OR 647: Queueing Theory, Spring 2021
Homework Assignment 3
Due Wed. Mar. 3, 2021

1. Problem 1.17

2. Problem 1.18

3. Problem 1.19

4. The table below gives the arrival and service times of a sequence of customers to an airline ticket counter. The counter has one line and customers are served in a first-come-first-served manner. Note: Use the same time horizon for parts (a) and (b) in computing the average number in queue.
    a. Assuming one server, compute the average wait in queue and the average number in queue.
    b. Assuming there are two servers who are working at half speed (i.e., double the service times), compute the average wait in queue and the average number in queue.
    c. Compare the answers in (a) and (b) and discuss why there is a difference.

Customer #	Arrival time (min)	Service time (min)
1	0	6
2	6	4
3	9	6
4	10	1
5	15	2
6	17	1
7	19	3
8	23	5
9	29	8
10	35	6

5. [10 points] Reconstruct the Excel spreadsheet given in class for airplane arrivals and departures to/from a single runway. Specifically, create columns containing the following elements: Operation #, inter-arrival time (min), actual arrival time (min), arrival or departure (A or D), runway hold time, queue waiting time, departure time, arrival count (1 for arrivals 0 for departures), departure count (0 for arrivals 1 for departures). The spreadsheet should also contain the following global parameters: % arrivals (0.5), arrival runway hold time (1 min), departure runway hold time (1.5 min), operation arrival rate (40 / hr), average service time, rho.
    a. Graph the queue-waiting time for a given sequence of 400 simulated operations.
    b. Vary λ from 5, 10, …, 45, 50. Calculate the average queue-waiting time W_q for each of these cases (based on simulation of 400 operations).
    c. Calculate the average queue-waiting time for each of these cases for an M/M/1 queue with the same λ and μ. Plot both values of W_q as a function

of λ and compare the results. How good is the M/M/1 queue as an approximation you're your simulation?

d. Repeat part (b) when 75% of operations are departures.

6. A hardware store has a "merchandise pickup" window where customers arrive to load previously purchased items into their car. Arrivals to the pickup window follow a Poisson process with rate 45 per hour. The time for a store employee to load merchandise into a customer's car is exponentially distributed with a mean of 3 minutes. There are 3 employees. What is the average time it takes for a customer to get his/her merchandise loaded into the car (wait in queue plus service time)?

7. A local polling location has 5 voting machines. People arrive to cast their votes according to a Poisson process with rate $\lambda = 75$ per hour. The time to cast a ballot is exponential with a mean of 2 minutes.
   a. What is the fraction of time that a given voting machine is being utilized?
   b. What is the fraction of time that 3 or fewer machines are in use?
   c. What is the average line length?

8. Arrivals to a queueing system follow a Poisson process with rate 18 per hour. Service times are exponential with rate 20 per hour. Each server is paid $40 per hour while working and $10 per hour while idle. The company determines that it loses $2 per customer per hour spent waiting in the queue, due to "ill-will" or lost customer loyalty.
   a. What is the hourly cost to the company if 1 server is employed?
   b. What is the hourly cost to the company if 2 servers are employed?
   c. If the company adds a 3rd server, would the cost go up or down compared with 2 servers? (Give a one- or two-sentence explanation without working the numbers out fully).

9. Problem 3.32

10. Problem 3.37