Advanced Methods on Computational Physics

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Exercise Set 10

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Run the programs by using 'main10.sh' bash script file, 'armadillo' and 'openblas' libraries are required.

1. Solved by using the relaxation method with the algorithm explained in the class
2. In finite difference method (explained in the previous exercise "readme9.pdf") the equation has been solved by putting

$$r(t) = 0, p(t) = -2, g(t) = -1$$
$$t_{final} = 1, t_{init} = 0, \alpha = 1, \beta = 3$$

3. In all of the below problems I use these symbolizations

$$p = y', q = y'', r = y^{(3)}$$

A:

$$y''(t) = f(t, y, y') = 2y^3 - 6y - 2t^3\ , \alpha = 2, \beta = \frac{5}{2}, a = 1, b = 2$$

To solve this problem with RKF45 method we need to know $y'(1) = ?$

$$p(1) = y'(1) = T$$

$$\frac{\partial y}{\partial T} = z \rightarrow T_{k+1} = T_k + \frac{\beta - y_k(b)}{z_k(b)}$$

$$p_z' = \frac{\partial^2 z}{\partial t^2} = \frac{\partial y''}{\partial T} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial T} + \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial T} = (6y - 6)z = g(y, z)$$

$$p_{z0} = \frac{\partial z}{\partial t}(1) = \frac{\partial y'}{\partial T}(1) = \frac{\partial T}{\partial T} = 1, z_0 = \frac{\partial y}{\partial T}(1) = \frac{\partial \alpha}{\partial T} = 0$$

$$T_0 = \frac{(\beta - \alpha)}{N}$$

Now we can solve these two initial value problems (I tried this by putting $T_0 = \frac{\beta - \alpha}{b - a}$ but with this assumption T doesn't converge)

B:

$$y^{(3)} = f(t, y, y', y'') = -yy'' + y'^2 - 1, \alpha = 0, \beta = 0, y'(0) = 0, a = 0, b = 1$$

To solve this problem with Euler method we need to know $y''(0) = ?$

$$q(0) = y''(0) = T$$

$$\frac{\partial y}{\partial T} = z \rightarrow T_{k+1} = T_k + \frac{\beta - y_k(b)}{z_k(b)}$$

$$p_z = z' = \frac{\partial y'}{\partial T} \quad , q_z = z'' = \frac{\partial y''}{\partial T}$$

$$\Rightarrow q_z' = z^{(3)} = \frac{\partial y^{(3)}}{\partial T} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial T} + \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial T} + \frac{\partial f}{\partial y''}\frac{\partial y''}{\partial T} = -y''z + 2y'z' - yz''$$

$$\rightarrow g(t, y, p, q, z, p_z, q_z) = -qz + 2pp_z - yq_z$$

$$pz_0 = \frac{\partial y'}{\partial T}(0) = 0, z_0 = 0, qz_0 = \frac{\partial y''}{\partial T}(0) = \frac{\partial T}{\partial T} = 1$$

$$T_0 = \frac{\beta - \alpha}{b - a}$$

C:

$$y^{(4)} = -y^2 + \frac{t^{-\frac{5}{2}}}{16}(9 + 30t + 105t^2) + t^3(1 - t^4) = f(t, y)$$

$$\alpha = 0, \beta = 0, y'(0) = \alpha' = 0, y'(1) = \beta' = 0$$

To solve this problem with Euler method we need to know $q_0 = y''(0) = ?$ and $r_0 = y^{(3)}(0) = ?$

$$q_0 = T, r_0 = T'$$

$$\frac{\partial y}{\partial T} = z \rightarrow T_{k+1} = T_k + \frac{\beta - y_k(b)}{z_k(b)}$$

$$\frac{\partial y'}{\partial T'} = z' \rightarrow T'_{k+1} = T'_k + \frac{\beta' - y'_k(b)}{z'_k(b)} = T'_k + \frac{\beta' - p_k(b)}{pz_k(b)}$$

$$p_z = z' = \frac{\partial y'}{\partial T} \quad , q_z = z'' = \frac{\partial y''}{\partial T} \quad , r_z = z^{(3)} = \frac{\partial y^{(3)}}{\partial T}$$

$$r_z' = \frac{\partial y^{(4)}}{\partial T} = -2yz = g(y, z)$$

$$pz_0 = \frac{\partial y'}{\partial T}(0) = 0, z_0 = \frac{\partial y}{\partial T}(0) = 0, qz_0 = \frac{\partial q}{\partial T}(0) = 1, rz_0 = \frac{\partial T'}{\partial T} = \frac{\partial}{\partial t}(1) = 0$$

$$T_0 = \frac{\beta - \alpha}{b - a} \quad , \qquad T'_0 = \frac{\beta' - \alpha'}{b - a}$$

Since $f(t, y)$ has a singularity in $t = 0$ I considered $t_{init} = a = 0.001$