Growth & decay models $\square$ Pass out H/O.

For any quantitative problem:
1) Identify appropriate model
2) Apply the model to the problem.
3) Solve the modeled problem
4) Interpret solution.

Problems where the time rate of change of some quantity is proportional to the level of that quantity are well-modeled by
$$y(t) = a e^{kt}$$
where
t = time
$y(t)$ = quantity of interest at time t
a = $y(0)$
k = constant ("$-k$" used for decay)

[$\square$ Soln to ODE]

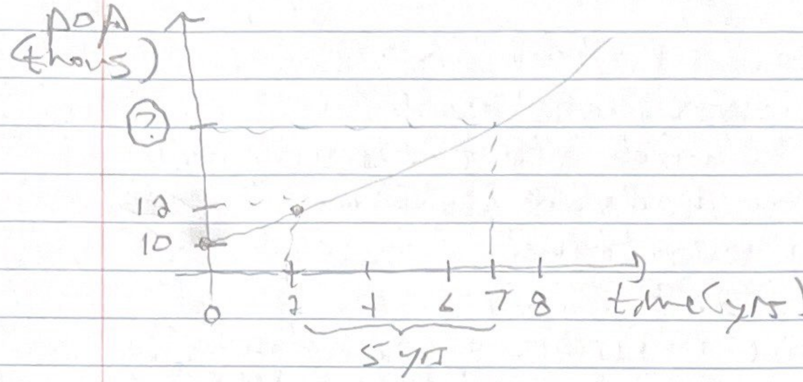
Important relation:
- If $y = e^x$, then $x = \ln y$.
I.e., ln is inverse function for exp.

Recall rules for logarithms:

$$\ln(a \cdot b) = \ln a + \ln b$$
$$\ln a^b = b \ln a$$

Rules for exponents:
$$e^{a+b} = e^a e^b$$
$$e^{ab} = (e^a)^b$$

Problem 1



1) Model:

$y(t) = ae^{kt}$

2) Apply:

t = time in yrs from pop at 10k

y(t) = pop at t

a = y(0) = 10000

? k unknown

3) Solve: i) for k, ii) for y(7).

i) $y(2) = y(0)\, e^{k\cdot 2}$

$12000 = 10000\, e^{k2} \Rightarrow 1.2 = e^{k2}$

$\ln(1.2) = \ln e^{k2} = k\cdot 2 \cancel{\ln e}^{1} = k\cdot 2$

$\Rightarrow k = \ln(1.2)/2 \quad (\approx 0.091161)$

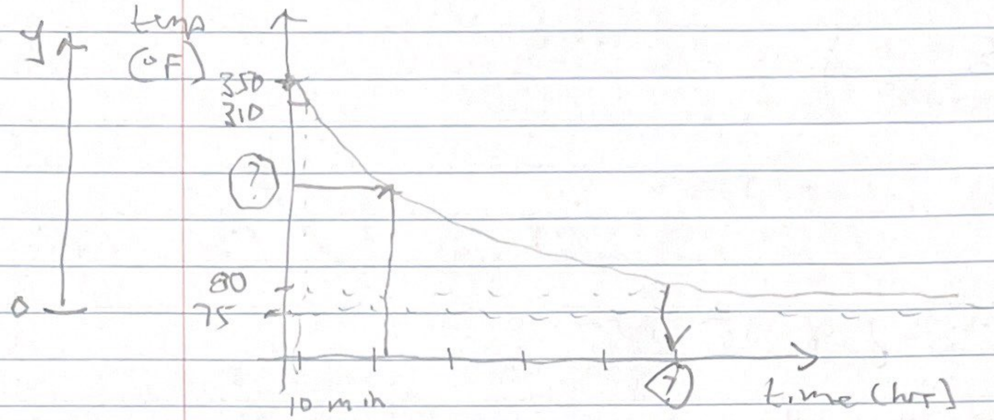
ii) $y(7) = \underbrace{y(0)\, e^{\frac{\ln(1.2)}{2}\cdot 7}}_{\text{compute}}$

$= \boxed{18929}$

4) Interpret

5 years from now, the town population is forecast to be 18929 people.

Problem 2



1) Model $y(t) = a e^{-kt}$

2) Apply
t = time in hrs
$y(t)$ = °F above ambient
$a = y(0) = 350$

3) Solve: i) for k
ii) part a) iii) part b)

i) $y(1/6) = y(0)\, e^{-k \cdot 1/6}$

$k = 6 \ln\left(\frac{(310 - 75)}{(350 - 75)}\right)$ $(310 - 75) = (310 - 75)\ \cancel{e^{k \cdot 1/6}}$

$\approx .943$

ii) $y(7/6) = \underbrace{y(0)\, e^{-k \cdot 7/6}}_{\text{compute}}$

$= 91.5$

$\Rightarrow$ temp $= 91.5 + 75 = 166$ °F

(iii) $\underbrace{y(t)}_{5} = (350-75)\, e^{-kt}$

$\Rightarrow \left(\frac{5}{350-75}\right) = e^{-kt}$

$\Rightarrow t = -\ln\left(\frac{5}{350-75}\right) / k$

$= 4.25$ hrs

$\Rightarrow$ time from now = $\boxed{4 \text{ hrs } 5 \text{ min}}$

4) Interpret

a. The temp in an hour will be ~166 °F.

b. To cool to 80°F will take ~4 hr 5 min