



# MACHINE FORECAST DISAGREEMENT AND EQUITY RETURNS

Turan G. Bali

Ran Chang

Bryan T. Kelly

Presented by Minjoo Kim

## INTRO

- "Disagreement" -> central to on some level everything that occurs in financial markets.
- Miller [1977] hypothesizes that stock prices are upward biased when there is a divergence of opinion among investors about the stock value and pessimistic investors face short-sale constraints.
- Empirical work is very limited -> "Difficult to measure investor beliefs"
- Previous literature -> rely on analyst forecast -> biased

# **MAIN FINDINGS**

 The author proposes a belief-generating model from which we build a statistical measure of investor disagreement. → simulate differences in beliefs across investors by endowing them with different machine learning models for forecasting returns from the same set of inputs.

 Find a significantly negative cross-sectional relation between investor disagreement and future stock returns

 Present evidence that this return predictability is driven by mispricing induced by information frictions, short-selling costs, and limits-to-arbitrage

#### EMPIRICAL MODEL OF DISAGREEMENT

$$E_{i,t}[r_{t+1}] = f_i(x_t)$$

Each investor i is represented by a model  $f_i$  and investors heterogeneity is considered by Random functions  $f_i$ .

<Random features machine learning>

$$f_i = a_i' z_{i,t}$$
, where  $z_{i,t} = [sin(w_1' x_t), cos(w_1' x_t), ..., sin(w_P' x_t), cos(w_P' x_t)]'$  and  $w_p \sim N(0, \sigma^2)$ .

Set N=100,000, Draw the random  $W_i$  from normal distribution (investor n's belief formation) and construct transformed feature  $Z_{i,t}$ 

$$r_{t+1} = a_i' z_{i,t} + e_{t+1}.$$

Ridge regression

$$sigma(E_{i,t}[r_{t+1}])$$

Machine forecast disagreement



## 1.1 UNIVARIATE PORTFOLIO LEVEL ANALYSIS

Form a long-short portfolio that takes a long position in the highest decile of MFD and a short position in the lowest decile of MFD

Panel A:	Returns and alpha	as on value-	weighted MFD	-sorted decile	portfolios (100	00 largest stock	is)
Rank	Excess Return	CAPM	FF6	SY	HXZ4	DHS	
P1	0.64** (2.17)	0.15 (0.71)	0.14 (0.64)	0.01 (0.28)	0.13 (0.68)	0.13 (0.71)	
P2	0.50* (1.68)	0.14 (0.38)	0.11 (0.43)	-0.02 (-0.06)	0.11 (0.64)	0.09 (0.33)	
P3	0.49* (1.68)	0.03 (0.72)	0.03 (0.18)	-0.14 (-0.05)	-0.02 (-0.31)	-0.08 (-0.24)	
P4	0.47 (1.60)	0.13 (0.35)	0.08 (0.42)	-0.05 (-0.06)	0.10 (0.32)	-0.02 (-0.26)	
P5	0.35* (1.70)	-0.05 (-0.79)	-0.10 (-0.23)	-0.14 (-0.65)	-0.14 (-0.18)	-0.27 (-0.76)	
P6	0.37 (0.92)	-0.09 (-0.81)	-0.11 (-0.41)	-0.16 (-0.82)	-0.27 (-0.36)	-0.32 (-1.51)	
P7	0.25 (0.42)	-0.31 (-1.28)	-0.34 (-1.26)	-0.34 (-1.00)	-0.39 (-1.61)	-0.41*	MFD indeed contains
P8	0.25 (0.76)	-0.21 (-1.33)	-0.25 (-0.63)	-0.19 (-0.79)	-0.37 (-1.54)	-0.29*	incremental predictive power beyond these well-
P9	0.27 (0.24)	-0.41* (-1.89)	-0.38 (-1.38)	-0.34 (-1.65)	-0.36* (-1.69)	0.41**	known
P10	0.20 (0.03)	-0.38* (-1.75)	-0.38 (-1.43)	-0.38* (-1.84)	-0.40** (-2.04)		equity return predictors.
L/S	-0.43*** (-2.70)	-0.52*** (-2.81)	-0.52*** (-2.63)	-0.39* (-1.94)	-0.54** (-2.00)	-0.50** (-2.38)	

The significant relation between the MFD and future returns is largely coming from the short leg of the arbitrage portfolio -> high MFD firms are overvalued due to disagreement as well as limits of arbitrage

#### 1.1. LONG TERM PREDICTIVE POWER

	t+2	t+3	t+4	t+5	t+6	t+7	t + 8	t+9	t + 10	t + 11	t + 12
P1	0.11	0.07	0.04	0.04	-0.01	-0.07	-0.16	-0.12	-0.10	-0.11	-0.15
	(0.71)	(0.41)	(0.35)	(0.49)	(-0.53)	(-0.45)	(-0.33)	(-0.17)	(-0.15)	(-0.17)	(-0.06)
P2	-0.05	0.01	0.00	0.04	-0.02	-0.08	-0.18	-0.16	-0.17	-0.16	-0.18
	(-0.66)	(0.31)	(0.18)	(0.05)	(-0.35)	(-0.24)	(-0.05)	(-0.16)	(-0.11)	(-0.08)	(-0.04)
P3	-0.13	-0.03	-0.06	-0.13	-0.18	-0.12	-0.22	-0.21	-0.22	-0.17	-0.18
	(-0.26)	(-0.46)	(-0.68)	(-0.39)	(-0.17)	(-0.29)	(-0.48)	(-0.04)	(-0.02)	(-0.04)	(-0.19)
P4	-0.13	-0.01	-0.05	-0.08	-0.19	-0.11	-0.19	-0.17	-0.18	-0.18	-0.16
	(-0.30)	(-0.05)	(-0.39)	(-0.03)	(-0.24)	(-0.07)	(-0.24)	(-0.09)	(-0.08)	(-0.03)	(-0.15)
P5	-0.14	-0.15	-0.24	-0.13	-0.24	-0.13	-0.25	-0.20	-0.20	-0.21	-0.17
	(-1.10)	(-1.25)	(-0.92)	(-0.50)	(-0.65)	(-0.49)	(-0.42)	(-0.12)	(-0.02)	(-0.24)	(-0.18)
P6	-0.14	-0.22	-0.26	-0.15	-0.23	-0.13	-0.29	-0.22	-0.21	-0.21	-0.18
	(-1.16)	(-1.34)	(-0.9)	(-0.54)	(-0.84)	(-0.78)	(-0.49)	(-0.31)	(-0.03)	(-0.27)	(-0.21)
P7	-0.36**	-0.33*	-0.36*	-0.31	-0.28	-0.18	-0.32	-0.29	-0.24	-0.29	-0.20
	(-2.40)	(-1.82)	(-1.82)	(-1.13)	(-1.32)	(-1.13)	(-0.72)	(-0.52)	(-0.15)	(-0.39)	(-0.24)
P8	-0.26**	-0.38*	-0.25	-0.28	-0.29	-0.15	-0.30	-0.25	-0.20	-0.28	-0.19
	(-2.50)	(-1.67)	(-1.32)	(-1.04)	(-1.18)	(-0.82)	(-0.63)	(-0.47)	(-0.09)	(-0.33)	(-0.24)
P9	-0.41**	-0.35*	-0.32*	-0.41	-0.34	-0.22	-0.38	-0.31	-0.26	-0.27	-0.23
	(-2.50)	(-1.90)	(-1.75)	(-1.62)	(-1.16)	(-1.34)	(-0.82)	(-0.49)	(-0.22)	(-0.55)	(-0.28)
P10	-0.42**	-0.40*	-0.43**	-0.39*	-0.37	-0.33	-0.43	-0.35	-0.28	-0.30	-0.24
	(-2.28)	(-1.90)	(-2.15)	(-1.85)	(-1.39)	(-1.25)	(-1.20)	(-0.76)	(-0.59)	(-0.61)	(-0.25)
L/S	-0.53***	-0.47**	-0.47**	-0.44*	-0.35**	-0.26	-0.27	-0.23	-0.17	-0.19	-0.09
t	(-2.90)	(-2.37)	(-2.10)	(-1.82)	(-2.04)	(-1.60)	(-1.35)	(-0.93)	(-0.97)	(-0.94)	(-0.52)

Results show that the negative cross-sectional relation between the MFD and future returns is not just a one-month affair and the underreaction to firm-specific uncertainty persists several months into the future, which is consistent with the theoretical evidence of the gradual diffusion of information into stock prices.

## 1.2. TESTING THE MISPRICING HYPOTHESIS

Provide direct empirical evidence that high-MFD stocks are indeed overvalued using stock-level mispricing measure (MISP) of Stambaugh et al. [2015]

Panel A: aver	Panel A: average mispricing score in MFD decile portfolio											
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
Misp	48.44	48.95	49.19	48.72	48.62	49.80	49.38	49.37	50.03	50.04	1.60***	(6.46)

high-MFD stocks indeed have a higher average mispricing score than the low-MFD stocks. - > High MFD stocks with greater investor disagreement are indeed overvalued

Panel B: mispr	Panel B: mispricing score and MFD-sorted alphas												
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat	
Misp 1	0.22	0.18	0.03	0.11	-0.11	-0.14	-0.36	-0.29	-0.43	-0.50	-0.72***	(-3.05)	
Misp 2	0.09	0.07	0.01	0.05	-0.06	-0.06	-0.15	-0.15	-0.24	-0.25	-0.34*	(-1.66)	
Misp 3	0.13	0.11	0.02	0.06	-0.08	-0.10	-0.27	-0.20	-0.30	-0.32	-0.45**	(-2.38)	
Misp 4	0.19	0.14	0.03	0.09	-0.12	-0.11	-0.37	-0.31	-0.40	-0.44	-0.63***	(-3.01)	
Misp 5	0.38	0.28	0.05	0.16	-0.22	-0.22	-0.71	-0.61	-0.81	-0.90	-1.28***	(-4.48)	
Misp 5 - Misp 1	0.16	0.10	0.02	0.05	-0.11	-0.08	-0.35	-0.32	-0.38	-0.40	-0.56**	(-2.04)	
Misp 5 - Misp 2	0.28	0.21	0.03	0.12	-0.16	-0.16	-0.56	-0.46	-0.57	-0.66	-0.94***	(-3.36)	
Misp 5 - Misp 3	0.25	0.17	0.03	0.10	-0.14	-0.11	-0.44	-0.41	-0.51	-0.58	-0.83**	(-2.49)	
Misp 5 - Misp 4	0.19	0.14	0.02	0.07	-0.10	-0.10	-0.34	-0.31	-0.41	-0.47	-0.65*	(-1.88)	

The disagreement premium is stronger for overpriced stocks, i.e., stocks with higher investor disagreements are subject to a higher degree of mispricing and lower subsequent return

## 1.3. AVERAGE PORTFOLIO CHARACTERISTICS

Investigate which firm characteristics can potentially explain the negative relation between the MFD and future stock returns.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
MFD	0.03	0.03	0.04	0.04	0.05	0.06	0.06	0.07	0.07	0.08	0.05***	(120.13)
SIZE	4.82	4.55	4.07	4.78	4.95	4.36	4.51	4.27	4.13	4.39	-0.43***	(-12.50)
$_{\mathrm{BM}}$	-0.64	-0.82	-0.53	-0.54	-0.60	-0.65	-0.72	-0.53	-0.57	-0.47	0.16***	(14.47)
$\mathbf{AG}$	1.12	1.42	1.18	1.22	1.23	1.35	1.62	1.37	1.30	1.55	0.43***	(5.15)
GP	0.44	0.38	0.39	0.42	0.42	0.41	0.39	0.40	0.32	0.34	-0.11***	(-57.07)
MOM	0.14	0.15	0.15	0.12	0.12	0.13	0.12	0.11	0.12	0.12	-0.03***	(-5.98)
STR	-0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02***	(5.05)
ILLIQ	0.13	0.22	0.15	0.17	0.20	0.20	0.27	0.30	0.29	0.25	0.12***	(4.88)
TURN	0.96	1.04	1.12	1.17	1.19	1.14	1.16	1.13	1.17	1.20	0.24***	(15.06)
SUE	0.15	0.12	0.09	0.10	0.06	0.06	0.03	0.03	0.00	-0.03	-0.18***	(-9.20)
IVOL	0.01	0.02	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.03***	(11.44)
MAX	0.01	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.03***	(7.59)
DAE	2.24	2.18	2.25	2.15	2.31	2.15	2.35	2.29	2.38	2.33	0.09***	(10.43)
DALG	1.06	1.12	1.11	1.14	1.14	1.17	1.11	1.14	1.16	1.13	0.07***	(10.42)

Dispersion in analysts' earnings forecasts (DAE), and dispersion in analysts' long-term growth forecasts (DALG)-> indicating a positive correlation between the MFD and the existing measures of divergence of opinion

Stocks with higher MFD (or higher firm-specific uncertainty) are indeed smaller, less liquid, and have higher idiosyncratic volatility and stronger lottery features.

#### 1.4 BIVARIATE PORTFOLIO-LEVEL ANALYSIS

Control for firm characteristics, shown to be relevant at previous table

	SIZE	$_{\mathrm{BM}}$	$\overline{AG}$	GP	MOM	STR	ILLIQ	TRUN	SUE	IVOL	MAX	DAE	DALG
P1	0.00	0.17	0.18	0.10	0.27	0.20	0.20	0.04	0.17	0.27	0.23	0.17	0.21
	(0.55)	(0.64)	(0.81)	(0.49)	(0.33)	(0.20)	(0.59)	(0.57)	(0.28)	(0.78)	(0.90)	(0.64)	(0.56)
P2	0.08	-0.15	0.06	0.26	-0.06	0.17	0.12	0.29	0.16	0.09	-0.03	0.09	0.13
	(0.94)	(-0.47)	(0.08)	(1.07)	(-0.58)	(0.03)	(0.38)	(0.57)	(0.78)	(0.03)	(-0.41)	(0.09)	(0.51)
P3	-0.01	-0.22	-0.19	0.09	-0.29**	0.03	-0.15	0.04	0.14	-0.31	0.05	-0.06	-0.48
	(-0.44)	(-0.98)	(-0.98)	(0.25)	(-2.18)	(0.75)	(-1.14)	(1.26)	(0.08)	(-0.57)	(0.41)	(-1.36)	(-0.52)
P4	-0.04	-0.21	-0.21	0.03	-0.32***	-0.07**	-0.25	0.03	0.15	-0.40*	-0.13	-0.12	-0.58
	(-0.31)	(-1.13)	(-1.16)	(0.01)	(-2.81)	(-2.03)	(-1.09)	(1.46)	(0.89)	(-1.74)	(-0.35)	(-1.34)	(-1.15)
P5	-0.16	-0.18	-0.14	-0.11	-0.09	0.12	0.04	-0.32*	0.10	-0.11	-0.13	0.09	0.02
	(-0.53)	(-0.11)	(-0.18)	(-0.23)	(-0.68)	(0.22)	(0.00)	(-1.79)	(1.01)	(-0.39)	(-0.96)	(0.73)	(0.23)
P6	-0.25	-0.27	-0.22	-0.17	-0.35***	-0.11**	-0.09	-0.30**	0.03	-0.12	-0.16*	0.02	-0.45
	(-1.3)	(-1.30)	(-1.33)	(-0.84)	(-2.63)	(-2.16)	(-0.68)	(-2.36)	(0.98)	(-0.57)	(-1.84)	(0.83)	(-0.23)
P7	-0.46	-0.31***	-0.26*	-0.20	-0.41***	-0.65**	-0.36*	-0.37**	-0.24	-0.47**	-0.18**	-0.27	-0.55
	(-1.64)	(-2.72)	(-1.84)	(-1.03)	(-4.04)	(-2.32)	(-1.73)	(-2.56)	(-1.17)	(-2.14)	(-2.08)	(-1.59)	(-1.4)
P8	-0.65**	-0.37***	-0.62**	-0.46***	-0.50***	-0.57**	-0.52**	-0.57***	-0.41**	-0.44**	-0.32***	-0.34*	-0.65**
	(-2.50)	(-3.93)	(-1.97)	(-3.60)	(-4.05)	(-2.33)	(-1.97)	(-3.54)	(-2.54)	(-2.23)	(-3.32)	(-1.95)	(-2.69)
P9	-0.45**	-0.31**	-0.24	-0.34***	-0.41***	-0.39**	-0.61***	-0.42***	-0.27**	-0.58***	-0.22***	-0.48***	-0.66**
	(-2.23)	(-2.18)	(-1.52)	(-3.01)	(-2.66)	(-2.06)	(-2.87)	(-2.74)	(-2.18)	(-3.37)	(-2.85)	(-2.93)	(-2.77)
P10	-0.67***	-0.57***	-0.63***	-0.59***	-0.64***	-0.73***	-0.60***	-0.68***	-0.53***	-0.53***	-0.61***	-0.55***	-0.57**
	(-4.05)	(-3.91)	(-2.65)	(-3.47)	(-3.74)	(-3.64)	(-4.18)	(-3.60)	(-2.72)	(-3.64)	(-3.69)	(-3.29)	(-3.81)
L/S	-0.67***	-0.74***	-0.81**	-0.69***	-0.91***	-0.93***	-0.8***	-0.72***	-0.70***	-0.80***	-0.85***	-0.72***	-0.77**
t-stat	(-4.00)	(-4.81)	(-2.51)	(-4.57)	(-4.64)	(-3.74)	(-4.41)	(-4.37)	(-2.81)	(-4.42)	(-4.63)	(-3.41)	(-4.43)

- Cross-sectional relation between the MFD and future returns remain economically large and significant even after controlling for these robust, most prominent return predictors.
- The significant alpha spread on MFD-sorted portfolios remains to be driven by the underperformance of high-MFD stocks

## 1.5 CROSS-SECTIONAL REGRESSION

Independent Variables	RET	RET	RET-INDRET	DGTW-adj. RET
MFD	-9.38***	-9.07***	-8.68***	-6.89***
	(-3.26)	(-3.13)	(-3.84)	(-4.46)
SIZE	-0.06**	-0.07*	-0.06*	-0.08**
	(-2.11)	(-1.92)	(-1.83)	(-2.21)
BM	0.11	0.08	0.09	0.06
	(1.50)	(1.15)	(1.10)	(0.86)
AG	-0.06	-0.05	-0.10	-0.08
	(-0.36)	(-0.41)	(-0.64)	(-0.67)
GP	0.19	0.22	0.30	0.36
	(0.45)	(0.72)	(0.60)	(1.04)
MOM	0.48	0.53	0.34	0.41
	(0.79)	(1.11)	(0.67)	(0.74)
STR	-1.14***	-1.30***	-1.17**	-1.24***
	(-2.97)	(-3.22)	(-2.33)	(-3.35)
ILLIQ	0.97	4.48	5.47	10.90
·	(0.05)	(0.20)	(0.30)	(0.51)
TURN	-0.40	-0.05	-0.40	-0.10
	(-1.06)	(-0.20)	(-1.09)	(-0.32)
SUE	0.08**	0.07**	0.06*	0.05**
	(2.03)	(2.33)	(1.91)	(1.99)
IVOL	-0.79	-0.82	-0.62	-0.84
	(-1.25)	(-1.48)	(-0.84)	(-1.06)
MAX	-1.49**	-1.52***	-1.20***	-1.05*
	(-2.56)	(-2.77)	(-2.60)	(-1.86)
DAE	-0.70	-0.53	-0.66	-0.66
	(-1.39)	(-1.20)	(-1.27)	(-0.98)
DALG	,	-0.30	-0.30	-0.30
		(-0.61)	(-0.88)	(-0.69)
Intercept	1.14*	1.00*	1.00	0.95
•	(1.85)	(1.81)	(1.58)	(1.54)
Industry FEs	Yes	Yes	No	Yes
N	2,085,442	1,662,360	1,662,360	1,654,261
Adj. R2	0.212	0.209	0.123	0.198

- -Consistent with the portfolio results, find a negative and significant relation between the MFD and one-month-ahead returns controlling for a large number of predictors.
- -Overall, these results indicate that the MFD provides incrementally value-relevant information.
- -The predictive power of the MFD is distinct and robust to the inclusion of other well-known return predictors and the existing measures of investor disagreement.

#### 2.1. SOURCE OF PREDICTABILITY - INFORMATION FRICTIONS

-Hirshleifer et al. [2013] who emphasize that investors would have more difficulty in processing information that is less tangible.

Based on that, the authors conjecture that the elusive nature of MFD thus makes investors face more severe informational frictions.

The stock market can underreact to uncertainty/disagreement proxy, and the informative signals provided by the MFD for stocks largely held by retail investors and less by institutional investors are not incorporated into prices quickly -> causing return predictability

	TATO											
Panel A: average	ge INS	I in M	FD de	cile po	rtfolio							
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
INST	0.37	0.29	0.29	0.33	0.27	0.28	0.29	0.26	0.27	0.27	-0.10***	(-5.56)
Panel B: INST	and M	FD-so	rted al	phas								
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
INST 1	0.32	0.26	0.06	0.15	-0.21	-0.21	-0.61	-0.53	-0.75	-0.84	-1.16***	(-4.97)
INST 2	0.23	0.23	0.04	0.14	-0.15	-0.19	-0.49	-0.39	-0.61	-0.70	-0.93***	(-4.32)
INST 3	0.21	0.18	0.04	0.09	-0.12	-0.13	-0.36	-0.30	-0.43	-0.56	-0.76***	(-3.71)
INST 4	0.13	0.12	0.02	0.06	-0.08	-0.09	-0.24	-0.21	-0.30	-0.32	-0.45*	(-1.83)
INST 5	0.09	0.06	0.01	0.04	-0.05	-0.06	-0.13	-0.11	-0.12	-0.09	-0.18	(-1.41)
INST 5 - INST 1	-0.23	-0.20	-0.04	-0.10	0.16	0.15	0.47	0.42	0.64	0.75	0.98***	(3.56)



#### 2.2. SOURCE OF PREDICTABILITY – SHORT SELLING COSTS

- Use short interest and option-implied lending fees to proxy for the short-sale costs
- First, the authors investigate whether the short-selling costs are higher for high-MFD stocks.
- Second, the authors test whether the magnitude of the negative relation between the MFD and future returns is stronger for those stocks in which short-selling costs are higher compared to those stocks in which short-selling costs are lower.

Panel A:	average	SI in I	MFD d	ecile po	rtfolio							
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
SI	0.00	0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.05	0.05***	(7.44)
Panel B:	SI and	MFD-s	orted a	lphas								
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
SI 1	0.28	0.24	0.12	-0.04	0.05	-0.05	-0.10	-0.13	-0.10	-0.08	-0.36**	(-2.25)
SI 2	0.20	0.19	0.12	-0.07	0.04	-0.08	-0.18	-0.19	-0.23	-0.27	-0.47**	(-2.34)
SI 3	0.17	0.16	0.08	-0.11	0.03	-0.10	-0.26	-0.32	-0.37	-0.51	-0.68**	(-2.51)
SI 4	0.12	0.09	0.05	-0.12	0.02	-0.14	-0.30	-0.41	-0.49	-0.56	-0.69***	(-2.81)
SI 5	0.07	0.05	0.04	-0.18	0.01	-0.18	-0.45	-0.56	-0.59	-0.77	-0.84***	(-3.13)
SI 5 - SI 1	-0.21	-0.19	-0.08	-0.14	-0.04	-0.13	-0.35	-0.44	-0.49	-0.69	-0.48**	(-2.40)

These results indicate that the disagreement premium is much stronger for equities with high short interest.

#### 2.3 SOURCE OF PREDICTABILITY – LIMITS OF ARBITRAGE

IVOL	Excess Return	CAPM	FF6	SY	HXZ4	DHS
Low	-0.21	-0.29	-0.37	-0.22	-0.31	-0.32
	(-1.47)	(-1.53)	(-1.38)	(-1.09)	(-1.24)	(-1.16)
Medium	-0.43***	-0.58***	-0.76***	-0.69***	-0.72***	-0.62**
	(-3.21)	(-4.08)	(-4.38)	(-3.10)	(-3.15)	(-2.38)
High	-0.61***	-0.92***	-0.96***	-0.81***	-0.95***	-0.93***
	(-4.53)	(-4.88)	(-5.01)	(-3.38)	(-3.83)	(-4.12)
High - Low	-0.39***	-0.63***	-0.59***	-0.59***	-0.65***	-0.61***
	(-2.86)	(-3.62)	(-3.75)	(-2.79)	(-2.97)	(-2.86)
ILLIQ	Excess Return	CAPM	FF6	SY	HXZ4	DHS
Low	-0.32*	-0.50**	-0.51*	-0.36	-0.55	-0.45
	(-1.74)	(-2.03)	(-1.95)	(-1.38)	(-1.59)	(-1.58)
Medium	-0.34***	-0.70***	-0.63***	-0.49***	-0.57**	-0.66***
	(-3.10)	(-3.12)	(-3.71)	(-2.70)	(-2.58)	(-3.18)
High	-0.55***	-0.90***	-0.82***	-0.65***	-0.83***	-0.81***
	(-3.52)	(-3.66)	(-4.04)	(-3.22)	(-2.92)	(-3.09)
High - Low	-0.23**	-0.40*	-0.31**	-0.29*	-0.27	-0.36*
	(-2.05)	(-1.85)	(-2.10)	(-1.73)	(-1.52)	(-1.88)
SIZE	Excess Return	CAPM	FF6	SY	HXZ4	DHS
Low	-0.56***	-1.06***	-0.96***	-0.75***	-1.06***	-0.90***
	(-4.52)	(-4.72)	(-4.67)	(-3.61)	(-4.28)	(-4.15)
Medium	-0.43***	-0.58***	-0.78***	-0.64**	-0.81***	-0.60***
	(-3.53)	(-2.79)	(-3.09)	(-2.25)	(-2.75)	(-2.90)
High	-0.23	-0.29*	-0.36	-0.30	-0.36	-0.38
_	(-1.59)	(-1.83)	(-1.65)	(-1.27)	(-1.52)	(-1.55)
High - Low	0.33***	0.77***	0.6***	0.45**	0.70***	0.51**
_	(2.85)	(2.81)	(3.01)	(2.52)	(2.85)	(2.29)

- If the predictive power of MFD is driven by mispricing to some extent, then we should expect the return predictability to be more pronounced for stocks with high arbitrage costs.

- Consistent with the limits-to-arbitrage hypothesis, Table shows that the return and alpha spreads on MFD-sorted portfolios are negative and larger in absolute magnitude, and statistically more significant for stocks with high arbitrage costs

#### 3. CONCLUSION

 Introduces a belief-generating model from which we build a novel measure of investor disagreement.

• Find a significantly negative cross-sectional relation between this newly proposed, statistical measure of investor disagreement and future stock returns.

- Disagreement premium is stronger for stocks with higher retail ownership, and higher short selling/arbitrage costs.
- To sum up, MFD-driven return predictability is likely due to mispricing induced by informational frictions, short-selling costs, and limits to arbitrage.

