The University of Texas at Austin
McCombs School of Business

First day – ML in Finance

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Goals for the class

→ Become better economists by understanding the usefuleness of ML tools in research (there are so many cool ideas beyond linear regression!)

→ Understand the power and pitfalls of using ML to investigate problems in economics

→ Be **skeptical** but well-informed researchers

Key concepts

→ Supervised learning and the bias-variance tradeoff

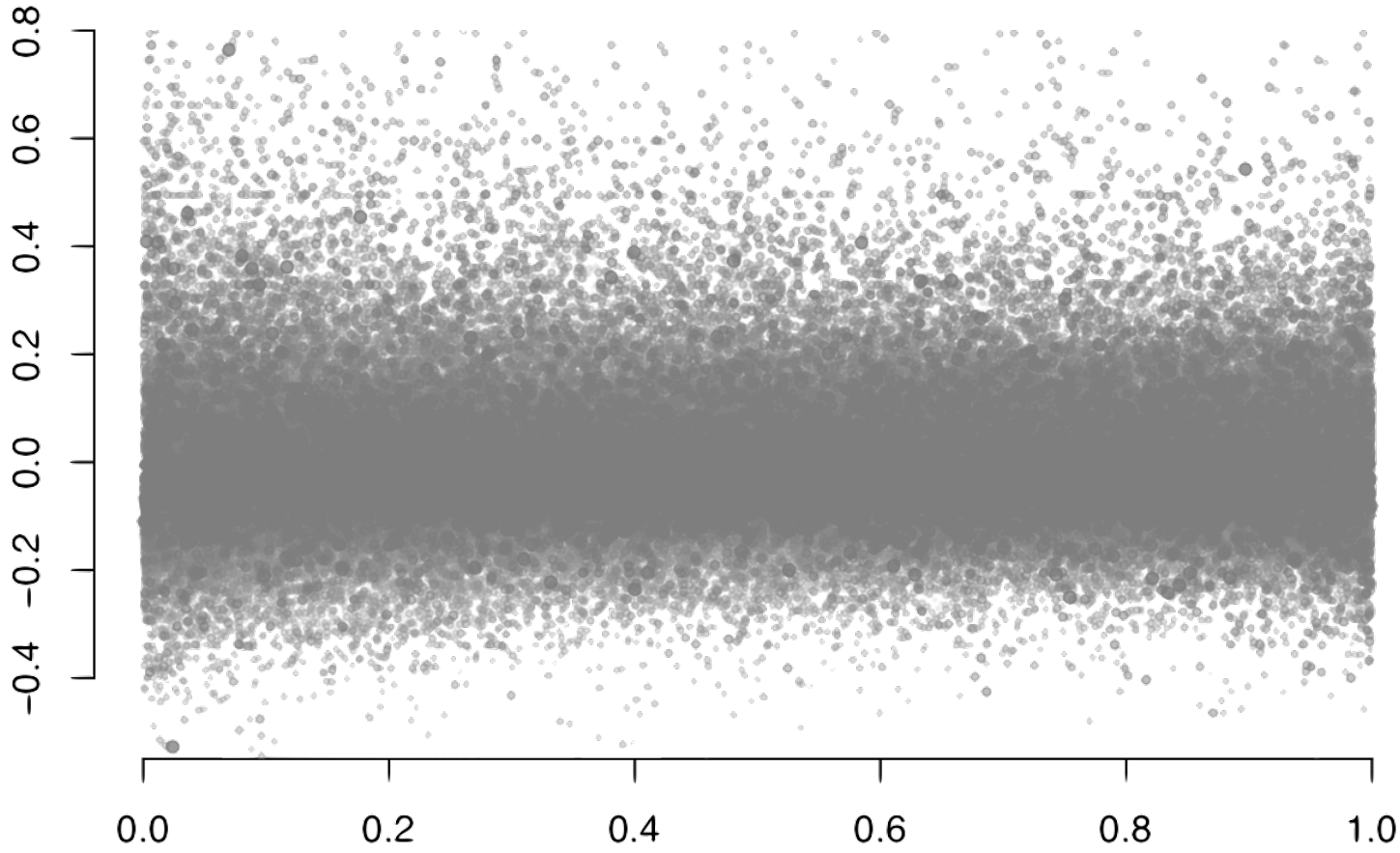
→ Using supervised learning to model the cross-section of returns: characteristic selection and the factor zoo – we'll spend a lot of time here.

→ Unsupervised learning

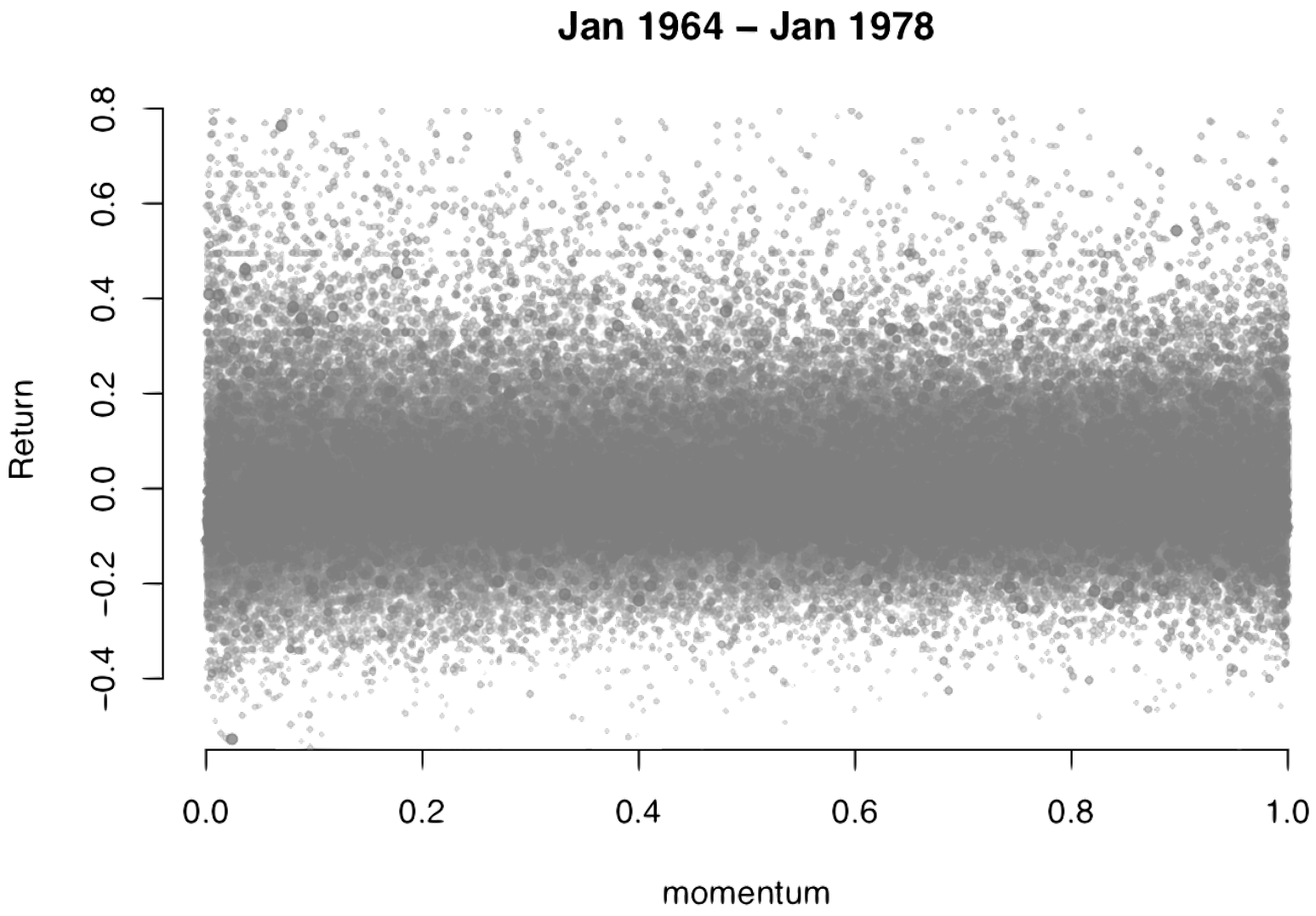
→ Advanced topics in causal inference: using ML models to estimate heterogenous causal effects in economic systems (OLS, RIP)

As an example for a paper presentation, let's turn to a fundamental problem in finance: how do firm characteristics relate to the firm's return?

The data



The data: Monthly returns



Given this observed data, our motivating question is:

What firm characteristics are predictive of its return?

There are two parts to the answer:

1. Building an appropriate (Bayesian) model.

2. Selecting characteristics (posterior summarization).

The object of interest

The conditional expectation of returns given observed characteristics

$$\mathbb{E}[R_{it} \mid \mathrm{X}_{it-1}] = f\left(\mathrm{X}_{it-1}\right)$$

R_{it}: excess return of firm i at time t
X_{it-1}: vector of characteristics of firm i at time t

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We would like to learn f
and which $\mathbf{X}_{it-1}^{k}$'s matter!

Step 1: A model for f

Portfolio sorts are one way ...

Jegadeesh and Titman (2001)

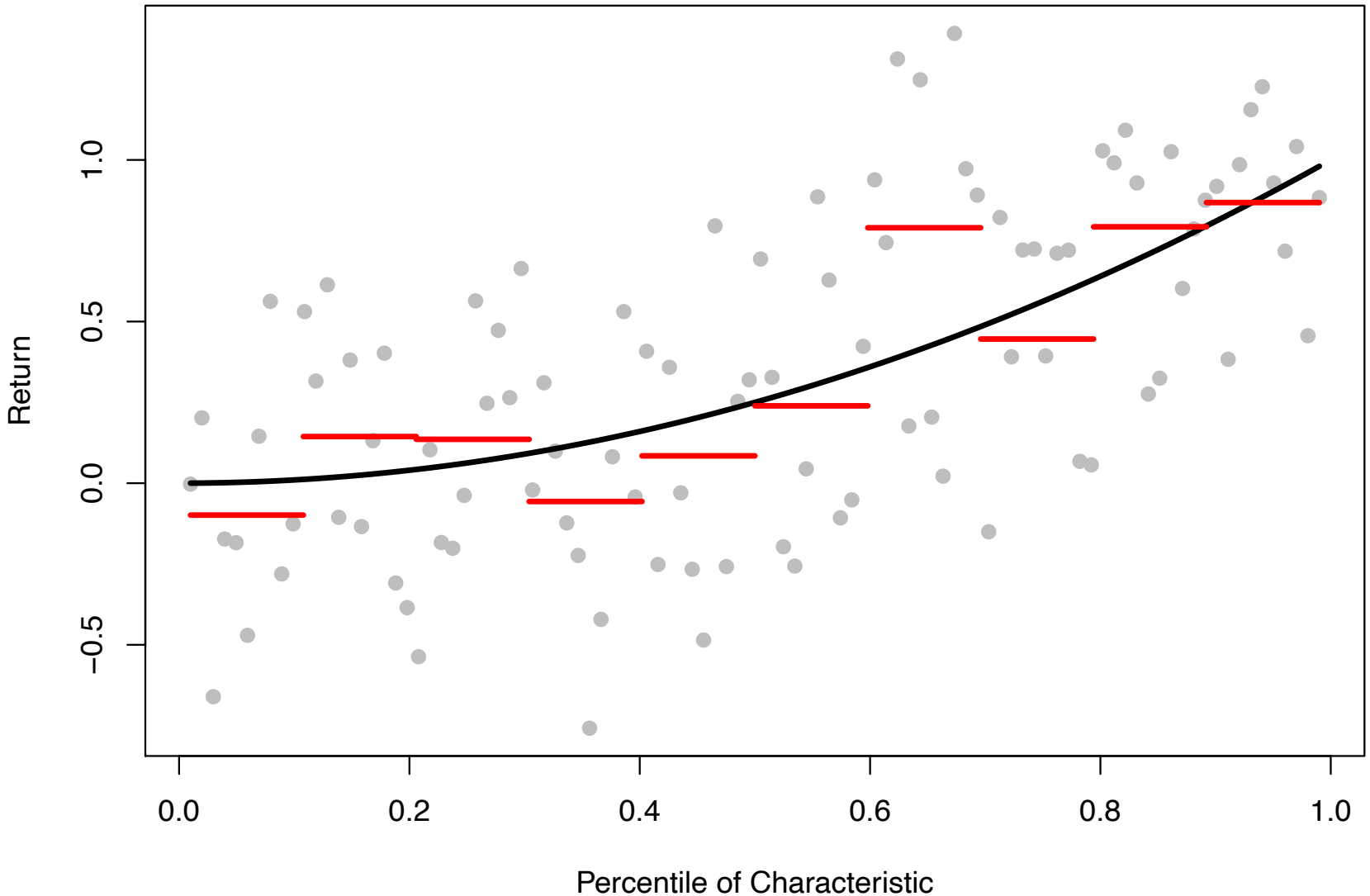
Table I

Momentum Portfolio Returns

This table reports the monthly returns for momentum portfolios formed based on past six-month returns and held for six months. P1 is the equal-weighted portfolio of 10 percent of the stocks with the highest returns over the previous six months, P2 is the equal-weighted portfolio of the 10 percent of the stocks with the next highest returns, and so on. The "All stocks" sample includes all stocks traded on the NYSE, AMEX, or Nasdaq excluding stocks priced less than $5 at the beginning of the holding period and stocks in the smallest market cap decile (NYSE size decile cutoff). The "Small Cap" and "Large Cap" subsamples comprise stocks in the "All Stocks" sample that are smaller and larger than the median market cap NYSE stock respectively. "EWI" is the returns on the equal-weighted index of stocks in each sample.

	All Stocks			Small Cap			Large Cap		
	1965–1998	1965–1989	1990–1998	1965–1998	1965–1989	1990–1998	1965–1998	1965–1989	1990–1998
P1 (Past winners)	1.65	1.63	1.69	1.70	1.69	1.73	1.56	1.52	1.66
P2	1.39	1.41	1.32	1.45	1.50	1.33	1.25	1.24	1.27
P3	1.28	1.30	1.21	1.37	1.42	1.23	1.12	1.10	1.19
P4	1.19	1.21	1.13	1.26	1.34	1.05	1.10	1.07	1.20
P5	1.17	1.18	1.12	1.26	1.33	1.06	1.05	1.00	1.19
P6	1.13	1.15	1.09	1.19	1.26	1.01	1.09	1.05	1.20
P7	1.11	1.12	1.09	1.14	1.20	0.99	1.09	1.04	1.23
P8	1.05	1.05	1.03	1.09	1.17	0.89	1.04	1.00	1.17
P9	0.90	0.94	0.77	0.84	0.95	0.54	1.00	0.96	1.09
P10 (Past losers)	0.42	0.46	0.30	0.28	0.35	0.08	0.70	0.68	0.78
P1–P10	1.23	1.17	1.39	1.42	1.34	1.65	0.86	0.85	0.88
t statistic	6.46	4.96	4.71	7.41	5.60	5.74	4.34	3.55	2.59
EWI	1.09	1.10	1.04	1.13	1.19	0.98	1.03	1.00	1.12

A generated example



Challenges and a solution

- X_{it-1} is multidimensional.
- No information sharing across the X space.

We propose modeling the CEF using additive quadratic splines (with monotonicity constraints *and* time variation):

$$\mathbb{E}[R_{it} \mid \mathsf{X}_{it-1}] = \alpha_t + \sum_{k=1}^{K} g_{kt}(x_{ki,t-1})$$

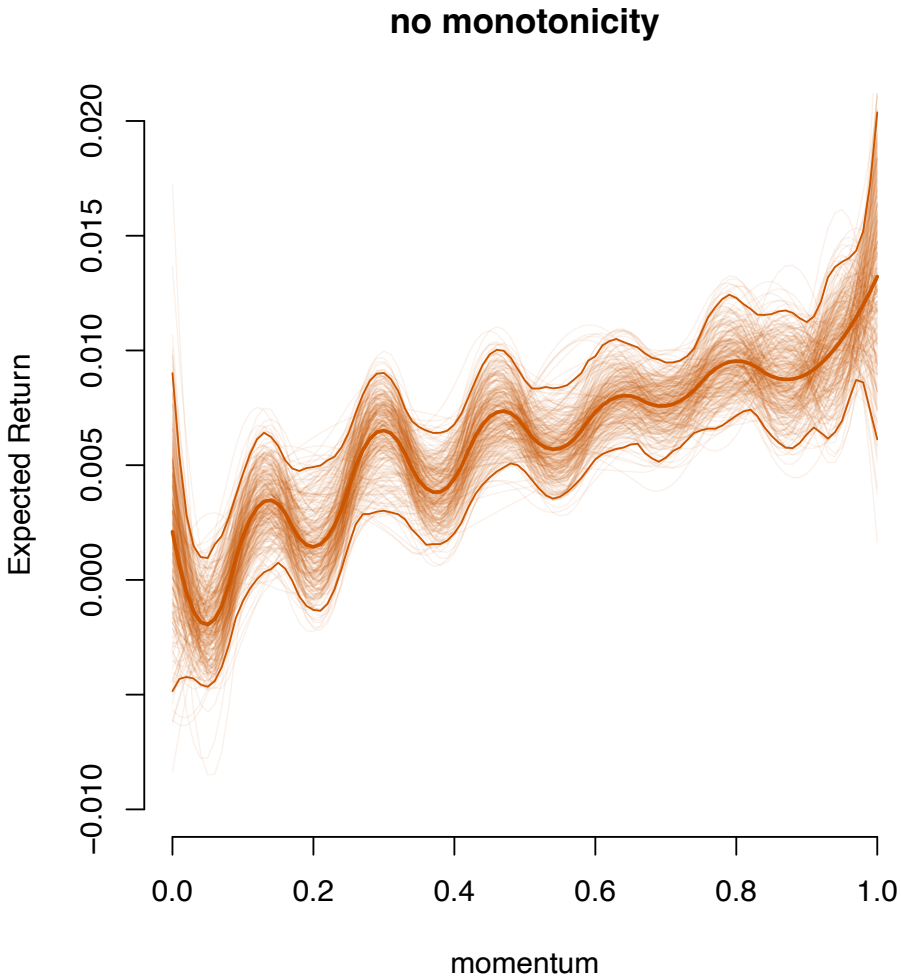
Features of our model

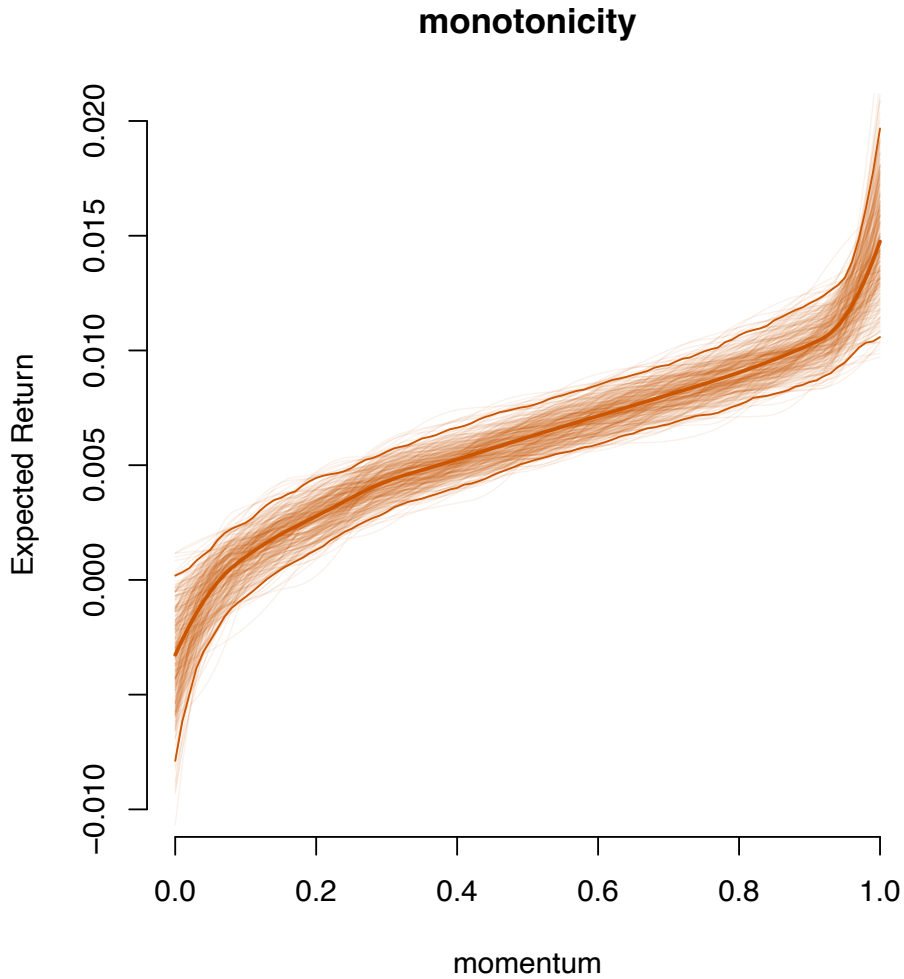
1. Monotonicity on partial effects are incorporated by constraint on the spline coefficients.
   $\rightarrow$ improvement over nonlinear models that fit to noise.

2. Time dynamics modeled using a power-weighting likelihood approach.
   $\rightarrow$ improvement over rolling window models.

Why monotonicity?

Estimated functions at January 1978

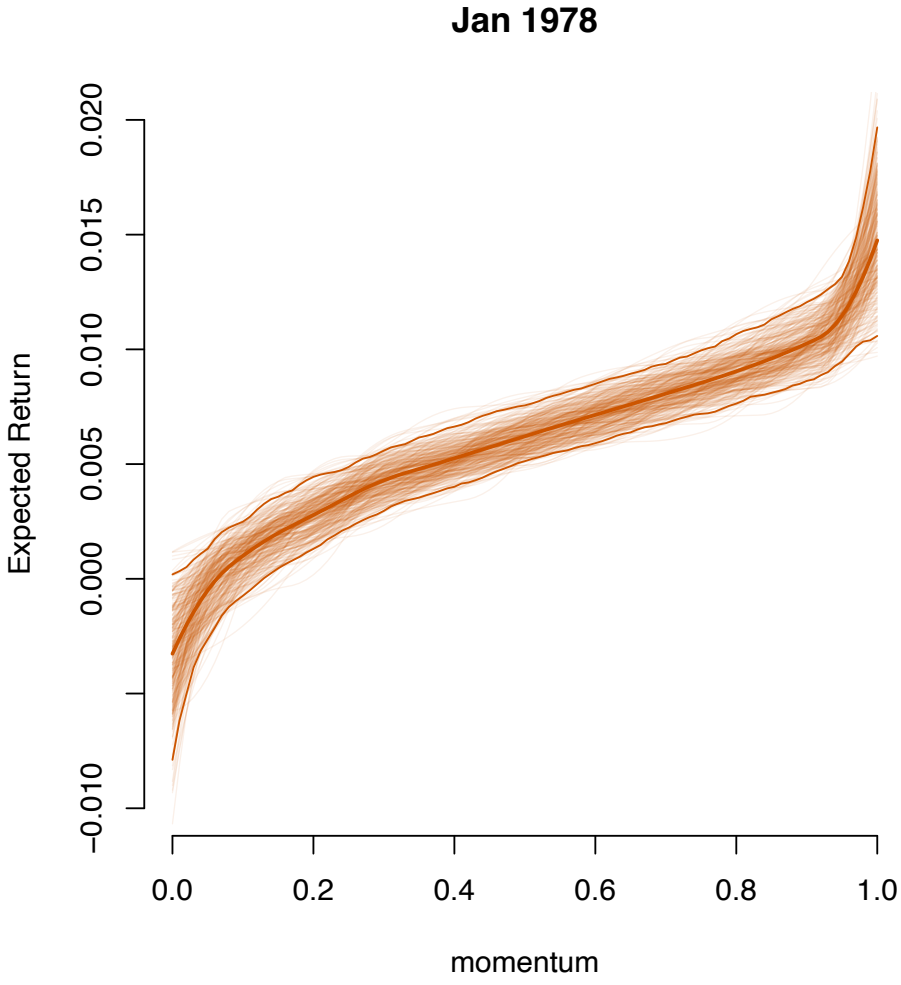


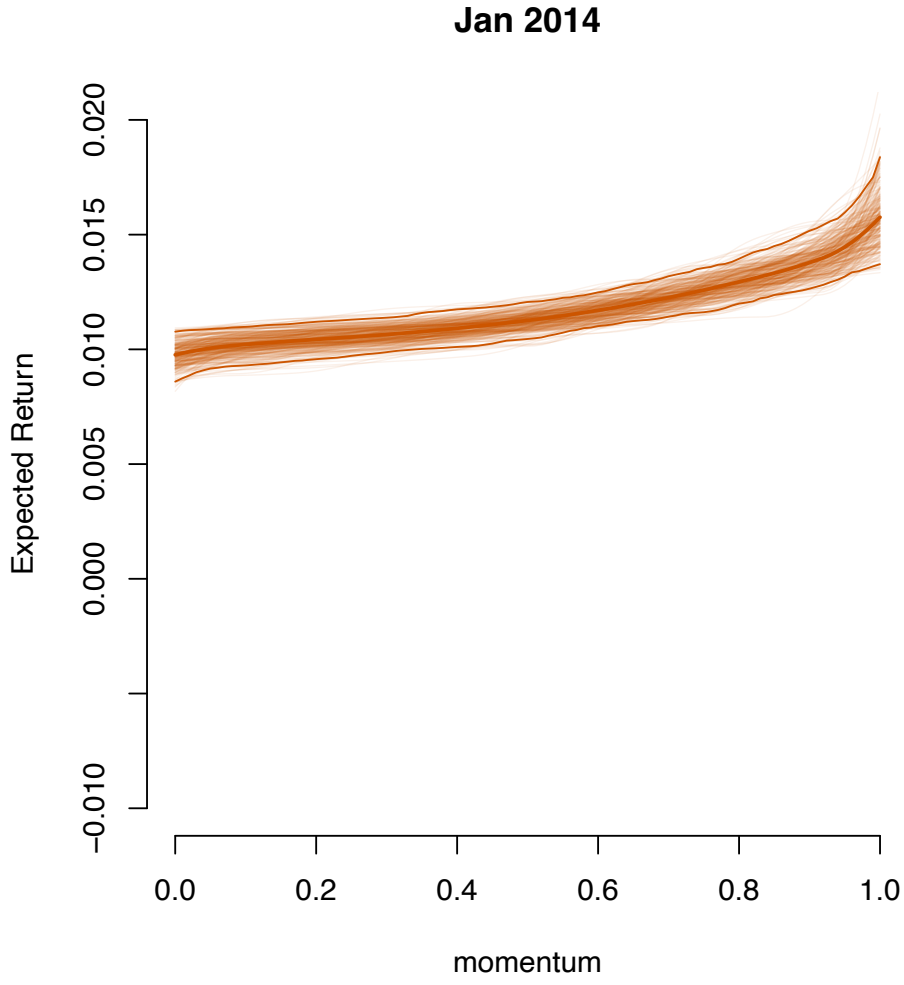


monotonicity is enforced by linear constraints on spline coefficients

Why time variation?

Estimated functions at January 1978 and 2014





dynamics are modeled by likelihood discounting, McCarthy and Jenson (2016)

Step 2: We have our posterior ... now what?

We now have a **complicated** posterior for f, from

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$\rightarrow$ variance and knot inclusion parameters
$\rightarrow$ everything is time-varying
$\rightarrow$ ...

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$\rightarrow$ ...

How do we understand which characteristics matter in the model?

Answer: Use a loss function

Characteristic selection is a decision problem. Therefore, we advocate for using a suitable loss function, **not** a more clever prior.

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Characteristic selection is a decision problem. Therefore, we advocate for using a suitable loss function, **not** a more clever prior.

We perform selection in a "post-inference world" by comparing models (sets of characteristics) based on utility.

Posterior summarization: Ingredients

Let b_t be a model decision, λ_t be a complexity parameter, Θ_t be a vector of model parameters, and $\tilde{R}_t$ be future data (posterior predictive).

1. Loss function $\mathcal{L}(b_t, \tilde{R}_t)$ – measures utility.
2. Complexity function $\Phi(\lambda_t, b_t)$ – measures sparsity.
3. Statistical model $\Pi(\Theta_t)$ – characterizes uncertainty.

Posterior summarization: Procedure

- Optimize $\mathbb{E}[\mathcal{L}(b_t, \tilde{R}_t) + \Phi(\lambda_t, b_t)]$, where the expectation is over $p(\tilde{R}_t, \Theta_t \mid \mathbf{R})$.

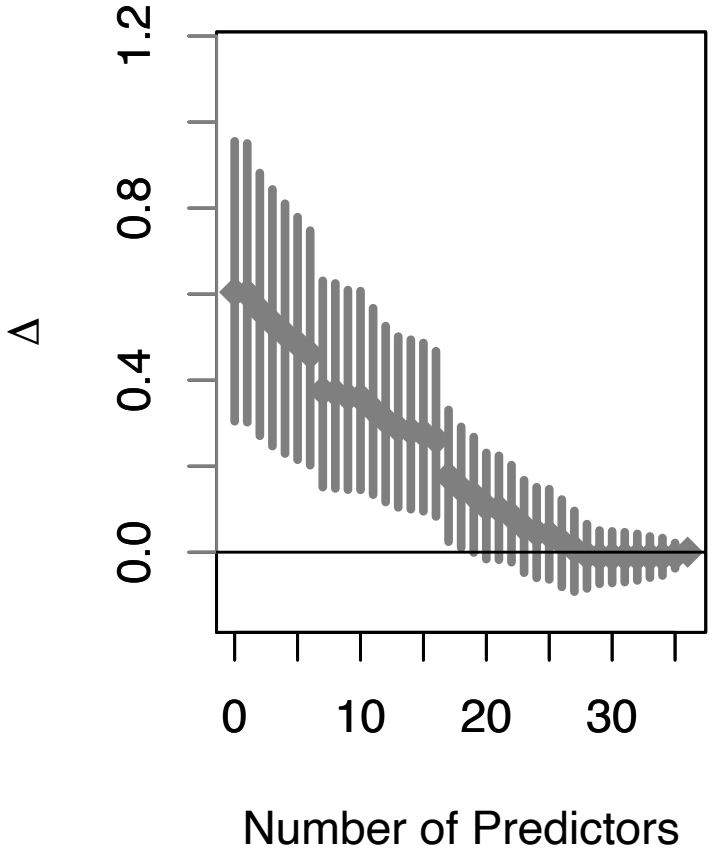
- Calculate "regret" versus a target b_t^* for decisions indexed by λ_t.

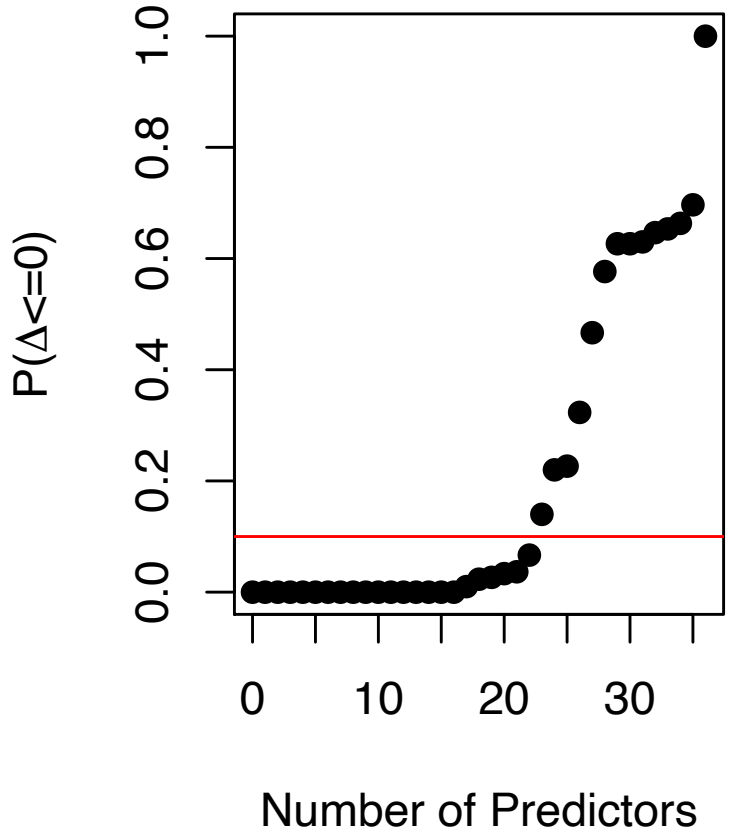
  $\rightarrow \quad \Delta(b_{\lambda_t}, b_t^*, \tilde{R}_t) = \mathcal{L}(b_{\lambda_t}, \tilde{R}_t) - \mathcal{L}(b_t^*, \tilde{R}_t)$

- Posterior summary: Look at graphical summaries of optimal simpler models, i.e.:

  $\rightarrow \quad \pi_{\lambda_t} = \mathbb{P}[\Delta(b_{\lambda_t}, b_t^*, \tilde{R}_t) < 0]$ (satisfaction probability)

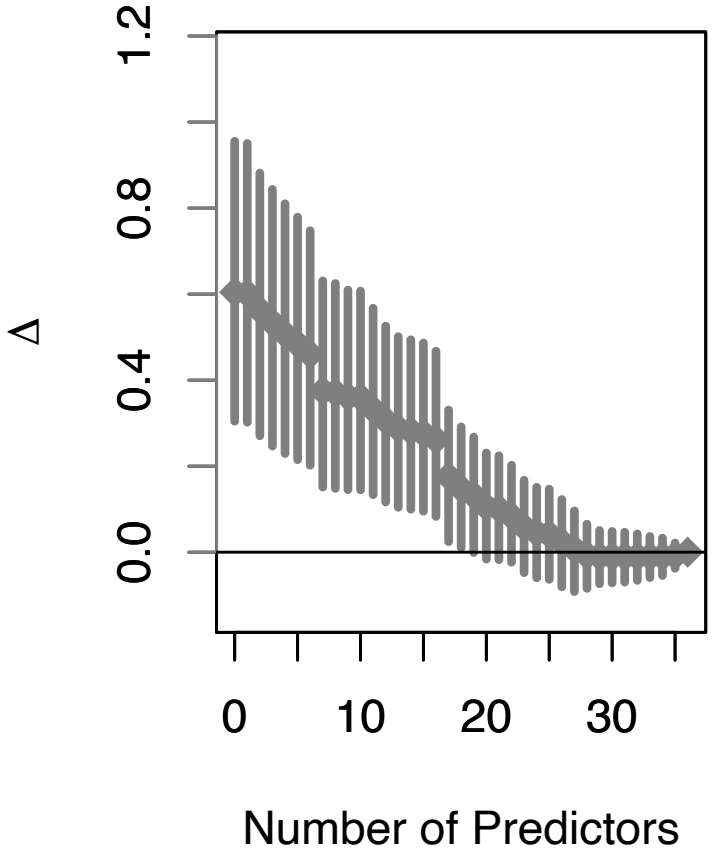
Procedure output: Posterior summary plots

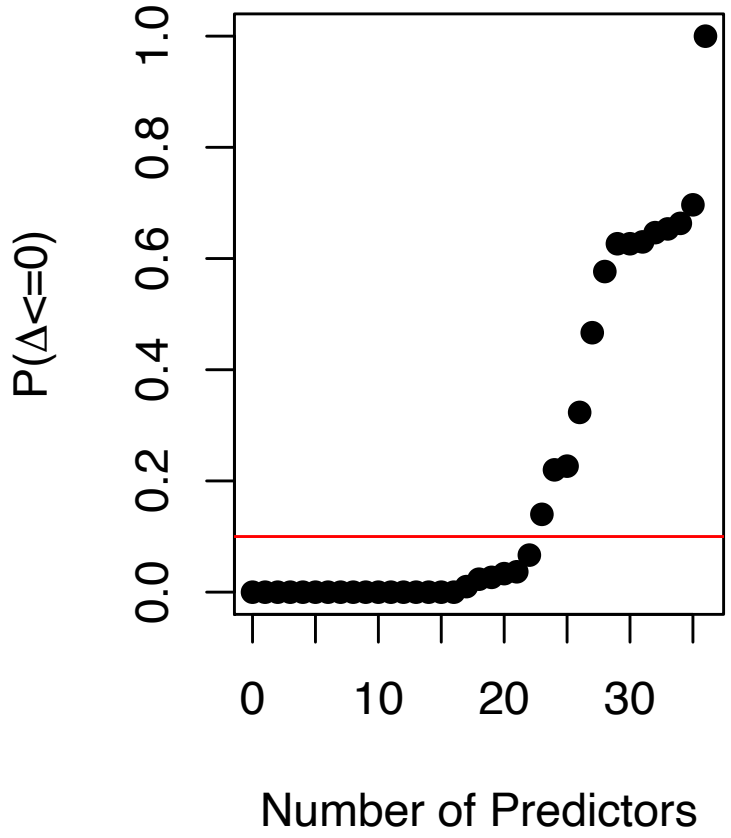




- Like a LASSO solution path, but better!

Procedure output: Posterior summary plots





- Like a LASSO solution path, but better!
- Predictive uncertainty bands surround each expected utility optimal model.

Results

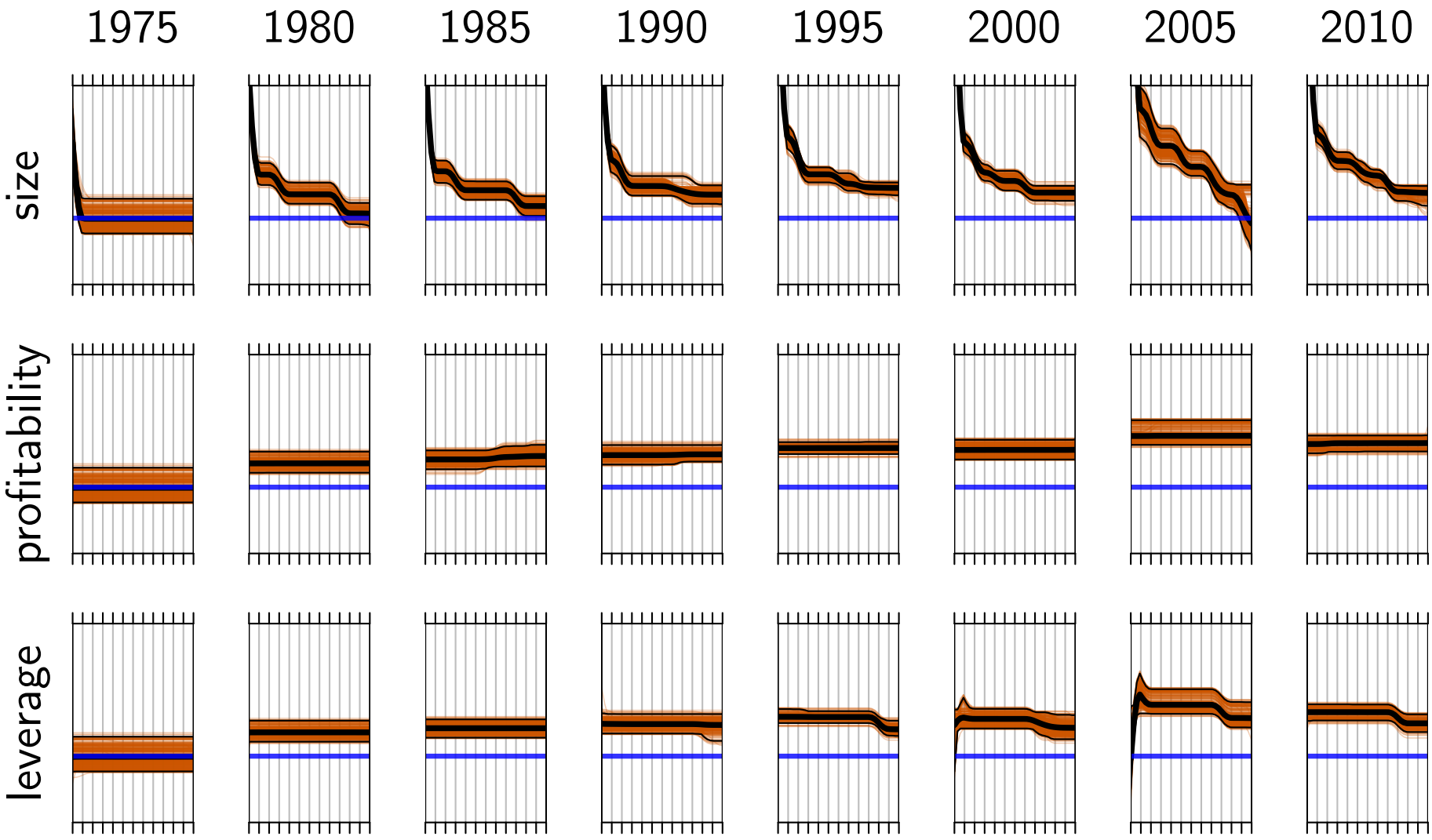
The data

Freyberger, Neuhierl, and Weber (2017)'s dataset:

- CRSP monthly stock returns for most US traded firms
- 36 characteristics from Compustat and CRSP, including size, momentum, leverage, etc.
- July 1962 - June 2014

Presence and direction of monotonicity is determined by important papers in the literature.

Dynamics of estimated functions



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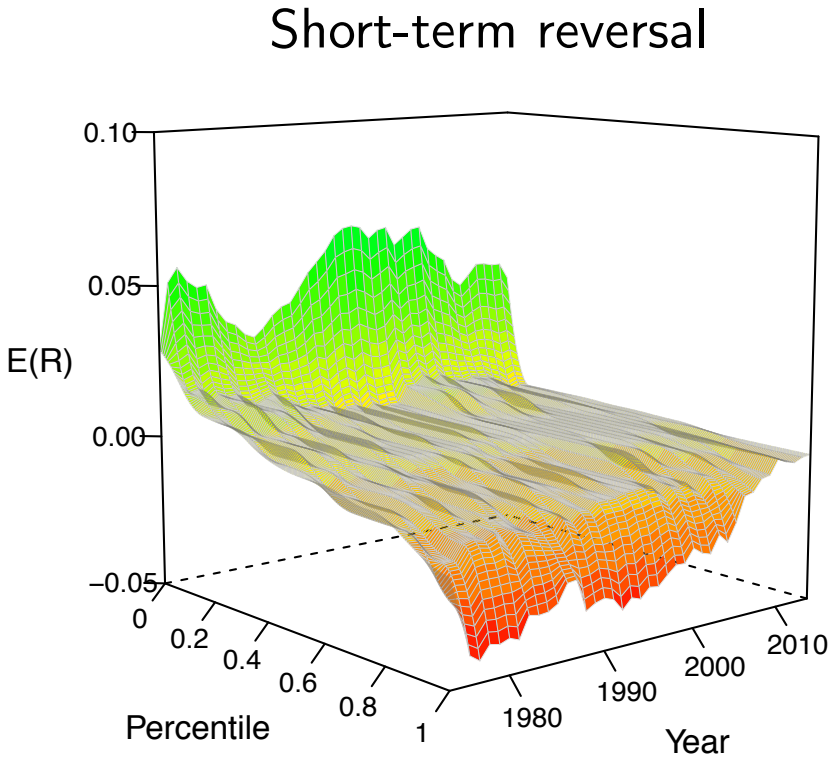
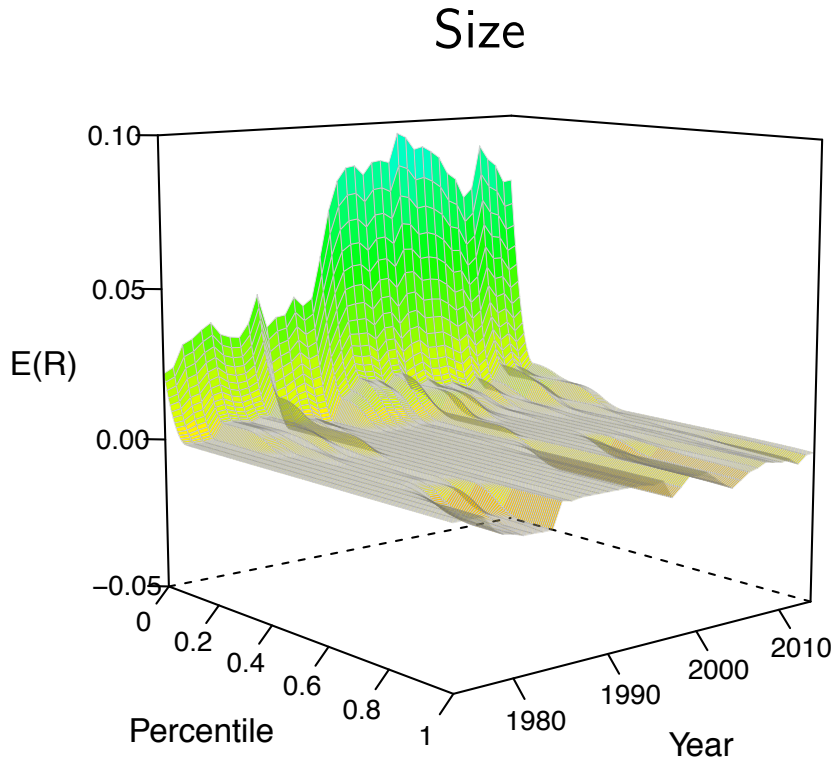
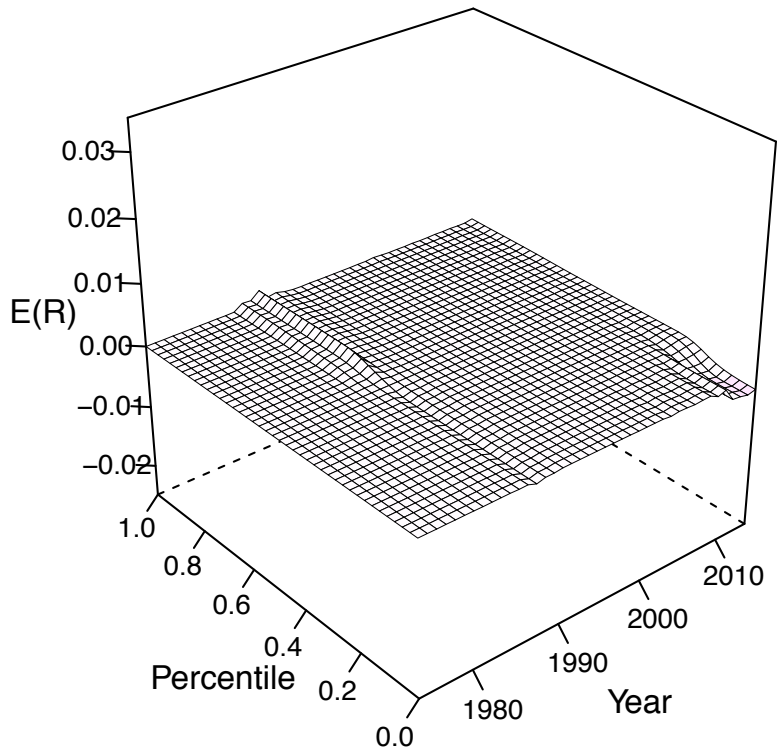




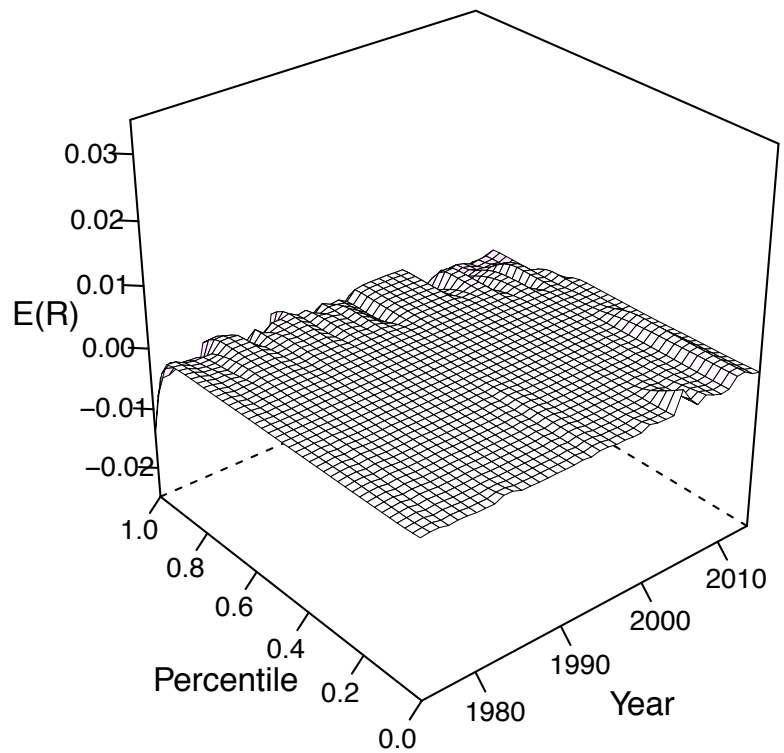
Partial effects of characteristics change over time

Dynamics of estimated functions

Profitability



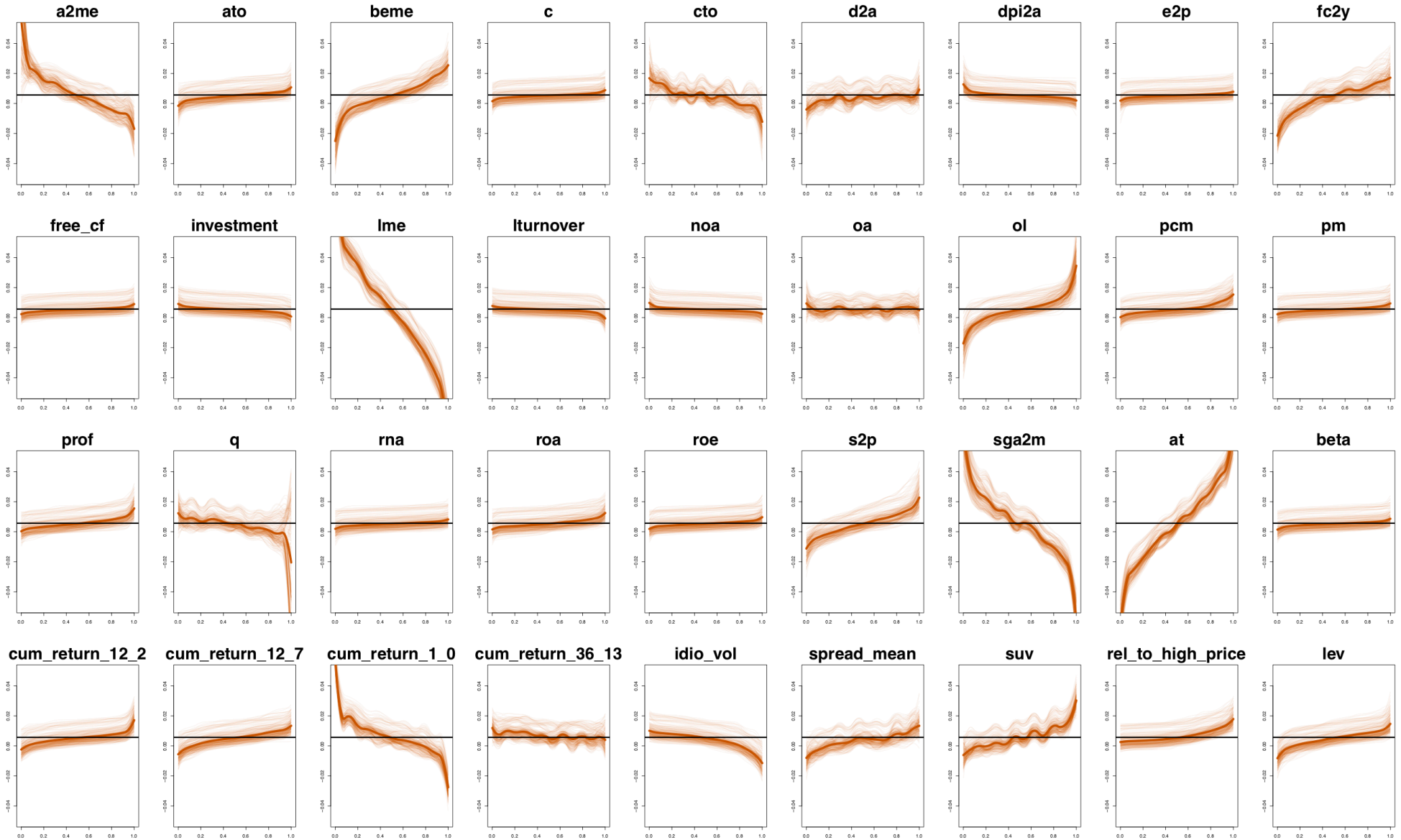
Investment



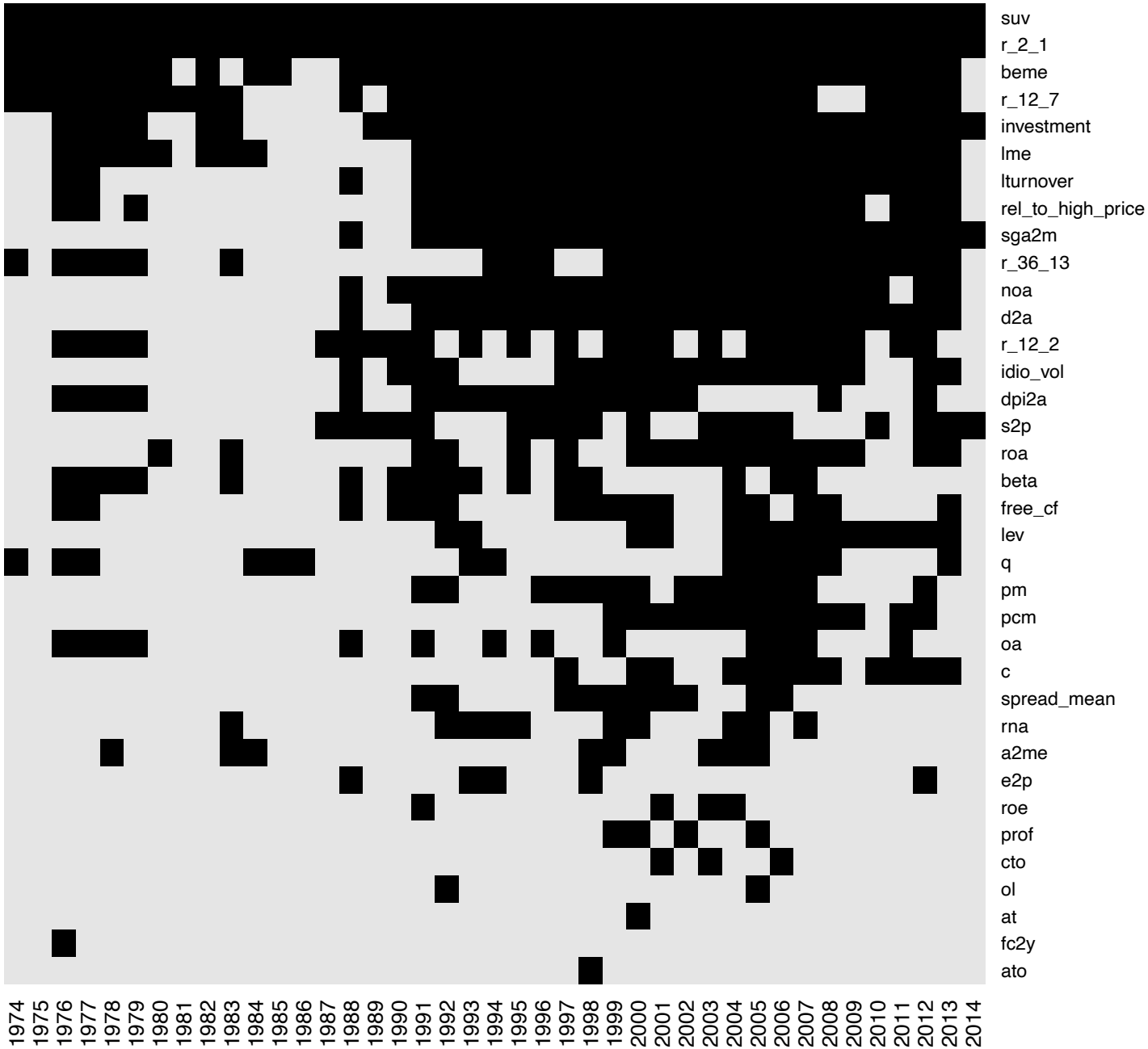
Not much evidence for the new Fama and French factors*

*conditional upon all other characteristics

Which characteristic matter?



Volatility and momentum strategies selected often



Machine learning vs. Monotonic splines

	Panel A: OOS R^2				Panel B: Sharpe Ratio			
	Window Size (months)				Window Size (months)			
	All	120	60	36	All	120	60	36
OLS	0.57	0.43	-0.07	-0.57	2.45	2.26	1.71	1.48
Random Forest	0.74	0.62	0.14	-0.43	3.11	2.61	1.89	1.45
BART	**1.22**	**0.98**	0.23	-0.58	**3.44**	**3.29**	**2.83**	2.35
Splines-0	0.87	0.68	0.24	-0.16	3.02	2.98	2.60	2.30
Splines-6	**0.87**	**0.68**	**0.25**	**-0.13**	3.05	2.99	2.64	**2.35**
Splines-24	0.81	0.67	**0.27**	**-0.09**	**3.14**	**3.22**	**2.71**	**2.41**

Monotonic splines are usually in between BART and Random Forest, but are also interpretable! Be wary of using default machine learning models with finance data.

Concluding thoughts, and thanks!

- **Utility functions can enforce inferential preferences that are not prior beliefs.**

- Statistical uncertainty can be used as a guide to avoid overfitting.

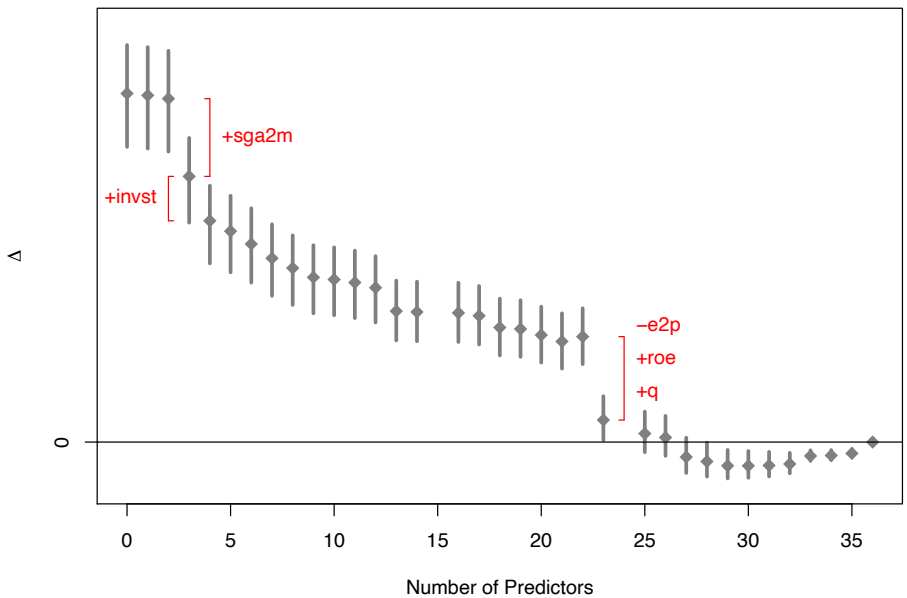
"Model interpretation through lower dimensional posterior summarization." *Journal of Computational and Graphical Statistics* (2020). S Woody, C Carvalho, J Murray.

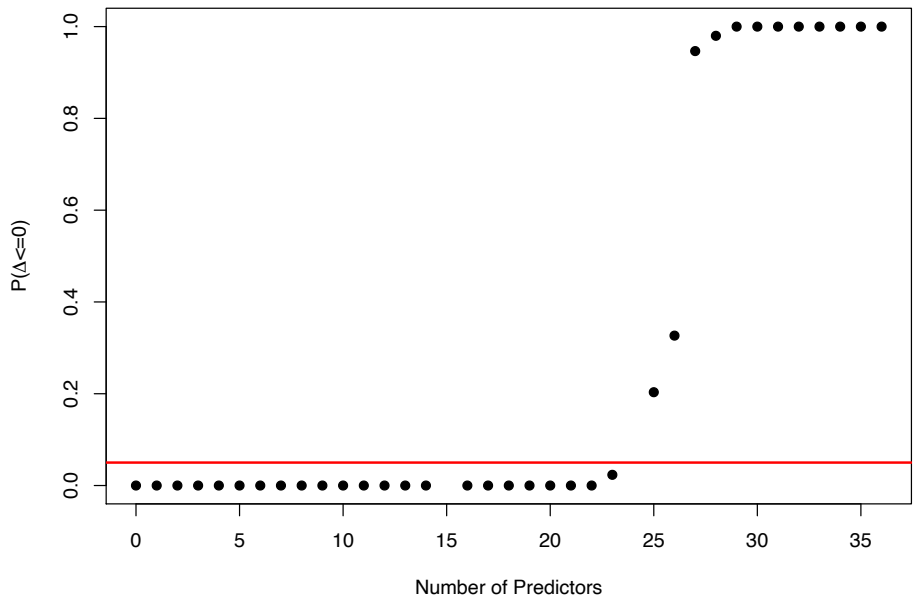
"Monotonic Effects of Characteristics on Returns." *Annals of Applied Statistics* (2020). J Fisher, DP, C Carvalho.

"Variable selection in seemingly unrelated regressions with random predictors." *Bayesian Analysis* (2017). DP, R Hahn, C Carvalho.

Extra slides

What characteristics matter over the entire period?

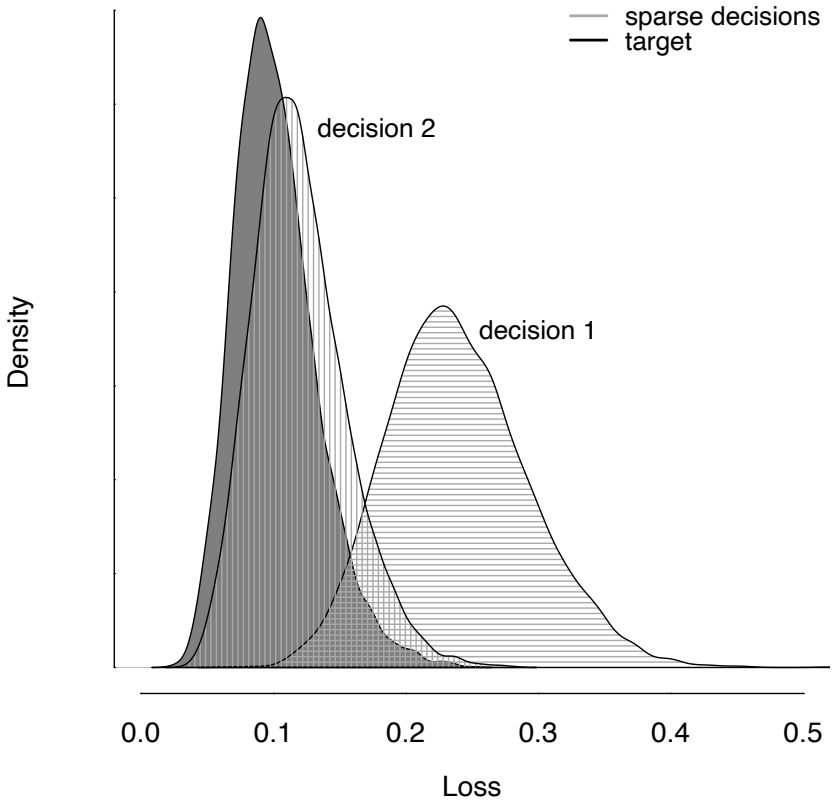


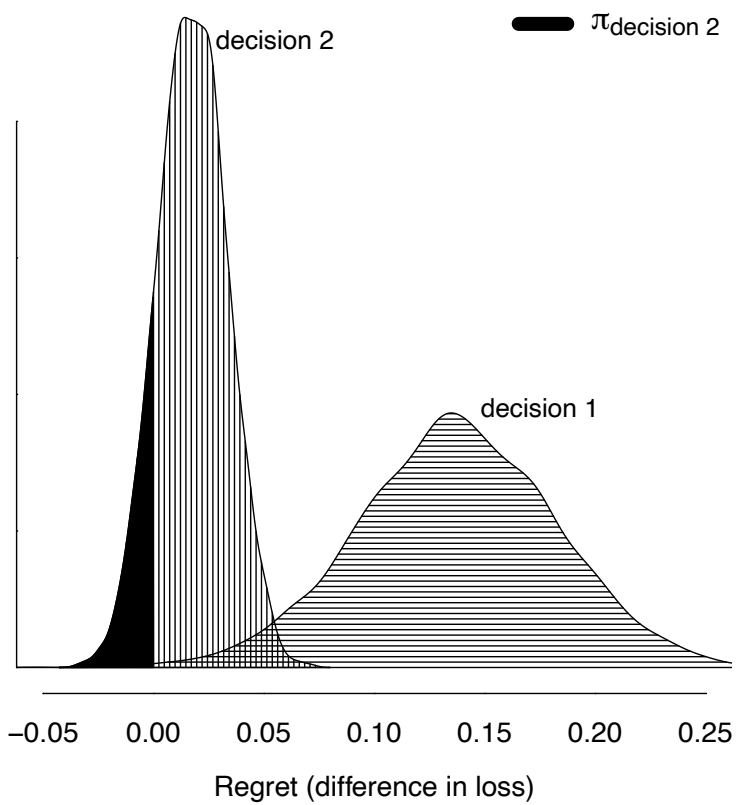


Regret-based selection: Illustration

d_λ : sparse decisions, d^* : target decision.

$\pi_\lambda = \mathbb{P}[\rho(d_\lambda, d^*, \tilde{Y}) < 0]$: probability of not regretting λ-decision.





UBS for Monotonic function estimation

The regression model is:

$$R_{it} = \alpha_t + \sum_{k=1}^{K} f_{kt}(x_{ki,t-1}) + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma^2)$$

Insight – with quadratic splines for all f_{kt}, this can be written as a predictive regression:

$$\mathrm{R}_t \sim \mathrm{N}\left(\mathbb{X}_{t-1}\mathbf{B}_t,\ \sigma_t^2 \mathbb{I}_{n_t}\right)$$

where

$$\mathbb{X}_{t-1} = \begin{bmatrix} \mathbf{1}_{n_t} & \mathbf{X}_{t-1} \end{bmatrix}, \quad \mathbf{B}_t = \begin{bmatrix} \alpha_t & \boldsymbol{\beta}_t \end{bmatrix}$$

$\mathbf{X}_{t-1}$ is matrix of size $n_t \times K(m+2)$, $\boldsymbol{\beta}_t$ is vector of size $K(m+2)$. Therefore, each firm is given a row in $\mathbf{X}_{t-1}$, and each $m+2$ block of $\boldsymbol{\beta}_t$ corresponds to the coefficients on the spline basis for a particular characteristic, k.

UBS for Monotonic function estimation

We can now proceed as Hahn and Carvalho (2015). The loss function is the negative log density of the regression plus a penalty function Φ with parameter λ_t. Also, let the "sparsified action" for the coefficient matrix $\mathbf{A}_t$.

$$\mathcal{L}_t(\tilde{\mathrm{R}}_t, \mathbf{A}_t, \Theta_t) = \frac{1}{2}(\tilde{\mathrm{R}}_t - \mathbb{X}_{t-1}\mathbf{A}_t)^T(\tilde{\mathrm{R}}_t - \mathbb{X}_{t-1}\mathbf{A}_t) + \Phi(\lambda_t, \mathbf{A}_t).$$

After integrating over $p(\tilde{\mathrm{R}}_t, \Theta_t)$, we obtain:

$$\mathcal{L}_{\lambda_t}(\mathbf{A}_t) = \left\|\mathbb{X}_{t-1}\mathbf{A}_t - \mathbb{X}_{t-1}\overline{\mathbf{B}}_t\right\|_2^2 + \Phi(\lambda_t, \mathbf{A}_t)$$

What does this look like for our problem?

Ingredients:

1. Loss: $\mathcal{L}(\tilde{\mathrm{R}}_t, \mathbf{A}_t, \Theta_t) = \frac{1}{2}(\tilde{\mathrm{R}}_t - \mathbb{X}_{t-1}\mathbf{A}_t)^T(\tilde{\mathrm{R}}_t - \mathbb{X}_{t-1}\mathbf{A}_t)$
2. Complexity: Group lasso penalty on the spline basis coefficients $\mathbf{A}_t$ defined as $\Phi(\lambda_t, \mathbf{A}_t)$
3. Model: Dynamic monotonic quadratic splines

Expected Loss:

Integrating over $p(\tilde{\mathrm{R}}_t, \Theta_t)$, we obtain:

$$\mathcal{L}_{\lambda_t}(\mathbf{A}_t) = \left\| \mathbb{X}_{t-1}\mathbf{A}_t - \mathbb{X}_{t-1}\overline{\mathbf{B}}_t \right\|_2^2 + \Phi(\lambda_t, \mathbf{A}_t)$$

Modeling Time-dynamics: McCarthy and Jensen (2016)

- Power-weighted likelihoods let information decay over time
- To estimate parameters at time τ, let $\delta_t = 0.99^{\tau - t}$, such that $\delta_1 \leq \delta_2 \leq ... \leq \delta_\tau = 1$, the likelihood at time $\tau \in \{1, ..., T\}$ is

$$p(R_1, ..., R_\tau | \Theta_\tau) = \prod_{t=1}^{\tau} p(R_t | \Theta_\tau)^{\delta_t}.$$

Model Summary

$$
\begin{aligned}
R_t|\cdot &\sim N\left(\alpha_t 1_{n_t} + \sum_{k=1}^{K} f_{kt}(x_{k,t-1}),\ \sigma_t^2 I_n\right)^{\delta_t} \\
f_{kt}(x_{k,t-1}) &= X_{k,t-1}\beta_{kt} = X_{k,t-1}L^{-1}L\beta_{kt} = W_{kt}\gamma_{kt} \\
\alpha_t &\sim N(0, 10^{-2}) \\
\sigma_t^2 &\sim U(0, 10^3) \\
(\gamma_{jkt}|I_{jkt} = 1, \sigma_t^2) &\sim N_+(0, c_k\sigma_t^2) \\
(\gamma_{jkt}|I_{jkt} = 0) &= 0 \\
I_{jkt} &\sim Bn(p_{jk} = 0.2).
\end{aligned}
$$

Our Contribution

If we are serious about understanding the functional form of these partial relationships, then we should have

1. Additive splines: flexible and can separate to marginal effects
2. Monotonicity: complement the flexibility of the splines with a priori known structure
3. A single intercept: identifiable and intuitive
4. Time-dynamics modeled, not just a rolling window
5. Separation between the shrinkage of coefficients and selection of characteristics

1 - Additive Model

$$\mathbb{E}(r_{it}|\boldsymbol{x}_{i,t-1}) = \alpha_t + \sum_{k=1}^{K} f_{kt}(x_{k,i,t-1})$$

- $x_{k,i,t-1} \in (0,1)$ is the empirical percentile of characteristic k for firm i at time $t-1$, ranked over all firms
- Note that there are no interactions built into the model, as the intention is to see the partial effect

2 - Monotonicity

- For m known knots, $\tilde{x}_1, ..., \tilde{x}_m$,

$$f(x) = \beta_1 x + \beta_2 x^2 + \beta_3 (x - \tilde{x}_1)_+^2 + ... + \beta_{m+2} (x - \tilde{x}_m)_+^2$$

2 - Monotonicity

- For m known knots, $\tilde{x}_1, ..., \tilde{x}_m$,

$$f(x) = \beta_1 x + \beta_2 x^2 + \beta_3 (x - \tilde{x}_1)_+^2 + ... + \beta_{m+2} (x - \tilde{x}_m)_+^2$$

- Nondecreasing if all first derivatives are nonnegative

2 - Monotonicity

- For m known knots, $\tilde{x}_1, ..., \tilde{x}_m$,

$$f(x) = \beta_1 x + \beta_2 x^2 + \beta_3 (x - \tilde{x}_1)_+^2 + ... + \beta_{m+2} (x - \tilde{x}_m)_+^2$$

- Nondecreasing if all first derivatives are nonnegative
- Shively, Sager and Walker (2009) claim this yields $m + 2$ linear constraints:

$$\boldsymbol{L}\boldsymbol{\beta} \geq 0$$

2 - Monotonicity

For m knots, there are $m+2$ conditions to satisfy:

$$
\begin{aligned}
0 &\leq f'_{kt}(0) = \beta_{1kt} \\
0 &\leq f'_{kt}(\tilde{x}_{1k}) = \beta_{1kt} + 2\beta_{2kt}\tilde{x}_{1k} \\
0 &\leq f'_{kt}(\tilde{x}_{2k}) = \beta_{1kt} + 2\beta_{2kt}\tilde{x}_{2k} + 2\beta_{3kt}(\tilde{x}_{2k} - \tilde{x}_{1k}) \\
&\vdots \\
0 &\leq f'_{kt}(1) = \beta_{1kt} + 2\beta_{2kt} + 2\beta_{3kt}(1 - \tilde{x}_{1k}) + ... + 2\beta_{m+2,kt}(1 - \tilde{x}_{mk})
\end{aligned}
$$

2 - Monotonicity

This can be vectorized as

$$\mathbf{0} \leq \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2\tilde{x}_{\mathbf{1}k} & 0 & \dots & 0 & 0 \\ 1 & 2\tilde{x}_{\mathbf{2}k} & 2(\tilde{x}_{\mathbf{2}k} - \tilde{x}_{\mathbf{1}k}) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 2 & 2(1 - \tilde{x}_{\mathbf{1}k}) & \dots & 2(1 - \tilde{x}_{m-\mathbf{1},k}) & 2\beta_{m+\mathbf{2},kt}(1 - \tilde{x}_{mk}) \end{bmatrix} \boldsymbol{\beta}_{kt}$$

2 - Monotonicity

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  and the correct prior on $\boldsymbol{\gamma} = \boldsymbol{L}\boldsymbol{\beta}$ will enforce monotonicity
- We use a modified version of their shrinkage prior:

$$\begin{aligned} (\gamma_j | I_j = 0) &\sim \delta_0 \\ (\gamma_j | I_j = 1) &\sim N_+(0, c\sigma^2) \\ I_j &\sim Bernoulli(0.2) \end{aligned}$$

3 - Intercept adjustment

Recall our additive model, with spline basis $\boldsymbol{X}_{i,k,t-1}$ and a single intercept

$$\mathbb{E}(r_{it}|\boldsymbol{x}_{i,t-1}) = \alpha_t + \sum_{k=1}^{K} \boldsymbol{X}_{i,k,t-1}\boldsymbol{\beta}_{kt}$$

$\Rightarrow$ α_t is the expected return for a firm with the minimum value for all characteristics, i.e. $\boldsymbol{X}_{i,k,t-1} = 0$, $\forall k$.
Problems:

1. Computationally challenging due to few and volatile data points
2. Intuitively unfavorable as a baseline
3. Cannot see the lower tail effects change over time

3 - Intercept adjustment

Proposal: let the intercept be the expected return for a firm that has the median value for all characteristics

- Requires transforming the splines such that they equal 0 at the median $x = 0.5$ and not $x = 0$
- This then requires carefully expand spline basis and the monotonicity constraint matrix L

4 - Time-dynamics: McCarthy and Jensen (2016)

- Power-weighted likelihoods let information decay over time
- To estimate parameters at time τ, let $\delta_t = 0.99^{\tau - t}$, such that $\delta_1 \leq \delta_2 \leq ... \leq \delta_\tau = 1$, the likelihood at time $\tau \in \{1, ..., T\}$ is

$$p(\boldsymbol{r}_1, ..., \boldsymbol{r}_\tau | \Theta_\tau) = \prod_{t=1}^{\tau} p(\boldsymbol{r}_t | \Theta_\tau)^{\delta_t}.$$

Empirics

- Freyberger, Neuhierl, and Weber (2017)'s dataset:
  - CRSP monthly stock returns for most US traded firms
  - 36 characteristics from Compustat and CRSP, including size, momentum, leverage, etc.
  - July 1962 - June 2014
- Model trained on 120 month rolling window
- Presence and direction of monotonicity is determined by important papers in the literature
  - "Fully" Monotonic model includes constraints on 24 of 36 predictors
  - "FF5" Monotonic model includes constraints on 6 predictors: size, book-to-market, investment, profitability, two horizons of momentum