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Hierarchical modeling of export share panel data

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CONTENTS

1 THE MODEL

$$y_{ijt} = \alpha_i + \sum_{c=1}^{C} \beta_{ij}^c x_{ijt}^c + \epsilon_{ijt} \tag{1.1}$$

$$\epsilon_{ijt} \sim N(0, \sigma^2) \tag{1.2}$$

where y_{ijt} is the response (change in export share) and $\{x_{ijt}^c\}_{c=1}^C$ is the set of C covariates. We need to estimate the following parameters: α_i, β_{ij}^c.

To form a hierarchical model, we put priors and hyperpriors on the parameters we need to estimate.

$$\beta_{ij}^c \sim N(\mu_c, \tau_c^2) \quad \forall c \tag{1.3}$$

$$\alpha_i \sim N(0, \sigma_\alpha^2) \quad \forall i \tag{1.4}$$

$$\sigma^2 \propto \frac{1}{\sigma^2} \tag{1.5}$$

$$\mu_c \sim N(0, \sigma_0^2) \quad \forall c \tag{1.6}$$

$$\tau_c^2 \propto \frac{1}{\tau_c} \quad \forall c \tag{1.7}$$

$$\sigma_0^2 = 50 \tag{1.8}$$

2 POSTERIORS

These priors and hyperpriors imply the following posterior distributions of the parameters. Let's suppose we have $T * I * J$ total observations in our panel. Here, T is the number of time points, I is the number of country i's and, J is the number of country j's. We re-write the model is matrix notation as:

$$y = \alpha + X\beta + \epsilon \tag{2.1}$$

$y \in \mathbb{R}^{T*I*J}$, $\alpha \in \mathbb{R}^{T*I*J}$ and satisfying $\alpha_n = \alpha_i$ if $y_n = y_{i..}$ $\forall n \in \{1, \cdots, T*I*J\}$.

$X \in \mathbb{R}^{(T*I*J)x(I*J*C)}$ is a sparse matrix.

$\beta \in \mathbb{R}^{I*J*C}$.

The complete conditionals are:

$$
\begin{aligned}
\beta|\cdot &\sim N(m_1, \Sigma_1) \\
\Sigma_1^{-1} &= \frac{1}{\sigma^2} X^T X + T^{-1} \\
m_1 &= \Sigma_1 \left(\frac{1}{\sigma^2} X^T (y - \alpha) + T^{-1} \mu \right)
\end{aligned} \tag{2.2}
$$

$$
\sigma^2|\cdot \sim Ga\left(\frac{T * I * J}{2} + 2, \quad \frac{(y - \alpha - X\beta)^T (y - \alpha - X\beta)}{2} \right) \tag{2.3}
$$

$$
\tau_c^2|\cdot \sim Ga\left(\frac{I * J + 1}{2} + 1, \quad \right) \tag{2.4}
$$