Name: ________________________

OR 647: Queueing Theory
Mid-Term Exam
March 16, 2016

This exam is part in class and part take home. There are 6 questions. You must answer 5 of the questions in class and the remaining question you may answer at home. You may choose which 5 questions you answer in class. The in-class part (5 questions) is due by the end of class (10:00 pm). The take-home part (1 question) is due Friday by 10:00 pm. Return the take-home part to my mailbox in the SEOR office or scan and e-mail me your exam.

Please circle/identify which 5 questions you are handing in for the "in-class" part.
1 2 3 4 5 6

On this exam, you may use the following: Lecture material associated with this class (your notes, class notes), your book, and a calculator. On this exam, you may ***not*** *use the following: Your friend, other textbooks, homework sets, other resources (for example, sites on the Internet). In addition, you may use a computer with QTS for the take-home part of the exam. Please do communicate with other students about the exam until Friday 10pm.*

1. [18 points] Arrivals to a queueing system follow a Poisson process with rate 18 per hour. Service times are exponential with rate 20 per hour. Each server is paid $40 per hour while working and $10 per hour while idle. The company determines that it loses $2 per customer per hour spent waiting in the queue, due to "ill-will" or lost customer loyalty.
   a. What is the hourly cost to the company if 1 server is employed?
   b. What is the hourly cost to the company if 2 servers are employed?
   c. If the company adds a 3rd server, would the cost go up or down compared with 2 servers? (Give a one- or two-sentence explanation without working the numbers out fully).

2. [18 points] Gamma Airlines has a large call center to handle customer service operations. Calls arrive to the center according to a Poisson process with rate 500 per hour. Call times are exponentially distributed with a mean of 12 minutes. The center has 115 representatives. It is observed that the fraction of callers who do not immediately connect with a representative is .095. Gamma Airlines makes the following promise to its customers: If a caller does not immediately connect with a representative, Gamma Airlines gives the caller a $5 travel voucher that can be applied to future purchases. Approximately 70% of customers eventually use the travel voucher, while 30% never use them. Representatives are paid $30 per hour.
   a. What is the approximate hourly cost to Gamma Airlines? State any assumptions you make.
   b. If the arrival rate increases to 600 calls per hour, what number of representatives would be needed to achieve the same level of service (in terms of fraction of callers who do not immediately connect)?
   c. With the original assumptions (500 calls per hour), if the number of representatives is reduced to 110, what would be the approximate fraction of callers who do not immediately connect? (Use the square-root law to answer the question.)

3. [12 points] A telecommunication line can handle a maximum of 5 calls at one time. Caller demand follows a Poisson process with a rate of 100 calls per hour. Call lengths are exponential with a mean of 6 minutes. When the system is full, arriving callers are blocked and do not return. Determine the fraction of customers blocked.

4. [19 points] A single server queue has Poisson arrivals with rate 10 per hour and exponential service times with a mean of 6 minutes. The maximum number of customers in the system is 5 (i.e., 4 in the queue and 1 in service; customers arriving when the system is full are lost). Further, customers in line may become impatient and leave the system. More specifically, any customer who is 2nd or 3rd or 4th in line will renege with rate 2 per hour. That is, the renege time for any such customer is exponential with mean ½ hr. These customers will not renege once they are 1st in line.
   a. Draw the rate transition diagram for the CTMC associated with this system.
   b. Suppose you know the stationary probabilities p_n. Give W_q in terms of these probabilities (where W_q is defined as the average time in queue among all customers who join the system, including those who renege).
   c. Find the fraction of time the server is busy (i.e., a numerical value).
   d. If the arrival rate were very large (instead of 10), what would be the approximate average time waiting in queue among customers who eventually receive service?

5. [14 points] A graduate program in systems engineering has full time students and part time students. The number of *full-time* students who join the program each year is a Poisson random variable with a mean of 30. The number of *part-time* students who join the program each year is a Poisson random variable with a mean of 20. 30% of full-time students graduate in 1.5 years and 70% graduate in 2 years. 50% of part-time students graduate in 3 years and 50% graduate in 4 years.
   a. What is the average number of students enrolled in the program?
   b. On average, at a given moment in time, what fraction of enrolled students are full-time and what fraction are part-time?

6. [19 points] A queueing system has 2 queues, a left queue and a right queue. Both queues are served by a single common server, who uses a "serve the longest queue first" discipline. This means that, following a service completion, the server chooses the next customer from the front of the longer of the two lines. As a tie-breaking rule, if both lines have the same length, the server takes a customer from the left queue. (If both queues are empty, the server takes the next arriving customer from either stream.)
   a. Determine $W_{q\mathrm{L}}$ (the average waiting time in queue among customers in the left queue), $W_{q\mathrm{R}}$ (the average waiting time in queue among customers in the right queue) and W_q (the average waiting time in queue among all customers).
   b. Find L_q. State the time horizon you are using to measure L_q.

Left Queue			Right Queue		
Customer	Arrival Time	Service Time	Customer	Arrival Time	Service Time
1	0	5	A	3	8
2	6	4	B	7	2
3	12	6	C	8	3
4	16	3	D	12	5
5	20	2	E	14	1
6	25	7	F	23	4

Name: ________________________

GMU Honor Code Certification. I have abided by the GMU honor code in taking this examination. The work on this exam is my own. I have received no assistance from other persons, nor have I given any assistance to any other persons, during the exam period (Wed night until 10 pm Fri).

___________________________________ _______________

Signature Date

Table of Standard Normal Distribution

beta	PDF	CDF
0	0.398942	0.5
0.05	0.398444	0.519939
0.1	0.396953	0.539828
0.15	0.394479	0.559618
0.2	0.391043	0.57926
0.25	0.386668	0.598706
0.3	0.381388	0.617911
0.35	0.37524	0.636831
0.4	0.36827	0.655422
0.45	0.360527	0.673645
0.5	0.352065	0.691462
0.55	0.342944	0.70884
0.6	0.333225	0.725747
0.65	0.322972	0.742154
0.7	0.312254	0.758036
0.75	0.301137	0.773373
0.8	0.289692	0.788145
0.85	0.277985	0.802337
0.9	0.266085	0.81594
0.95	0.254059	0.828944
1	0.241971	0.841345
1.05	0.229882	0.853141
1.1	0.217852	0.864334
1.15	0.205936	0.874928
1.2	0.194186	0.88493
1.25	0.182649	0.89435
1.3	0.171369	0.9032
1.35	0.160383	0.911492
1.4	0.149727	0.919243
1.45	0.139431	0.926471
1.5	0.129518	0.933193
1.55	0.120009	0.939429
1.6	0.110921	0.945201
1.65	0.102265	0.950529
1.7	0.094049	0.955435
1.75	0.086277	0.959941
1.8	0.07895	0.96407
1.85	0.072065	0.967843
1.9	0.065616	0.971283
1.95	0.059595	0.974412
2	0.053991	0.97725