1 Grid-search outputs gallery (fixed paths)

1.1 Stage 1 backbone diagnostics

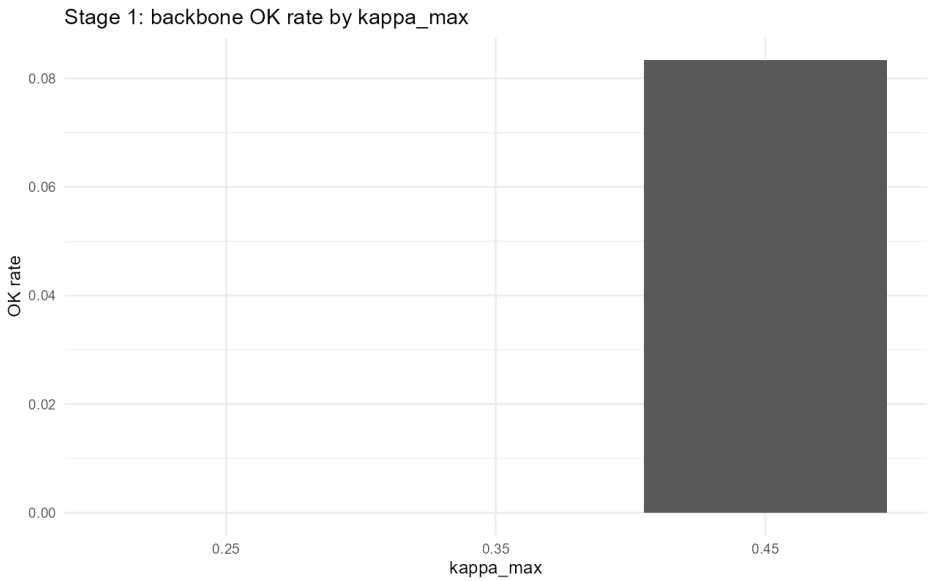

Figure 1: Backbone OK rate by $\kappa_{\max}$.

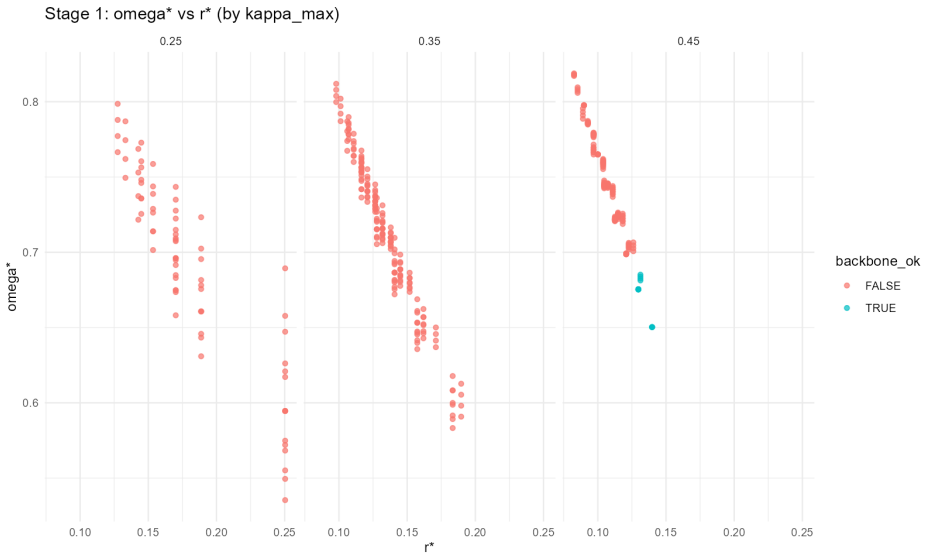

Figure 2: Stage 1: ω^* vs r^* (faceted by $\kappa_{\max}$).

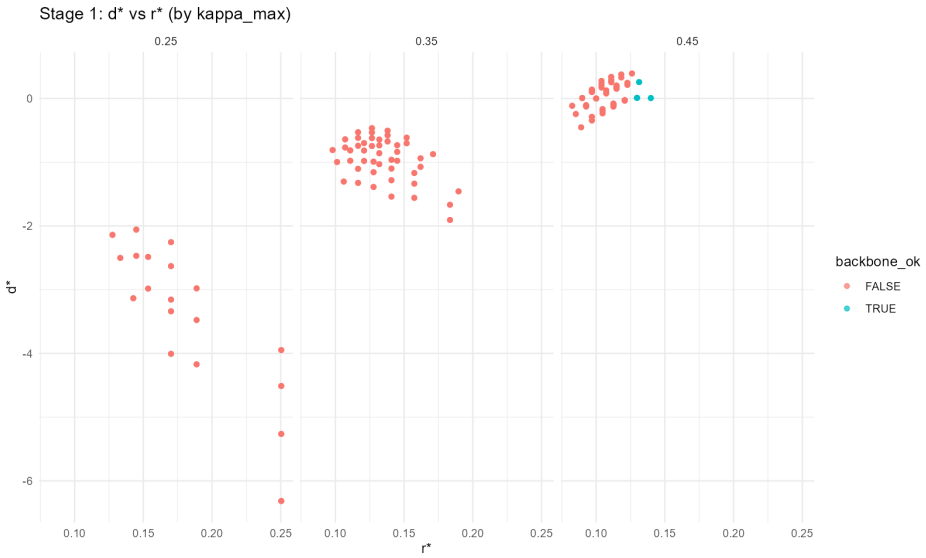

Figure 3: Stage 1: d^* vs r^* (faceted by $\kappa_{\max}$).

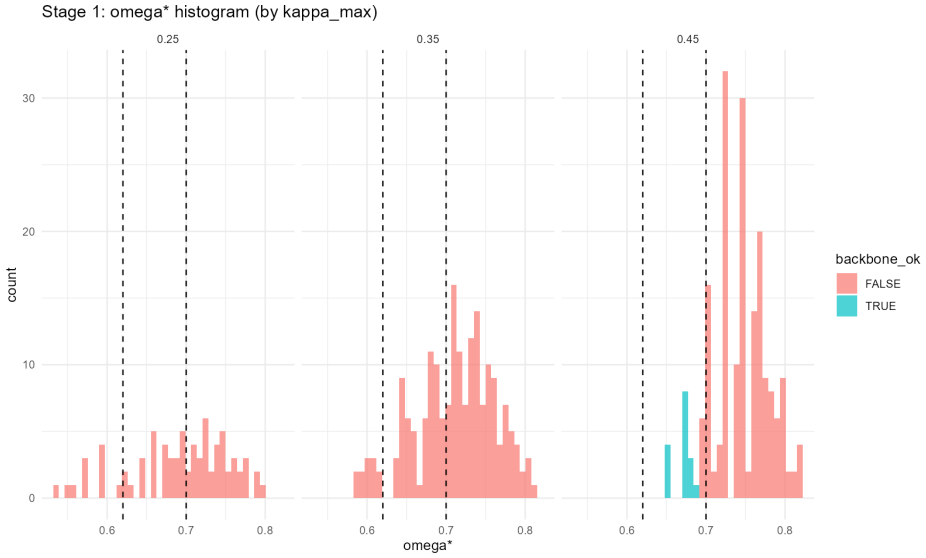

Figure 4: Stage 1: histogram of ω^* (faceted by $\kappa_{\max}$).

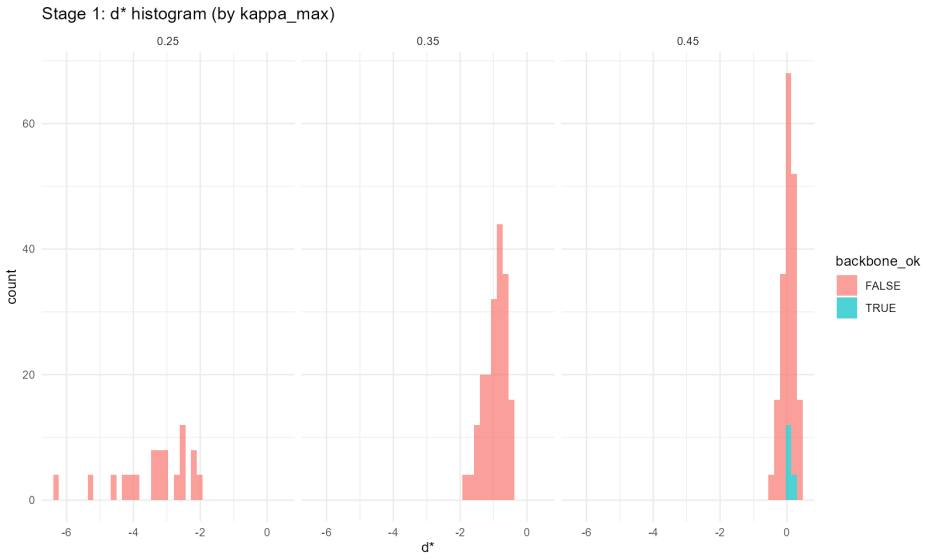

Figure 5: Stage 1: histogram of d^* (faceted by $\kappa_{\max}$).

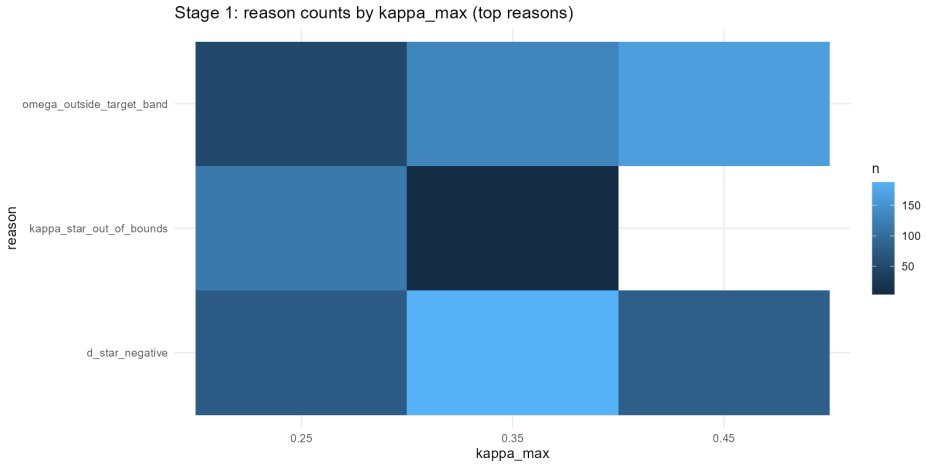

Figure 6: Stage 1: failure reasons by $\kappa_{\max}$ (tile).

1.2 Stage 2 finance/discipline diagnostics

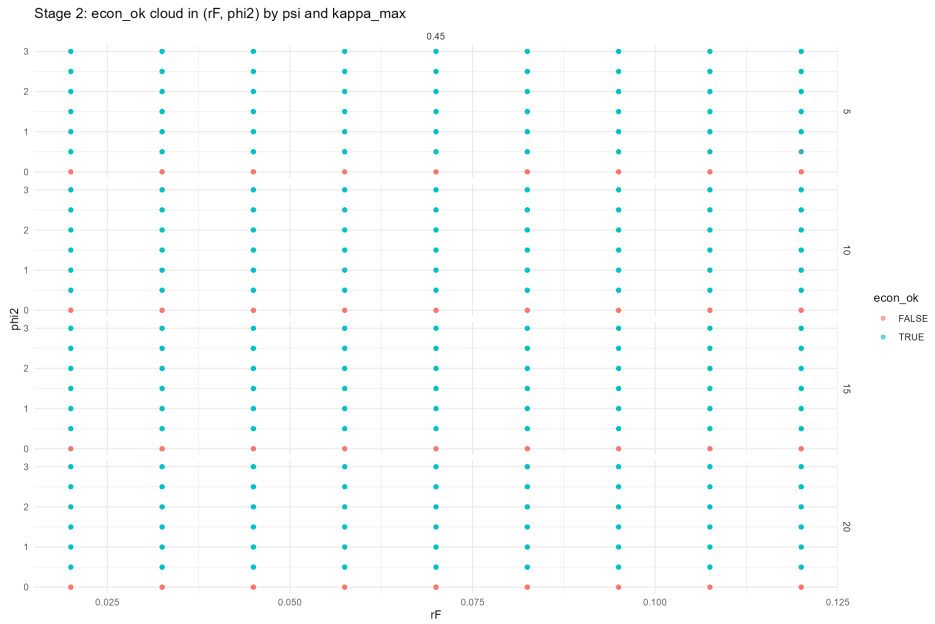

Figure 7: Stage 2: econ_ok cloud in (r_F, ϕ_2), faceted by ψ and $\kappa_{\max}$.

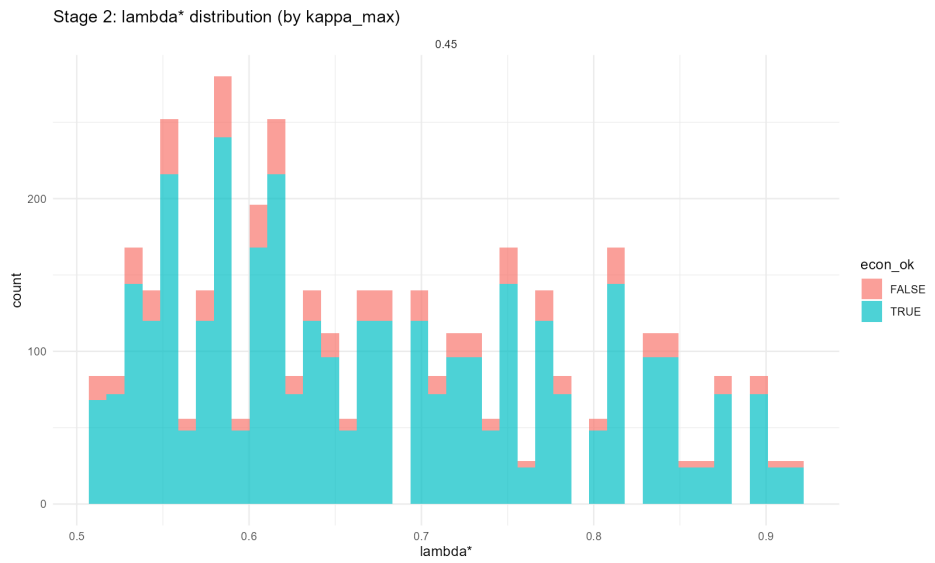

Figure 8: Stage 2: λ^* distribution (faceted by $\kappa_{\max}$).

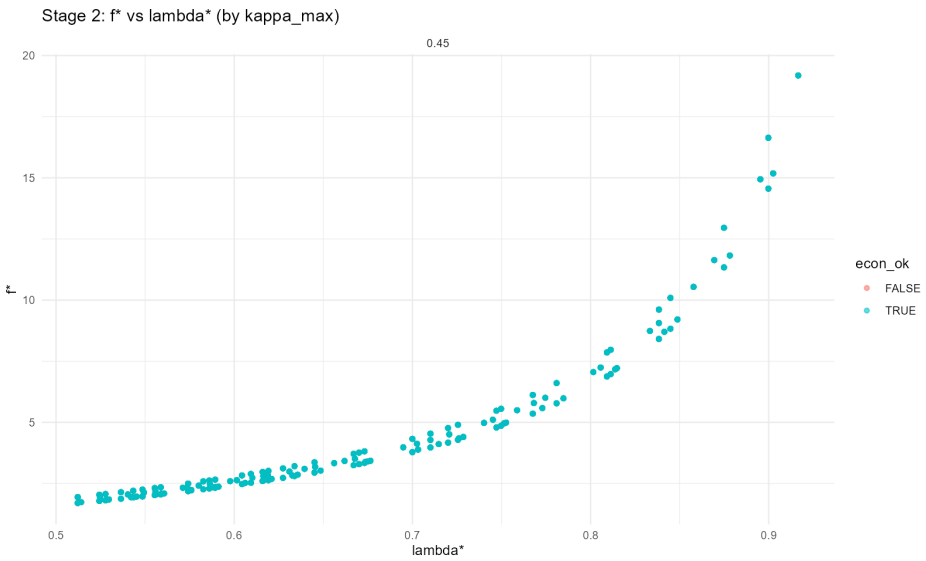

Figure 9: Stage 2: f^* vs λ^* (faceted by $\kappa_{\max}$).

1.3 Stage 3 stability and Hopf diagnostics

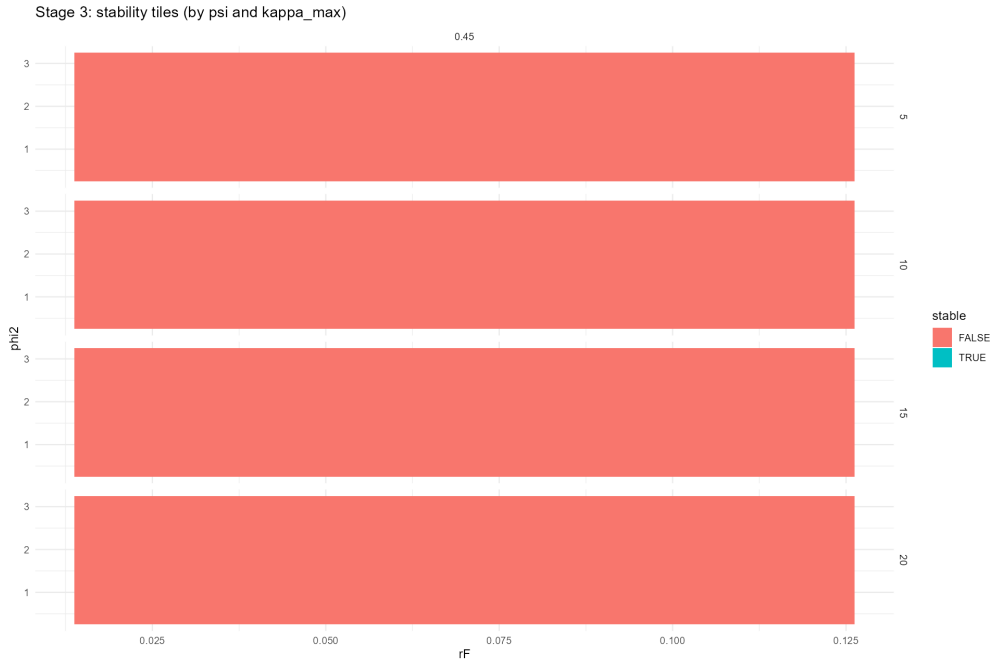

Figure 10: Stage 3: stability tiles (faceted by ψ and $\kappa_{\max}$).

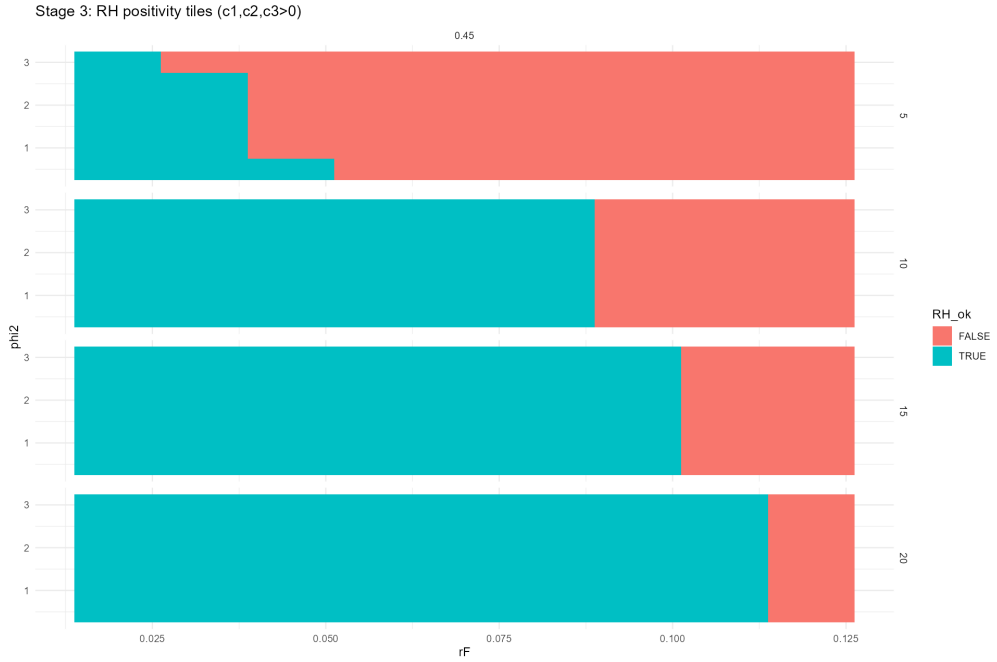

Figure 11: Stage 3: RH positivity tiles ($c_1, c_2, c_3 > 0$), faceted by ψ and $\kappa_{\max}$.

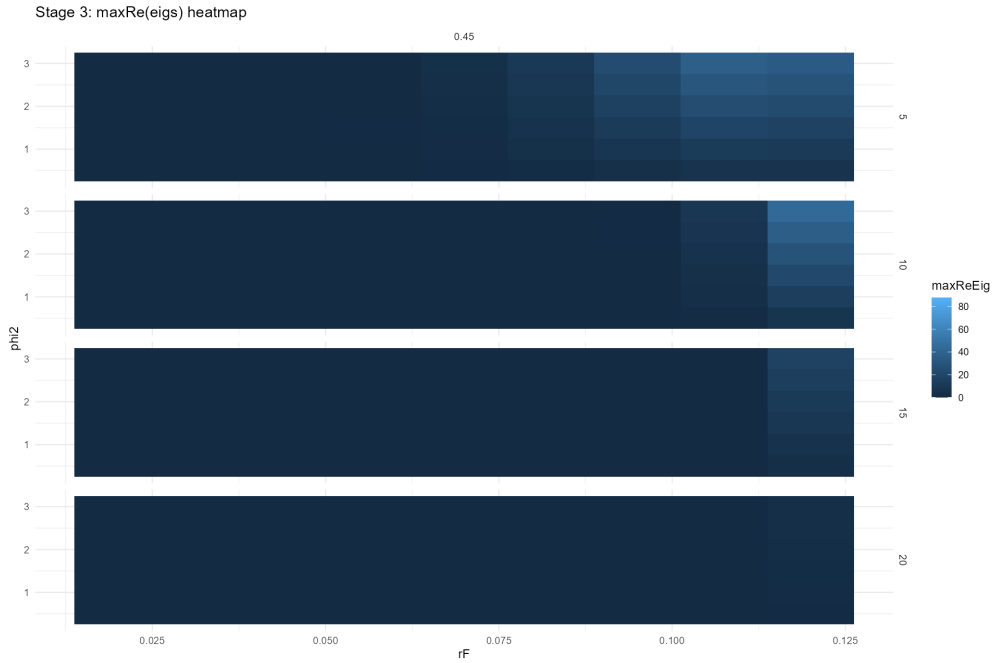

Figure 12: Stage 3: $\max \Re(\lambda)$ heatmap, faceted by ψ and $\kappa_{\max}$.

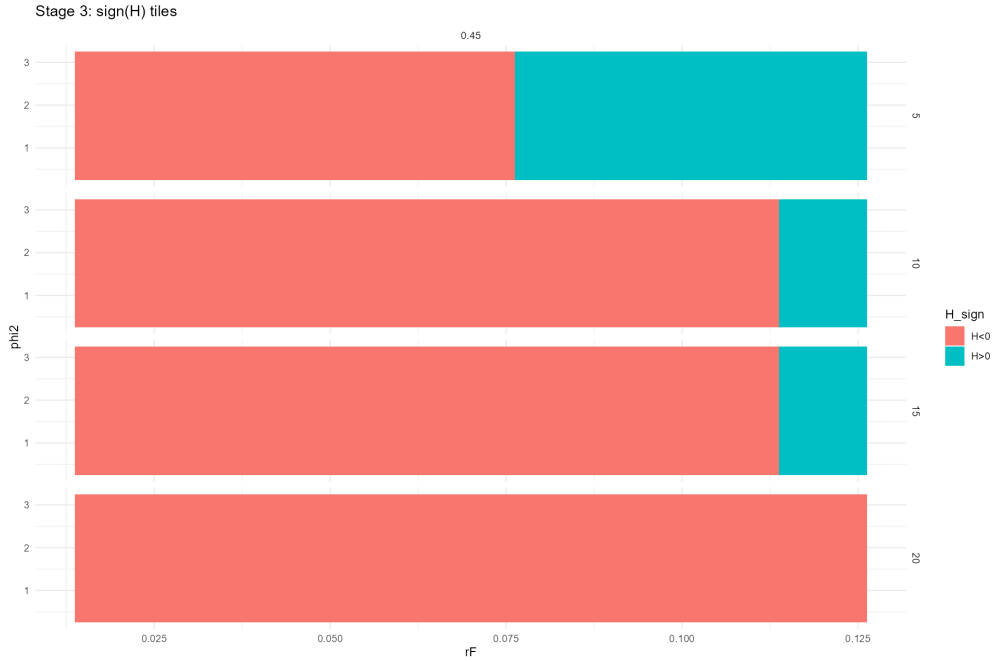

Figure 13: Stage 3: $\operatorname{sign}(H)$ tiles, faceted by ψ and $\kappa_{\max}$.

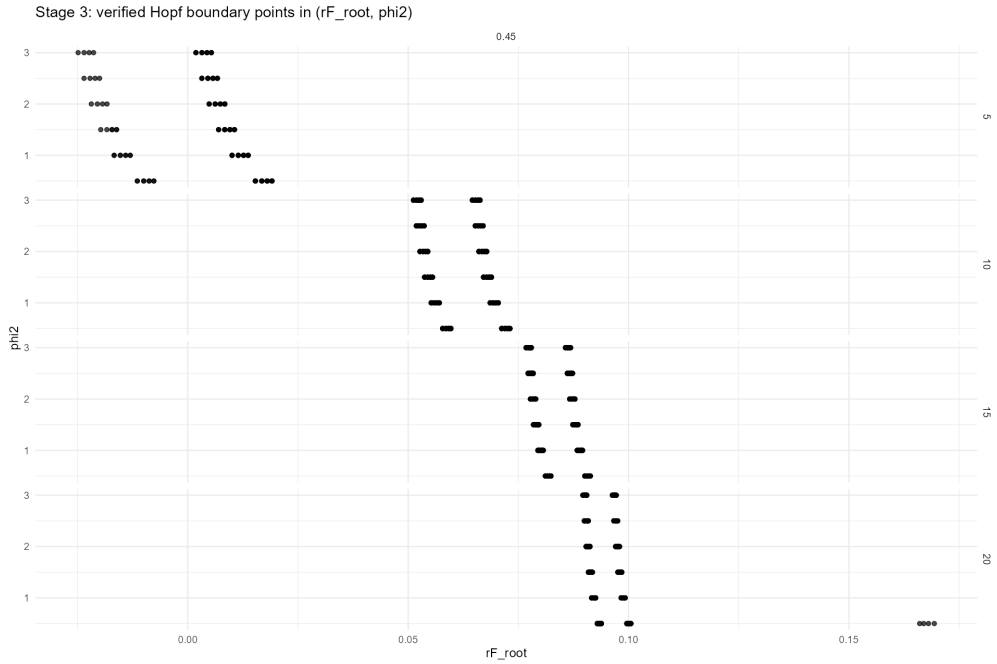

Figure 14: Stage 3: verified Hopf boundary points in (r_F^{root}, ϕ_2) (faceted by ψ and $\kappa_{\max}$).

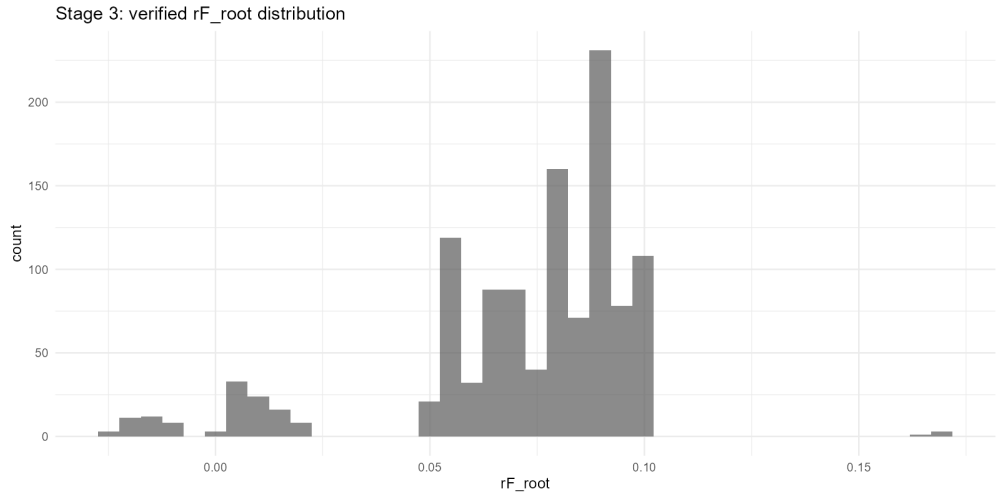

Figure 15: Stage 3: histogram of verified Hopf root locations (r_F^{root}).

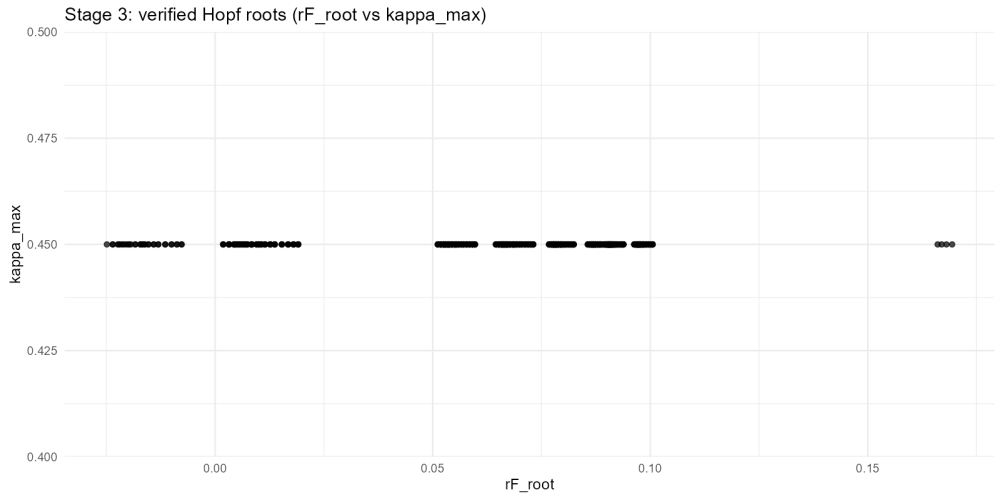

Figure 16: Stage 3: verified Hopf roots scatter (r_F^{root} vs $\kappa_{\max}$).

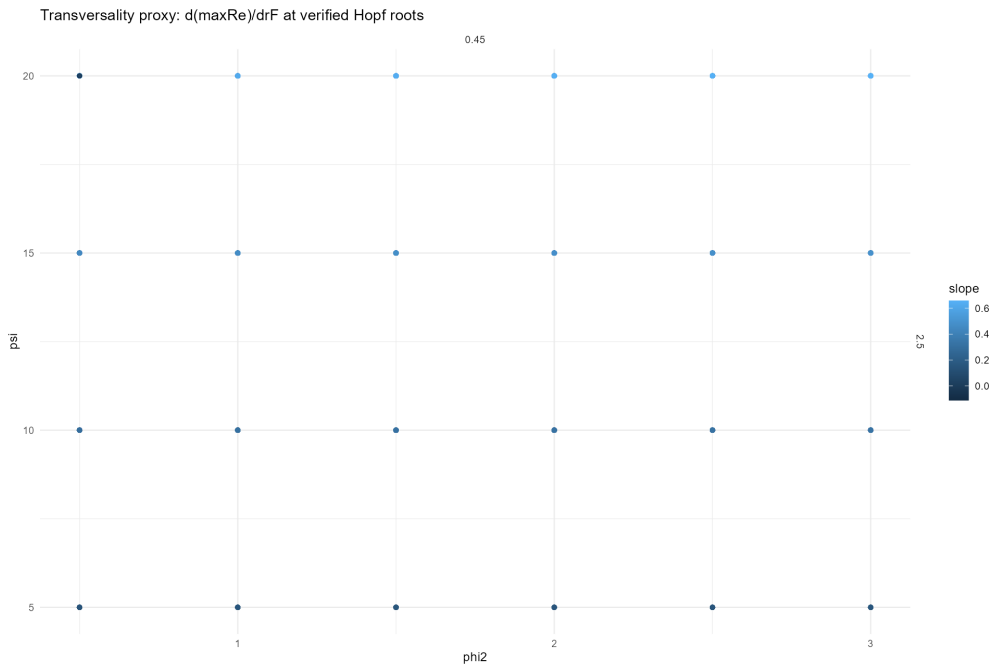

Figure 17: Stage 3: transversality proxy map (finite-difference slope $d(\max \Re)/dr_F$ at root).

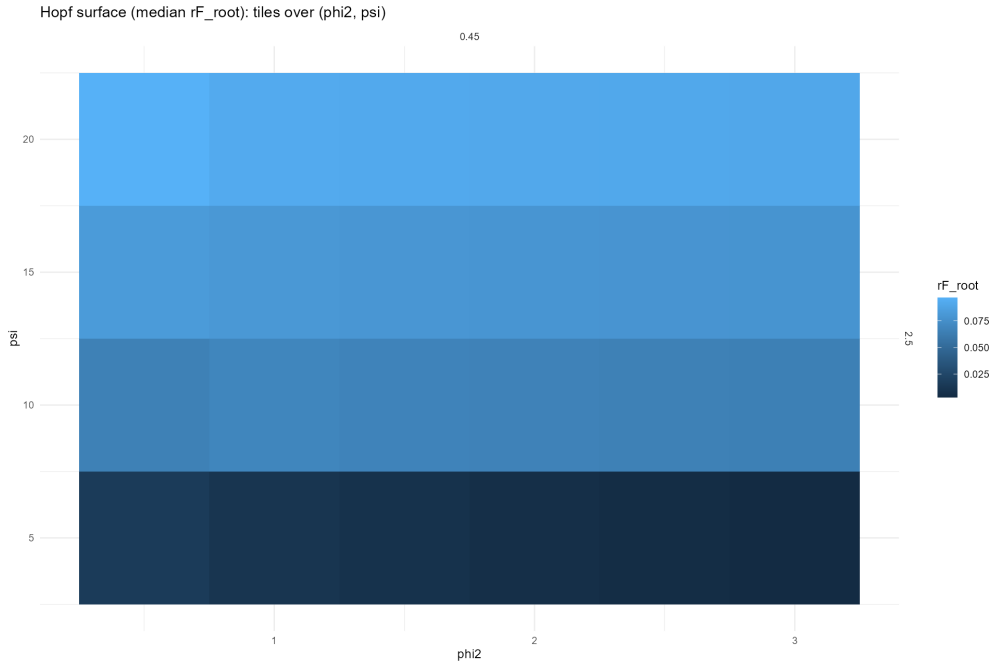

Figure 18: Hopf surface: median r_F^{root} as tiles over (ϕ_2, ψ), faceted by $(\sigma, \kappa_{\max})$.

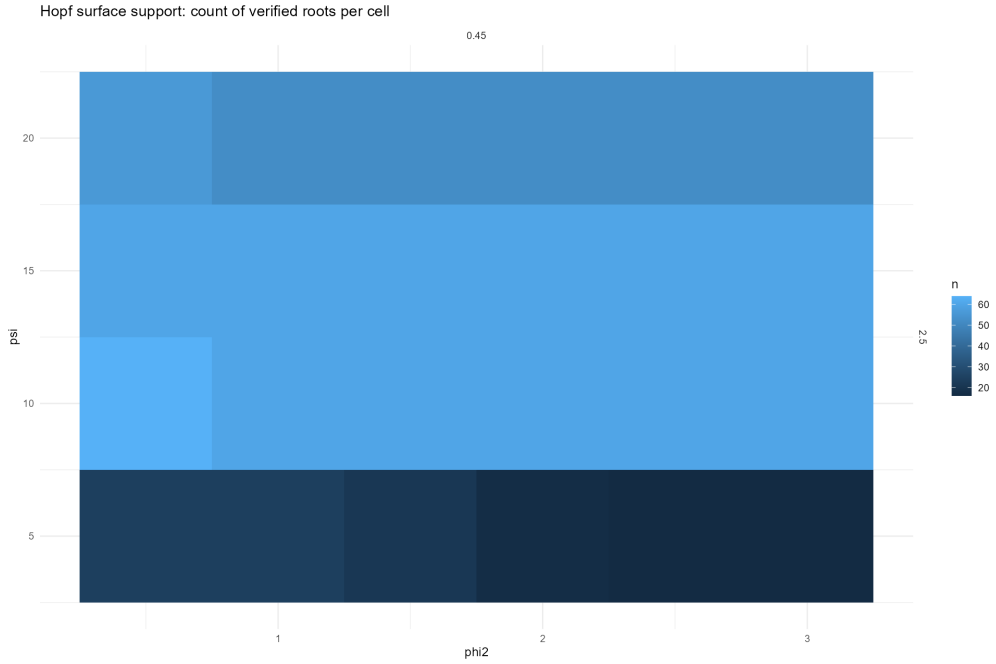

Figure 19: Hopf surface support: number of verified roots per cell over (ϕ_2, ψ).

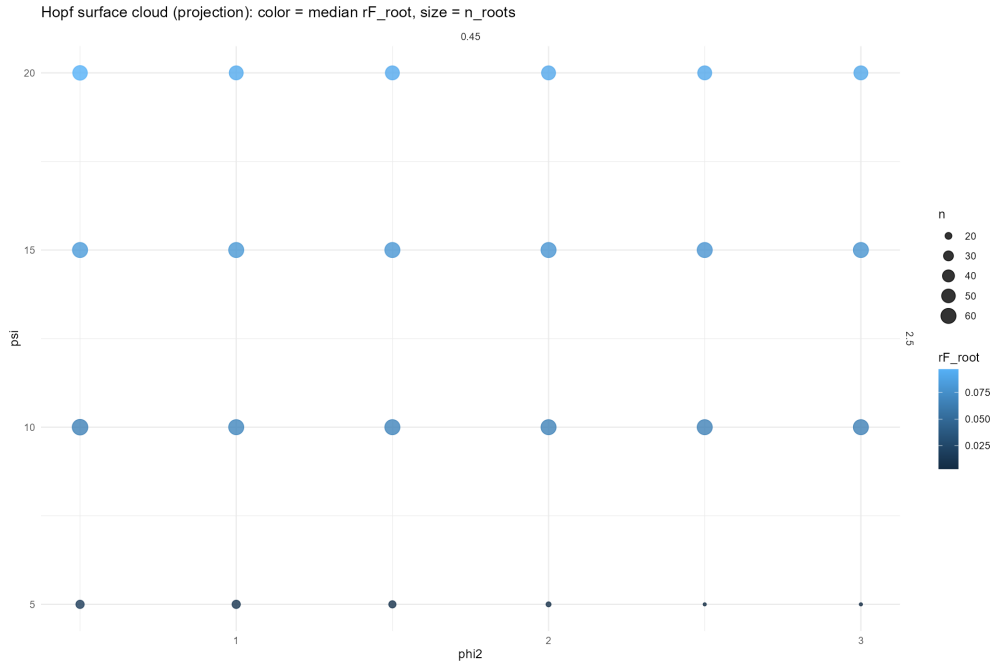

Figure 20: Hopf surface cloud (projection): color = median r_F^{root}, size = support count.

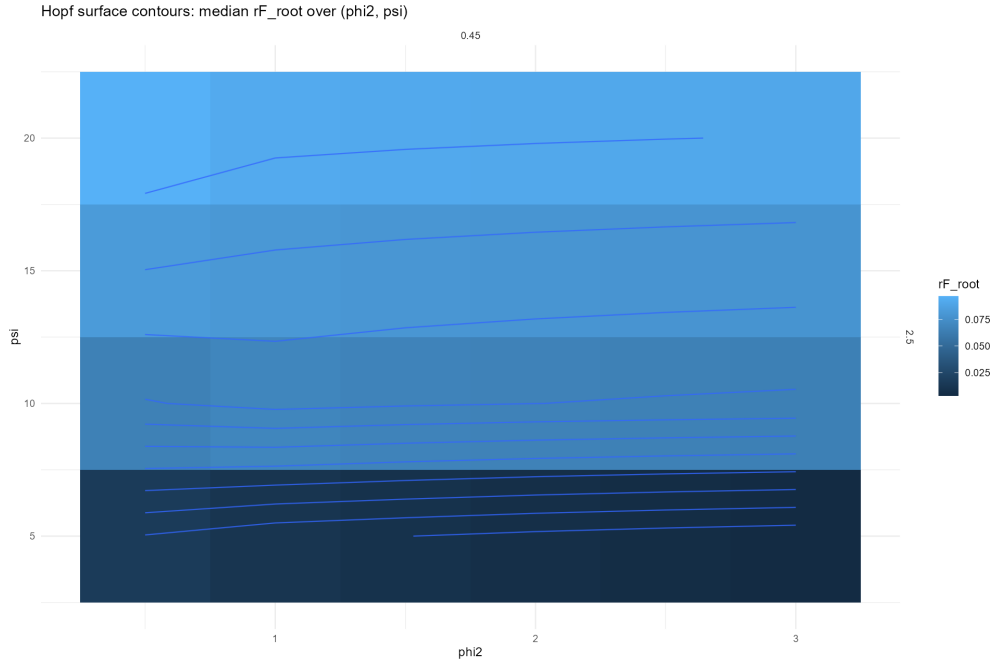

Figure 21: Hopf surface contours (projection): median r_F^{root} over (ϕ_2, ψ).

1.4 Stage 3 summary tables (CSV-backed to avoid alignment issues)

Table 1: Stage 3 summary by $(\psi, \kappa_{\max})$ (from CSV export).

psi	kappa˙max	n	n˙stable	share˙stable	n˙RH˙ok	share˙RH˙ok
5	0.45	860	0	0	144	0.17
10	0.45	864	177	0.2	512	0.59
15	0.45	864	264	0.31	651	0.75
20	0.45	864	312	0.36	704	0.81

Table 2: Verified Hopf root summary by $(\psi, \kappa_{\max})$ (from CSV export).

psi	kappa˙max	n˙verified	rF˙root˙med	rF˙root˙p25	rF˙root˙p75	slope˙med
5	0.45	118	0.01	−0.01	0.01	0.13
10	0.45	364	0.07	0.05	0.07	0.25
15	0.45	360	0.08	0.08	0.09	0.38
20	0.45	316	0.09	0.09	0.1	0.5

1.5 Stage 4 scoring and shortlists

1.5.1 Gate diagnostics (CSV)

Table 3: Stage 4 gate diagnostics (from `stage4_scoring/gate_diagnostics.csv`).

n˙total	n˙RH˙ok	n˙complex	n˙hopf	n˙omega˙le˙cap
3,452	2,011	2,444	1,724	864

1.5.2 Score diagnostics (plots)

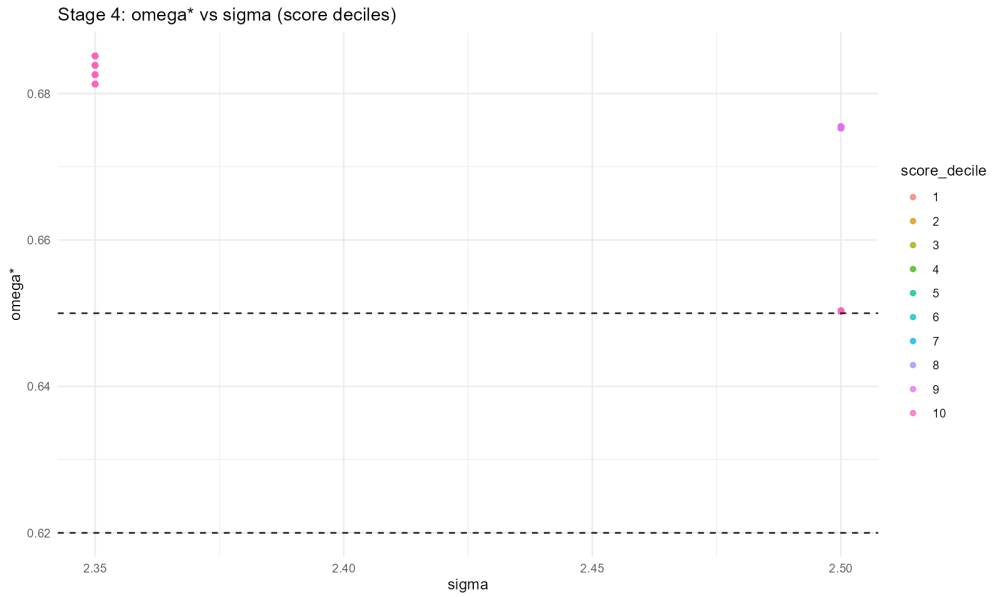

Figure 22: Stage 4: ω^* vs σ colored by score deciles (lower score = preferred).

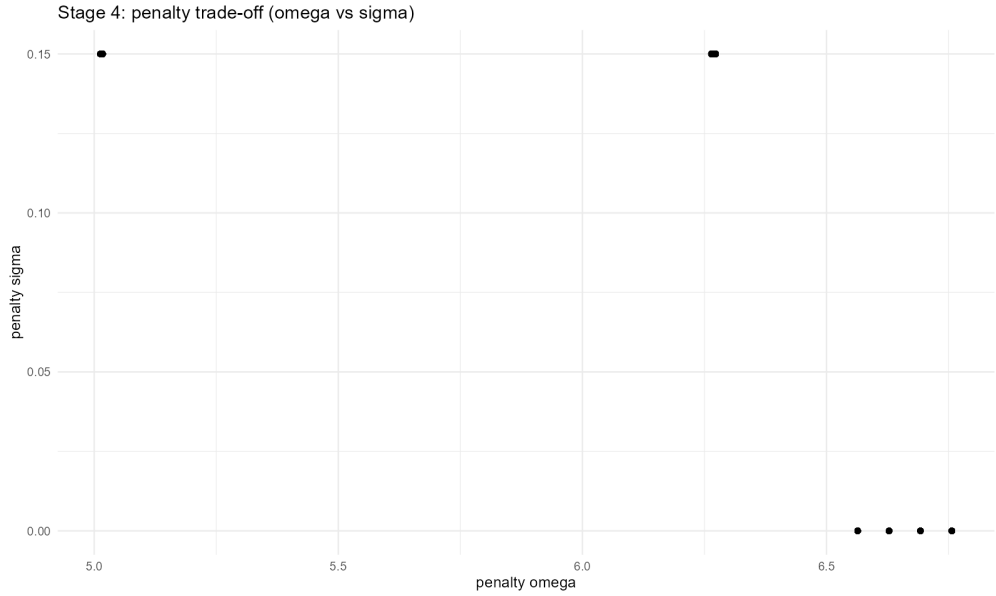

Figure 23: Stage 4: penalty tradeoff diagnostic (ω^* vs σ).

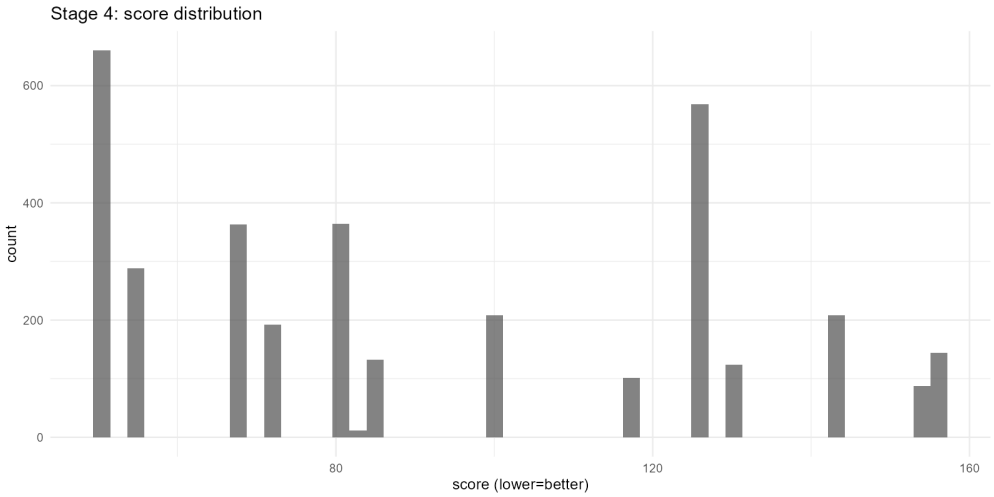

Figure 24: Stage 4: score distribution (lower = better).

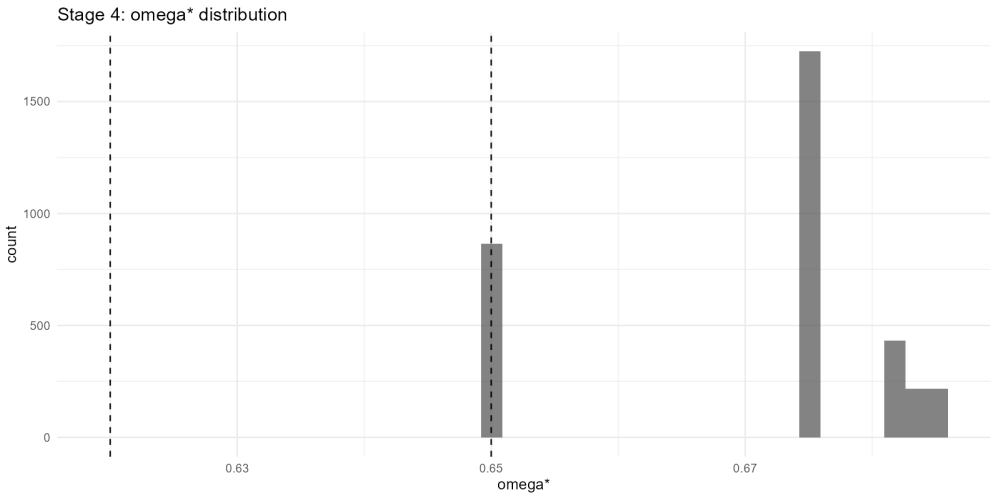

Figure 25: Stage 4: ω^* distribution in the scored candidate set.

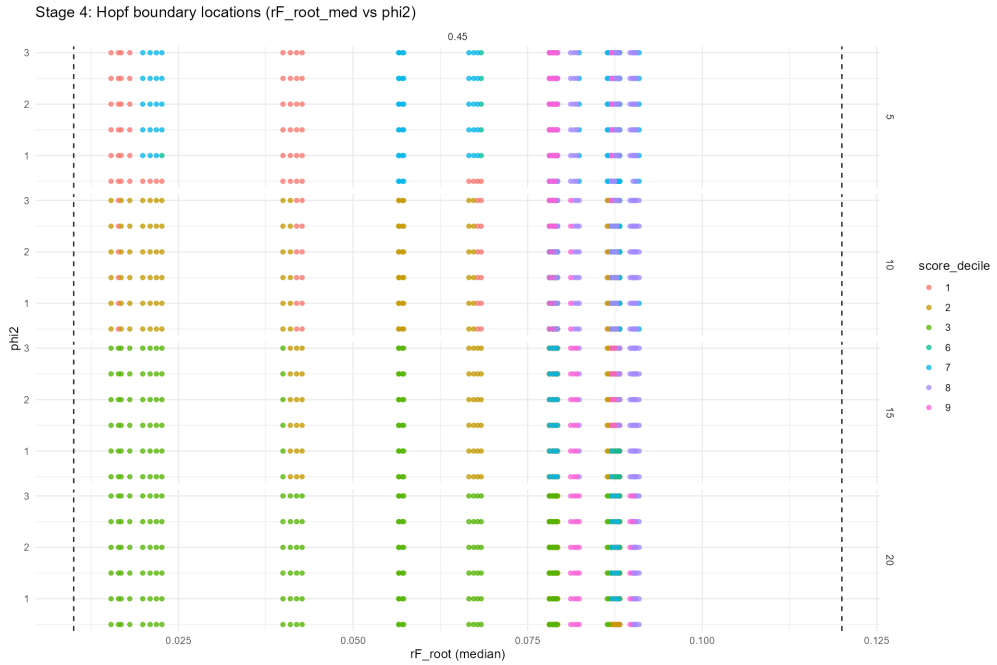

Figure 26: Stage 4: Hopf boundary locations (r_F^{root} vs ϕ_2), colored by score deciles (faceted).

1.5.3 Stage 4 shortlist tables (`\input` with underscore-safe catcode)

Table 4: Stage 4 shortlist (top 20 by score; rounded to 2 decimals).

try id	cand id	psi	kappa max	sigma	omega star	rF	has hopf	rF root med	score
576	16	5	0.45	2.5	0.65	0.04	F	NA	67.39
564	14	5	0.45	2.5	0.65	0.04	F	NA	67.40
564	14	15	0.45	2.5	0.65	0.11	F	NA	67.41
576	16	5	0.45	2.5	0.65	0.03	F	NA	67.41
576	16	5	0.45	2.5	0.65	0.03	F	NA	67.41
576	16	5	0.45	2.5	0.65	0.03	F	NA	67.41
576	16	5	0.45	2.5	0.65	0.03	F	NA	67.41
576	16	5	0.45	2.5	0.65	0.03	F	NA	67.41
552	12	5	0.45	2.5	0.65	0.04	F	NA	67.42
576	16	10	0.45	2.5	0.65	0.08	F	NA	67.42
576	16	10	0.45	2.5	0.65	0.08	F	NA	67.42
576	16	10	0.45	2.5	0.65	0.08	F	NA	67.42
576	16	10	0.45	2.5	0.65	0.08	F	NA	67.42
576	16	10	0.45	2.5	0.65	0.08	F	NA	67.42
576	16	10	0.45	2.5	0.65	0.08	F	NA	67.42
552	12	15	0.45	2.5	0.65	0.11	F	NA	67.42
564	14	5	0.45	2.5	0.65	0.03	F	NA	67.42
564	14	5	0.45	2.5	0.65	0.03	F	NA	67.42
564	14	5	0.45	2.5	0.65	0.03	F	NA	67.42
564	14	5	0.45	2.5	0.65	0.03	F	NA	67.42

Table 5: Stage 4 shortlist (top 20 by score, gated; rounded to 2 decimals).

try˙id	cand˙id	psi	kappa˙max	sigma	omega˙star	rF	has˙hopf	rF˙root˙med	rF˙sim˙low	rF˙sim˙high