Colouring aperiodic tiling patterns...

...we designed the JuliaCon 2019 T-shirt in Julia!

juliacon
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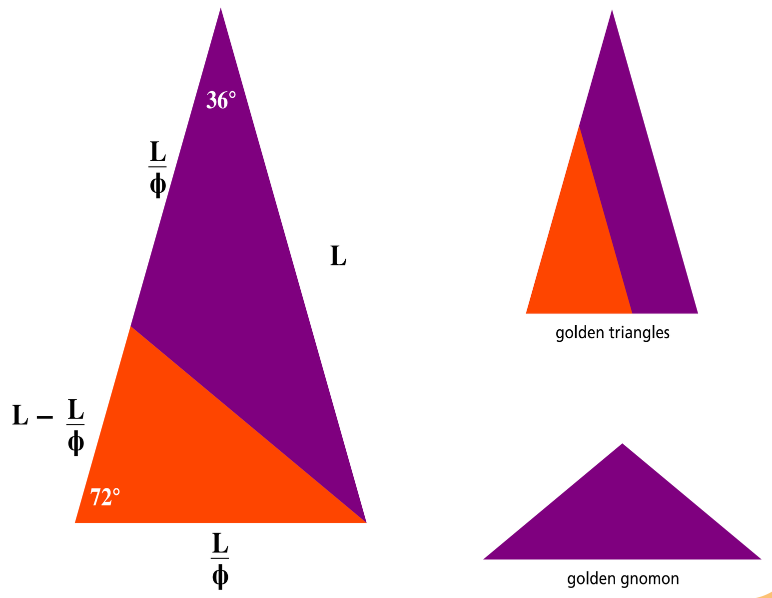
David P Sanders
Department of Physics, Faculty of Sciences
Universidad Nacional Autónoma de México (UNAM)

cormullion
Bricoleur and enthusiastic code artist

Code at https://github.com/dpsanders/JuliaCon2019_tshirt

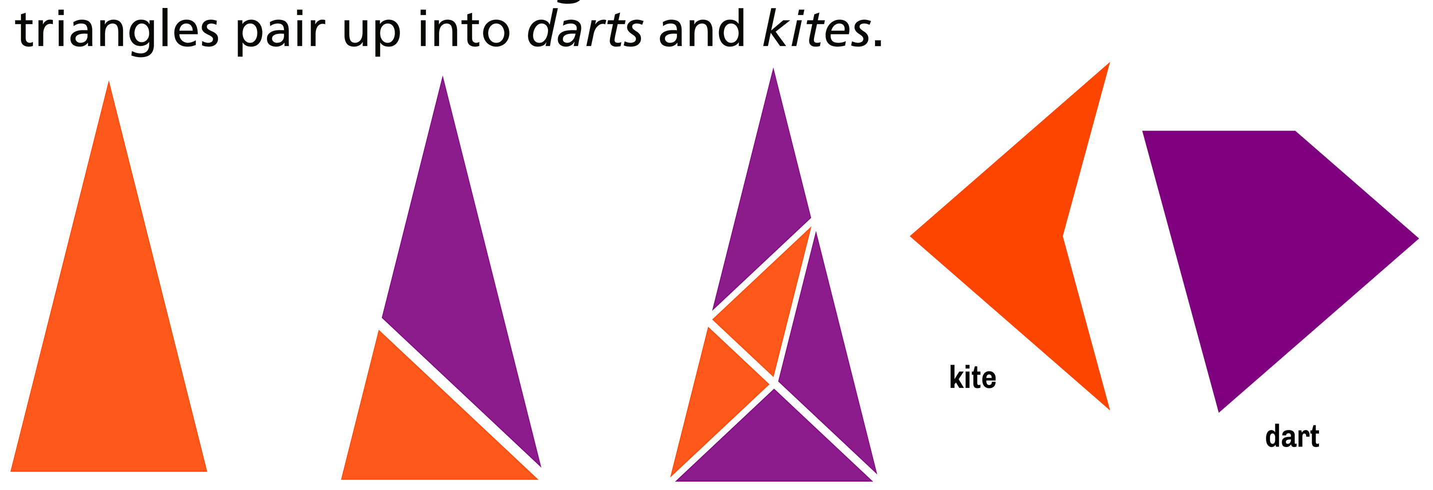
Golden triangles

The golden triangle and golden gnomon are the lesser known cousins of the golden rectangle. They also use the golden ratio ϕ (1.618:1). You can divide a golden triangle into a smaller golden triangle and a golden gnomon.



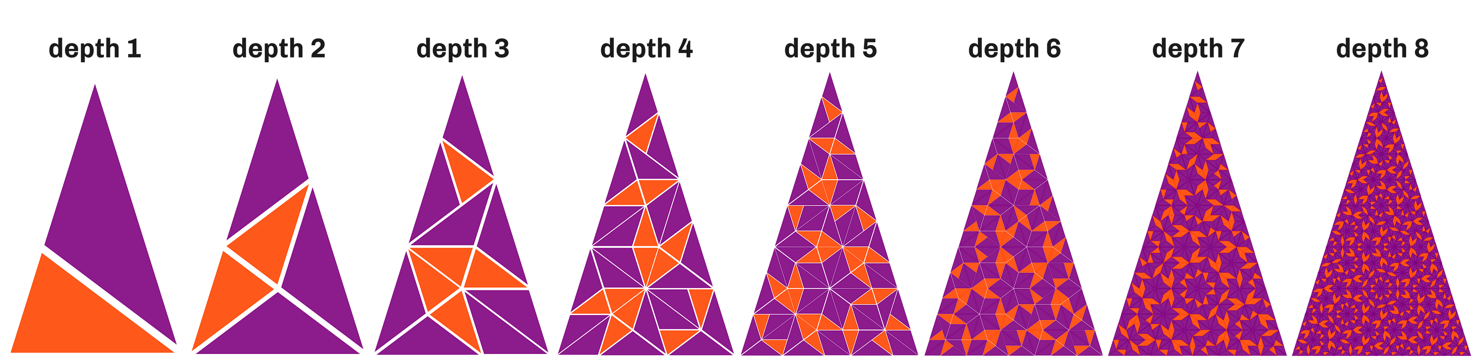
Recursive subdivision

By recursively subdividing golden triangles, you get smaller golden triangles and gnomons. And you can divide gnomons into smaller gnomons and triangles. The golden triangles are called **Robinson triangles** in the Penrose literature. Robinson triangles pair up into *darts* and *kites*.



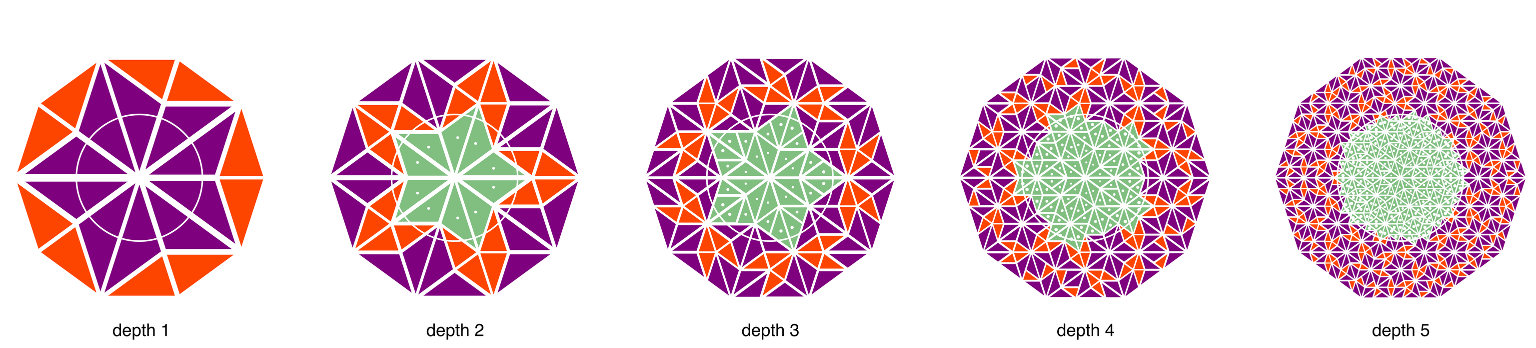
Making aperiodic (Penrose) tiling

The aperiodic tiling that results from this recursive subdivision is one of the well-known **Penrose** tilings.



Center removal

Triangles less than a set distance from the center can be removed from the final design.

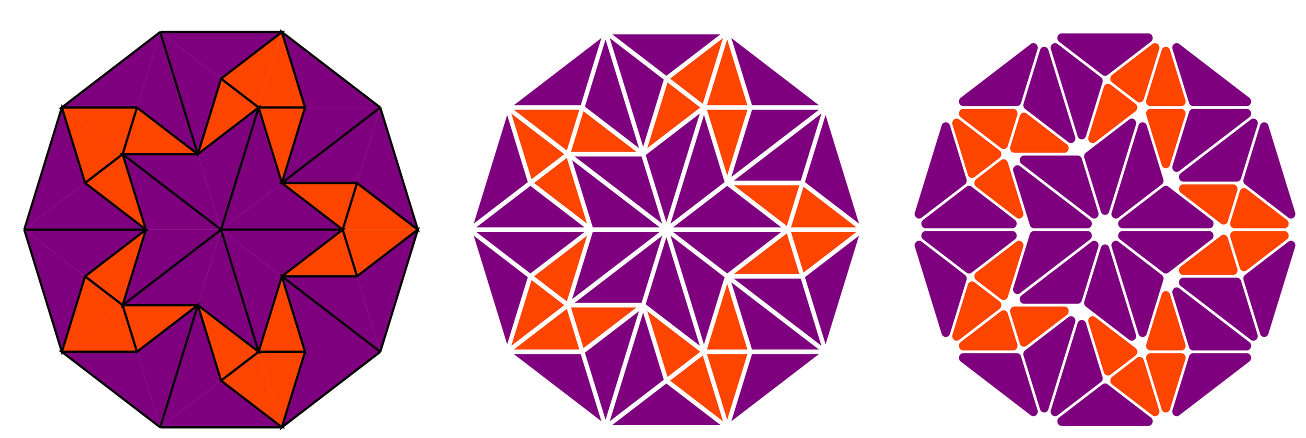


Colouring all the things

There are many ways to colour the triangles. Random colours, slowly shifting gradients from ColorSchemes.jl, even a single colour. How about the Julia colours...?!

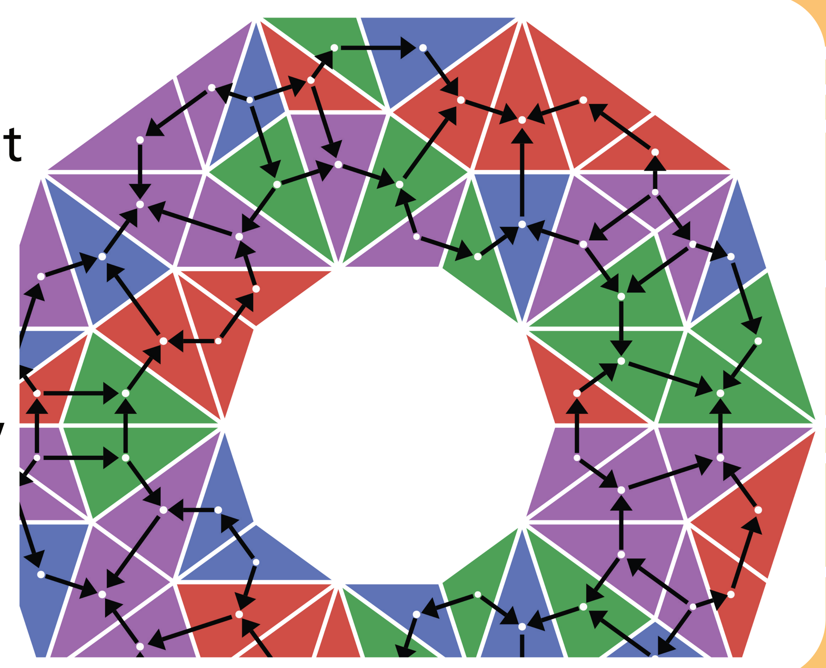
Tidier triangles

Graphics were created with Luxor.jl. There are useful functions such as `polysmooth()` and `offsetpoly()` which help make cleaner graphics by rounding corners and offsetting the edges of the polygons in or out.



Four colours suffice

A Penrose tiling can be thought of as a planar map. The Four-colour Theorem says that any map of the plane can be coloured using at most 4 colours. By spooky coincidence, there are four colours in the Julia logo. And we don't want adjacent tiles to be the same colour. So...



Graphs to the rescue

We use graphs to treat the collection of triangles as a graph or network—a collection of vertices joined by edges. We find the (symmetric) **adjacency matrix** of the graph, a matrix A_{ij} which contains 1 if nodes **i** and **j** are connected, and 0 if not.

0	1	1	0	0	0	0	0	0	1
1	0	0	0	1	1	0	0	0	0
1	0	0	0	1	1	0	0	0	1
0	0	0	0	0	0	0	0	1	0
0	1	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	1	0	0
1	0	1	0	0	0	0	0	0	0

Greedy colouring

Finally, we can traverse the graph of tile connectivity by finding one of the **graph colourings** which have the property that adjacent nodes must not share the same colour. It turns out that for the Penrose tiling, there are rather few nodes that use the fourth colour.