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Laplace Transforms: Expressions with Bessel and Modified Bessel Functions

No	Original function, $f(x)$	Laplace transform, $\widetilde{f}(p) = \int_0^\infty e^{-px} f(x)\, dx$
1	$J_0(ax)$	$\dfrac{1}{\sqrt{p^2 + a^2}}$
2	$J_\nu(ax), \qquad \nu > -1$	$\dfrac{a^\nu}{\sqrt{p^2 + a^2}\,\big(p + \sqrt{p^2 + a^2}\,\big)^\nu}$
3	$x^n J_n(ax), \qquad n = 1, 2, \ldots$	$1 \cdot 3 \cdot 5 \ldots (2n-1) a^n \big(p^2 + a^2\big)^{-n-1/2}$
4	$x^\nu J_\nu(ax), \qquad \nu > -\frac{1}{2}$	$2^\nu \pi^{-1/2} \Gamma\big(\nu + \frac{1}{2}\big) a^\nu \big(p^2 + a^2\big)^{-\nu-1/2}$
5	$x^{\nu+1} J_\nu(ax), \qquad \nu > -1$	$2^{\nu+1} \pi^{-1/2} \Gamma\big(\nu + \frac{3}{2}\big) a^\nu p \big(p^2 + a^2\big)^{-\nu-3/2}$
6	$J_0\big(2\sqrt{ax}\,\big)$	$\dfrac{1}{p} e^{-a/p}$
7	$\sqrt{x}\, J_1\big(2\sqrt{ax}\,\big)$	$\dfrac{\sqrt{a}}{p^2} e^{-a/p}$
8	$x^{\nu/2} J_\nu\big(2\sqrt{ax}\,\big), \qquad \nu > -1$	$a^{\nu/2} p^{-\nu-1} e^{-a/p}$
9	$J_0\big(a\sqrt{x^2 + bx}\,\big)$	$\dfrac{1}{\sqrt{p^2 + a^2}} \exp\big(bp - b\sqrt{p^2 + a^2}\,\big)$
10	$I_0(ax)$	$\dfrac{1}{\sqrt{p^2 - a^2}}$
11	$I_\nu(ax), \qquad \nu > -1$	$\dfrac{a^\nu}{\sqrt{p^2 - a^2}\,\big(p + \sqrt{p^2 - a^2}\,\big)^\nu}$
12	$x^\nu I_\nu(ax), \qquad \nu > -\frac{1}{2}$	$2^\nu \pi^{-1/2} \Gamma\big(\nu + \frac{1}{2}\big) a^\nu \big(p^2 - a^2\big)^{-\nu-1/2}$
13	$x^{\nu+1} I_\nu(ax), \qquad \nu > -1$	$2^{\nu+1} \pi^{-1/2} \Gamma\big(\nu + \frac{3}{2}\big) a^\nu p \big(p^2 - a^2\big)^{-\nu-3/2}$
14	$I_0\big(2\sqrt{ax}\,\big)$	$\dfrac{1}{p} e^{a/p}$
15	$\dfrac{1}{\sqrt{x}} I_1\big(2\sqrt{ax}\,\big)$	$\dfrac{1}{\sqrt{a}}\big(e^{a/p} - 1\big)$
16	$x^{\nu/2} I_\nu\big(2\sqrt{ax}\,\big), \qquad \nu > -1$	$a^{\nu/2} p^{-\nu-1} e^{a/p}$

No	Original function, $f(x)$	Laplace transform, $\widetilde{f}(p) = \int_0^\infty e^{-px} f(x)\, dx$
17	$Y_0(ax)$	$-\dfrac{2}{\pi} \dfrac{\operatorname{arcsinh}(p/a)}{\sqrt{p^2 + a^2}}$
18	$K_0(ax)$	$\dfrac{\ln\left(p + \sqrt{p^2 - a^2}\,\right) - \ln a}{\sqrt{p^2 - a^2}}$

Notation: $J_\nu(z)$ is the Bessel function of the first kind, $Y_\nu(z)$ is the Bessel function of the second kind, $I_\nu(z)$ is the modified Bessel function of the first kind, $K_\nu(z)$ is the modified Bessel function of the second kind.

References

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