Colouring aperiodic tiling patterns...

cormullion
bricoleur and enthusiastic code artist

&

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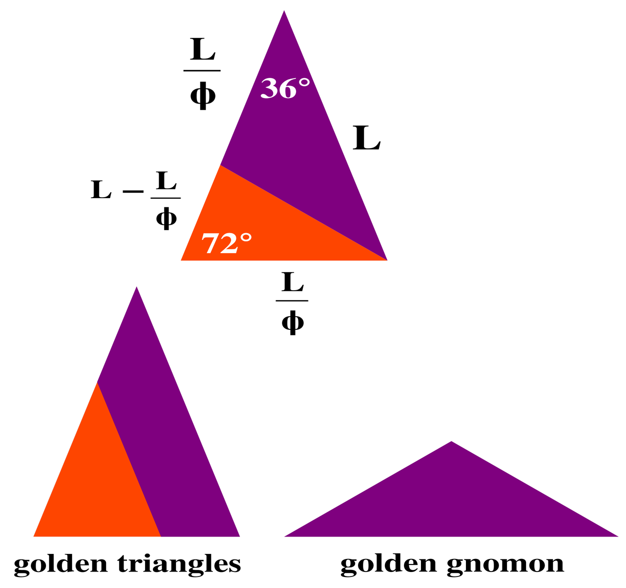
Golden Triangles

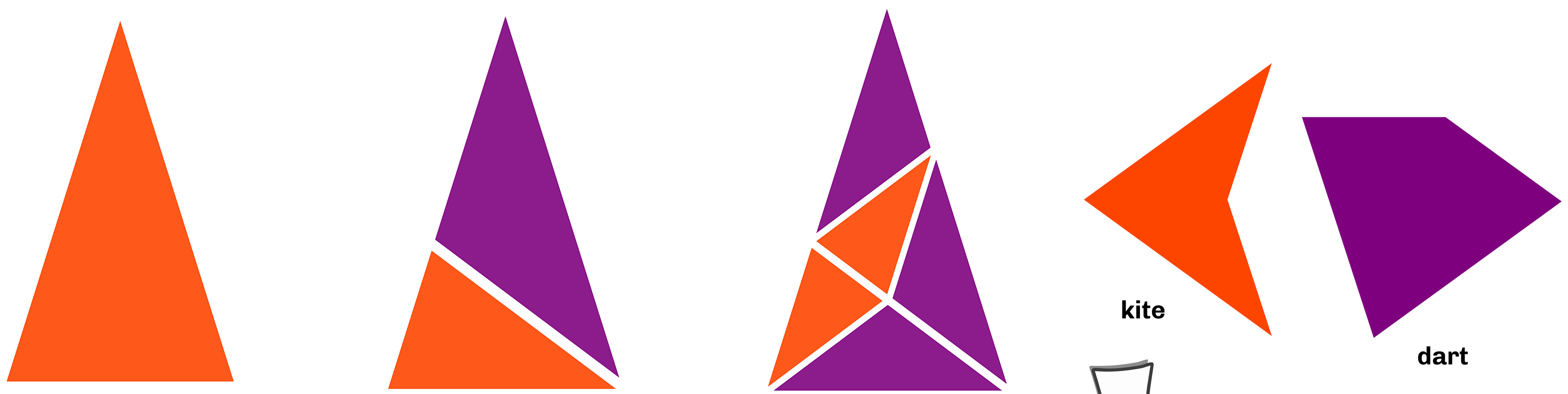
The golden triangle and golden gnomon are the lesser known cousins of the golden rectangle. They also use the golden ratio ϕ (1.618:1). You can divide a golden triangle into a triangle and a golden gnomon.



...we designed the JuliaCon 2019 t-shirt in Julia!

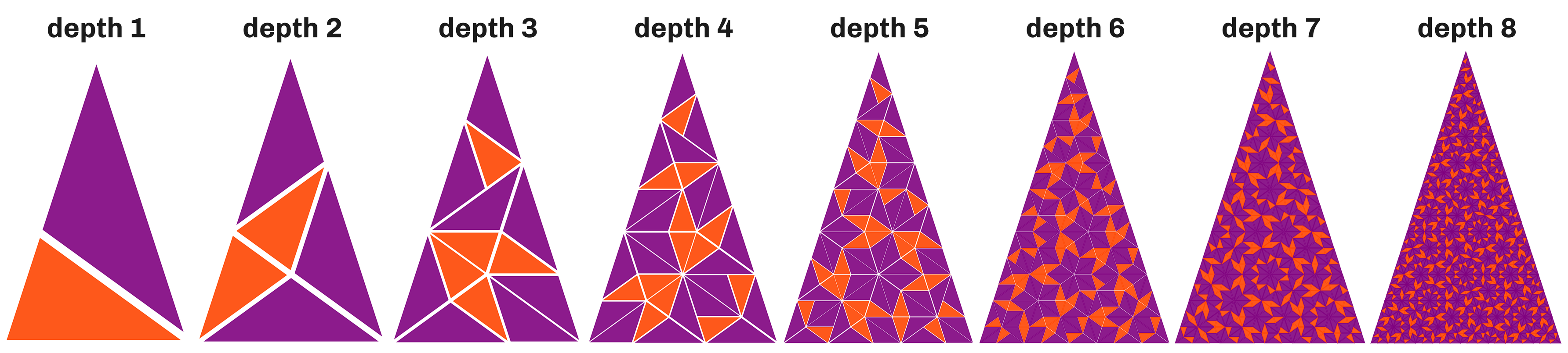
Recursive subdivision

By recursively subdividing a golden triangle, you get a smaller triangles and gnomons. And you can divide gnomons into smaller gnomons and triangles. The golden triangles are called **Robinson triangles** in the Penrose literature. Robinson triangles pair up into *darts* and *kites*.



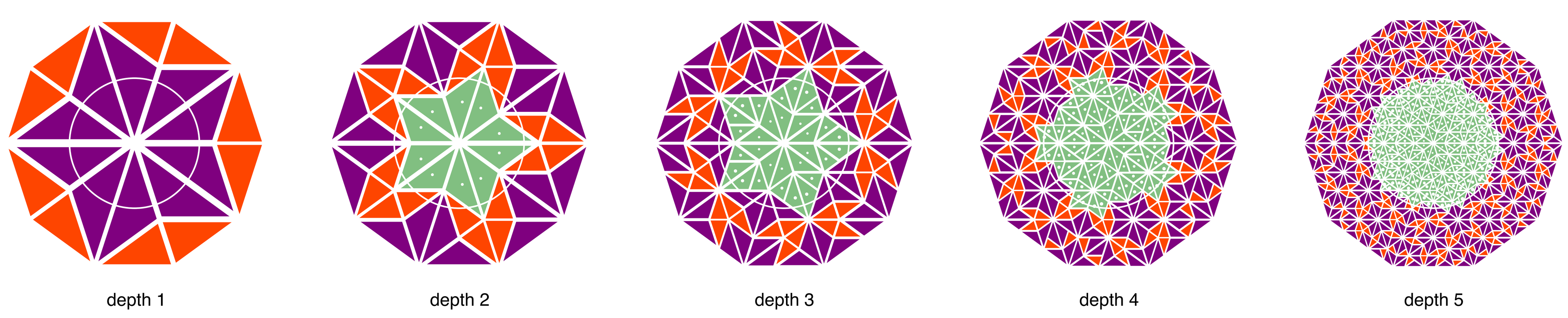
Making aperiodic (Penrose) tiling

The aperiodic tiling that results from this recursive subdivision is one of the well-known **Penrose** tilings.



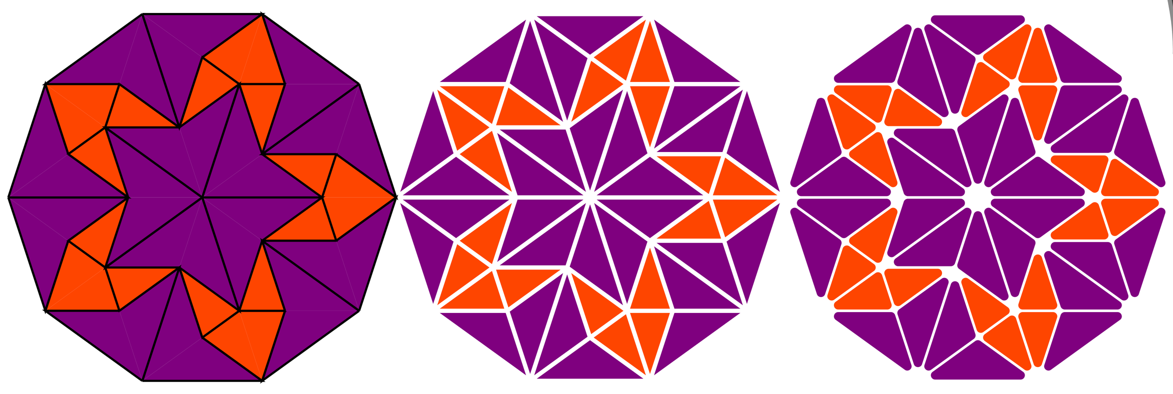
Center removal

Triangles less than a set distance from the center are removed from the final design.



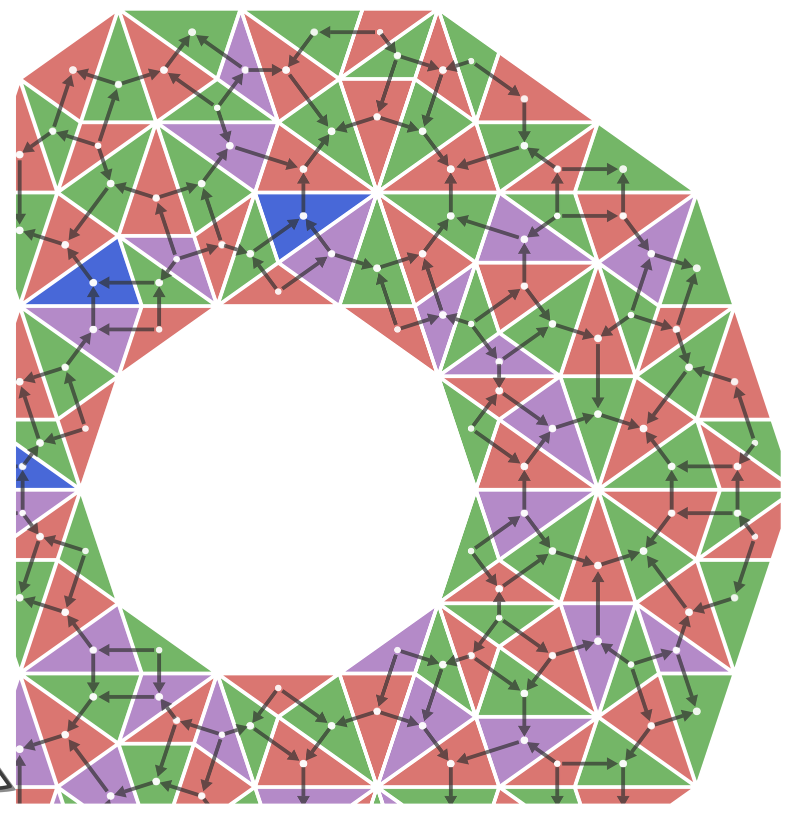
Tidier triangles

Luxor.jl has useful functions `polysmooth()` and `offsetpoly()` which help make cleaner graphics by rounding corners and offsetting the sides of the polygons in or out.



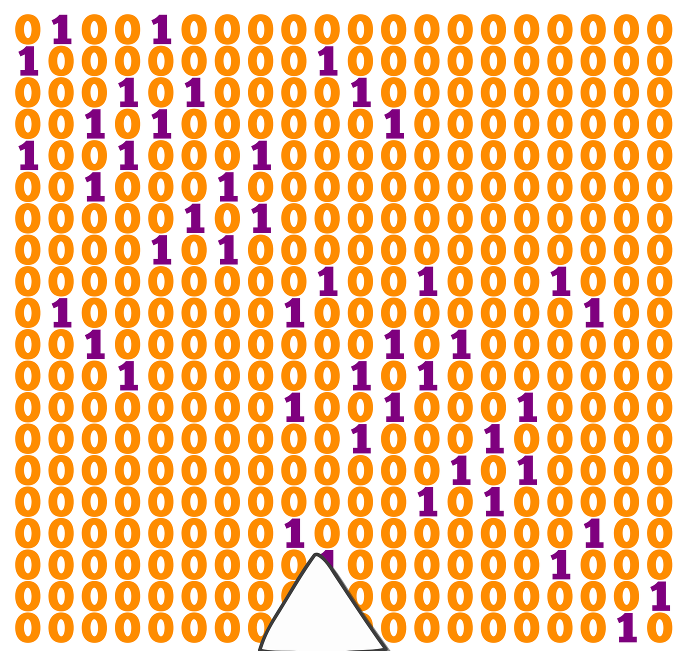
Greedy coloring

Finally, we can traverse the graph by finding the **graph colouring** with the property that adjacent nodes do not share the same colour. It turns out that for the Penrose tiling, there are rather few nodes that use the fourth colour (i.e. that have 3 neighbouring nodes).



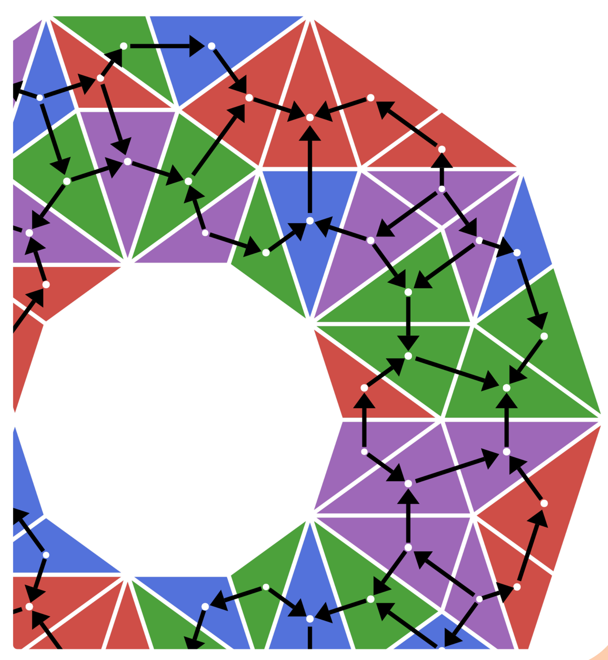
Graphs to the rescue

We use Graphs to treat the collection of triangles as a graph or network—a collection of vertices joined by edges. We find the **adjacency matrix** of the graph, a matrix `Aij` which contains 1 if nodes `i` and `j` are connected, and 0 if not.



Four colours suffice

A Penrose tiling can be thought of as a planar map. The Four-colour Theorem says that any map of the plane can be coloured using at most 4 colours. By spooky coincidence, there are four colours in the Julia logo. And we don't want adjacent tiles to be the same colour. So...



Colouring all the things

There are many ways to colour the triangles. Random colours, slowly shifting gradients from ColorSchemes.jl, a single colour. But...