Appendix
Laplace Transforms Involving Fractional and Irrational Operations

As the cases of integer-order systems, Laplace transform and its inverse are very important. In this appendix, the definition is given first. Then some of the essential special functions are described. Finally, an inverse Laplace transform table involving fractional and irrational-order operators is given.

A.1 Laplace Transforms

For a time-domain function $f(t)$, its Laplace transform, in s-domain, is defined as

$$\mathscr{L}[f(t)] = \int_0^\infty f(t)\mathrm{e}^{-st}\mathrm{d}t = F(s), \tag{A.53}$$

where $\mathscr{L}[f(t)]$ is the notation of Laplace transform.

If the Laplace transform of a signal $f(t)$ is $F(s)$, the inverse Laplace transform of $F(s)$ is defined as

$$f(t) = \mathscr{L}^{-1}[F(s)] = \frac{1}{\mathrm{j}2\pi}\int_{\sigma-\mathrm{j}\infty}^{\sigma+\mathrm{j}\infty} F(s)\mathrm{e}^{st}\mathrm{d}s, \tag{A.54}$$

where σ is greater than the real part of all the poles of function $F(s)$.

A.2 Special Functions for Laplace Transform

Since the evaluation for some fractional-order is difficult, special functions may be needed. Here some of the special functions are introduced and listed in Table A.1.

A.3 Laplace Transform Tables

An inverse Laplace transform table involving fractional and irrational operators is collected in Table A.2 [86, 300].

Table A.1 Some special functions

Special functions	Definition
Mittag-Leffler	$\mathscr{E}^{\gamma}_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{(\gamma)_k}{\Gamma(\alpha k+\beta)}\frac{z^k}{k!},\ \mathscr{E}_{\alpha,\beta}(z)=\mathscr{E}^{1}_{\alpha,\beta}(z),\ \mathscr{E}_{\alpha}(z)=\mathscr{E}_{\alpha,1}(z)$
Dawson function	$\mathrm{daw}(t)=\mathrm{e}^{-t^2}\int_0^t \mathrm{e}^{\tau^2}\mathrm{d}\tau$
erf function	$\mathrm{erf}(t)=\frac{2}{\sqrt{\pi}}\int_0^t \mathrm{e}^{-\tau^2}\mathrm{d}\tau$
erfc function	$\mathrm{erfc}(t)=\frac{2}{\sqrt{\pi}}\int_t^{\infty} \mathrm{e}^{-\tau^2}\mathrm{d}\tau=1-\mathrm{erf}(t)$
Hermit polynomial	$\mathscr{H}_n(t)=\mathrm{e}^{t^2}\frac{\mathrm{d}^n}{\mathrm{d}t^n}\mathrm{e}^{-t^2}$
Bessel function	$\mathscr{J}_\nu(t)$ is the solution to $t^2\ddot{y}+t\dot{y}+(t^2-\nu^2)y=0$
Extended Bessel function	$\mathscr{I}_\nu(t)=\mathrm{j}^{-\nu}\mathscr{J}_\nu(\mathrm{j}t)$

Table A.2 Inverse Laplace transforms with fractional and irrational operators

$F(s)$	$f(t)=\mathscr{L}^{-1}[F(s)]$	$F(s)$	$f(t)=\mathscr{L}^{-1}[F(s)]$
$\frac{s^{\alpha\gamma-\beta}}{(s^\alpha+a)^\gamma}$	$t^{\beta-1}\mathscr{E}^{\gamma}_{\alpha,\beta}\left(-at^\alpha\right)$	$\frac{1}{s^n\sqrt{s}}, n=1,2,\cdots$	$\frac{2^n t^{n-\frac{1}{2}}}{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}$
$\frac{k}{s^2+k^2}\coth\left(\frac{\pi s}{2k}\right)$	$\lvert\sin kt\rvert$	$\arctan\frac{k}{s}$	$\frac{1}{t}\sin kt$
$\log\frac{s^2-a^2}{s^2}$	$\frac{2}{t}(1-\cosh at)$	$\frac{1}{s\sqrt{s}}\mathrm{e}^{-k\sqrt{s}}$	$2\sqrt{\frac{t}{\pi}}\mathrm{e}^{-\frac{1}{4t}k^2}-k\ \mathrm{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
$\log\frac{s^2+a^2}{s^2}$	$\frac{2}{t}(1-\cos at)$	$\frac{\mathrm{e}^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})}$	$\mathrm{e}^{ak}e^{a^2t}\mathrm{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)$
$\frac{(1-s)^n}{s^{n+\frac{1}{2}}}$	$\frac{n!}{(2n)!\sqrt{\pi t}}\mathscr{H}_{2n}\left(\sqrt{t}\right)$	$\frac{1}{\sqrt{s+b}(s+a)}$	$\frac{1}{\sqrt{b-a}}\mathrm{e}^{-at}\mathrm{erf}\left(\sqrt{(b-a)t}\right)$
$\frac{1}{\sqrt{s^2+a^2}}$	$\mathscr{J}_0(at)$	$\frac{(1-s)^n}{s^{n+\frac{3}{2}}}$	$-\frac{n!}{(2n+1)!\sqrt{\pi}}\mathscr{H}_{2n+1}\left(\sqrt{t}\right)$
$\frac{1}{\sqrt{s^2-a^2}}$	$\mathscr{I}_0(at)$	$\frac{(a-b)^k}{(\sqrt{s+a}+\sqrt{s+b})^{2k}}$	$\frac{k}{t}\mathrm{e}^{-\frac{1}{2}(a+b)t}\mathscr{I}_k\left(\frac{a-b}{2}t\right),\ k>0$
$\frac{\sqrt{s+2a}-\sqrt{s}}{\sqrt{s+2a}+\sqrt{s}}$	$\frac{1}{t}\mathrm{e}^{-at}\mathscr{I}_1(at)$	$\frac{\sqrt{s+2a}-\sqrt{s}}{\sqrt{s+2a}+\sqrt{s}}$	$\frac{1}{t}\mathrm{e}^{-at}\mathscr{I}_1(at)$
$\frac{(\sqrt{s^2+a^2}-s)^\nu}{\sqrt{s^2+a^2}}$	$a^\nu\mathscr{J}_\nu(at), \nu>-1$	$\frac{1}{(s^2-a^2)^k}$	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}}\mathscr{I}_{k-\frac{1}{2}}(at)$
$\frac{(\sqrt{s^2-a^2}+s)^\nu}{\sqrt{s^2-a^2}}$	$a^\nu\mathscr{I}_\nu(at),\ \nu>-1$	$\frac{1}{(\sqrt{s^2+a^2})^k}$	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}}\mathscr{J}_{k-\frac{1}{2}}(at)$
$(\sqrt{s^2+a^2}-s)^k$	$\frac{ka^k}{t}\mathscr{J}_k(at), k>0$	$\log\frac{s-a}{s-b}$	$\frac{1}{t}\left(\mathrm{e}^{bt}-\mathrm{e}^{at}\right)$
$\frac{1}{s+\sqrt{s^2+a^2}}$	$\frac{\mathscr{J}_1(at)}{at}$	$\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$\mathrm{e}^{-\frac{1}{2}(a+b)t}\mathscr{I}_0\left(\frac{a-b}{2}t\right)$

Table A.2 (continued)

$F(s)$	$f(t)=\mathscr{L}^{-1}[F(s)]$	$F(s)$	$f(t)=\mathscr{L}^{-1}[F(s)]$
$\dfrac{1}{(s+\sqrt{s^2+a^2})^N}$	$\dfrac{N\mathscr{J}_N(at)}{at},\ N>0$	$\dfrac{b^2-a^2}{(s-a^2)(\sqrt{s}+b)}$	$\mathrm{e}^{a^2t}\left[b-a\ \mathrm{erf}\left(a\sqrt{t}\right)\right]-b\mathrm{e}^{b^2t}\mathrm{erfc}\left(b\sqrt{t}\right)$
$\sqrt{s-a}-\sqrt{s-b}$	$\dfrac{1}{2\sqrt{\pi t^3}}\left(\mathrm{e}^{bt}-\mathrm{e}^{at}\right)$	$\dfrac{\sqrt{s+2a}-\sqrt{s}}{\sqrt{s}}$	$a\mathrm{e}^{-at}\left[\mathscr{I}_1(at)+\mathscr{I}_0(at)\right]$
$\dfrac{1}{s}\mathrm{e}^{-k/s}$	$\mathscr{J}_0\left(2\sqrt{kt}\right)$	$\dfrac{1}{\sqrt{s}}\mathrm{e}^{-k/s}$	$\dfrac{1}{\sqrt{\pi t}}\cos 2\sqrt{kt}$
$\dfrac{1}{\sqrt{s}}\mathrm{e}^{k/s}$	$\dfrac{1}{\sqrt{\pi t}}\cosh 2\sqrt{kt}$	$\dfrac{1}{s\sqrt{s}}\mathrm{e}^{-k/s}$	$\dfrac{1}{\sqrt{\pi k}}\sin 2\sqrt{kt}$
$\dfrac{1}{s\sqrt{s}}\mathrm{e}^{k/s}$	$\dfrac{1}{\sqrt{\pi k}}\sinh 2\sqrt{kt}$	$\dfrac{1}{s^\nu}\mathrm{e}^{-k/s}$	$\left(\dfrac{t}{k}\right)^{\frac{1}{2}(\nu-1)}\mathscr{J}_{\nu-1}\left(2\sqrt{\text{[illegible]}}\right),\nu>0$
$\mathrm{e}^{-k\sqrt{s}}$	$\dfrac{k}{2\sqrt{\pi t^3}}\mathrm{e}^{-\frac{1}{4k}k^2}$	$\dfrac{1}{s^\nu}\mathrm{e}^{k/s}$	$\left(\dfrac{t}{k}\right)^{\frac{1}{2}(\nu-1)}\mathscr{I}_{\nu-1}\left(2\sqrt{kt}\right)$
$\dfrac{1}{s}\mathrm{e}^{-k\sqrt{s}}$	$\mathrm{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$	$\dfrac{1}{s\sqrt{s}}\mathrm{e}^{-\sqrt{s}}$	$2\sqrt{\dfrac{t}{\pi}}\mathrm{e}^{-\frac{1}{4t}}-\mathrm{erfc}\left(\dfrac{1}{2\sqrt{t}}\right)$
$\dfrac{1}{\sqrt{s}}\mathrm{e}^{-k\sqrt{s}}$	$\dfrac{1}{\sqrt{\pi t}}\mathrm{e}^{-\frac{1}{4t}k^2}$	$\dfrac{\mathrm{e}^{-\sqrt{s}}}{\sqrt{s}(\sqrt{s}+1)}$	$\mathrm{e}^{t+1}\mathrm{erfc}\left(\sqrt{t}+\dfrac{1}{2\sqrt{t}}\right)$
$\dfrac{1}{(s+a)^\alpha}$	$\dfrac{t^{\alpha-1}}{\Gamma(\alpha)}\mathrm{e}^{-at}$	$\dfrac{1}{s^\alpha+a}$	$t^{\alpha-1}\mathscr{E}_{\alpha,\alpha}\left(-at^\alpha\right)$
$\dfrac{a}{s(s^\alpha+a)}$	$1-\mathscr{E}_\alpha\left(-at^\alpha\right)$	$\dfrac{s^\alpha}{s(s^\alpha+a)}$	$\mathscr{E}_\alpha\left(-at^\alpha\right)$
$\dfrac{1}{s^\alpha(s-a)}$	$t^\alpha\mathscr{E}_{1,1+\alpha}(at)$	$\dfrac{s^\alpha}{s-a}$	$-t^\alpha\mathscr{E}_{1,1-\alpha}(at),\ 0<\alpha<1$
$\dfrac{1}{\sqrt{s}}$	$\dfrac{1}{\sqrt{\pi t}}$	$\dfrac{1}{s\sqrt{s}}$	$2\sqrt{\dfrac{t}{\pi}}$
$\dfrac{1}{\sqrt{s}(s+1)}$	$\dfrac{2}{\sqrt{\pi}}\mathrm{daw}\left(\sqrt{t}\right)$	$\dfrac{\sqrt{s}}{s+1}$	$\dfrac{1}{\sqrt{\pi t}}-\dfrac{2}{\sqrt{\pi}}\mathrm{daw}\left(\sqrt{t}\right)$
$\dfrac{1}{\sqrt{s}(s+a^2)}$	$\sqrt{t}\ \mathscr{E}_{1,3/2}\left(-a^2t\right)$	$\dfrac{s}{(s-a)\sqrt{s-a}}$	$\dfrac{1}{\sqrt{\pi t}}\mathrm{e}^{at}(1+2at)$
$\dfrac{\sqrt{s}}{s+a^2}$	$\dfrac{1}{\sqrt{t}}\mathscr{E}_{1,1/2}\left(-a^2t\right)$	$\dfrac{1}{\sqrt{s}+a}$	$\dfrac{1}{\sqrt{\pi t}}-a\mathrm{e}^{a^2t}\mathrm{erfc}\left(a\sqrt{t}\right)$
$\dfrac{1}{s\sqrt{s+1}}$	$\mathrm{erf}\left(\sqrt{t}\right)$	$\dfrac{\sqrt{s}}{s-a^2}$	$\dfrac{1}{\sqrt{\pi t}}+a\mathrm{e}^{a^2t}\mathrm{erf}\left(a\sqrt{t}\right)$
$\dfrac{1}{\sqrt{s}(s-a^2)}$	$\dfrac{1}{a}\mathrm{e}^{a^2t}\mathrm{erf}\left(a\sqrt{t}\right)$	$\dfrac{1}{\sqrt{s}(s+a^2)}$	$\dfrac{2}{a\sqrt{\pi}}\mathrm{e}^{-a^2t}\displaystyle\int_0^{a\sqrt{t}}\mathrm{e}^{\tau^2}\mathrm{d}\tau$
$\dfrac{1}{\sqrt{s}(\sqrt{s}+a)}$	$\mathrm{e}^{a^2t}\mathrm{erfc}\left(a\sqrt{t}\right)$	$\dfrac{s\sqrt{s}}{s+1}$	$2\sqrt{\dfrac{t}{\pi}}-\dfrac{2}{\sqrt{\pi}}\mathrm{daw}\left(\sqrt{t}\right)$
$\dfrac{1}{\sqrt{s+1}}$	$\dfrac{\mathrm{e}^{-t}}{\sqrt{\pi t}}$	$\dfrac{1}{\sqrt{s}(s-1)}$	$\mathrm{e}^t\mathrm{erf}\left(\sqrt{t}\right)$
$\dfrac{\sqrt{s}}{s-1}$	$\dfrac{1}{\sqrt{\pi t}}+\mathrm{e}^t\mathrm{erf}\left(\sqrt{t}\right)$	$\dfrac{k!}{\sqrt{s}\pm\lambda}$	$t^{(k-1)/2}\mathscr{E}^{(k)}_{1/2,1/2}\left(\mp\lambda\sqrt{t}\right),\Re(s)>\lambda^2$
$\dfrac{1}{s^\alpha}$	$\dfrac{t^{\alpha-1}}{\Gamma(\alpha)}$	$\dfrac{s^{\alpha-1}}{s^\alpha\pm\lambda}$	$\mathscr{E}_\alpha\left(\mp\lambda t^\alpha\right),\ \Re(s)>\lvert\lambda\rvert^{1/\alpha}$

Table A.2 (continued)

$F(s)$	$f(t)=\mathscr{L}^{-1}[F(s)]$
$\dfrac{1}{\sqrt{s(s+a)}(\sqrt{s+a}+\sqrt{s})^{2\nu}}$	$\dfrac{1}{a^{\nu}}\mathrm{e}^{-at/2}\mathscr{I}_{\nu}\left(\dfrac{a}{2}t\right), k>0$
$\dfrac{\Gamma(k)}{(s+a)^k(s+b)^k}$	$\sqrt{\pi}\left(\dfrac{t}{a-b}\right)^{k-\frac{1}{2}}\mathrm{e}^{-\frac{1}{2}(a+b)t}\mathscr{I}_{k-\frac{1}{2}}\left(\dfrac{a-b}{2}t\right)$
$\dfrac{1}{\sqrt{s^2+a^2}(s+\sqrt{s^2+a^2})^N}$	$\dfrac{J_N(at)}{a^N}$
$\dfrac{1}{\sqrt{s^2+a^2}(s+\sqrt{s^2+a^2})}$	$\dfrac{J_1(at)}{a}$
$\dfrac{b^2-a^2}{\sqrt{s}(s-a^2)(\sqrt{s}+b)}$	$\mathrm{e}^{a^2t}\left[\dfrac{b}{a}\mathrm{erf}\left(a\sqrt{t}\right)-1\right]+\mathrm{e}^{b^2t}\mathrm{erfc}\left(b\sqrt{t}\right)$
$\dfrac{a\mathrm{e}^{-k\sqrt{s}}}{s(a+\sqrt{s})}$	$-\mathrm{e}^{ak}\mathrm{e}^{a^2t}\mathrm{erfc}\left(a\sqrt{t}+\dfrac{k}{2\sqrt{t}}\right)+\mathrm{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
$\dfrac{1}{\sqrt{s+a}(s+b)\sqrt{s+b}}$	$t\mathrm{e}^{-\frac{1}{2}(a+b)t}\left[\mathscr{I}_0\left(\dfrac{a-b}{2}t\right)+\mathscr{I}_1\left(\dfrac{a-b}{2}t\right)\right]$
$\dfrac{\mathrm{e}^{-\sqrt{s}}}{\sqrt{s}+1}$	$\dfrac{\mathrm{e}^{-\frac{1}{4k}}}{\sqrt{\pi t}}-\mathrm{e}^{t+1}\mathrm{erfc}\left(\sqrt{t}+\dfrac{1}{2\sqrt{t}}\right)$
$\dfrac{\mathrm{e}^{-\sqrt{s}}}{s(\sqrt{s}+1)}$	$\mathrm{erfc}\left(\dfrac{1}{2\sqrt{t}}\right)-\mathrm{e}^{t+1}\mathrm{erfc}\left(\sqrt{t}+\dfrac{1}{2\sqrt{t}}\right)$

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