Homework 1

CALCULUS I (STM 1001)

due: **Friday, January 16, 2026 at 11:30a**

Instructions: Please submit solutions on Populi as a knitted markdown pdf. For handwritten work, please scan your solution pages and include in the final pdf. Homeworks can be completed individually or in groups of up to three people. Names of all group members must be included on the writeups.

Problem 1: MMAC Exercises

Chapter 1.3 Exercises: 2, 4, 6, 8, 51–58

Chapter 1.4 Exercises: 16, 46, 48

Chapter 1.6 Exercises: 65–68

Chapter 1.7. On your own, please read through the basics of trigonometry!

Problem 2: Pattern-Book Functions in R

This problem will get you comfortable with defining and visualizing pattern-book functions in R using the `mosaicCalc` package.

a. Define each of the following pattern-book functions in R using `makeFun()`:

- Constant: $f(x) = 5$
- Line: $g(x) = 2x + 3$
- Power: $h(x) = x^3$
- Exponential: $j(x) = e^{2x}$
- Logarithm: $k(x) = \ln(x)$ (remember the domain restriction)
- Sinusoid: $\ell(x) = 3\sin(2x) + 1$
- Hump: $m(x) = e^{-x^2}$
- Sigmoid: $n(x) = \frac{1}{1+e^{-x}}$
- Reciprocal: $p(x) = \frac{1}{x}$

b. Create plots of each function over an appropriate domain. For each function, choose a domain that clearly shows its key features (e.g., for the reciprocal, show both positive and negative regions, avoiding $x = 0$).

c. Identify which pattern-book functions are:

- Monotonically increasing on their entire domain
- Monotonically decreasing on their entire domain
- Neither (not monotonic)

Justify your answers.

d. Which pattern-book functions have:

- Horizontal asymptotes? If so, what are they?

- Vertical asymptotes? If so, where are they?

e. The sigmoid function $n(x) = \frac{1}{1+e^{-x}}$ is commonly used in machine learning. Evaluate $n(0)$, $n(1)$, and $n(-1)$. What is the range of this function? Why is this useful for modeling probabilities?

Problem 3: Describing Functions

This problem will help you practice using the vocabulary for describing function features.

a. Consider the function $f(x) = x^3 - 3x + 2$.

- Plot this function over the domain $[-3, 3]$.
- Identify all intervals where the function is:
    - Increasing (positive slope)
    - Decreasing (negative slope)
- Identify all intervals where the function is:
    - Concave up
    - Concave down
- Find all local extrema (local maxima and minima). At what x-values do they occur?
- Does this function have any asymptotes? If so, describe them.

b. Consider the function $g(x) = \frac{x^2 - 4}{x - 2}$.

- Simplify this function algebraically. What is the domain of the original function? What is the domain after simplification?
- Plot both the original function and the simplified version. What do you notice?
- Is this function continuous? If not, where are the discontinuities?
- Does this function have any asymptotes? If so, describe them.

c. Consider the function $h(x) = \sin(x) + \cos(2x)$.

- Plot this function over the domain $[0, 4\pi]$.
- Is this function periodic? If so, what is its period?
- Identify at least two local maxima and two local minima in the plotted domain.
- Is this function monotonic on any interval? If so, identify such intervals.

d. Create your own function that demonstrates each of the following properties (you can use different functions for each):

- A function that is continuous but not monotonic
- A function with both a horizontal and vertical asymptote
- A function that is periodic with period π
- A function that has exactly two local extrema

For each function, provide the formula, a plot, and a brief explanation of why it has the stated property.