Hierarchical modeling of export share panel data

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CONTENTS

1 THE MODEL

$$y_{ijt} = \alpha_i + \sum_{c=1}^{C} \beta_{ij}^c x_{ijt}^c + \epsilon_{ijt} \tag{1.1}$$

$$\epsilon_{ijt} \sim N(0, \sigma^2) \tag{1.2}$$

where y_{ijt} is the response (change in export share) and $\{x_{ijt}^c\}_{c=1}^{C}$ is the set of C covariates. We need to estimate the following parameters: α_i, β_{ij}^c.

To form a hierarchical model, we put priors and hyperpriors on the parameters we need to estimate.

$$\beta_{ij}^c \sim N(\mu_c, \tau_c^2) \quad \forall c \tag{1.3}$$

$$\alpha_i \sim N(0, \sigma_\alpha^2) \quad \forall i \tag{1.4}$$

$$\sigma^2 \propto \frac{1}{\sigma^2} \tag{1.5}$$

$$\mu_c \sim N(0, \sigma_0^2) \quad \forall c \tag{1.6}$$

$$\tau_c^2 \propto \frac{1}{\tau_c} \quad \forall c \tag{1.7}$$

$$\sigma_0^2 = 50 \tag{1.8}$$

2 POSTERIORS

These priors and hyperpriors imply the following posterior distributions of the parameters. Let's suppose we have $T * I * J$ total observations in our panel. Here, T is the number of time points, I is the number of country i's and, J is the number of country j's. We re-write the model is matrix notation as:

$$y = \alpha + X\beta + \epsilon \tag{2.1}$$

$y \in \mathbb{R}^{T*I*J}$, $\alpha \in \mathbb{R}^{T*I*J}$ and satisfying $\alpha_n = \alpha_i$ if $y_n = y_{i..}$ $\forall n \in \{1, \cdots, T * I * J\}$.

$X \in \mathbb{R}^{(T*I*J) x (I*J*C)}$ is a sparse matrix.

$\beta \in \mathbb{R}^{I*J*C}$.