Analytical Receiver Collisions Performance Modeling of a Multi-channel Network

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***Abstract*— In this paper, an accurate analytical performance study of a multi-channel network is derived, based on the probability theory. Especially, we mathematically model the performance of a multi-channel network, such as a wireless or an optical WDM network, assuming both the collisions over the multiple channels and the destination conflicts. The receiver collisions phenomenon is extensively studied, while a general analytical formula is derived that provides the probability of the rejection events at destination, depending on the buffer capacity of the station receiver. The performance measures of throughput and delay are provided based on a Markovian model analysis for finite population. Numerical results are studied for diverse numbers of receivers per station.**

Keywords— Multi-channel network; receiver collisions; rejection probability

I. INTRODUCTION

With an abstract point of view, many modern network technologies may be assumed as multi-channel networks. By this terminology we refer to a network technology that divides the access medium bandwidth into several parallel logical channels, each of which is distinguishable from the others based on a multiplexing method. For example, the high speed optical and the wireless networks that use the Wavelength Division Multiplexing (WDM) [1] and the Code Division Multiplexing (CDM) [2] technique respectively are well-known multi-channel network examples that are broadly used.

The performance of a multi-channel network mainly depends on the effect of two parameters. First, on the collisions on the channels that occur if more than one packets are concurrently transmitted over the same channel. Second, on the receiver collisions which occur if there is a packet that is successfully transmitted through the multi-channel network but it cannot be picked up by the destination station since its receiver buffer is full by the time of the packet arrival [3-4]. Both the channel and the receiver collisions cause packets loss and significantly affect the network performance, by reducing the system throughput and increasing the total delay.

In a multi-channel network, the access scheme followed critically differentiates the above performance parameters effect. Especially, in synchronous transmission Medium Access Control (MAC) protocols, a possible packets channel collision is the result of the packets transmission on the same channel at the same time instant. Similar, a possible packets collision at destination is the result of the transmission at the same time instant over different channels, of packets with the same destination [3-6]. In these cases, the packets collision is total. On the contrary, in asynchronous transmission MAC protocols the collision has different sense and may be partial. Especially in this case, both the channel and the receiver collisions refer to packet transmissions that may differ by time interval less than the packet transmission time [6-8].

In literature, most of the studies for multi-channel protocols assume an infinite receiver buffer capacity [9]. This assumption leads to the faulty result that a destination station is capable of concurrently receiving an infinite number of packets that are destined to it. Some other studies, assume a receiver buffer size of one packet at each destination station, providing more realistic results [6]. Finally, recent studies for WDM networks extent the receiver buffer size assumption to more than one packets, to improve the performance [10-12].

In this paper, we assume at each station a receiver buffer with capacity of more than one packets. Thus, we assume that each station is able to concurrently receive up to F ($F \geq 2$) packets from different channels of the multi-channel network. So, we achieve to significantly improve the throughput and to decrease the packet rejection probability at destination.

Especially, in this study we adopt a synchronous transmission MAC protocol that considers both the channel collisions and the destination conflicts. We analytically study the effect of the receiver collisions phenomenon, providing a closed mathematical formula for the rejection probability at destination based on statistics of finite population. A Markovian model is adopted for the multi-channel system performance study, while the performance measures of average throughput, delay, number of backlogged stations and rejection probability at destination are analytically derived. The proposed protocol performance is extensively studied for several receiver buffer sizes, number of channels and stations.

The paper is organized as follows. Section II presents the network model and the assumptions. The analysis is provided in Section III. Numerical results and comments are studied in Section IV. The conclusion is outlined in Section V. Finally, the Appendix gives the proof of the closed formula for the rejection probability at destination.

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II. NETWORK MODEL AND ASSUMPTIONS

We consider a multi-channel network that consists of N parallel logical channels of the same capacity, like in Fig. 1. The multi-channel network interconnects a finite number M of stations. Each station is connected to each channel with separate network interface. The channels are error-free without capture phenomena. The packet transmission time is defined as time unit and is called cycle, as Fig. 1 shows. The packet carries its source and destination address information. Time axis is divided into contiguous cycles of equal length and stations are synchronized for transmission at the beginning of a cycle. At each station, the transmitter buffer capacity is one packet, while the receiver buffer capacity is F ($1 \le F \le N$) packets. If the transmitter buffer is empty the station is said to be free, otherwise it is backlogged. If a station is backlogged and generates a new packet, the packet is lost. Free stations that unsuccessfully transmit on the channels or in case of rejection at destination due to receiver collisions during a cycle, are getting backlogged on the next cycle. A backlogged station is getting free at the next cycle if it manages to retransmit without collision over a channel and its packet retransmission is not aborted due to receiver collisions. The propagation delay is assumed to be smaller than the cycle duration. Thus, the transmitting station is informed by the end of the cycle weather its transmission was successful or not.

Packets are generated independently at each station following a geometric distribution with birth probability p. A backlogged station retransmits the unsuccessfully transmitted packet following a geometric distribution with probability p_1.

The transmission phase consists of two stages, as Fig. 2 shows. In the first stage, at the beginning of each cycle, if a station attempts transmission, it chooses randomly one of the N channels. The transmitted packets compete according to the Slotted Aloha scheme to gain access over the N multi-channel system. Thus, if x packets are (re)transmitted at the beginning of a cycle, only y of them manage successful transmission over the N channels due to the channel collisions, ($0 \le x \le M$) and ($0 \le y \le x$). In the second stage, the successfully (re)transmitted packets through the N channels, are uniformly distributed to the M stations with equal and constant probability $1/M$ (for the sake of simplicity we suppose that a station may send to and receive from itself). If more than F stations (re)transmit successfully through different channels and their packets are destined for the same station, the destination station accepts only F packets and rejects all the others, as Fig. 2 illustrates. In other words, only z packets are correctly received by their destinations at the end of the cycle, due to the receiver collisions, ($0 \le z \le y$), as Fig. 2 shows.

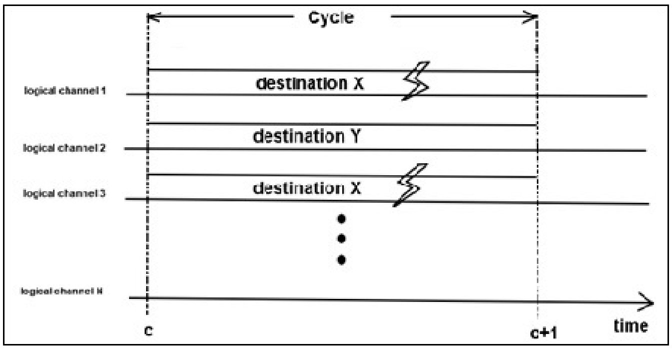

Fig. 1. Multi-channel network, cycle definition and receiver collision case.

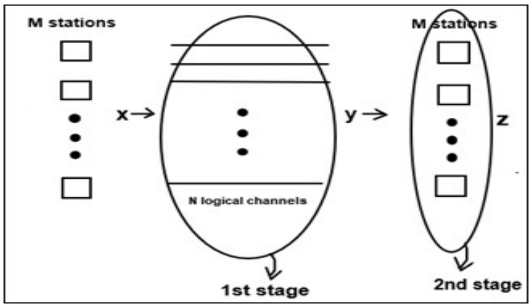

Fig. 2. Trasnmission stages during a cycle.

III. ANALYSIS

The examined system performance can be described by a discrete time Markov chain. We denote the state of the system by X_t, t=1,2… where X_t=0,1…M is the number of backlogged stations at the beginning of a cycle. Let:

H_c =The number of new packets arrivals at the beginning of a cycle, c=0,1,2,…

$A_{c,F}$ = The number of the correctly received packets by the destination at the end of a cycle, c=0,1,2…

$S(x)$=The number of successfully transmitted packets over the N channels conditional that x (re)transmissions occurred during a cycle, c=0,1,2,….

The probability $\Pr[S(x)=y]$ of y successes over the N channels from x (re)transmissions during a cycle is [9]:

$$\Pr[S(x)=y] = \frac{(-1)^y N!x!}{N^x y!} \sum_{j=y}^{\min(N,x)} \frac{(-1)^j (N-j)^{x-j}}{(j-y)!(N-j)!(x-j)!} \quad (1)$$

and $0 \le y \le \min(N,x)$ and x-$y \ne 1$

Also, let $A(y)$=The number of correctly received packets by the destination, conditional that y successful (re)transmissions occurred over the N channels during a cycle.

The probability $\Pr[A(y)=z]$ of z correctly received packets by the destination, conditional that y successful (re)transmissions occurred over the N channels during a cycle, is proven in the Appendix and it is:

$$\Pr[A(y)=z]=\begin{cases}0, & \text{if}:(y<z)\text{ OR }(y>z\text{ AND }F>z)\\ 1, & \text{if}:(y=z\text{ AND }F\geq y\\ prob(y,z,F), & \text{if}:(y\geq z\text{ AND }F<y)\end{cases} \quad (2)$$

We define the function $\Phi_N(x,y,z)$ as the product:

$$\Phi_N(x,y,z)=\Pr[S(x)=y]\Pr[A(y)=z] \quad (3)$$

Also, we define the conditional probability $q_{i,n}$ that i out of n backlogged stations attempt to retransmit with probability p_1 during the cycle. $q_{i,n}$ is given by: $q_{i,n}=bin(n,i,p_1)$ (4)

Similar, the conditional probability $Q_{i,n}$ that i out of $(M\text{-}n)$ free stations attempt to transmit with probability p during the cycle, is defined as: $Q_{i,n}=bin(M-n,i,p)$ (5)

where: $$bin(i,j,p)=\binom{i}{j}p^j(1-p)^{i-j}, \qquad i\geq j \quad (6)$$

The Markov chain X_t t=1,2.. is homogeneous, aperiodic and irreducible. The one step transition probabilities are [13]:

$$P_{ij}=(X_{t+1}=j\mid X_t=i) \quad (7)$$

where:

Case A: if $j<i\text{-}N$ then: $P_{ij}=0$ (8)

Case B: if $j=i-N$ then: $P_{ij}=Q_{0,i}q_{N,i}\Phi_N(N,N,N)$ (9)

Case C: if $i-N<j<i$ then: $P_{ij}=$

$$=\begin{cases}\sum_{n=i-j}^{N}q_{n,i}\sum_{m=0}^{\min(M-i,N-n)}Q_{m,i}\sum_{s=0}^{n+j-i}\Phi_N(n+m,i-j+m+s,m+i-j)+ \\ +\sum_{n=i-j+2}^{N}q_{n,i}\sum_{m=N-n+1}^{\min(M-i,N+j-i-1)}Q_{m,i}\sum_{s=0}^{N-m+j-i-1}\Phi_N(n+m,i-j+m+s,m+i-j)+ \\ +\sum_{n=N+1}^{i}q_{n,i}\sum_{m=0}^{\min(M-i,N+j-i-1)}Q_{m,i}\sum_{s=0}^{N-m+j-i-1}\Phi_N(n+m,i-j+m+s,m+i-j)\end{cases} \quad (10)$$

Case D: if $j=i$ then:

$$P_{ij}=\begin{cases}Q_{0,i}\sum_{n=2}^{i}q_{n,i}\Phi_N(n,0,0)+q_{0,i}\sum_{m=0}^{N}Q_{m,i}\Phi_N(m,m,m)+ \\ +\sum_{m=1}^{\min(M-i,N-1)}Q_{m,i}\sum_{n=1}^{N-m}q_{n,i}\sum_{s=0}^{N-m}\Phi_N(n+m,m+s,m)+ \\ +\sum_{m=1}^{\min(M-i,N-1)}Q_{m,i}\sum_{n=N-m+1}^{i}q_{n,i}\sum_{s=0}^{N-m-1}\Phi_N(n+m,m+s,m)\end{cases} \quad (11)$$

Case E: if $j>i$ then:

$$P_{ij}=\begin{cases}Q_{j-i,i}\sum_{n=0}^{i}q_{n,i}\Phi_N(j-i+n,0,0)+ \\ +\sum_{m=1}^{\min(M-j,N-j+i)}Q_{j-i+m,i}\sum_{n=0}^{N-m-j+i}q_{n,i}\sum_{s=0}^{N-m}\Phi_N(j-i+n+m,m+s,m)+ \\ +\sum_{m=1}^{\min(M-j,N-1)}Q_{j-i+m,i}\sum_{n=N-m-j+i+1}^{i}q_{n,i}\sum_{s=0}^{N-m-1}\Phi_N(j-i+n+m,m+s,m)\end{cases} \quad (12)$$

Performance measures

Since the Markov chain X_t t=1,2.. is ergodic, the steady state probabilities can be obtained by solving the system of the following linear equations:

$$\boldsymbol{\pi}=\boldsymbol{\pi}\ \mathbf{P} \quad (13) \qquad \text{and:} \qquad \sum_{i=0}^{M}\pi_i=1 \quad (14)$$

where **P** is the transition matrix with elements the probabilities P_{ij} and $\boldsymbol{\pi}$ is a row vector of the steady state probabilities π_i.

The conditional throughput $S_F(i)$ is the expected value of the successful data packet receptions by the destination during a cycle, given that the number of the backlogged stations at the beginning of the cycle is i when the receiver buffer capacity is F packets, i.e:

$$S_F(i)=\begin{cases}\sum_{k=1}^{N}k\sum_{m=0}^{\min(M-i,N)}Q_{m,i}\sum_{\substack{n=0\\ m+n\geq k}}^{\min(N-m,i)}q_{n,i}\sum_{s=0}^{N-k}\Phi_N(n+m,k+s,k)+ \\ +\sum_{k=1}^{N-1}k\sum_{m=0}^{\min(M-i,N)}Q_{m,i}\sum_{n=N+1}^{i}q_{n,i}\sum_{s=0}^{N-k-1}\Phi_N(n+m,k+s,k)+ \\ +\sum_{k=1}^{N-1}k\sum_{m=N+1}^{M-i}Q_{m,i}\sum_{n=0}^{i}q_{n,i}\sum_{s=0}^{N-k-1}\Phi_N(n+m,k+s,k)\end{cases} \quad (15)$$

Thus, the steady state average throughput S_F when the receiver buffer capacity is F packets, is given by:

$$S_F=E[S_F(i)]=\sum_{i=0}^{M}S_F(i)\pi_i \quad (16)$$

The steady state average number B_F of backlogged stations when the receiver buffer capacity is F packets, is given by:

$$B_F=E[i]=\sum_{i=0}^{M}i\pi_i \quad (17)$$

The average delay D_F is defined as the average number of cycles that a packet has to wait until its successful transmission, when the receiver buffer capacity is F packets. According to the Little's formula, it is:

$$D_F=1+\frac{B_F}{S_F} \quad (18)$$

Rejection Probability

We define the average rejection probability $P_{rej}(F)$ at destination when the receiver buffer size per station is F, as the ratio of the expected number of packets rejections per cycle due to receiver collisions to the expected number of successfully transmitted packets over the multi-channel system per cycle in steady state:

$$P_{rej}(F) = \frac{S_0 - S(F)}{S_0} \quad (19)$$

where: S_0 is the throughput of the data multi-channel system.

IV. Performance Evaluation

For the numerical solutions, we consider that: L=10 time units, R=5 time units and p_1 =0.3. In order to verify the Markovian analysis results, we adopted a simulation model based on Prolog programming that simulates the transmission stages during a cycle. The analytical results have been proven to have one to one correspondence to the simulation ones. Finally, in order to study the performance of the proposed MAC protocol with the multiple packet size receiver buffer, we compare it with the relative one of [13] which uses the same MAC protocol but it has a single packet receiver buffer size per station. The comparison provides the benefits offered by the receiver buffer size increase.

Fig. 3 shows the average throughput S_F per cycle versus the birth probability p, for M=50 stations, N=20, 30, 50 channels, and F=1, 2 packets receiver buffer size. As it can be noticed, the S variation is similar for F=1, 2, for all values of N. This means that the S is an increasing function of N. For example for F=2 and for p=0.9, the S is: 7.5 packets/cycle for N=20, 10.8 packets/cycle for N=30, and 15.5 packets/cycle for N=50. This is because as the number N increases, the probability of a control packet successful transmission over the multi-channel network increases too. Thus, the probability of a correct packet reception at destination is getting higher, providing higher S values.

As Fig. 3 shows, if the receiver buffer size increases from F=1 to F=2 packets, the S values are significantly increased. Particularly, for p=0.9, the S improvement is: 6.8% for N=20, 11.2 % for N=30, and 17.8% for N=50. The explanation is based on the fact that as N increases, the probability of a data packet rejection at destination increases too, due to the higher probability of a successful transmission over the multi-channel network. In case that we increase the value F from F=1 to F=3 packets, the S improvement rises to: 7.2% for N=20 and 19.5% for N=50. This behavior is also illustrated in Fig. 4 that depicts the percentage change of S versus birth probability p, for M=50, N=20, 30, 50, when F=1 increases to F=2 and F=3.

The P_{rej} variation when the number N varies is studied in Fig. 5. Especially, Fig. 5 presents P_{rej} versus the birth probability p, for M=50 stations, N=20, 30, 50 channels, and F=2 packets receiver buffer size. As it is shown, the P_{rej} is an increasing function of N. This is because, as the number N increases for given values of M and p, the probability of an arrival at a destination station of a successfully transmitted packet increases too giving rise to the rejections at destination.

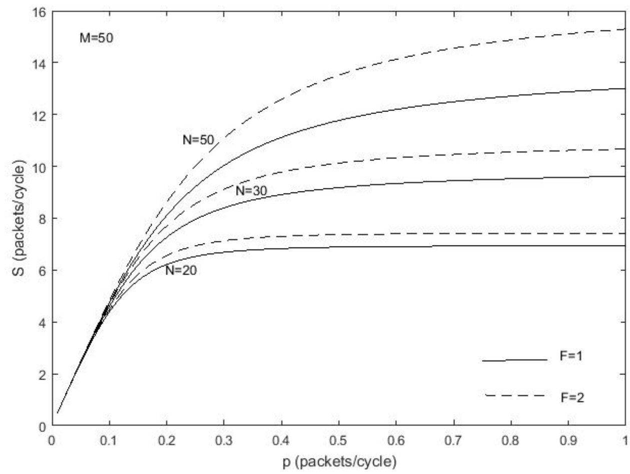

Fig. 3. Throughput S_2 versus birth probability p, for M=50, N=20, 30, 50 and F=1, 2.

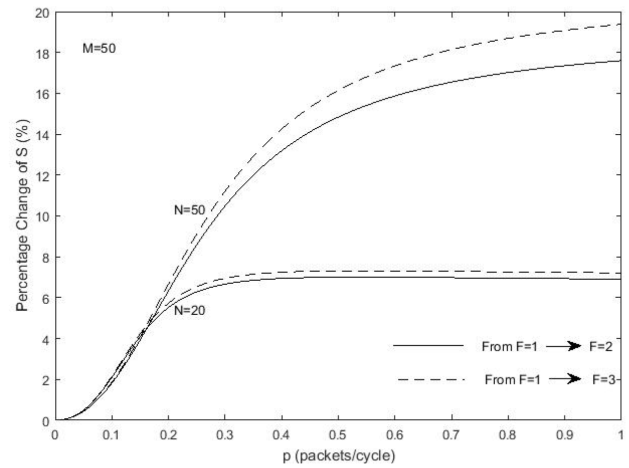

Fig. 4. Percentage change of throughput versus birth probability p, for M=50, N=20, 30, 50, when F=1 increses to F=2 and F=3.

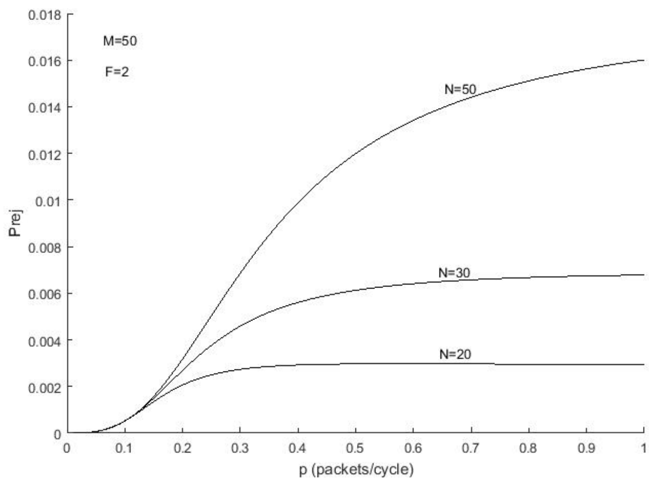

Fig. 5. Rejection probability P_{rej} versus birth probability p, for M=50, N=20, 30, 50 and F= 2.

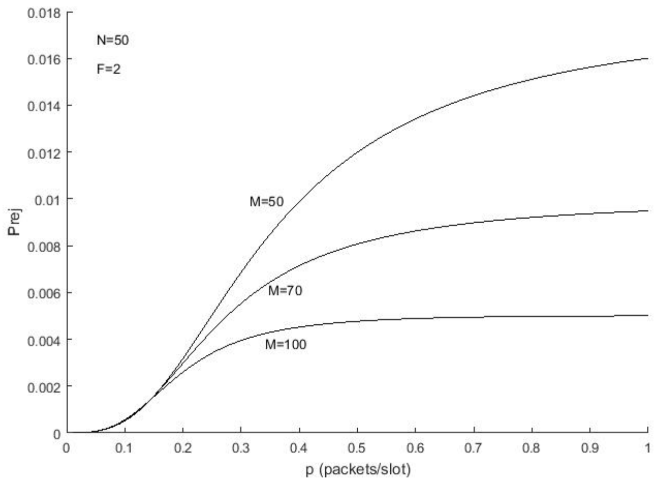

Fig. 6. Rejection probability P_{rej} versus birth probability p, for M=50, 70, 100, N=50 and F= 2.

Finally, Fig. 6 depicts the P_{rej} versus the birth probability p, for M=50, 70, 100 stations, N=50 channels and F= 2 packets receiver buffer size. As it is shown, the P_{rej} is a decreasing function of M. This is because, as the number M increases for given values of N and p, the sum of successfully transmitted packets through the multi-channel network are distributed to a higher number of destination stations, providing lower values of rejection events at destination.

V. Conclusion

In this paper, the main investigation goal is the analytical formulation of the packet rejection at destination due to the receiver collisions in multi-channel multi-access networks, where the stations have a finite receiver buffer size larger than one packet. The statistical analysis for finite population provides a closed formula for the rejection probability function. Based on this analytical tool, a Markovian model is adopted in order to derive the performance measures of average throughput and delay, in order to estimate the performance of a synchronous transmission MAC protocol that takes under consideration both the channel collisions and the destination conflicts. The analytical results are validated by simulation. Comparative results prove that the assumption of a station receiver buffer size larger than one packet provides significant throughput improvement and rejection probability reduction.

Appendix

We explore the probability $\Pr[A(y)=z]$ of z correctly received packets by the destination, conditional that y successful (re)transmissions occurred over the N channels during a cycle, in case that: $(y \geq z \text{ AND } F < y)$. The other cases of (2) are obvious. Let:

E_d = The number of destination stations with exactly d successfully transmitted packets destined to each of them by the end of a cycle, where: d, $E_d \in [0,M]$.

M_d = The number of destination stations with at least d successfully transmitted packets destined to each of them by the end of a cycle, where: d, $M_d \in [0,M]$. Let be:

$$a_s = \begin{cases} E_s, & \text{if} : s \in [2, F-1] \\ M_s, & \text{if} : s = F \end{cases} \tag{20}$$

J = The number of ways to choose the destination stations so as at least one packet to be destined to each of them. It is:

$$J = \binom{M}{z - \sum_{l=2}^{F} (l-1) a_l} \tag{21}$$

K = The number of ways in which the packets can be chosen in order each of them to be destined to a station with no other packet destined to it. It is:

$$K = \binom{y}{z - \sum_{l=2}^{F} l a_l} \tag{22}$$

L = The number of possible replacements of packets that are destined to a destination station with no other packet destined to it. It is:

$$L = \left(z - \sum_{l=2}^{F} l a_l \right)! \tag{23}$$

P = The number of ways in which the destination stations can be chosen in order at least F packets to be destined to each of them. It is:

$$P = \binom{z - \sum_{l=2}^{F} (l-1) a_l}{a_F} \tag{24}$$

Q = The number of ways in which (y-z+Fa_F) packets can be distributed to a_F destination stations in order at least F packets to be destined to each of them. It is:

$$Q = (a_F)!\, SN_F\left(y - z + F a_F, a_F\right) \tag{25}$$

where the function $SN_F(n,k)$ denotes the F-associated Stirling number of the second kind and provides the number of ways n packets are distributed to k destination stations, in order at least F packets to be destined to each of them. $SN_F(n,k)$ is given by the retroactive equation [14]:

$$SN_F(n,k) = k SN_F(n-1,k) + \binom{n-1}{F-1} SN_F(n-F, k-1) \tag{26}$$

where: $SN_F(0,0) = 1$

R = The number of ways to choose a_w destination stations in order exactly w packet to be destined to each of them, $w \in [2,F\text{-}1]$. It is:

$$R=\binom{z-\sum_{l=2}^{F}(l-1)a_l-\sum_{l=w+1}^{F}a_l}{a_w} \tag{27}$$

T = The number of ways in which the packets can be chosen in order each of them to be destined to a station where exactly w packets are destined to it, $w \in [2,F\text{-}1]$. It is:

$$T=\binom{y-\left(z-\sum_{l=2}^{F}la_l\right)-\sum_{l=w+1}^{F-1}la_l}{wa_w} \tag{28}$$

V = The number of ways in which wa_w packets can be distributed to a_w destination stations in order exactly w packets to be destined to each of them, $w \in [2,F\text{-}1]$. It is:

$$V=\frac{(wa_w)!}{(w)!^{a_w}} \tag{29}$$

Thus, in case that: $(y \geq z \text{ AND } F < y)$, it is:

$$\Pr[A(y)=z]=prob(y,z,F)=\frac{1}{M^y}\sum_{a_F=u}^{\left\lfloor \frac{z}{F}\right\rfloor}\ \sum_{a_{F-1}=0}^{\left\lfloor \frac{z-Fa_F}{F-1}\right\rfloor}\cdots\sum_{a_s=0}^{\left\lfloor \frac{z-\sum_{l=s+1}^{F}la_l}{s}\right\rfloor}\ \sum_{a_2=0}^{\left\lfloor \frac{z-\sum_{l=3}^{F}la_l}{2}\right\rfloor}\left\{J\times K\times L\times P\times Q\times\prod_{w=2}^{F-1}\{R\times T\times V\}\right\} \tag{30}$$

Substituting (21)-(29) to (30), in the case: $(y \geq z \text{ AND } F < y)$ we get:

$$\Pr[A(y)=z]=prob(y,z,F)=$$

$$=\frac{1}{M^y}\sum_{a_F=u}^{\left\lfloor \frac{z}{F}\right\rfloor}\ \sum_{a_{F-1}=0}^{\left\lfloor \frac{z-Fa_F}{F-1}\right\rfloor}\cdots\sum_{a_s=0}^{\left\lfloor \frac{z-\sum_{l=s+1}^{F}la_l}{s}\right\rfloor}\cdots\sum_{a_2=0}^{\left\lfloor \frac{z-\sum_{l=3}^{F}la_l}{2}\right\rfloor}\left\{\binom{M}{z-\sum_{l=2}^{F}(l-1)a_l}\times\right.$$

$$\binom{y}{z-\sum_{l=2}^{F}la_l}\left(z-\sum_{l=2}^{F}la_l\right)!\times$$

$$\binom{z-\sum_{l=2}^{F}(l-1)a_l}{a_F}(a_F)!\,SN_F\left(y-z+Fa_F,a_F\right)\times \tag{31}$$

$$\prod_{w=2}^{F-1}\left\{\binom{z-\sum_{l=2}^{F}(l-1)a_l-\sum_{l=w+1}^{F}a_l}{a_w}\times\right.$$

$$\left.\left.\binom{y-\left(z-\sum_{l=2}^{F}la_l\right)-\sum_{l=w+1}^{F-1}la_l}{wa_w}\frac{(wa_w)!}{(w)!^{a_w}}\right\}\right\}$$

where:

$$u=\begin{cases}1, & \text{if}: y>z\\ 0, & \text{if}: y=z\end{cases} \tag{32}$$

Finally, we denote as $\left\lfloor \frac{a}{b}\right\rfloor$ the integer part of the division $\frac{a}{b}$.

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