

```
(%i1) load(drawdf);
```

```
(%o1)
```

```
C:/maxima-5.44.0/share/maxima/5.44.0/share/diffequations/drawdf.mac
```

```
(%i2) alias(W, lambert_w);
```

```
(%o2) [W]
```

```
(%i3) alias(WW, generalized_lambert_w);
```

```
(%o3) [WW]
```

```
(%i4) sit(iv, g):=block([i0:iv[1], it0:iv[2]],  
    [it0/i0+g, i0]  
);
```

```
(%o4) sit(iv, g):=block([i0:iv_1, it0:iv_2], [it0/i0+g, i0])
```

```
(%i5) sit([1.9,5],2);
```

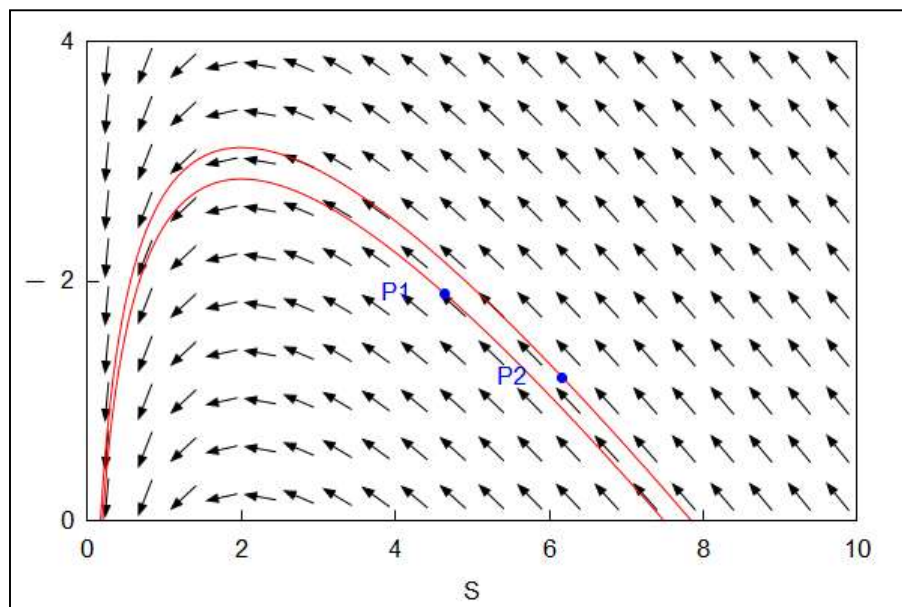
```
(%o5) [4.631578947368421, 1.9]
```

```
(%i6) sit([1.2,5],2);
```

```
(%o6) [6.166666666666667, 1.2]
```

```
(%i7) wxdrawdf([-x*y, x*y-2*y], [x,0,10], [y,0,4], [xtics=2, ytics=2], [xlabel=S, ylabel=I], solns_at(sit([1.9,5],2), sit([1.2,5],2))
```

```
(%t7)
```



```
(%o7) 0
```

```
(%i8) sit([1.1,2.0],12);
```

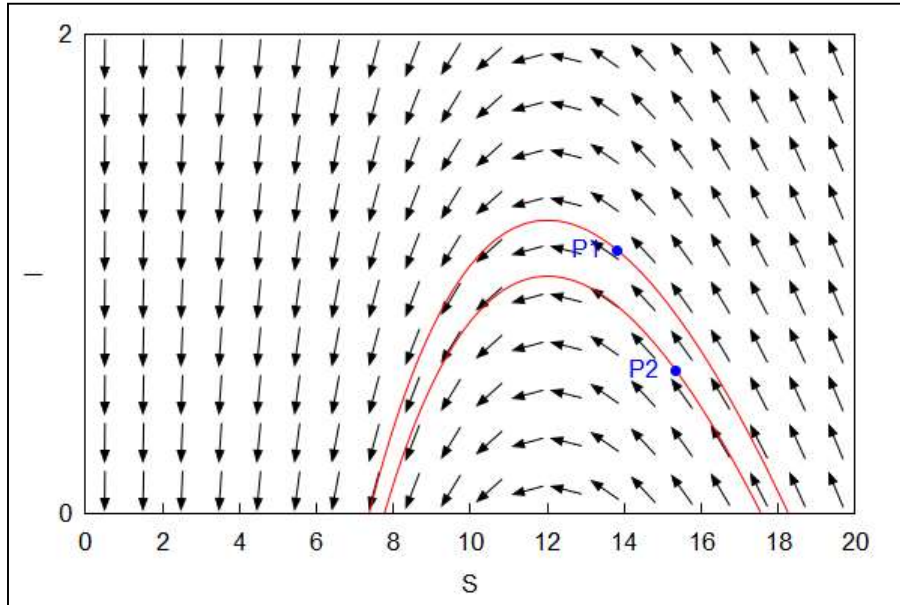
```
(%o8) [13.81818181818182, 1.1]
```

```
(%i9) sit([0.6,2.0],12);
```

```
(%o9) [15.33333333333333, 0.6]
```

```
(%i10) wxdrawdf([-x*y, x*y-12*y], [x,0,20], [y,0,2], [xtics=2, ytics=2],[xlabel=S, ylabel =I],solns_at( sit(
```

```
(%t10)
```



```
(%o10) 0
```

Lambertian differential equation

```
(%i11) idiffi(i,iv, g):=block([i0:iv[1], s0:iv[2], im],
  im: i0+s0-g- g*log(s0/g),
  if not numberp(i) then return (- g* i * (W(-s0/g*exp((i - (i0+s0))/g)) +1) ),
  if i <=im then
    - g* i * (WW(-1,-s0/g*exp((i - (i0+s0))/g)) +1)
  else
    - g* i * (W(-s0/g*exp((i - (i0+s0))/g)) +1)
);
```

```
(%o11) idiffi ( i , iv , g ) := block
```

```
(%i12) deq0:idiffi(i,[i0, s0],g);
```

$$(deq0) \quad -g \, i \left(W \left(- \frac{s0 \, \%e^{\frac{-s0 - i0 + i}{g}}}{g} \right) + 1 \right)$$

```
(%i13) idiffi1(i,iv, g):=block([i0:iv[1], it0:iv[2], s0, im],
  s0:it0/i0+g,
  im: i0+s0-g- g*log(s0/g),
  if not numberp(i) then return (- g* i * (W(-s0/g*exp((i - (i0+s0))/g)) +1) ),
  if i <=im then
    - g* i * (WW(-1,-s0/g*exp((i - (i0+s0))/g)) +1)
  else
    - g* i * (W(-s0/g*exp((i - (i0+s0))/g)) +1)
);
```

```
(%o13) idiffi1 ( i , iv , g ) := block
```

```
(%i14) idiffi1(2,[1.9, 5],2.0);
```

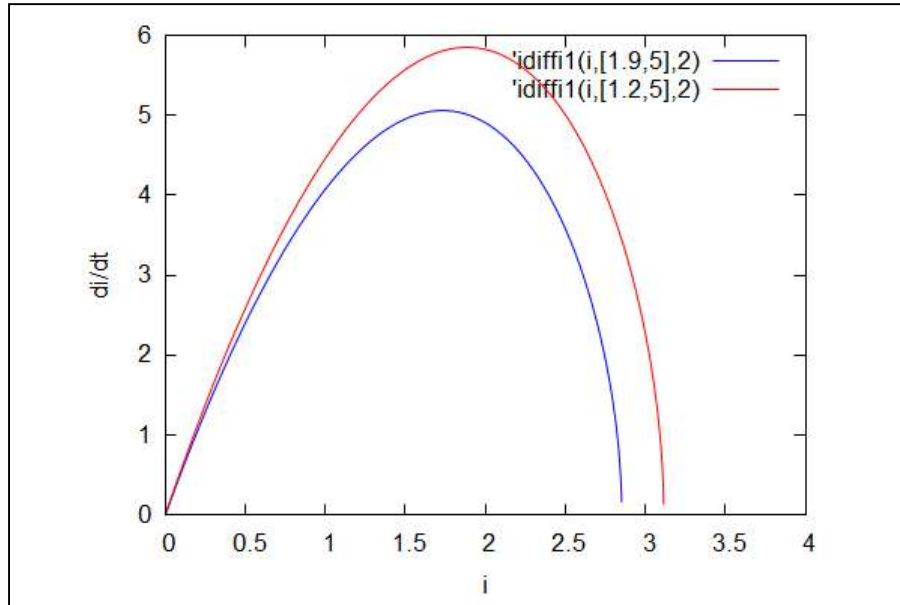
```
(%o14) 4.905779723767649
```

```
(%i15) wxplot2d(['idiffi1(i,[1.9, 5],2), 'idiffi1(i,[1.2, 5],2) ], [i,0,4], [xlabel, "i"], [ylabel, "di/dt"])$
```

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

```
(%t15)
```



```
(%i16) si1(i,iv, g, br):=block([i0:iv[1], it0:iv[2], s0, im],
s0:it0/i0+g,
im: i0+s0-g- g*log(s0/g),
if not numberp(i) then return (- g *W(-s0/g*exp((i - (i0+s0))/g))) ),
if i <=im then
if br<0 then
- g* WW(-1,-s0/g*exp((i - (i0+s0))/g))
else
- g* W(-s0/g*exp((i - (i0+s0))/g))
else 'nan
);
```

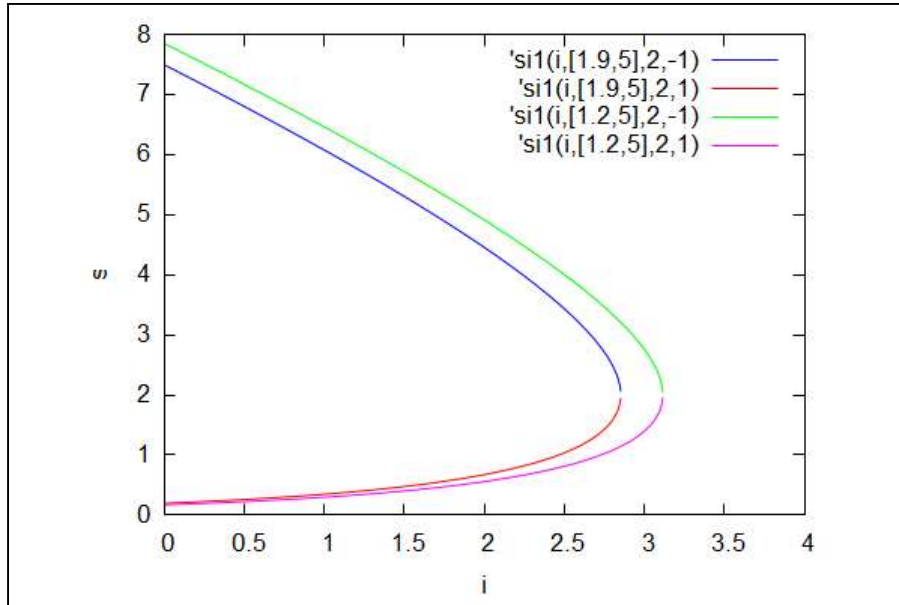
```
(%o16) si1(i,iv,g,br):=block
```

```
(%i17) si1(i, [i0, it0], g,1);
```

$$(\%o17) -g W\left(\frac{\left(-\frac{it0}{i0} - g\right) e^{\frac{-\frac{it0}{i0} - i0 + i - g}{g}}}{g}\right)$$

(%i18) wxplot2d(['si1(i,[1.9, 5],2, -1)', 'si1(i,[1.9, 5],2, 1)', 'si1(i,[1.2, 5],2,-1)', 'si1(i,[1.2, 5],2,+1)'], [i,0,4],
 plot2d: expression evaluates to non-numeric value somewhere in plotting range.
 plot2d: expression evaluates to non-numeric value somewhere in plotting range.
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 plot2d: expression evaluates to non-numeric value somewhere in plotting range.

(%t18)



(%i19) ri1(i,iv, g, br):=block([i0:iv[1], it0:iv[2], s0, im],
 s0:it0/i0+g,
 im: i0+s0-g- g*log(s0/g),
 if not numberp(i) then return (g *W(-s0/g*exp((i - (i0+s0))/g)) - g* WW(-1,-s0/g*exp((- (i0+s0))/g))
 if i <=im then
 if br<0 then
 g* WW(-1,-s0/g*exp((i - (i0+s0))/g)) - g* WW(-1,-s0/g*exp((- (i0+s0))/g)) - i
 else
 g* W(-s0/g*exp((i - (i0+s0))/g)) - g* WW(-1,-s0/g*exp((- (i0+s0))/g)) - i
 else 'nan
);

(%o19) ri1(i,iv,g,br):=block

(%i21) ri1(i, [i0, it0], g,1);

$$g W \left(\frac{\left(-\frac{it0}{i0} - g \right) \%e^{\frac{-\frac{it0}{i0} - i0 + i - g}{g}}}{g} \right) - i -$$

$$WW \left(-1, \frac{\left(-\frac{it0}{i0} - g \right) \%e^{\frac{-\frac{it0}{i0} - i0 - g}{g}}}{g} \right) g$$

(%i22) wxplot2d(['ri1(i,[1.9, 5],2, -1)', 'ri1(i,[1.9, 5],2, 1)', 'ri1(i,[1.2, 5],2, -1)', 'ri1(i,[1.2, 5],2, 1)'], [i,0,4], [

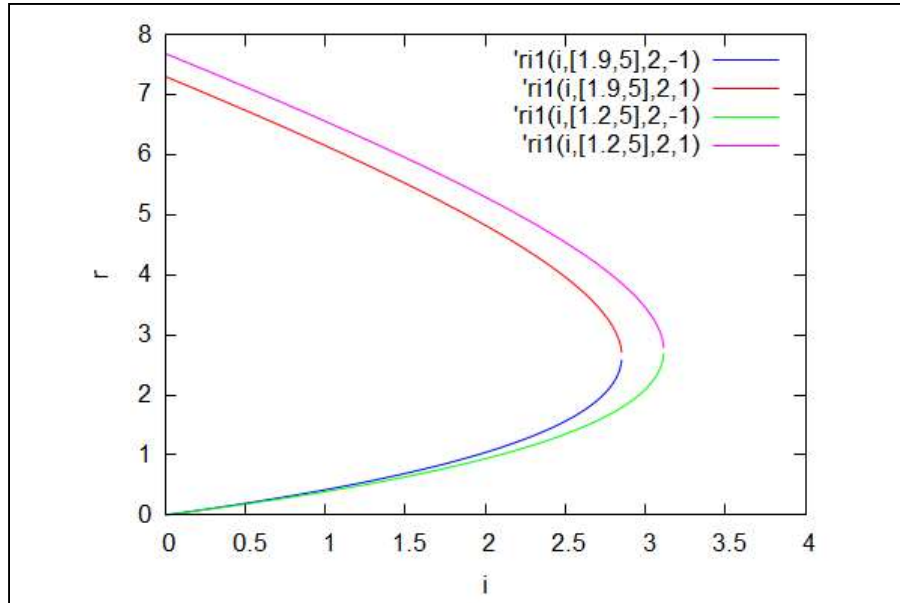
plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

(%t22)

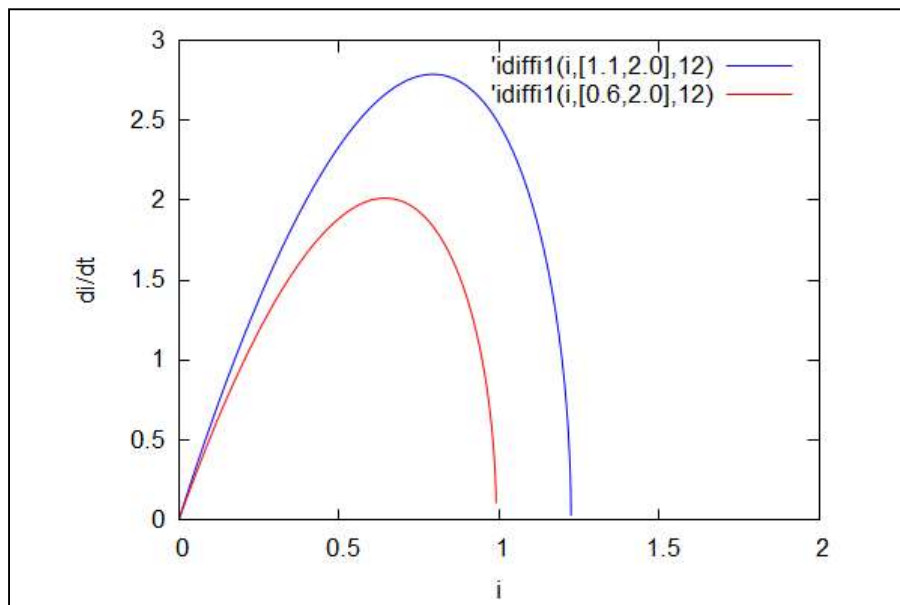


(%i23) wxplot2d(['idiffi1(i,[1.1, 2.0],12)', 'idiffi1(i,[0.6, 2.0],12)'], [i,0,2], [xlabel, "i"], [ylabel, "di/dt"])

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

(%t23)



(%i24) wxplot2d(['si1(i,[1.1, 2.0],12,-1)', 'si1(i,[1.1, 2.0],12,1)', 'si1(i,[0.6, 2.0],12,-1)', 'si1(i,[0.6, 2.0],12,1)'],

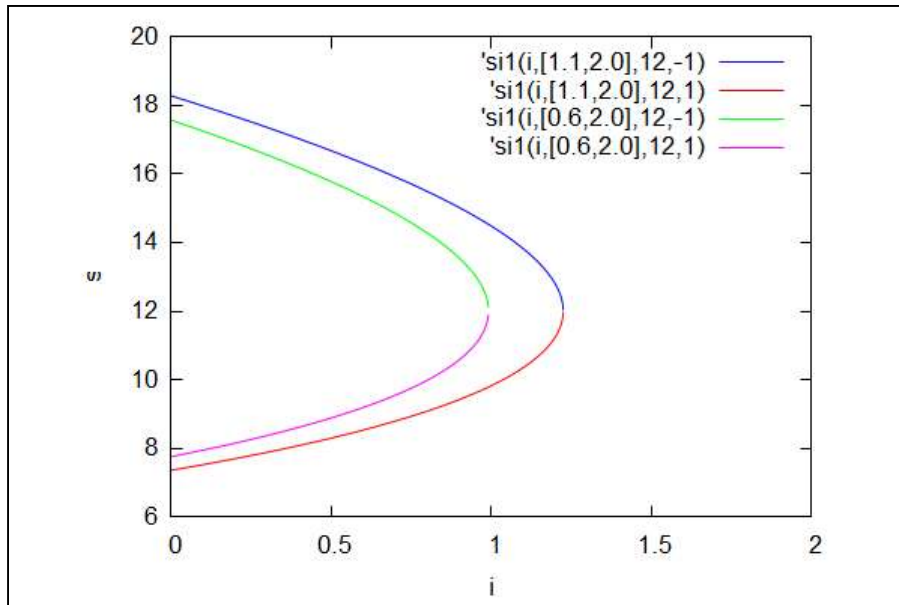
plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

(%t24)



(%i25) wxplot2d(['ri1(i,[1.1, 2.0],12,-1)', 'ri1(i,[1.1, 2.0],12,1)', 'ri1(i,[0.6, 2.0],12,-1)', 'ri1(i,[0.6, 2.0],12,1)'],

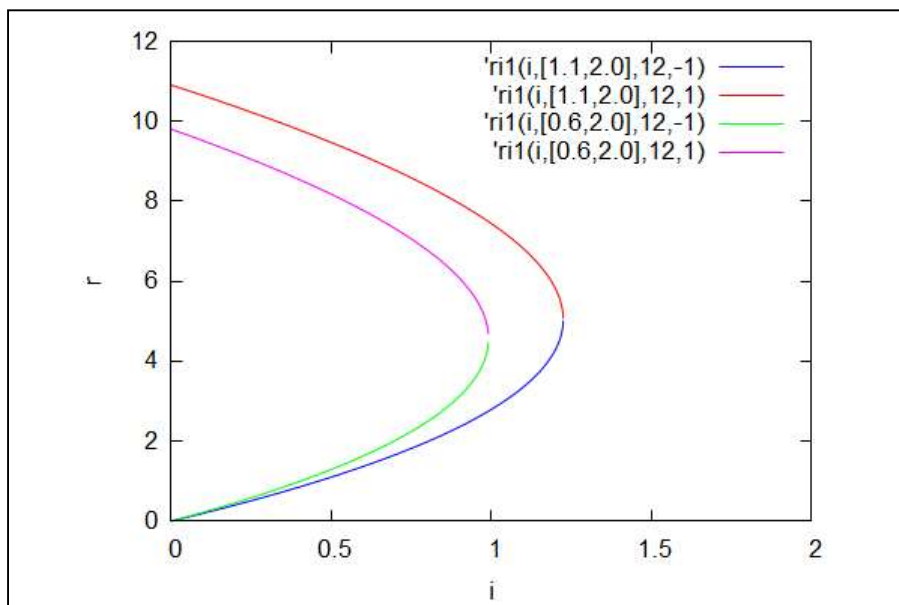
plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

(%t25)



(%i26) depends([I, S, R], t);

(%o26) [I(t), S(t), R(t)]

Kudryashov system

(%i27) deq02: diff(I,t)=I*S-g*I;

(deq02) $\frac{d}{dt} I = I S - I g$

(%i28) deq01: diff(S,t)=-I*S;

(deq01) $\frac{d}{dt} S = -I S$

(%i29) deq03:diff(R,t)=g*I;

(deq03) $\frac{d}{dt} R = I g$

First integral

(%i30) deq01+deq02+deq03;

(%o30) $\frac{d}{dt} S + \frac{d}{dt} R + \frac{d}{dt} I = 0$

(%i31) deq4:deq03, R=1-S-I,diff;

(deq4) $-\frac{d}{dt} S - \frac{d}{dt} I = I g$

(%i32) sol4:solve(deq4, 'diff(S,t,1));

(sol4) $\left[\frac{d}{dt} S = -I g - \frac{d}{dt} I \right]$

(%i33) sol0:solve(deq01, S);

(sol0) $\left[S = -\frac{\frac{d}{dt} S}{I} \right]$

(%i34) sol01:sol0, sol4,expand;

(sol01) $\left[S = g + \frac{\frac{d}{dt} I}{I} \right]$

(%i35) deq2:diff(deq02,t);

(deq2) $\frac{d^2}{dt^2} I = -\left(\frac{d}{dt} I \right) g + I \left(\frac{d}{dt} S \right) + \left(\frac{d}{dt} I \right) S$

(%i36) deq3:deq2,deq01;

(deq3) $\frac{d^2}{dt^2} I = -\left(\frac{d}{dt} I \right) g + \left(\frac{d}{dt} I \right) S - I^2 S$

```
(%i37) deq4:deq3, sol01;
```

$$(\text{deq4}) \quad \frac{d^2}{dt^2} l = \left(\frac{d}{dt} l \right) \left(g + \frac{\frac{d}{dt} l}{l} \right) - l^2 \left(g + \frac{\frac{d}{dt} l}{l} \right) - \left(\frac{d}{dt} l \right) g$$

```
(%i38) deq4:deq4,expand;
```

$$(\text{deq4}) \quad \frac{d^2}{dt^2} l = -l^2 g + \frac{\left(\frac{d}{dt} l \right)^2}{l} - l \left(\frac{d}{dt} l \right)$$

```
(%i39) deq10: l*diff(l,t,2)-diff(l,t)^2+a*l^2*diff(l,t)+a*b*l^3;
```

$$(\text{deq10}) \quad l^3 a b + l^2 \left(\frac{d}{dt} l \right) a + l \left(\frac{d^2}{dt^2} l \right) - \left(\frac{d}{dt} l \right)^2$$

```
(%i41) sol1:solve( deq10, ('diff(l,t,2)));
```

$$(\text{sol1}) \quad \left[\frac{d^2}{dt^2} l = - \frac{l^3 a b + l^2 \left(\frac{d}{dt} l \right) a - \left(\frac{d}{dt} l \right)^2}{l} \right]$$

```
(%i44) sol1[1],l=x,'diff(l,t)=y,expand;
```

$$(\%o44) \quad \frac{d^2}{dt^2} x = \frac{y^2}{x} - a x y - a b x^2$$

```
(%i45) sol2:sol1[1], expand,a=1, b=g;
```

$$(\text{sol2}) \quad \frac{d^2}{dt^2} l = -l^2 g + \frac{\left(\frac{d}{dt} l \right)^2}{l} - l \left(\frac{d}{dt} l \right)$$

```
(%i46) sol2, l=K*exp(-g*t);
```

$$(\%o46) \quad \frac{d^2}{dt^2} (K e^{-g t}) = \frac{e^{g t} \left(\frac{d}{dt} (K e^{-g t}) \right)^2}{K} - K e^{-g t} \left(\frac{d}{dt} (K e^{-g t}) \right) - K^2 g e^{-2 g t}$$

```
(%i47) %,diff;
```

$$(\%o47) \quad K g^2 e^{-g t} = K g^2 e^{-g t}$$


```
(%i48) sol3:sol2, 'diff(l,t)=-diff(y,t), l=x,'diff(l,t)=y, expand;
```

```
(sol3) 
$$\frac{d}{dt} y = \frac{y^2}{x} - x y - g x^2$$

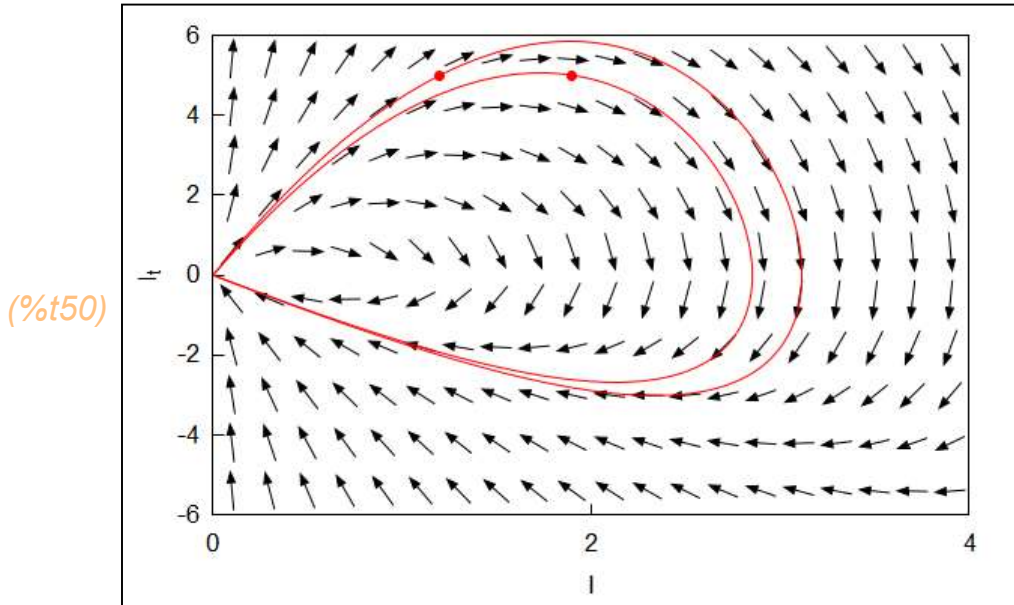
```

```
(%i49) sol4: 'diff(x,t)=y;
```

```
(sol4) 
$$\frac{d}{dt} x = y$$

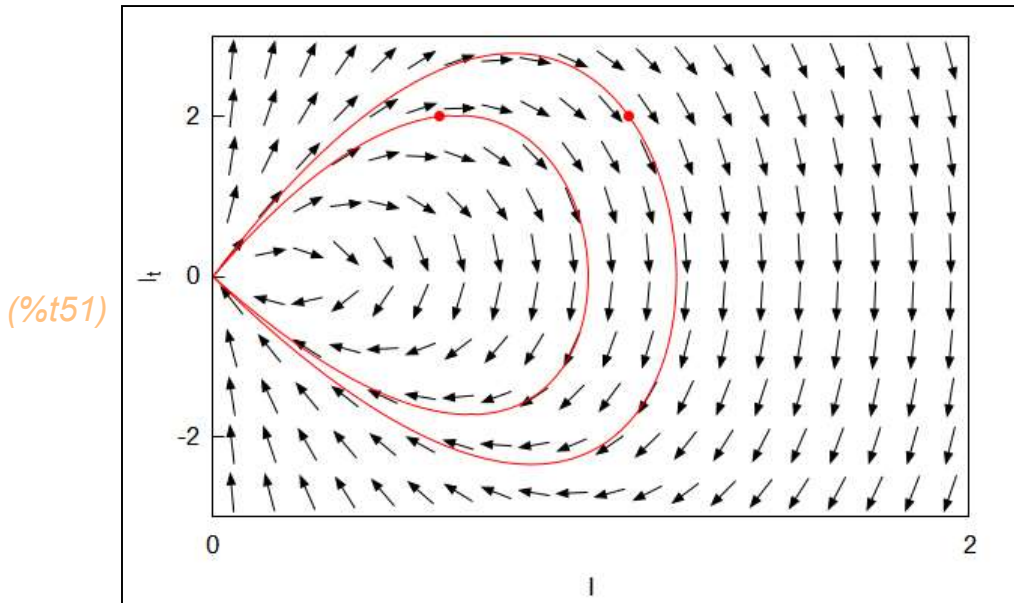
```

```
(%i50) wxdrawdf([y, y^2/x-x*y-2*x^2 ], [x,0,4], [y,-6,6], [xtics =2, ytics=2],[xlabel=l, ylabel =l_t], [trajecto
```



```
(%o50) 0
```

```
(%i51) wxdrawdf([y, y^2/x-x*y-12*x^2 ], [x,0,2], [y,-3,3], [xtics =2, ytics=2],[xlabel=l, ylabel =l_t], [trajecto
```



```
(%o51) 0
```

→