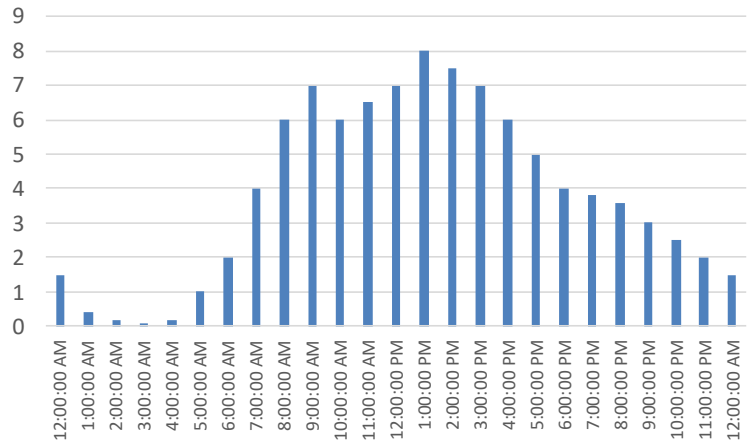
OR 647: Queueing Theory, Spring 2021
Homework Assignment 4
Due Wed. Mar. 24, 2021

1. Problem 3.32 [from homework 3]

2. Problem 3.37 [from homework 3]

3. A call center has 400 service representatives. The offered load is 360. Assume the call center can be modeled as an M/M/c queue. The company believes that its current level of service (as measured by the fraction of customers who immediately connect to a representative) is good. However, demand has been increasing, so the company decides to hire 50 more representatives. If demand increases by 5% every week, approximately how many weeks will it take before the service level (with 450 representatives) drops below its current level (with 400 representatives).

4. A company owns a group license for a particular optimization software package. The license allows for *n* employees to use the package simultaneously. When an employee logs in to use the software, if *n* people are currently using the package, then the employee is denied access to the software. Requests for use of the software follow a Poisson process, where the mean arrival rate (per hour) varies by time of day, as shown in the graph below. The time a user spends using the software is exponentially distributed with a mean of 30 minutes. The company wants to buy enough licenses *n* so that, during the busiest hour of the day, at least 95% of people who log in to use the software are able to immediately use it. What is the minimum number *n* of licenses needed?



5. Problem 3.50

6. Problem 3.55

7. Consider an M/M/c/K queue with $\lambda = 20$, $\mu = 5$, $c = 4$, $K = 7$
    a. For this system, find p_n for $n = 0, 1, 2, \ldots, 7$.
    b. Find the probability that an arriving customer is able to enter service immediately.
    c. Find the average number in queue L_q.
    d. Find the average waiting time in queue among customers who enter the system.

8. Problem 5.13

9. Problem 5.14

10. Widgets arrive to a production facility according to a Poisson process with rate γ=10 / hr. To complete production, a widget must be processed by four machines (A, B, C, and D; each of the four stations has a single machine). The processing time at each machine is exponential with rate $\mu_A = \mu_B = 15$ / hr and $\mu_C = \mu_D = 12$ / hr. After being processed by machine B, a widget is inspected: 10% of widgets are discarded, 20% must complete re-work starting from machine A, and 70% move on to machine C. Similarly, widgets are inspected after processing by machine D: 20% are discarded, 10% must complete re-work starting from machine C, 10% must complete re-work starting from machine A and 60% are completed. The inspection time is assumed to be instantaneous (no queues forms at the inspection station).
    a. What is the rate that widgets complete production (i.e., widgets completed per time, not counting widgets that are discarded)?
    b. What is the average number of widgets in the system?
    c. What is the average time a widget spends in the system?