Financial Literacy and Economic Outcomes

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Abstract

We explore the relationship between financial literacy and self-reported economic outcomes using survey data from the United States. Our dataset includes a large number of covariates from the National Financial Capability Study, and we use a new econometric technique developed by Hahn et al. (2018a) to test whether changes in financial literacy are positively related to changes in economic outcomes while controlling for appropriate confounders. The estimate of the treatment parameter on financial literacy is positive, and the regularized regression treatment effect estimates provide more confidence about this finding than OLS.

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1. Introduction

Is financial literacy associated with a household achieving more financial happiness? There is a natural presumption that basic literacy is good and gradations of increasing financial literacy are better. Interest and education in financial literacy in the United States has become widespread and institutionalized by the Federal Reserve that has taken an active role in creating financial information for individuals.¹ Books on personal finance, investing, and wealth creation have become as ubiquitous as books on health and weight loss.

One objective of investing in financial knowledge is clear: a household will have a better chance at optimizing their living standard if they are financially adept. Specialized knowledge appears to be important and just being educated does not enable competence about personal finance. Mitchell and Lusardi (2015) find that increased education and financial literacy are positively correlated, yet they find less than 50% of college-educated students can successfully answer three key financial literacy questions and less than 64% of students with a post-graduate education can answer all three questions correctly. Perhaps "interventionist" activity is needed. Campbell (2016) has argued that important questions about financial regulations surrounding household finance are ripe for the attention of economists because there is not much known about how consumer financial regulatory costs would be offset by its benefits.

There is a large body of more recent research summarized by Hastings et al. (2013) and Lusardi and Mitchell (2014) that trace many of the key questions around a more financially literate populace to education choices, the timing of education delivery during an individuals life-cycle, public policy prescriptions, and regulatory intervention. What is less well understood is whether being personally financially literate matters to economic outcomes? Anecdotally, it is nearly impossible to argue that some personal financial knowledge is unimportant to a household's economic outcome. However, individuals do rely on the specialist knowledge of others to diagnose illnesses, construct legal documents, even to prepare a good meal. We heed the call of others to strengthen our understanding of the connection between financial

¹As an example of financial education sources the U.S Treasury department has compiled a list. See https://www.treasury.gov/resource-center/financial-education/Documents/OFE-CFAP-Resources.pdf, and the Chicago FED lists educational offerings of banks in the system. See https://www.chicagofed.org/region/community-development/cedric/federal-reserve-financial-education-initiatives.

competence and financial outcomes.²

We begin by isolating economic outcomes from behaviors utilizing the data generated from FINRA's complete 2015 National Financial Capability Survey study. We identify five survey questions that reveal actual respondent economic outcomes. Further, we categorize a number of good and and bad financial practice behaviors and include numerous other respondent and household factors that are plausibly related to economic outcomes, including a financial literacy treatment variable. In this study we are interested in a primary question:

Is there a systematic relationship between individuals who experience a negative financial outcome and their financial literacy?

It is clear there is a potential for bias in any estimation because the literacy treatment variable is correlated with a dependent variable and a set of covariates. Additionally, the dimensions of the final sample from the NCFS study are characterized by a large number of covariates and a relatively limited observations count making more traditional estimation approaches problematic.³ To resolve the statistical problem we apply the work of Hahn et al. (2018a) who call on recent research in treatment effect estimation and machine learning. Hahn et al. (2018a) describe a data-driven approach to identify confounders and mitigate treatment effect estimation bias.⁴

After a review of the literature, the study proceeds by defining a new variable, a household's economic outcome, from information in the NFCS dataset. Following a brief discussion of the controls, we provide background on the estimation approach and report both OLS and regularized regression results that show a positive relationship between financial literacy and economic outcome for a model that is specified *a priori*. We contrast these findings with results from a dataset with the same number of observations but a much larger set of covariates, that

²As noted by Lusardi and Mitchell (2014), p. 34, "Though it is challenging to establish a causal link between financial literacy and economic behavior, both instrumental variables and experimental approaches suggest that financial literacy plays a role in influencing financial decision making, and the causality goes from knowledge to behavior." Two noteworthy contributions along this path are Calvet et al. (2007) and Agarwal et al. (2009) ³Indeed, most empirical questions surrounding financial literacy are characterized by these problems.

⁴These authors propose jointly modeling the treatment and response $Y, Z \mid X$ by first modeling the treatment variable as a function of covariates $Z \mid X$, and then modeling the response $Y \mid Z, X$. The first likelihood provides information on the propensity of being treated as a function of covariates, and the second utilizes this information to mitigate endogeneity when estimating the partial effect of Z on Y. Importantly, their procedure provides a way to "shrink-away" irrelevant covariates using Bayesian shrinkage priors.

is, a model the includes all available covariates and has the ability to select the most meaningful ones. The finding that higher levels of financial literacy are related to higher levels of economic outcome remains robust in the regularized regression approach, but is no longer evident with OLS.

1.1. Literature Review

It is well-documented that U.S. citizens have low levels of financial knowledge and make financial "mistakes." Calvet et al. (2009) created an index of financial sophistication from mistakes related to under diversification, risky share turnover and the disposition effect and find that less sophistication is related to individuals with less wealth, smaller family size, less education and less financial experience.⁵ Choi et al. (2011) found that more than a third of employees do not take advantage of an employer match to a 401(k) plan when it is clearly to their benefit to do so.⁶ Keys et al. (2016) found that 20% of households do not refinance their mortgage even when it is to their benefit. Recently, Agarwal et al. (2017) found the individuals who opt for points in their mortgages, a poor financial choice in their analysis, are less responsive to interest rate changes and preferred refinancing behavior.

The literature suggests that investments in financial education may not be helpful in solving the problem of poor financial choices, contrary to intuition. Willis (2008) confronts the idea that financial education is inherently a good idea by taking the view that a "financial regulation-through-education policy model" imposes costs on those aspiring to be financial literate that are significantly higher than the benefits from the financial literacy gained.⁷ Policymakers who promote financial literacy as important intend, at least partly, to have the individual bear responsibility for the management of his or her financial future. Indeed, Willis (2011) asserts that financial regulation replaced by financial education is a "fundamental fal-

⁵Odean (1998) defined the disposition effect as the tendency for investors who hold losers to hold them too long, and investors who own winners to have sold them too quickly.

⁶Choi et al. (2011) sample included employees who were older than 59.5 who were unconstrained by withdrawal penalties: they could have simply withdrew employer contributions but chose not to take advantage of it. Even among a subsequent experiment, the researchers find conclude that low financial literacy and poor choice about a matching contribution are positively related.

⁷See Willis (2008), p. 204. Willis interprets policymakers promotion of financial literacy as an ineffective substitute for financial regulation that places too high a burden on non-expert consumers.

lacy."⁸ The individual would always be chasing the details of new product innovation and once the consumer shortens their information disadvantage, Willis argues that the industry would "outmaneuver" them. Willis (2011) notes that empirical work to date is replete with evidence that "biases, heuristics, and other nonrational influences" circumvent good financial decision-making. Lusardi et al. (2017) develop a model that includes the prospect of financial knowledge investment, and illustrate conditions under which less investment is preferred. The implication is that if financially literate consumers do not make better financial decisions, then personal investments in financial knowledge are best not incurred.

If education would not be helpful, then how do consumers plan well for their financial futures? Calcagno and Monticone (2015) offer a different perspective. They start with a premise that those who are less financially literate may benefit from more personal finance advice or derive more value from a financial adviser. They construct a theoretical model that considers an adviser who has the ability to sell investments and is compensated by a proportional commission linked to the size of the investment. The adviser's customer may be asymmetrically informed about the attributes of candidate investments. Considering incentives, penalties and information costs, their model predicts that those who are better informed are more likely to invest in risky assets and utilize a financial adviser. Indeed, the authors use bank survey data from Italy to find empirically that utilization of financial advisers is higher among those who are already financially literate, and that financial literacy is positively related to the probability of investing in risky assets. In a different yet relevant study, Balasubramnian and Brisker (2016) used 2012 NFCS survey data and an instrumental variables approach to mostly corroborate Calcagno and Monticone's empirical results. Balasubramnian and Brisker defined advisers by their role with a survey participant (e.g., Investment adviser, Debt Counselor, Tax adviser and so forth) if such a relationship existed at all. 10 The researchers found a positive relationship between working with an investment financial adviser and financial literacy, although they found a negative relationship for those who worked with a debt consolidation adviser.

⁸See Willis (2011) p. 429.

⁹Almenberg and Dreber (2015) link financial literacy and investing in the stock market with the intent to explore how investing varies between men and women when financial literacy is controlled. The authors measure financial literacy by identifying basic and advanced financial skills. While the authors find that men have higher probabilities of investing in the stock market, controlling for financial literacy skills reduces the probability differences between men and women substantially, and makes a "gender gap," inconsequential.

¹⁰53.4% of Balasubramnian and Brisker's sample used one of the defined advisers.

The literature to date supports the conclusions that households error in their decision-making, more highly educated individuals are not inclined to be better at making financial decisions, and that investments in financial literacy have low payoffs. More sophisticated individuals may be more inclined to hire financial advisers, but is that a good idea, and do individuals of any sophistication level know there are differences in how advisers are paid and the incentives that drive their recommendations? There are good reasons for consideration of a third-party who can force guidance on consumers in the spirit of the arguments presented by Campbell (2016). Promoting higher levels of financial literacy, however delivered, suggests that not enough is yet known about whether financial literacy can create good economic outcomes. The answer to this question is of primary interest. Secondarily, we want to know whether financial behaviors commonly thought as "good" or "bad" have the expected relationship with economic outcome. Evidence that literacy, behavior, or both are important to household wealth would have implications for education and policy choices and the allocation of resources to these activities.

2. The Study

What is an Economic Outcome? The NFCS survey provides reasonable proxies of economic outcomes based on answers given by respondents to survey questions sprinkled throughout the questionnaire.¹¹ We identified five questions used in the 2015 National Financial Capability Study, that are representations by respondents about their current financial circumstance.¹² They are the following:

- 1. "Overall, thinking of your assets, debts and savings, how satisfied are you with your current personal financial condition?"
- 2. "In the last 12 months, have you [or your spouse/partner] taken a hardship withdrawal from your retirement account(s)?"
- 3. "Are you concerned that you might not be able to pay off your student loans?"
- 4. "In a typical month, how difficult is it for you to cover your expenses and pay all your bills?"

¹¹Generally, the economic outcome for a household at any point in time is its economic net worth; that is, assets including household human capital less debt.

¹²Studies were conducted in 2009, 2012 and 2015. See http://www.usfinancialcapability.org.

5. "How strongly do you agree or disagree with the following statement? - I have too much debt right now"

Defining an economic outcome from survey data that does not contain financial metrics can, at first glance, pose a challenge. However, the NFCS survey questions listed above are asking respondents to reflect on their economic comfort which takes the judgment about how an economic outcome might be defined by a researcher and places the judgment within the direct purview of the respondent. Answers to the five survey questions are ordinal, and we use a principal component analysis to aggregate answers to multiple questions with ordinal outcomes to create a single economic outcome measure.

The Data and Initial Analysis. We start with the full data set from the 2015 NFCS study made available to us by FINRA. There are 27,564 observations and 149 continuous and categorical variables. Many observations contain variables with answers coded as missing, "Don't know" or "Prefer not to say." Analogous to other standard modeling approaches, we omit observations that have these uninformative answers. The largest set of complete data for which we are able to include a sufficiently rich subset of survey variables includes 973 observations and 41 continuous and categorical variables. These covariates are described in Table 4. When the categorical variables are expanded into their equivalent binary variables representation, our set of covariates numbers 144.

Dependent Variable. Our dependent variable of interest is an economic outcome index that is created from the responses to the economic outcome questions which are summarized in Table 1. Questions 1,4, and 5 are measured on integer scales from 1 to 10, 1 to 3, and 1 to 7, respectively. The mean, minimum, and maximum values are displayed on the left in Table 1. Questions 2 and 3 are binary variables, so we display their summary statistics on the right in Table 1.

	Q1	$\mathbf{Q4}$	Q5
mean	6.7	2.4	5.1
min	1	1	1
max	10	3	7
s.d.	2.5	0.7	1.8

	$\mathbf{Q2}$	Q3
% No	72%	59%
% Yes	28%	41%

Table 1: (left) Responses to questions used to construct the economic outcome (dependent) variable: Summary statistics for quantitative variables across the NFCS sample of 973 observations. (right) Responses to questions used to construct the economic outcome (dependent) variable: Summary statistics for binary variables across the NFCS sample of 973 observations.

These five questions comprise our measures of economic outcomes for individuals in the sample. In Section 3, we discuss how we combine this information from multiple outcomes into a single, meaningful measure.

Financial Literacy Variable. The strength of the connection between economic outcomes and financial literacy is tested with a measure for financial literacy that is the total number of correct answers to six financial literacy questions included in the 2015 survey and reported in Table 2.¹³ The third column of Table 2 summarizes the percentage of correct answers for each of the six financial literacy questions across the sample. Relatively higher proportions of respondents were successful at answering correctly the savings account growth question ("Interest rate") and the mortgage payment and mortgage interest paid ("Mortgage") question. More than 60% of respondents did not understand the relationship between a bond's value and market interest rates ("Bond price"), and about half of the respondents proved adept at questions related to inflation, the rule of 72 and risk. The six questions taken together across the final sample showed a mean number of correct answers equal to 3.72.

¹³The interest rate, inflation and risk questions were designed by Olivia Mitchell and Annamaria Lusardi. See Lusardi and Mitchell (2014). According to the 2015 NFCS national report, the Rule of 72 question was added as an additional interest rate question to "to test the concept of interest compounding in the context of debt."

Concept addressed	Question	% Correct
Interest rate	Suppose you had \$100 in a savings account and the interest rate was 2% per year. After 5 years, how much do you think you would have in the account if you left the money to grow?	82%
Inflation	Imagine that the interest rate on your savings account was 1% per year and inflation was 2% per year. After 1 year, how much would you be able to buy with the money in this account?	59%
Bond price	If interest rates rise, what will typically happen to bond prices?	36%
Rule of 72	Suppose you owe \$1,000 on a loan and the interest rate you are charged is 20% per year compounded annually. If you didnt pay anything off, at this interest rate, how many years would it take for the amount you owe to double	47%
Mortgage	15-year mortgage typically requires higher monthly payments than a 30-year mortgage, but the total interest paid over the life of the loan will be less. Measurement Level: Nominal	92%
Risk	Buying a single company's stock usually provides a safer return than a stock mutual fund.	55%

Table 2: Financial Literacy Questions considered from the FINRA data set. Displayed are the financial concept addressed, question asked, and the percentaged that answered correctly within our sample.

Additional Control Variables. Generally, descriptive statistics of the NFCS data have been reported by many researchers including the NFCS itself, and we follow accordingly for the data used in this paper.¹⁴ For discussion, we segregate the controls into socio-economic factors and financial behaviors. The socio-economic factors are numerous and include information about the respondent's age, education, and marital status along with the other variables listed in Table 4.

We present the listing of variables associated with the survey questions and answers that are indicative of financial behaviors that we categorize as good practice (Panel A) and bad practice (Panel B). We undertake this exercise to more easily frame the analysis as it relates to an economic outcome. In Panel A we included the set of variables based on questions a positive answer to which would more clearly indicate that the respondent is more active in thinking about financial planning. Having emergency funds, a budget, long-term financial goals, an implemented savings plans and the pay down of a credit card bill are indicative of both planning and financial practice. By contrast, in Panel B of Table 4 a longer list of variables, positive answers to which are more indicative of financial stress. Many of these variables indicate higher debt loads, or consumption greater than current resources.

¹⁴See http://www.usfinancialcapability.org/results.php?region=US.

NFCS	Panel A Description: Good practice financial behaviors
	Do you have emergency funds that can cover 3 months of expenses?
J6	Are you saving for your children's college education?
J31	Does household have a budget?
J33_2	I set long-term financial goals and try to achieve them
F2_1	Over the past 12 months have you always paid your credit card in full?
C5	Do you or your spouse regularly contribute to a thrift plan, 401(k) or IRA
	Panel B Description: Bad practice financial behaviors
B4	Do you overdraw from your checking on occasion?
B30	How often do you use a reloadable prepaid debit card
E15	How many times have you been late with your mortgage payment?
E20	Do you owe more on your home than it is worth?
F2_2	Over the past 12 months have you carried a balance and were charged interest?
F2_3	Over the past 12 months, in some months I paid the minimum payment only
F2_4	Over the past 12 months, I incurred credit card late fee
F2_5	Over the past 12 months, I was charged an over the limit fee for exceeding my credit line
F2_6	Over the past 12 months, I used my card for a cash advance
$G25_{-1}$	In the past 5 years, how many times have you taken out an auto title loan?
G25_2	In the past 5 years, how many times have you taken out a payday loan?
$G25_{-4}$	In the past 5 years, how many times have you used a pawn shop?
G25_5	In the past 5 years, how many times have you used a rent-to-own store?
	Panel C Description: Other controls
A3	Gender of respondent
log(A3A)	The log of the age of the respondent
A4A	Ethnicity subcategory
A5_2015	Highest level of education
A7	Living arrangement in the household
A7A	Marital status that includes a living with partner choice
A8	Household's income
A9	Best describes your current employment or work status
A10	Best describes your spouse/partner work status
A11	Children who are financially dependent
AM21	Armed forces member?
B4	Do you overdraw from your checking on occasion?
B30	How often do you use a reloadable prepaid debit card
E15_2015	How many times have you been late with your mortgage payment?
E20	Do you owe more on your home than it is worth?
G1	Do you have an auto loan?
G20	Do you have any unpaid medical bills?
J2	When thinking of your financial investments, how willing are you to take risks?
J3	Would you say your spending was less than, more than, or about equal to your income?
J31	Does household have a budget?
J33_2	I set long-term financial goals and try to achieve them
STATEQ	The state of the respondent

Table 3: This table presents the additional control variables that we consider in our emepirical analysis. They are categorized into three buckets, (i) good practice financial behaviors, (ii) bad practice financial behaviors, and (iii) other controls.

3. Empirical Analysis

3.1. Economic Index Construction

We construct a single economic outcome index for each observation combining the answers to the five questions summarized in Table 1 in a rigorous way. These five variables provide self-reported proxies for the economic health as well as perceived future economic health of the observations. Answers to these questions are certainly correlated. Since there is overlapping information present in each variable, our first methodological step is to extract the relevant variation among the original five variables, and we use principal component analysis (PCA) to accomplish this dimension reduction task. Specifically, the variables of interest are assembled into a 973×5 matrix \mathbf{Y} . Mathematically, PCA rotates this original data matrix \mathbf{Y} into an orthogonal space to produce:

$$\mathbf{Y}^{\text{rot}} = \mathbf{Y}\mathbf{W} \tag{1}$$

where **W** is a 5×5 matrix that contain the eigenvectors of the square matrix $\mathbf{Y}^T\mathbf{Y}$. Since the resulting columns of \mathbf{Y}^{rot} are formed from linear combinations of the eigenvectors, they are uncorrelated with each other by construction.

The columns of \mathbf{W} represent the dimensions that successively capture the most variance of the original data. In other words, the first eigenvector contained in the first column of \mathbf{W} is the "first principal component." Along this dimension, the original data contained in \mathbf{Y} varies the most. When the data is projected onto \mathbf{W} , we obtain \mathbf{Y}^{rot} . Crucially, the first column in this rotated data is now our univariate variable of interest. It corresponds to the original data rotated onto the first principal component, and it is a linear combination of the original five dependent variables by construction.

The usefulness of this procedure is its production of a univariate variable from the original five measures of economic outcome that preserves as much information as possible (in terms of variance). In the subsequent analysis where we estimate the effect of literacy on economic

¹⁵For example, a person who is not satisfied with her current personal financial condition (Question 1) is also likely to have difficulty paying bills every month (Question 4).

outcome, we use this univariate variable – taken from the first column of \mathbf{Y}^{rot} – as our dependent variable.

Since this variable is intended to serve as our measure of economic outcome for each observation in our data set, we can scrutinize the link between a respondent's answer to a distinct literacy question and the constructed index. Figure 1 displays this analysis. Each of the boxplots corresponds to the five questions discussed in Section 2. The economic outcome index is shown on each vertical axis and is scaled to have zero mean and unit variance. Positive values of the index correspond to favorable economic outcomes, and negative values correspond to unfavorable outcomes. Question 1 (Q1) deals with how satisfied the respondent is with their financial condition. Low levels of satisfaction given by answer on the far left of the x-axis are associated with (on average) negative values for the index. As satisfaction increases, the conditional expectation of the index (given by the black lines in the center of the boxplots) increases. Interestingly, the conditional distributions of the index for answers of "9" or "Extremely satisfied" do not follow the monotone trend and are centered around large negative values. We conjecture this is due to observations that are "blissfully ignorant" of their personal balance sheets and choose to answer very confidently to Q1.

Q2 displays conditional distributions of the index whose results are intuitive. Taking a loan from a personal retirement account is often an early signal of financial distress, and we observe that observations answering "No" have positive index values, and observations answering "Yes" have negative index values. Plots in Q3 and Q4 exhibit similar intuitive conditional distributions of the index. Q3 gauges the observation's perceived ability to pay off her student loans. An answer of "Yes" signals personal financial instability, and we observe the conditional distribution centered over negative index values. In contrast, respondents who are not concerned about paying off student loans take on more positive index values. The fourth question measures an respondent's ability to cover expenses and pay bills rated on the three-level scale. There are separations in the conditional distributions here as well: The "Very difficult" respondents possess negative index values, while the "Not at all difficult" respondents have markedly higher, and on average positive, index values.

The final question measures agreement with the following statement: "I have too much debt right now." There are 7 levels of agreement. As the levels go from "Strongly agree" to

"Strong disagree," we observe a monotonic trend from negative index values to positive index values – especially in expectation. On average, respondents who feel that their personal debt is high are those with negative economic indices. Taken together, the conditional distributions displayed in Figure 1 provide evidence that the economic index is measuring what we want: negative index values imply economic instability and financial distress and positive values imply economic stability.

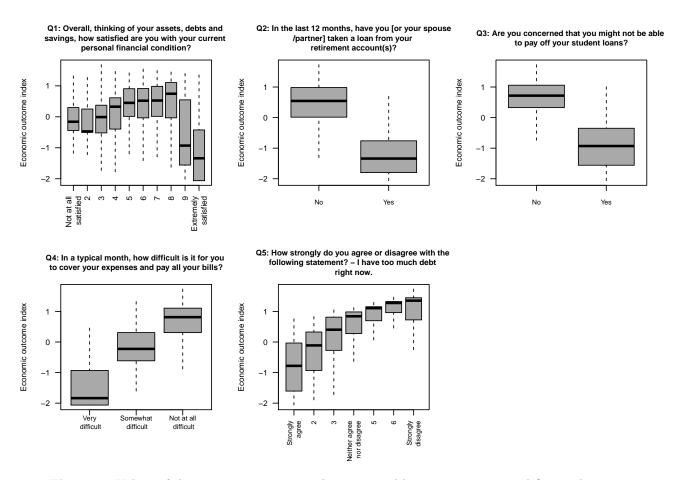


Figure 1. Values of the economic outcome index separated by answers to original financial outcome questions. Each subplot corresponds to a single question, and the individual boxplots display the distribution of the economic outcome index for the set of respondents that answered accordingly.

3.2. Hahn et al. (2018a) Estimation Approach

The establishment of an economic outcome index from the NFCS question set permits us to focus on the question of whether financial literacy has an effect on economic outcome. The problem is framed in terms of the following regression model for observation i:

$$Y_i = \alpha Z_i + X_i^t \beta + \nu_i, \tag{2}$$

where X_i is a vector of control variables, β is a vector of the control effects, Z_i is a continuous scalar treatment variable and α is a scalar regression coefficient. We assume the errors, ν_i , are normally distributed with zero mean and unknown variance. In our application, Z_i is a measure of financial literacy – the number of correct answers to the six questions outlined in Section 2. The dependent variable Y_i is the economic outcome index described in Section 3.1. The control variables (covariates) X_i are the 41 additional characteristics of the observations displayed in Table 4. For ease of interpretation, we scale Z_i and X_i to have zero mean and unit variance. This allows us to estimate Model 2 without an intercept.

 α is the parameter of interest. Accurate estimation of α must be done by including the "right" controls, where the formal definition for "right" is:

$$cov(Z_i, \nu_i | X_i) = 0. (3)$$

This condition ensures the desired counter-factual interpretation of α as the amount that economic outcome (Y) would change if literacy (Z) were increased by one unit. Written in terms of expectations, we have $\alpha = \mathbb{E}(Y \mid Z = z + 1, X) - \mathbb{E}(Y \mid Z = z, X)$. Ordinary least squares (OLS) is the most common approach to estimating Model 2. It is easy to implement, and by construction the condition described in 3 is satisfied – the residual vector produced by OLS is uncorrelated with the treatment vector.

However, it is not a priori clear which control variables to include in the model. If too few are included, then condition 3 will not be satisfied. If too many are included, unsatisfyingly imprecise treatment effect estimates will likely result (Leamer, 1978, 1983). For our analysis, we, like other researchers, quickly encounter the question: Which controls from the NFCS study should we select, and how does that selection impact the estimation of α ? Traditionally, one selects multiple sets of plausible control variables. "Plausibility" for inclusion in the model may be based on economic theory, anecdotal observation, or past empirical findings, but the lack of rigor in this approach is well known (Leamer (1983)). In personal financial survey data,

such as the NFCS data set, there is a large set of potential controls where variable inclusion can be informed from statistical regularization. We follow the approach outlined in Hahn et al. (2018a) and describe briefly below.

Regularized Regression for Treatment Effect Estimation. Statistical regularization is a broad set of model estimation methods using carefully placed biasing to improve prediction. One example of regularization is the use of shrinkage priors in Bayesian linear regression models. These priors are designed to shrink coefficients associated with unnecessary variables towards zero – in this case, the coefficient estimates are biased towards zero. This regularization helps navigate the bias-variance trade off by decreasing the variance in model predictions and, ideally, prediction errors.

Regularization has recently found use in inferential tasks like treatment effect estimation. This follows from the fact that in order to estimate treatment effects, one must understand (in fact, predict) the counter-factual outcomes. For example, in the problem at hand, what would have happened to a financial literate individual if, in an alternate state, they were financially illiterate? Hahn et al. (2018a) provide a methodology to use regularization for the specific task of treatment effect estimation. It relies on a two equation variant of Model 2 coupled with a novel reparameterization. To do this, we couple Model 2 with a model for financial literacy (Z) conditional upon the covariates:

Selection Eq.:
$$Z = X\gamma + \epsilon$$
, $\epsilon \sim N(0, \sigma_{\epsilon}^2)$,
Response Eq.: $Y = \alpha Z + X\beta + \nu$, $\nu \sim N(0, \sigma_{\nu}^2)$. (4)

The first model attempts to predict literacy using observation characteristics. Importantly, the residual error σ_{ϵ}^2 will gauge the degree of confounding – how well the characteristics of the observations describe the resulting financial literacy.

Hahn et al. (2018a) describe how jointly estimating the model for $Z \mid X$ and $Y \mid Z, X$ can lead to better inferences about α . The intuition follows in two steps. First, learn about which X's "matter" for Z from the selection equation. Second, use that information to properly "control" for those X's in the response equation. Regularization is the tool that is used to determine the meaningful X's. This is made clear by Hahn et al. (2018a) in the reparameterization of

Model 4 they consider:

Selection Eq.:
$$Z = X\beta_c + \epsilon$$
, $\epsilon \sim N(0, \sigma_{\epsilon}^2)$,
Response Eq.: $Y = \alpha(Z - X\beta_c) + X\beta_d + \nu$, $\nu \sim N(0, \sigma_{\nu}^2)$. (5)

In addition to the residual variances, the parameters of interest here are β_d , β_c , and the treatment effect α . Hahn et al. (2018a) refer to β_c as the confounding effect and β_d as the direct effect. Importantly, once we know β_c and X, we can think of $Z - X\beta_c$ as our randomized experiment that we use to infer α . This quantity can be thought of as the financial literacy of an observation that is *not explained* by their individual characteristics contained in X.

Hahn et al. (2018a) discuss how regularizing γ and β in Model 4 will lead to significant bias in the estimate of α – a phenomena called regularization-induced confounding (RIC). Additionally, they show how Model 5 avoids RIC, and β_c and β_d are able to be regularized with standard Bayesian shrinkage priors. Therefore, we use Model 5 to estimate α in our empirical study of the NFCS data.¹⁶

3.3. Results

The relationship between financial literacy and economic outcome is first explored by estimating a model which we describe as "hand-picked," one in which a smaller set of controls are posited a priori. To better shed light on the relationship between the literacy treatment and economic outcome we follow with an estimation of a model where a very large set of controls for the selection equation are determined from the data using Hahn's regularization approach. The sequence of our presentation is purposeful for it sheds light on the differences in the inferences from regularization and OLS when literacy is the treatment variable. In due course, more clarity on the relationship between financial literacy and economic outcome is given.

The Initial Estimation. The hand-picked model includes financial literacy and a small set

¹⁶We use the same set of priors and estimation methodology employed in the empirical studies of Hahn et al. (2018a). The shrinkage priors placed on the coefficients are closely related to the horseshoe priors of Carvalho et al. (2010) and estimation is undertaken using a variant of the elliptical slice sampler developed in Hahn et al. (2018b). 100,000 MCMC draws from the posterior are generated, and the first 50,000 are discarded as burn-in. The 95% intervals for the regularized approach are computed as the empirical quantiles of this posterior distribution.

of controls posited to be related to economic outcome. The number of variables The listing of these variables is in Table 4.

NFCS	Description
A3A	Age
STATEQ	State of respondent
A3	Gender
AM21	Military service
A9	Current employment status
A4A	Ethnic subcategory
A11	Number of children financial dependent
A10	Spouse/partners current employment status
J2	How willing are you to take risks

Table 4: Hand-picked controls.

For this empirical model, variables are primarily categorical, include 48 states, and Age is the only continuous variable. Total controls number p = 84 in this model and there are 973 observations. Table 5 reports the estimate of financial literacy for both OLS and the regularized regression and the 95% interval around the estimate. In this case, the estimated coefficient is positive indicating that higher levels of financial literacy are related to higher levels of economic outcomes. Moreover, the regularized regression gives a credible interval similar to the OLS confidence interval and the interval lengths are comparable and remain within the positive domain. The similarity between the OLS and regularized estimates and confidence intervals comes from the the fact that our hand-picked model is already "sparse." In other words, the number of control variables included is already small. Thus, shrinkage in the regularized model is minimal, and the inferences essentially coincide.

	α estimate	2.5%	97.5%	interval length
OLS	0.228	0.171	0.285	0.114
Regularized regression	0.232	0.175	0.292	0.117

Table 5: Credible/confidence intervals (95%) hand-picked set of controls ($p=84,\,n=973$)

¹⁷OLS estimates are available from the authors upon request.

A Different Estimation. The initial model is illustrating the traditional approach to variable selection, and one that has been important to financial literacy researchers: which set of controls should be used to properly estimate the literacy treatment effect? Rather than restrict controls to any hand-picked subset of available variables, we use Hahn et al. (2018a) approach to a "large set of controls" model that includes all variables in Table 4 plus several interactions terms. The regularization approach is not burdened by any need to defend the use of specific controls. Indeed, it informs the process of identifying appropriate controls to obtain a more confident interpretation that financial literacy is positively related to a household's economic outcome. From the data, total control variables number p = 221 in this model and there are 973 observations.

Table 6 reports the estimate of financial literacy for both OLS and the regularized regression and the 95% interval around the estimate. Noteworthy is the confidence from which an inference about the relationship between literacy and economic outcome can be taken. While both estimates of the literacy parameter in the OLS and the regularized regressions are positive, the OLS estimate is smaller than the regularized regression by 0.008, and the OLS confidence interval includes zero. By contrast, the positive coefficient estimate from the regularized regression is not surrounded by a confidence interval that includes zero. Higher levels of financial literacy are related to higher levels of economic outcomes.

	α estimate	2.5%	97.5%	interval length
OLS	0.038	-0.013	0.088	0.101
Regularized regression	0.046	0.001	0.092	0.091

Table 6: Credible/confidence intervals (95%) for large set of controls (p = 221, n = 973)

¹⁸The interaction terms number seven and include the interaction between log(Age) and the following variables: state (STATEQ), military service (AM21), current employment status (A9), ethnic sub-category (A4A), number of financially dependent children (A11), spouses's employment status (A10) and the answer to the question "how willing are you to take risks," (J2).

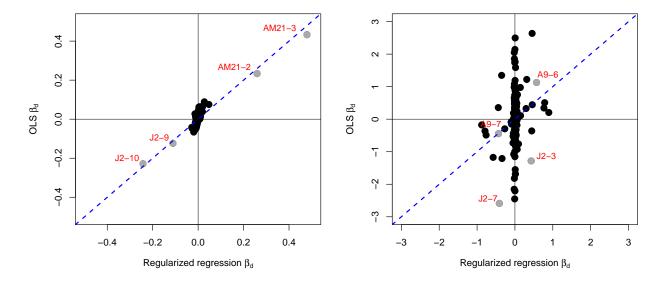


Figure 2. (Left) Estimated β_d from the hand-picked regression model. (Right) Estimated β_d from regression model with the large set of controls. The y-axis displays the OLS estimates and the x-axis displays the regularized regression estimates of the coefficients.

Figure 2 shows plots of the estimated coefficients of the direct effect β_d from the OLS (y-axis) and regularized regressions (x-axis). In other words, these plots show the coefficient vector values on the control variables X. The left plot displays the coefficients for the hand-picked regression model, and the right plot displays the coefficients from the regularized model.

Recalling the previous analysis of the hand-picked model (left plot), little regularization is needed since the specified model is already sparse. Thus, we observe that the coefficients from both estimation methods line up closely along y = x (given by the dashed blue line). The regularized regression plot (right plot) visually shows a significant amount of shrinkage. The OLS coefficient values on the y-axis are spread out while many of the regularized coefficient values shrink to zero as demonstrated by the gathering of dots along the x = 0 line. For illustration, we highlight a few variables with large absolute value of coefficients in both plots and they are displayed on the graph by their NFCS label. Given that these are coefficients from the direct effect, they can be thought of as prognostic variables for economic outcome. In other words, they are predictive of economic outcome despite conditioning upon our large set of controls. Indeed, they are describing variation in our economic outcome variable beyond that of the literacy variable. These variables measure variants of risk taking, armed services participation, and employment status, and the survey questions are given below.

- J2: When thinking of your financial investments, how willing are you to take risks?
- AM21: Have you ever been a member of the U.S. Armed Services, either in the active or reserve component?
- A9: Which of the following best describes your current employment or work status?

Finally, what if anything might be said about financial practice behaviors and economic outcome? To address this question we revert back to the OLS estimates. For brevity, Table 7 reports inferences from the estimations focusing on good and bad financial practice behaviors with statistics detailed in the Appendix.

Among good practice behaviors, setting aside funds for emergencies and engaging in longterm financial planning behaviors are related to those households with higher economic outcomes. Budgeting is not. Part of the explanation for these findings likely lies in the fact that households with more restricted finances are more keenly aware of the need to budget to make ends meet, and are not likely afforded the opportunity to set funds aside for any purpose including emergencies.

The classification of bad practice behaviors does yield insights consistent with expectation. Not overdrawing a checking account, infrequent use of a prepaid debit card and credit card minimum payments, not exceeding a credit limit, and less frequent late payment activity on a mortgage are consistent with higher economic outcomes. Negative home equity is related to a lower economic outcome which is likely the result of a household accepting permissive lending practices in residential real estate markets that subsequently struggled.

4. Conclusion

There is a general consensus that financial literacy is important to households and questions about how that literacy is acquired has been a topic of a growing body of research. This paper addresses a different type of question, that is, whether literacy matters to the economic well-being of a household. Our findings support the proposition that financial literacy is valuable to a household's perception of their economic well-being.

NFCS	Panel A: Good practice financial behaviors	Inference
J5	Do you have emergency funds that can cover 3 months of expenses?	setting aside funds related to higher economic outcome
	Are you saving for your child's college education?	insignificant
J31	Does household have a budget?	budgeting related to lower economic outcome
J33_2	I set long-term financial goals and try to achieve them	higher levels of agreement about financial goal-setting related to higher economic outcome
F2_1	Over the past 12 months have you always paid your credit card in full?	insignificant
C5	Do you or your spouse regularly contribute to a thrift plan, $401(k)$ or IRA	insignificant
NFCS	Panel B: Bad practice financial behaviors	Inference
B4	Do you overdraw from your checking on occasion?	not overdrawing related to higher economic outcome
B30	How often do you use a reloadable prepaid debit card	infrequent use related to higher economic outcome
E15	How many times have you been late with your mortgage payment?	more frequent late payments related to lower economic outcome
E20	Do you owe more on your home than it is worth?	negative equity related to lower economic outcome
F2_2	Over the past 12 months have you carried a balance and were charged interest?	insignificant
F2_3	Over the past 12 months, in some months I paid the minimum payment only	minimum payments only related to lower economic outcome
F2_4	Over the past 12 months, I incurred credit card late fee	insignificant
F2_5	Over the past 12 months, I was charged an over the limit fee for exceeding my credit line	exceeding credit limit related to lower economic outcome
F2_6	Over the past 12 months, I used my card for a cash advance	insignificant
G25_1	In the past 5 years, how many times have you taken out an auto title loan?	insignificant
G25_2	In the past 5 years, how many times have you taken out a payday loan?	insignificant
G25_4	In the past 5 years, how many times have you used a pawn shop?	insignificant
G25_5	In the past 5 years, how many times have you used a rent-to-own store?	insignificant

 $\textbf{Table 7:} \ \ \textbf{Inferences from OLS estimates in the model with a large set of controls.}$

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A. Appendix: OLS estimates for all controls

Variable Name	Estimate	Std. Error	t value	Pr> t
Literacy	0.0375	0.0258	1.45	0.1461
J2-2	-0.4943	0.4443	-1.11	0.2663
J2-3	-1.2826	0.5957	-2.15	0.0316
J2-4	-1.1525	0.6736	-1.71	0.0875
J2-5	-1.6849	0.7454	-2.26	0.0241
J2-6	-1.6881	0.8332	-2.03	0.0431
J2-7	-2.5884	0.9595	-2.70	0.0071
J2-8	-2.4504	1.0013	-2.45	0.0146
J2-9	-2.2051	0.8390	-2.63	0.0088
J2-10	-2.1529	0.9715	-2.22	0.0270
J3-2	-0.0877	0.0252	-3.48	0.0005
J3-3	-0.0458	0.0234	-1.96	0.0502
J6-2	-0.0342	0.0239	-1.43	0.1527
J5-2	-0.0794	0.0262	-3.03	0.0025
J31-2	0.0520	0.0239	2.18	0.0296
B30-2	0.0448	0.0308	1.45	0.1466
B30-3	0.0868	0.0455	1.91	0.0567
E15_2015-2	-0.0983	0.0249	-3.94	0.0001
E15_2015-3	-0.0874	0.0287	-3.05	0.0024
E20-2	0.1113	0.0305	3.65	0.0003
A4A-2	-0.6191	0.3584	-1.73	0.0845
A4A-3	0.2079	0.3494	0.60	0.5520
A4A-4	0.0409	0.2936	0.14	0.8892
A4A-5	0.0797	0.5041	0.16	0.8744
A7A-2	0.0029	0.0215	0.13	0.8936
J33_2-	0.0572	0.0254	2.25	0.0247
A9-2	1.5868	0.6661	2.38	0.0174
A9-3	0.9847	0.5043	1.95	0.0512
A9-4	0.4877	0.5410	0.90	0.3676
A9-5	-0.9936	0.6151	-1.62	0.1066
A9-6	1.1275	0.5132	2.20	0.0283
A9-7	-0.4437	0.4631	-0.96	0.3383
A9-8	-0.0512	0.7871	-0.07	0.9481
A8-2	-0.0912	0.0574	-1.59	0.1127
A8-3	-0.1045	0.1022	-1.02	0.3069
A8-4	-0.1263	0.1332	-0.95	0.3434
A8-5	-0.1947	0.2544	-0.77	0.4442
A8-6	-0.2099	0.2747	-0.76	0.4450
A8-7	-0.1821	0.2879	-0.63	0.5272
A8-8	-0.1027	0.2110	-0.49	0.6266
A5_2015-2	-0.0235	0.1374	-0.17	0.8640
A5_2015-3	-0.1010	0.1037	-0.97	0.3302
A5_2015-4	-0.1257	0.2480	-0.51	0.6123
$A5_{-}2015_{-}5$	-0.0425	0.2112	-0.20	0.8406
A5_2015-6	-0.1340	0.3093	-0.43	0.6650

$A5_2015-7$	-0.1578	0.2797	-0.56	0.5728
$F2_{-}1-2$	-0.0333	0.0349	-0.95	0.3408
$F2_{-}2-2$	0.0335	0.0295	1.13	0.2568
$F2_{-}3-2$	0.0675	0.0251	2.69	0.0073
$F2_{-}4-2$	0.0423	0.0270	1.57	0.1173
$F2_{-}6-2$	-0.0097	0.0281	-0.35	0.7292
$F2_{-}5-2$	0.0583	0.0304	1.92	0.0558
A7-2	0.0576	0.0264	2.18	0.0293
A7-3	0.0219	0.0226	0.97	0.3329
A7-4	0.0202	0.0231	0.88	0.3814
A11-2	0.4297	0.3692	1.16	0.2449
A11-3	-0.3202	0.3839	-0.83	0.4045
A11-4	0.1025	0.3949	0.26	0.7952
A10-2	-0.3753	0.5591	-0.67	0.5023
A10-3	-0.1525	0.4490	-0.34	0.7342
A10-4	0.0514	0.4599	0.11	0.9110
A10-5	1.8557	1.6507	1.12	0.2613
A10-6	-0.3369	0.4502	-0.75	0.4545
A10-7	-0.5997	0.4303	-1.39	0.1638
A10-8	-0.0785	0.3440	-0.23	0.8196
AM21-2	-0.7618	0.5114	-1.49	0.1368
AM21-3	-0.8121	0.6028	-1.35	0.1783
A3-2	0.0886	0.3874	0.23	0.1103
B4-2	0.0903	0.0277	3.26	0.0132
G1-2	0.0903 0.0179	0.0217	0.83	0.0012 0.4070
G20-2	0.1083	0.0210 0.0285	3.80	0.0002
C5_2012-2	0.1063	0.0283 0.0222	1.21	0.0002 0.2270
G25_1-2	-0.0056	0.0222 0.0255	-0.22	0.2270
G25_1-2 G25_1-3	-0.0030	0.0255 0.0311	-0.22 -0.28	0.8209 0.7771
G25_1-3 G25_1-4	0.0073	0.0311 0.0318	0.23	0.7771
G25_1-4 G25_1-5	-0.0751	0.0318 0.0357	-2.10	0.8173 0.0359
G25_1-5 G25_2-2	-0.0751	0.0397 0.0298	-2.10 -1.04	0.0359 0.3000
G25_2-2 G25_2-3		0.0298 0.0331		0.3000 0.2531
G25_2-3 G25_2-4	0.0378	0.0331 0.0342	$1.14 \\ 0.46$	0.2331 0.6447
	0.0158			
G25_2-5	-0.0367	0.0338	-1.08	0.2790
G25_4-2	-0.0109	0.0255	-0.43	0.6704
G25_4-3	-0.0535	0.0319	-1.68	0.0938
G25_4-4	-0.0479	0.0313	-1.53	0.1264
G25_4-5	-0.0306	0.0340	-0.90	0.3675
G25_5-2	-0.0219	0.0301	-0.73	0.4674
G25_5-3	-0.0368	0.0357	-1.03	0.3036
G25_5-4	-0.0195	0.0376	-0.52	0.6041
G25_5-5	-0.0032	0.0363	-0.09	0.9306
logA3A-	-0.5322	0.2610	-2.04	0.0418
STATEQ-2	-0.4633	0.5514	-0.84	0.4011
STATEQ-3	-0.6862	0.5397	-1.27	0.2040
STATEQ-4	-0.3402	0.4988	-0.68	0.4953
STATEQ-5	-1.0771	0.7233	-1.49	0.1368
STATEQ-6	-0.1798	0.5591	-0.32	0.7478
STATEQ-7	0.0047	0.5169	0.01	0.9928
STATEQ-8	-0.5339	0.5556	-0.96	0.3369

STATEQ-9	-0.2079	0.5110	-0.41	0.6843
STATEQ-10	-0.6082	0.4620	-1.32	0.1885
STATEQ-11	-0.5366	0.4657	-1.15	0.2496
STATEQ-12	-0.0515	0.5320	-0.10	0.9229
STATEQ-13	-0.6371	0.6333	-1.01	0.3148
STATEQ-14	-0.4720	0.6770	-0.70	0.4859
STATEQ-15	-0.5883	0.6267	-0.94	0.3482
STATEQ-16	-0.0550	0.5650	-0.10	0.9225
STATEQ-17	-0.6874	0.5052	-1.36	0.1740
STATEQ-18	-0.3186	0.5385	-0.59	0.5543
STATEQ-19	0.2057	0.4626	0.44	0.6567
STATEQ-20	-0.1944	0.5774	-0.34	0.7364
STATEQ-21	0.2419	0.4493	0.54	0.5905
STATEQ-22	0.1001	0.4514	0.22	0.8246
STATEQ-23	-0.0753	0.5847	-0.13	0.8976
STATEQ-24	-1.2083	0.5606	-2.16	0.0314
STATEQ-25	0.3432	0.5078	0.68	0.4993
STATEQ-26	-0.7303	0.5699	-1.28	0.2004
STATEQ-27	-0.4773	0.5734	-0.83	0.4054
STATEQ-28	-0.8415	0.6007	-1.40	0.1616
STATEQ-29	0.3282	0.5296	0.62	0.5356
STATEQ-30	-0.3619	0.6127	-0.59	0.5549
STATEQ-31	0.2631	0.5435	0.48	0.6285
STATEQ-32	0.0279	0.4560	0.46	0.9512
STATEQ-32 STATEQ-33	0.0275 0.1167	0.4800 0.6833	0.00 0.17	0.8644
STATEQ-34	-0.3962	0.4335	-0.91	0.3610
STATEQ-34 STATEQ-35	-0.3604	0.4555 0.5561	-0.65	0.5010 0.5171
STATEQ-36	-0.5076	0.5301 0.5182	-0.98	0.3276
STATEQ-30 STATEQ-37	-0.5228	0.5182 0.5090	-1.03	0.3270 0.3047
STATEQ-37 STATEQ-38	-0.5259	0.5030 0.5332	-0.99	0.3047 0.3243
STATEQ-39	-0.3239	0.5552 0.5559	-0.38	0.5245 0.7011
STATEQ-39 STATEQ-40	-0.2133 -0.3257	0.5601	-0.58	0.7611
STATEQ-40 STATEQ-41	-0.0466	0.4150	-0.33	0.9105
STATEQ-41 STATEQ-42	-0.3007	0.4150 0.6347	-0.11 -0.47	0.9103 0.6359
STATEQ-42 STATEQ-43	0.0089	0.0547 0.4643	0.02	0.0339
STATEQ-43 STATEQ-44	-0.5991	0.4043 0.6129	-0.98	0.9840 0.3286
STATEQ-45	-0.6811 -0.6648	$0.6917 \\ 0.5587$	-0.98 -1.19	0.3252 0.2345
STATEO 47				
STATEO 49	-0.7144	0.5885	-1.21	0.2251
STATEQ-48	0.5075	0.5133	0.99	0.3231
STATEQ-49	0.4697	0.5377	0.87	0.3826
STATEQ-50	0.0238	0.4963	0.05	0.9617
STATEQ-51	-0.7293	0.4900	-1.49	0.1371
logA3A:STATEQ-2	0.4534	0.5540	0.82	0.4134
logA3A:STATEQ-3	0.7362	0.5447	1.35	0.1769
logA3A:STATEQ-4	0.3467	0.4831	0.72	0.4732
logA3A:STATEQ-5	1.0791	0.7243	1.49	0.1367
logA3A:STATEQ-6	0.1926	0.5694	0.34	0.7353
logA3A:STATEQ-7	0.0363	0.5268	0.07	0.9450
logA3A:STATEQ-8	0.5559	0.5609	0.99	0.3219
logA3A:STATEQ-9	0.1864	0.5083	0.37	0.7139

logA3A:STATEQ-10	0.6043	0.4570	1.32	0.1865
logA3A:STATEQ-11	0.5683	0.4673	1.22	0.2243
logA3A:STATEQ-12	0.1042	0.5510	0.19	0.8501
logA3A:STATEQ-13	0.6753	0.6457	1.05	0.2959
logA3A:STATEQ-14	0.5437	0.6934	0.78	0.4333
logA3A:STATEQ-15	0.5852	0.6283	0.93	0.3519
logA3A:STATEQ-16	0.0594	0.5712	0.10	0.9172
logA3A:STATEQ-17	0.7360	0.5161	1.43	0.1543
logA3A:STATEQ-18	0.3396	0.5316	0.64	0.5231
logA3A:STATEQ-19	-0.1753	0.4625	-0.38	0.7047
logA3A:STATEQ-20	0.2189	0.5842	0.37	0.7079
logA3A:STATEQ-21	-0.1938	0.4571	-0.42	0.6718
logA3A:STATEQ-22	-0.0724	0.4645	-0.16	0.8762
logA3A:STATEQ-23	0.0981	0.5882	0.17	0.8676
logA3A:STATEQ-24	1.2226	0.5703	2.14	0.0324
logA3A:STATEQ-25	-0.3631	0.5077	-0.72	0.4747
logA3A:STATEQ-26	0.7547	0.5714	1.32	0.1870
logA3A:STATEQ-27	0.4778	0.5752	0.83	0.4064
logA3A:STATEQ-28	0.8762	0.6055	1.45	0.1483
logA3A:STATEQ-29	-0.2953	0.5259	-0.56	0.5746
logA3A:STATEQ-30	0.3920	0.6260	0.63	0.5314
logA3A:STATEQ-31	-0.1889	0.5573	-0.34	0.7347
logA3A:STATEQ-32	-0.0018	0.4561	-0.00	0.9968
logA3A:STATEQ-33	-0.0926	0.6828	-0.14	0.8922
logA3A:STATEQ-34	0.4215	0.4406	0.96	0.3391
logA3A:STATEQ-35	0.3569	0.5551	0.64	0.5204
logA3A:STATEQ-36	0.5131	0.5220	0.98	0.3259
logA3A:STATEQ-37	0.5537	0.5120	1.08	0.2798
logA3A:STATEQ-38	0.5366	0.5242	1.02	0.3064
logA3A:STATEQ-39	0.2218	0.5678	0.39	0.6961
logA3A:STATEQ-40	0.3435	0.5592	0.61	0.5392
logA3A:STATEQ-41	0.0423	0.4142	0.10	0.9186
logA3A:STATEQ-42	0.3025	0.6232	0.49	0.6276
logA3A:STATEQ-43	0.0197	0.4703	0.04	0.9667
logA3A:STATEQ-44	0.6251	0.6136	1.02	0.3086
logA3A:STATEQ-45	0.7001	0.6860	1.02	0.3078
logA3A:STATEQ-46	0.6579	0.5651	1.16	0.2447
logA3A:STATEQ-47	0.7471	0.6007	1.24	0.2140
logA3A:STATEQ-48	-0.4877	0.5083	-0.96	0.3377
logA3A:STATEQ-49	-0.4355	0.5522	-0.79	0.4305
logA3A:STATEQ-50	-0.0001	0.4996	-0.00	0.9998
logA3A:STATEQ-51	0.7493	0.4940	1.52	0.1297
A3-2:logA3A	-0.1360	0.3867	-0.35	0.7251
AM21-2:logA3A	0.8532	0.5307	1.61	0.1083
AM21-3:logA3A	0.9751	0.6309	1.55	0.1226
A9-2:logA3A	-1.5497	0.6620	-2.34	0.0195
A9-3:logA3A	-0.9251	0.4957	-1.87	0.0624
A9-4:logA3A	-0.4012	0.5266	-0.76	0.4463
A9-5:logA3A	0.9439	0.5968	1.58	0.1142
A9-6:logA3A	-1.1731	0.5061	-2.32	0.0207
A9-7:logA3A	0.4444	0.3661 0.4647	0.96	0.3392
110 1.10811011	0.1111	0.1011	0.00	0.0002

A9-8:logA3A	0.0185	0.8003	0.02	0.9816
A4A2:logA3A	0.6397	0.3583	1.79	0.0746
A4A-3:logA3A	-0.2107	0.3494	-0.60	0.5467
A4A-4:logA3A	-0.0615	0.2960	-0.21	0.8353
A4A-5:logA3A	-0.1065	0.5065	-0.21	0.8335
A11-2:logA3A	-0.4035	0.3691	-1.09	0.2747
A11-3:logA3A	0.3865	0.3830	1.01	0.3132
A11-4:logA3A	-0.0641	0.3949	-0.16	0.8710
A10-2:logA3A	0.3777	0.5541	0.68	0.4956
A10-3:logA3A	0.1211	0.4465	0.27	0.7862
A10-4:logA3A	-0.0142	0.4607	-0.03	0.9755
A10-5:logA3A	-1.8189	1.6348	-1.11	0.2662
A10-6: $logA3A$	0.2896	0.4431	0.65	0.5137
A10-7:logA3A	0.6048	0.4328	1.40	0.1627
A10-8:logA3A	0.1167	0.3558	0.33	0.7431
J2-2:logA3A	0.5320	0.4526	1.18	0.2401
J2-3:logA3A	1.3474	0.5998	2.25	0.0250
J2-4:logA3A	1.1913	0.6789	1.75	0.0797
J2-5:logA3A	1.7605	0.7536	2.34	0.0198
J2-6:logA3A	1.7429	0.8452	2.06	0.0395
J2-7:logA3A	2.6370	0.9682	2.72	0.0066
J2-8:logA3A	2.4970	1.0110	2.47	0.0137
J2-9:logA3A	2.1447	0.8187	2.62	0.0090
J2-10:logA3A	2.0509	0.9442	2.17	0.0302