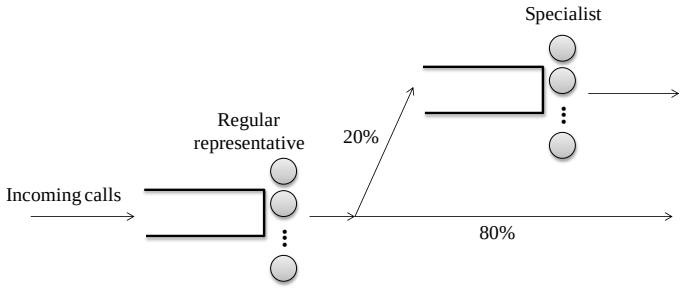
OR 647: Queueing Theory, Spring 2021
Homework Assignment 1
Due Wed. Feb. 3, 2021

1. Problem 1.7

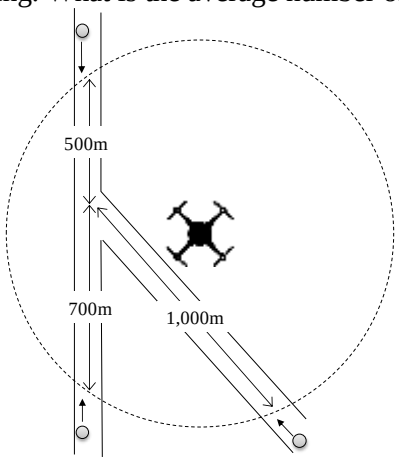
2. The length of time that a person owns a car before buying a new one has an Erlang-3 distribution with a mean of 5 years. Suppose that there are approximately 150 million cars in the United States.
    a. Assuming that a person's old car is destroyed when he or she buys a new car, how many cars does the auto industry expect to sell each year?
    b. Now assume that a person's old car is sold to somebody else when that person buys a new car. The person who buys the used car keeps it for period of time following an Erlang-3 distribution with a mean of 7 years. When that person buys another used car, his or her previous used car is assumed to be destroyed. Under the same previous assumptions, how many new cars does the auto industry expect to sell each year?

3. A graduate program in systems engineering has full time students and part time students. The number of full-time students who join the program each year is a Poisson random variable with a mean of 30. The number of part-time students who join the program each year is a Poisson random variable with a mean of 20. 30% of full-time students graduate in 1.5 years and 70% graduate in 2 years. 50% of part-time students graduate in 3 years and 50% graduate in 4 years.
    a. What is the average number of students enrolled in the program?
    b. On average, at a given moment in time, what fraction of enrolled students are full-time and what fraction are part-time?

4. Customers who have purchased a Delta laptop may call a customer support center to get technical help. Initially, a call is handled by a regular service representative. If the problem cannot be handled by a regular service representative, the call is transferred to a specialist. 20% of all calls are transferred to a specialist. On average, there are 40 customers being served or waiting to be served by a regular representative. On average, there are 10 customers being served or waiting to be served by a specialist. The average rate of incoming calls is 100 per hour. There are 30 regular representatives and 10 specialists.
    a. What is the average time spent in the system for an arbitrary customer? State any assumptions you make to answer this question.
    b. What is the average time spent in the system for a customer who needs to talk to a specialist?



5. Someone is using a hovering drone to take aerial pictures in a park. If the drone fails (e.g., a rotor breaks), it might fall anywhere in a circle of some radius (see top view in figure below). The drone operator is concerned about people who might be hit if the drone falls. The drone is hovering near several paths. People enter each of the three points on the circle according to a Poisson process with rate 1 every 5 minutes. Each person walks at a rate of 1.1 m/s. The path segment lengths are shown in the figure. At the intersection, each person is equally likely to continue on either of the other two paths.
   a. What is the average number of people at risk to a potential drone failure at any given time (i.e., what is the average number of people in the circle at any point in time)?
   b. Now suppose that 20% of all people stop at the intersection for 3 minutes to read a sign before continuing. What is the average number of people exposed in this case?



6. Is it better to split service into separate tasks (a tandem queueing system, like Starbucks with a cashier and barista) or have each server complete all service tasks (e.g., have 2 servers that do both jobs)? Identify strengths and weaknesses for each option (using common sense arguments, no math).