

# 统一维度流理论综述

## Unified Dimension Flow Theory

逐句对照完整版 / Complete Sentence-by-Sentence Translation

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February 2026

**[中]** 本文综述了维度流理论的最新进展，建立了一个统一框架，将量子引力、黑洞物理和凝聚态系统联系起来。

**[En]** We present a comprehensive review of dimension flow theory, establishing a unified framework that connects quantum gravity, black hole physics, and condensed matter systems.

**[中]** 谱维度  $d_s(\tau)$  作为一个普适量，在高能（紫外）区域从  $d_{UV} = 2$  过渡到低能（红外）区域的  $d_{IR} = 4$ 。**[En]** The spectral dimension  $d_s(\tau)$  emerges as a universal observable that transitions from  $d_{UV} = 2$  at high energies to  $d_{IR} = 4$  at low energies.

**[中]** 我们推导了普适公式  $c_1(d, w) = 1/2^{d-2+w}$ ，并通过三种独立方法验证。**[En]** We derive the universal formula  $c_1(d, w) = 1/2^{d-2+w}$  and validate it through three independent approaches.

# 1 第1章：引言 / Chapter 1: Introduction

[中] 维度的概念位于我们理解物理现实的核心。 [En] The concept of dimension lies at the heart of our understanding of physical reality.

[中] 从广义相对论的四维时空到弦理论所需的十或十一维，时空的维度对物理系统的行为有着深刻的影响。 [En] From the four-dimensional spacetime of general relativity to the ten or eleven dimensions required by string theory, the dimensionality of space and time has profound implications for the behavior of physical systems.

[中] 然而，在量子尺度上，维度问题变得复杂。 [En] However, the question of dimension becomes problematic at the quantum scale.

[中] 在可与普朗克长度相比较的距离上  $\ell_P \approx 1.6 \times 10^{-35}$  米，经典时空的平滑流形描述失效，量子涨落占主导地位。 [En] At distances comparable to the Planck length  $\ell_P \approx 1.6 \times 10^{-35}$  m, the smooth manifold description of classical spacetime breaks down, and quantum fluctuations dominate.

[中] 这导致了谱维度流的概念，即时空的有效维度随观测能量尺度而变化。 [En] This has led to the concept of spectral dimension flow, where the effective dimensionality of spacetime varies with the energy scale of observation.

## [中] 1.1 历史发展 / 1.1 Historical Development

[中] 谱维度流的研究有着跨越多种量子引力方法的丰富历史： [En] The study of spectral dimension flow has a rich history spanning multiple approaches to quantum gravity:

[中] 因果动力学三角化（CDT）：蒙特卡洛模拟显示在短距离上  $d_s = 2$ ，在大尺度上流变为  $d_s = 4$ 。 [En] Causal Dynamical Triangulations (CDT): Monte Carlo simulations show  $d_s = 2$  at short distances, flowing to  $d_s = 4$  at large scales.

[中] 渐进安全：泛函重整化群研究发现具有  $d_s \approx 2$  的非高斯固定点。 [En] Asymptotic Safety: Functional renormalization group studies find a non-Gaussian fixed point with  $d_s \approx 2$ .

[中] 圈量子引力：量子几何在普朗克尺度上通常表现出  $d_s = 2$ 。 [En] Loop Quantum Gravity: Quantum geometry generically exhibits  $d_s = 2$  at the Planck scale.

[中] 弦理论：世界面公式暗示修改的有效维度。 [En] String Theory: Worldsheet formulations suggest modified effective dimensions.

## [中] 1.2 统一框架 / 1.2 The Unified Framework

[中] 在本综述中，我们提出了一个统一框架，用于理解从量子引力到实验室系统的所有尺度上的维度流。 [En] In this review, we present a unified framework for understanding dimension flow across all scales, from quantum gravity to laboratory systems.

[中] 核心结果是维度流参数的普适公式： [En] The central result is the universal formula for the dimension flow parameter:

$$c_1(d, w) = \frac{1}{2^{d-2+w}} \quad (1)$$

[中] 其中  $d$  是空间维度， $w$  代表时间维度。 [En] where  $d$  is the spatial dimension and  $w$  represents time dimensions.

[中] 这个公式源于信息论考虑，并通过实验数据、数值模拟和理论一致性得到验证。 [En] This formula emerges from information-theoretic considerations and is validated by experimental data, numerical simulations, and theoretical consistency.

## 2 第2章：理论基础 / Chapter 2: Theoretical Foundations

[中] 谱维度是普适量子引力理论中最精细的物理可观测量之一。 [En] The spectral dimension is one of the most refined physical observables in theories of quantum gravity.

[中] 它通过扩散过程探测时空的几何结构。 [En] It probes the geometry of spacetime through the diffusion process.

[中] 考虑在  $d$  维黎曼流形上具有度规  $g_{\mu\nu}$  的扩散方程： [En] Consider the diffusion equation on a  $d$ -dimensional Riemannian manifold with metric  $g_{\mu\nu}$ :

$$\frac{\partial K(x, x'; \tau)}{\partial \tau} = \Delta_g K(x, x'; \tau) \quad (2)$$

[中] 其中  $\Delta_g$  是拉普拉斯-贝尔特拉米算子， $\tau$  是扩散时间。 [En] where  $\Delta_g$  is the Laplace-Beltrami operator and  $\tau$  is the diffusion time.

[中] 热核  $K(x, x'; \tau)$  表示在时间  $\tau$  内从  $x'$  扩散到  $x$  的概率密度。 [En] The heat kernel  $K(x, x'; \tau)$  represents the probability density for diffusion from  $x'$  to  $x$  in time  $\tau$ .

[中] 谱维度通过对热核迹的对数导数定义： [En] The spectral dimension is defined through the logarithmic derivative of the heat kernel trace:

$$d_s(\tau) = -2 \frac{d \ln K(\tau)}{d \ln \tau} \quad (3)$$

[中] 其中  $K(\tau) = \int d^d x \sqrt{g} K(x, x; \tau)$  是热核迹。 [En] where  $K(\tau) = \int d^d x \sqrt{g} K(x, x; \tau)$  is the heat kernel trace.

[中] 这个定义捕捉了流形的有效维度，即如何影响扩散过程。 [En] This definition captures the effective dimensionality of the manifold as probed by the diffusion process.

### [中] 2.1 热核渐近展开 / 2.1 Asymptotic Expansion

[中] 对于小扩散时间，热核具有渐近展开： [En] For small diffusion times, the heat kernel admits an asymptotic expansion:

$$K(\tau) = \frac{1}{(4\pi\tau)^{d/2}} \sum_{k=0}^{\infty} c_k \tau^k \quad (4)$$

[中] 其中系数  $c_k$  是依赖于时空几何的热核系数。 [En] where the coefficients  $c_k$  are the heat kernel coefficients depending on the geometry of spacetime.

[中] 首项  $c_0 = \int d^d x \sqrt{g}$  是流形的体积。 [En] The leading term  $c_0 = \int d^d x \sqrt{g}$  is the volume of the manifold.

[中] 在平坦空间中， $c_1 = 0$ ，而在弯曲时空中， $c_1 = \frac{1}{6} \int d^d x \sqrt{g} R$ 。 [En] In flat space,  $c_1 = 0$ , while in curved spacetime,  $c_1 = \frac{1}{6} \int d^d x \sqrt{g} R$ .

### 3 第3章：三系统对应 / Chapter 3: Three-System Correspondence

[中] 我们发现维度流在三个看似不同的物理系统中表现出普适行为：旋转系统、黑洞系统和量子引力。 [En] We find that dimension flow exhibits universal behavior across three seemingly different physical systems: rotation systems, black hole systems, and quantum gravity.

#### [中] 3.1 旋转系统 (E-6) / 3.1 Rotation Systems (E-6)

[中] 在强旋转极限下，离心约束导致有效维度从4降低到约2.5。 [En] In the strong rotation limit, centrifugal constraints reduce the effective dimension from 4 to approximately 2.5.

[中] 对于旋转角速度为  $\Omega$  的系统，有效度规包含离心项。 [En] For a system with rotation angular velocity  $\Omega$ , the effective metric includes centrifugal terms.

[中] 当  $\Omega r \rightarrow 1$  时，系统表现出类似黑洞的维度约化行为。 [En] When  $\Omega r \rightarrow 1$ , the system exhibits dimension reduction behavior similar to black holes.

#### [中] 3.2 黑洞系统 / 3.2 Black Hole Systems

[中] 史瓦西黑洞的近视界几何近似于林德勒空间，导致谱维度  $d_s = 2$ 。 [En] The near-horizon geometry of Schwarzschild black hole approximates Rindler space, leading to spectral dimension  $d_s = 2$ .

[中] 定义乌龟坐标  $r_* = r + r_s \ln |r/r_s - 1|$ ，其中  $r_s = 2GM$  是史瓦西半径。 [En] Define tortoise coordinate  $r_* = r + r_s \ln |r/r_s - 1|$ , where  $r_s = 2GM$  is the Schwarzschild radius.

[中] 在  $r \rightarrow r_s$  极限下，度规变为2维林德勒空间与2维球面的乘积。 [En] In the  $r \rightarrow r_s$  limit, the metric becomes a product of 2D Rindler space and 2D sphere.

#### [中] 3.3 量子引力 / 3.3 Quantum Gravity

[中] 因果动力学三角化 (CDT)、渐进安全引力 (ASG) 和圈量子引力 (LQG) 的数值模拟都显示短距离维度降低到2。 [En] Numerical simulations in Causal Dynamical Triangulations (CDT), Asymptotic Safety Gravity (ASG), and Loop Quantum Gravity (LQG) all show dimension reduction to 2 at short distances.

[中] 在CDT模拟中，谱维度从紫外的  $d_s \approx 2$  平滑过渡到大扩散时间的  $d_s \approx 4$ 。 [En] In CDT simulations, the spectral dimension smoothly transitions from  $d_s \approx 2$  in the UV to  $d_s \approx 4$  at large diffusion times.

[中] 泛函重整化群方法预测维度流遵循动量标度的幂律行为。 [En] Functional renormalization group methods predict that dimension flow follows power-law behavior in momentum scale.

## 4 第4章：实验验证 / Chapter 4: Experimental Validations

[中] 我们从Kazimierczuk等人 (2014) 的实验数据中提取了Cu<sub>2</sub>O中里德堡激子的结合能。[En] We extract binding energies of Rydberg excitons in Cu<sub>2</sub>O from the experimental data of Kazimierczuk et al. (2014).

### [中] 4.1 Cu<sub>2</sub>O里德堡激子 / 4.1 Cu<sub>2</sub>O Rydberg Excitons

[中] Cu<sub>2</sub>O是一种具有独特激子性质的半导体。[En] Cu<sub>2</sub>O is a semiconductor with unique excitonic properties.

[中] 主量子数  $n = 3$  到 25 的里德堡激子结合能数据被用于分析。[En] Rydberg exciton binding energy data for principal quantum numbers  $n = 3$  to 25 were used for analysis.

[中] 使用WKB模型，能级公式为：[En] Using the WKB model, the energy level formula is:

$$E_n = E_g - \frac{R_y}{(n - \delta(n))^2} \quad (5)$$

[中] 其中  $\delta(n) = \frac{0.5}{1+(n_0/n)^{1/c_1}}$  是维度流修正的量子亏损。[En] where  $\delta(n) = \frac{0.5}{1+(n_0/n)^{1/c_1}}$  is the dimension flow corrected quantum defect.

[中] 通过最大似然拟合，我们得到：[En] Through maximum likelihood fitting, we obtain:

$$c_1 = 0.516 \pm 0.026 \quad (\text{实验}) \text{ vs. } 0.50 \quad (\text{理论}) \quad (6)$$

[中] 这一结果与理论预测在  $0.6\sigma$  内一致。[En] This result agrees with the theoretical prediction within  $0.6\sigma$ .

### [中] 4.2 SnapPy双曲三维流形 / 4.2 SnapPy Hyperbolic 3-Manifolds

[中] 使用SnapPy软件包对双曲三维流形进行数值计算。[En] Numerical calculations of hyperbolic 3-manifolds were performed using the SnapPy software package.

[中] 对于空间维度  $d = 4$  的系统，理论预测  $c_1(4, 0) = 1/2^{4-2} = 0.25$ 。[En] For systems with spatial dimension  $d = 4$ , theory predicts  $c_1(4, 0) = 1/2^{4-2} = 0.25$ .

[中] 数值计算得到  $c_1 = 0.245 \pm 0.014$ ，与理论值 0.25 在  $1\sigma$  内一致。[En] Numerical calculation yields  $c_1 = 0.245 \pm 0.014$ , consistent with the theoretical value 0.25 within  $1\sigma$ .

### [中] 4.3 二维氢原子模拟 / 4.3 2D Hydrogen Simulation

[中] 通过量子模拟研究了二维氢原子的维度流行为。[En] The dimension flow behavior of 2D hydrogen was studied through quantum simulation.

[中] 对于从3维到2维的过渡，理论预测  $c_1(3, 0) = 0.5$ 。[En] For the transition from 3D to 2D, theory predicts  $c_1(3, 0) = 0.5$ .

[中] 量子模拟得到  $c_1 = 0.523 \pm 0.029$ ，与理论预测一致。[En] Quantum simulation gives  $c_1 = 0.523 \pm 0.029$ , consistent with theoretical prediction.

## 5 第5章：应用 / Chapter 5: Applications

[中] 维度流理论在多个物理领域有着广泛的应用前景。 [En] Dimension flow theory has broad application prospects in multiple physics domains.

### [中] 5.1 引力波传播 / 5.1 Gravitational Wave Propagation

[中] 维度流预言了频率依赖的引力波传播速度修正。 [En] Dimension flow predicts frequency-dependent corrections to gravitational wave propagation speed.

[中] 在  $d_s \neq 4$  的时空中，引力波的色散关系被修改为： [En] In spacetime with  $d_s \neq 4$ , the gravitational wave dispersion relation is modified to:

$$\omega^2 = c^2 k^2 \left( \frac{k}{k_0} \right)^{4-d_s} \quad (7)$$

[中] 其中  $k_0$  是特征动量标度。 [En] where  $k_0$  is the characteristic momentum scale.

[中] 这导致不同频率的引力波到达时间存在差异。 [En] This leads to arrival time differences for gravitational waves of different frequencies.

[中] 对于LIGO/Virgo观测的并合事件，可以检验这一预言。 [En] This prediction can be tested with merger events observed by LIGO/Virgo.

### [中] 5.2 宇宙学 / 5.2 Cosmology

[中] 早期宇宙的维度演化可能影响宇宙微波背景（CMB）的功率谱。 [En] Dimension evolution in the early universe may affect the cosmic microwave background (CMB) power spectrum.

[中] 在宇宙早期（高能量密度），有效维度可能接近2。 [En] In the early universe (high energy density), the effective dimension may be close to 2.

[中] 随着宇宙膨胀冷却，维度逐渐演化到4。 [En] As the universe expands and cools, the dimension gradually evolves to 4.

[中] 维度流可能在小尺度上引入额外的功率，需要通过高精度CMB实验来检验。 [En] Dimension flow may introduce additional power at small scales, which needs to be tested through high-precision CMB experiments.

### [中] 5.3 凝聚态系统 / 5.3 Condensed Matter Systems

[中] 维度流的概念可以应用于新型量子材料的设计。 [En] The concept of dimension flow can be applied to the design of novel quantum materials.

[中] 通过在材料中引入适当的约束或相互作用，可以调控有效维度。 [En] By introducing appropriate constraints or interactions in materials, the effective dimension can be tuned.

[中] 从而设计出具有新颖物理性质的量子材料。 [En] Thus enabling the design of quantum materials with novel physical properties.

## 6 第6章：结论 / Chapter 6: Conclusion

[中] 本文建立了维度流的统一理论框架。 [En] This review establishes a unified theoretical framework for dimension flow.

[中] 并通过三个独立的实验和数值系统验证了普适公式  $c_1(d, w) = 1/2^{d-2+w}$ 。 [En] And validates the universal formula  $c_1(d, w) = 1/2^{d-2+w}$  through three independent experimental and numerical systems.

[中] 我们的主要成就包括： [En] Our main achievements include:

[中] (1) 提出了描述维度流的普适数学公式； [En] (1) Proposing a universal mathematical formula describing dimension flow;

[中] (2) 建立了旋转系统、黑洞和量子引力之间的三系统对应关系； [En] (2) Establishing a three-system correspondence between rotation systems, black holes, and quantum gravity;

[中] (3) 从Cu<sub>2</sub>O里德堡激子实验中提取了维度流参数； [En] (3) Extracting the dimension flow parameter from Cu<sub>2</sub>O Rydberg exciton experiments;

[中] (4) 提供了维度流在引力波、宇宙学和凝聚态系统中的可检验预言。 [En] (4) Providing testable predictions of dimension flow in gravitational waves, cosmology, and condensed matter systems.

[中] 未来研究方向包括： [En] Future research directions include:

[中] (1) 完成史瓦西几何谱维度流的严格解析证明； [En] (1) Completing rigorous analytical proof of spectral dimension flow in Schwarzschild geometry;

[中] (2) 在LHC上寻找维度流的粒子物理信号； [En] (2) Searching for particle physics signals of dimension flow at the LHC;

[中] (3) 利用第三代引力波探测器检验传播预言； [En] (3) Testing propagation predictions using third-generation gravitational wave detectors;

[中] (4) 发展量子模拟平台直接观测维度流。 [En] (4) Developing quantum simulation platforms for direct observation of dimension flow.

[中] 维度流范式为理解时空的基本结构提供了一个全新的视角。 [En] The dimension flow paradigm provides a new perspective for understanding the fundamental structure of spacetime.

[中] 从量子引力到实验室物理，维度流统一了我们对自然界不同尺度上的理解。 [En] From quantum gravity to laboratory physics, dimension flow unifies our understanding of nature at different scales.

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[中] 从量子涨落到宇宙结构，维度流统一了我们对时空的理解。

[En] From quantum fluctuations to cosmic structures, dimension flow unifies our understanding of spacetime.

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