06 0.00e+00 1.00e+00  -1.7 8.10e+05  -5.9 1.00e+00 1.00e+00f  1
   6 -3.6399990e+06 0.00e+00 1.00e+00  -1.7 2.43e+06  -6.4 1.00e+00 1.00e+00f  1
   7 -1.0929999e+07 0.00e+00 1.00e+00  -1.7 7.29e+06  -6.9 1.00e+00 1.00e+00f  1
   8 -3.2799999e+07 0.00e+00 1.00e+00  -1.7 2.19e+07  -7.3 1.00e+00 1.00e+00f  1
   9 -9.8409999e+07 0.00e+00 1.00e+00  -1.7 6.56e+07  -7.8 1.00e+00 1.00e+00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  10 -2.9524000e+08 0.00e+00 1.00e+00  -1.7 1.97e+08  -8.3 1.00e+00 1.00e+00f  1
  11 -8.8573000e+08 0.00e+00 1.00e+00  -1.7 5.90e+08  -8.8 1.00e+00 1.00e+00f  1
  12 -2.6572000e+09 0.00e+00 1.00e+00  -1.7 1.77e+09  -9.2 1.00e+00 1.00e+00f  1
  13 -7.9716100e+09 0.00e+00 1.00e+00  -1.7 5.31e+09  -9.7 1.00e+00 1.00e+00f  1
```

```
  14 -2.3914840e+10 0.00e+00 1.00e+00  -1.7 1.59e+10 -10.2 1.00e+00 1.00e+00f  1
  15 -7.1744530e+10 0.00e+00 1.00e+00  -1.7 4.78e+10 -10.7 1.00e+00 1.00e+00f  1
  16 -2.1523360e+11 0.00e+00 1.00e+00  -1.7 1.43e+11 -11.2 1.00e+00 1.00e+00f  1
  17 -6.4570081e+11 0.00e+00 1.00e+00  -1.7 4.30e+11 -11.6 1.00e+00 1.00e+00f  1
  18 -1.9371024e+12 0.00e+00 1.00e+00  -1.7 1.29e+12 -12.1 1.00e+00 1.00e+00f  1
  19 -5.8113073e+12 0.00e+00 1.00e+00  -1.7 3.87e+12 -12.6 1.00e+00 1.00e+00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  20 -1.7433922e+13 0.00e+00 1.00e+00  -1.7 1.16e+13 -13.1 1.00e+00 1.00e+00f  1
  21 -5.2301766e+13 0.00e+00 1.00e+00  -1.7 3.49e+13 -13.5 1.00e+00 1.00e+00f  1
  22 -1.5690530e+14 0.00e+00 1.00e+00  -1.7 1.05e+14 -14.0 1.00e+00 1.00e+00f  1
  23 -4.7071589e+14 0.00e+00 1.00e+00  -1.7 3.14e+14 -14.5 1.00e+00 1.00e+00f  1
  24 -1.4121477e+15 0.00e+00 1.00e+00  -1.7 9.41e+14 -15.0 1.00e+00 1.00e+00f  1
  25 -4.2364430e+15 0.00e+00 1.00e+00  -1.7 2.82e+15 -15.5 1.00e+00 1.00e+00f  1
  26 -1.2709329e+16 0.00e+00 1.00e+00  -1.7 8.47e+15 -15.9 1.00e+00 1.00e+00f  1
  27 -3.8127987e+16 0.00e+00 1.00e+00  -1.7 2.54e+16 -16.4 1.00e+00 1.00e+00f  1
  28 -1.1438396e+17 0.00e+00 1.00e+00  -1.7 7.63e+16 -16.9 1.00e+00 1.00e+00f  1
  29 -3.4315189e+17 0.00e+00 1.00e+00  -1.7 2.29e+17 -17.4 1.00e+00 1.00e+00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  30 -1.0294557e+18 0.00e+00 1.00e+00  -1.7 6.86e+17 -17.8 1.00e+00 1.00e+00f  1
  31 -3.0883670e+18 0.00e+00 1.00e+00  -1.7 2.06e+18 -18.3 1.00e+00 1.00e+00f  1
  32 -9.2651009e+18 0.00e+00 1.00e+00  -1.7 6.18e+18 -18.8 1.00e+00 1.00e+00f  1
  33 -2.7795303e+19 0.00e+00 1.00e+00  -1.7 1.85e+19 -19.3 1.00e+00 1.00e+00f  1
  34 -8.3385908e+19 0.00e+00 1.00e+00  -1.7 5.56e+19 -19.7 1.00e+00 1.00e+00f  1
  35 -1.8338591e+20 0.00e+00 1.00e+00  -1.7 1.00e+20 -20.0 1.00e+00 1.00e+00f  1

Number of Iterations....: 35

                                   (scaled)                 (unscaled)
Objective...............:  -1.8338590849833298e+20   -1.8338590849833298e+20
Dual infeasibility......:   1.0000000000000000e+00    1.0000000000000000e+00
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   1.0000000000000000e+00    1.0000000000000000e+00


Number of objective function evaluations             = 36
Number of objective gradient evaluations             = 36
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 35
Total seconds in IPOPT                               = 2.768

EXIT: Iterates diverging; problem might be unbounded.
  Warning: could not parse Ipopt log file. SystemError("opening file \"C:\\\\Users\\\\dawut\\\\
  @ NLPModelsIpopt C:\Users\dawut\.julia\packages\NLPModelsIpopt\OGzSv\src\NLPModelsIpopt.jl:29
```

```
======= IPOPT =======
Statut     : unknown
Itérations: -1
x*         : [-1.8338590849833298e20]
f(x*)      : -1.8338590849833298e20
||g(x*)||   : 0.0
|| L||      : Inf
```

Commentez le statut final d'IPOPT, les résidus d'optimalité, ainsi que la solution finale identifiée. Ajoutez vos propres commentaires concernant ce problème d'optimisation.

Commentaires

Dans le cadre de la modélisation d'un problème dégénéré, le résumé IPOPT affichant status = unknown, iter = –1, un pas gigantesque vers -1.8×10^{2} et un résidu primal nul révèle que la contrainte xˆ2=0 n'a pas été transmise au modèle : IPOPT s'est donc retrouvé sans aucune restriction, a constaté que l'objectif linéaire f(x)=x est non borné vers $-\infty$ et a quitté aussitôt avec un gradient infini.