Blue Origin Interview

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About Me

- Mechanical/aerospace engineer by training
- Expert problem solver, with experience across a wide breadth of technical domains
- Experienced hands-on manager, leading teams to excellent outcomes regardless of technology stack

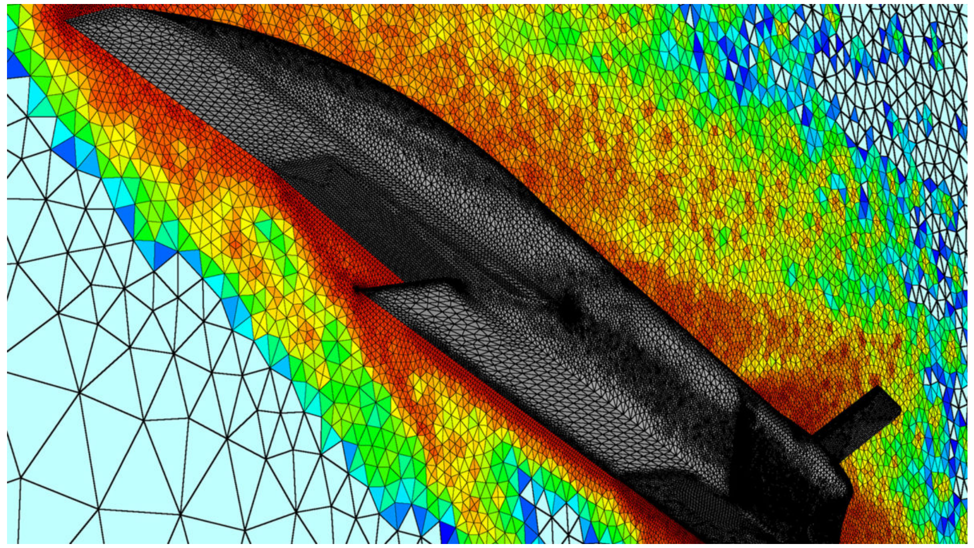
Significant Accomplishments

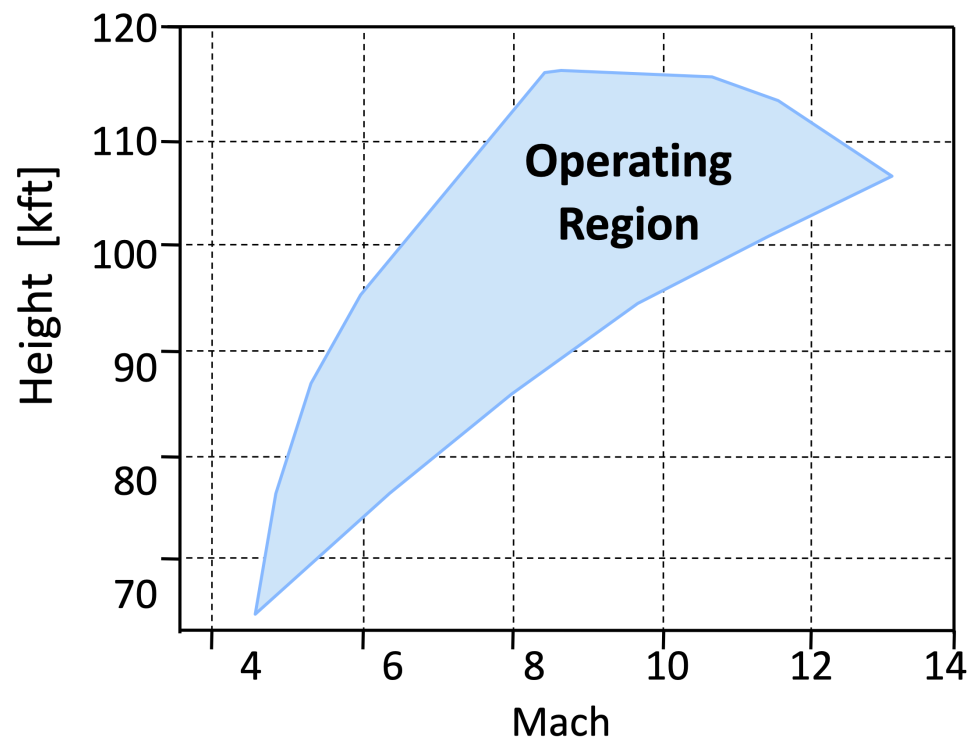
- **Completing a PhD in mechanical engineering at MIT**
  - Developed new heirarchical approach to adaptive output feedback control
  - Learned fundamental skills to think deeply, solve difficult problems, and communicate
- **Founding, running, and selling Humon**
  - Hardware/software engineering, signal processing and data science, manufacturing, IP, supply chain, and more
  - Learned a career's worth of skills in 3.5 years
- **Leading R&D at Whoop**
  - Hired and led a team of high-performing, multidisciplinary engineers and scientists
  - Drove excellent outcomes while maintaining leading eNPS

Section 2: Technical Presentation

Airbreathing Hypersonic Vehicles: Control Challenges

- **Highly open-loop unstable**
- **Difficult to model**
  - Poor CFD models
  - Limited wind tunnel data
  - Complex shock interactions
- **Must operate over a large flight envelope**
  - Largely varying dynamic pressure
- **Aerodynamic and propulsive coupling**
  - Unstart can create abrupt changes to moments
- **Unable to accurately obtain incidence angle measurements**



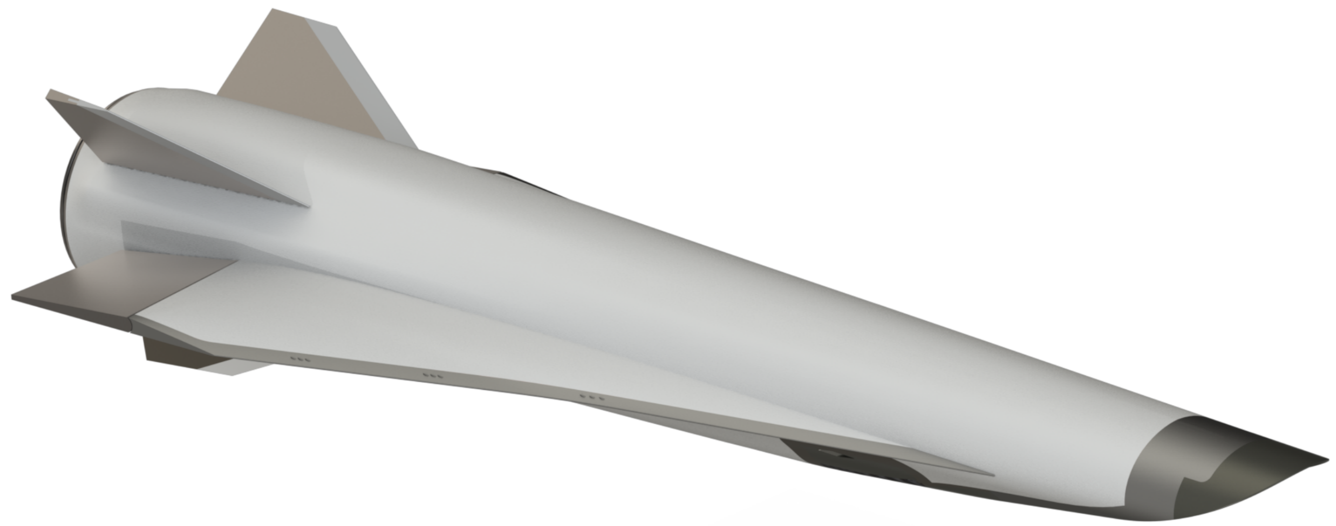


Problem Statement

Design a controller for a hypersonic vehicle that can

- Accommodate high levels of **model uncertainty**
- Be **robust** to delays and unmodeled dynamics
- Enable **agressive maneuvering**
- **Avoid unstart**
- Ensure satisfactory **command tracking**

Plant Overview (1)



Parameter	Unit	Value
Gross weight	[lbm]	1220.3
Empty weight	[lbm]	993.3
Vehicle length	[in]	175.9
Span	[in]	58.6
Nose diameter	[in]	11.0
Tail diameter	[in]	18.8

- The Generic Hypersonic Vehicle model is a variant of the HiFIRE 6 vehicle
- Equations of motion are standard flat-earth, nonlinear 12-state, 6-DOF equations
    - No rotating turbomachinery terms or fuel sloshing, vehicle is a rigid body

TABLE 2.5-1 The Flat-Earth, Body-Axes 6-DoF Equations

Force equations

$$\dot{U} = RV - QW - g_D \sin\theta + (X_A + X_T)/\mathrm{m}$$
$$\dot{V} = -RU + PW + g_D \sin\phi \cos\theta + (Y_A + Y_T)/\mathrm{m}$$
$$\dot{W} = QU - PV + g_D \cos\phi \cos\theta + (Z_A + Z_T)/\mathrm{m}$$

Kinematic equations

$$\dot{\phi} = P + \tan\theta\,(Q \sin\phi + R \cos\phi)$$
$$\dot{\theta} = Q \cos\phi - R \sin\phi$$
$$\dot{\psi} = (Q \sin\phi + R \cos\phi)/\cos\theta$$

Moment equations

$$\Gamma\dot{P} = J_{xz}\,[J_x - J_y + J_z]\,PQ - [J_z(J_z - J_y) + J_{xz}^2]\,QR + J_z\ell + J_{xz}n$$
$$J_y\dot{Q} = (J_z - J_x)PR - J_{xz}(P^2 - R^2) + m$$
$$\Gamma\dot{R} = [(J_x - J_y)J_x + J_{xz}^2]\,PQ - J_{xz}[J_x - J_y + J_z]\,QR + J_{xz}\ell + J_x n$$
$$\cdot\ \Gamma = J_xJ_z - J_{xz}^2$$

Navigation equations

$$\dot{p}_N = Uc\theta c\psi + V(-c\phi s\psi + s\phi s\theta c\psi) + W(s\phi s\psi + c\phi s\theta c\psi)$$
$$\dot{p}_E = Uc\theta s\psi + V(c\phi c\psi + s\phi s\theta s\psi) + W(-s\phi c\psi + c\phi s\theta s\psi)$$
$$\dot{h} = Us\theta - Vs\phi c\theta - Wc\phi c\theta$$

Plant Overview (2)

Control inputs U:

- δ_{th} - throttle
- δ_{elv} - elevator
- δ_{ail} - aileron
- δ_{rud} - rudder

Plant state X:

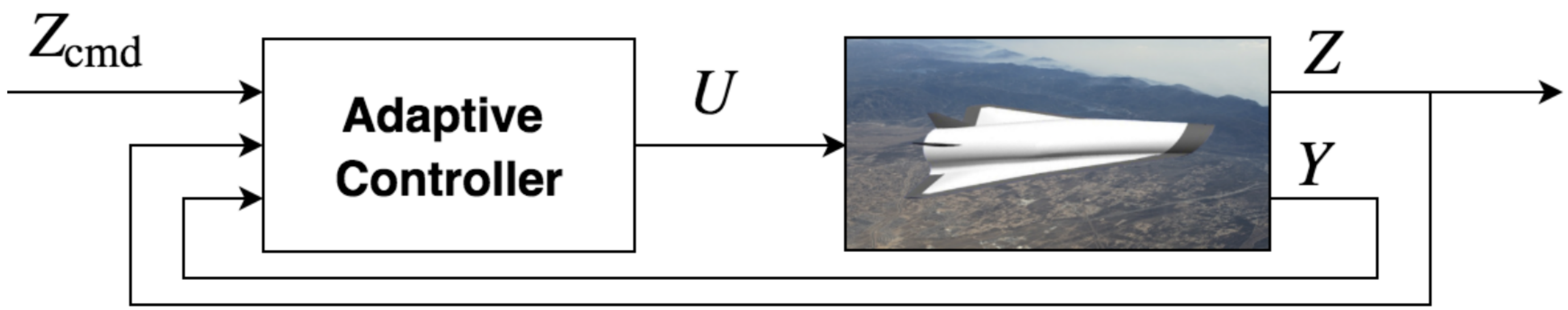
- v - velocity
- α - angle of attack
- q - pitch rate
- θ - pitch angle
- h - altitude
- β - sideslip angle
- p - roll rate
- r - yaw rate
- ϕ - roll angle
- ψ - heading angle

Sensor outputs $Y, \ Z$:

- v - velocity
- q - pitch rate
- h - altitude
- p - roll rate
- r - yaw rate
- ψ - heading angle

Incidence angles are not measurable

Control Problem



The equations of motion that describe the 6-DOF GHV model can be written as

$$\dot{X} = f(X, U)$$
$$Y = g(X, U)$$
$$Z = h(X, U)$$

Where the *measured output* Y, and *regulated output* Z are given by

$$Y = \begin{bmatrix} v & q & h & p & r & \psi \end{bmatrix}^\top$$
$$Z = \begin{bmatrix} h & \psi \end{bmatrix}^\top$$

Track heading and altitude

The *control goal* is to design the input U for this system such that the **regulated output Z tracks the command Z_{cmd}** with bounded errors using only the system outputs Z and Y.

Simplified Model for Controller Synthesis

$$
\begin{aligned}
\dot{X} &= f(X, U) \\
Y &= g(X, U) \\
Z &= h(X, U)
\end{aligned}
$$

Linearize and decouple

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_p \\ \dot{x}_g \end{bmatrix} &= \begin{bmatrix} A_p + B_p \Psi_p^\top & 0 \\ B_{gp} & A_g \end{bmatrix} \begin{bmatrix} x_p \\ x_g \end{bmatrix} + \begin{bmatrix} B_p \Lambda \\ 0 \end{bmatrix} u \\
\begin{bmatrix} y_p \\ y_g \end{bmatrix} &= \begin{bmatrix} C_p & 0 \\ 0 & C_g \end{bmatrix} \begin{bmatrix} x_p \\ x_g \end{bmatrix} \\
\begin{bmatrix} z_p \\ z_g \end{bmatrix} &= \begin{bmatrix} C_{pz} + D_{pz} \Psi_p^\top & 0 \\ 0 & C_{gz} \end{bmatrix} \begin{bmatrix} x_p \\ x_g \end{bmatrix} + \begin{bmatrix} D_{pz} \Lambda \\ 0 \end{bmatrix} u
\end{aligned}
$$

1. Design an **inner-loop** controller so that z_p tracks $z_{p,\mathrm{cmd}}$
2. Design an **outer-loop** controller that generates $z_{p,\mathrm{cmd}}$ so that z_g tracks $z_{g,\mathrm{cmd}}$.

- The nonlinear equations of motion are **linearized**
- Modal analysis is use to reduce the linear model into **several lower-order models**
- The reduced linear models are **further partitioned**
- **Uncertainty** manifests itself as shown above (more on this later)

Simplified Model for Controller Synthesis

- Linearize the 12-state model about a desired trim condition

$$f(X,U) = f(X_{\text{eq}},U_{\text{eq}}) + \left.\frac{\partial f(X,U)}{\partial X}\right|_{\text{eq}} x + \left.\frac{\partial f(X,U)}{\partial U}\right|_{\text{eq}} u + \epsilon$$

  - Validate linearity assumption

- Decouple 12-state *linear model* into several lower-order models
  - Velocity, longitudinal, lateral-directional dynamics

- Further simplify models by removing navigation and orientation dynamics

- These three linear inner-loop subsystems are represented by linear models

$$\begin{aligned}
\dot{x}_p(t) &= A_p x_p(t) + B_p u(t) \\
y_p(t) &= C_p x_p(t) \\
z_p(t) &= C_{pz} x_p(t) + D_{pz} u(t)
\end{aligned}$$

Simplified Model for Controller Synthesis

- The sensitivity matrix allows comparison of modes across state variables whose units differ

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
	-2.24	-4.87	1.89	1.37 ± 0.76j		0 ± 0.12j		-0.0039	-0.0272
V_T	3.44E-05	1.22E-13	6.57E-05	1.34E-10	1.34E-10	0.0022	0.0022	**0.9955**	2.46E-09
α	**0.3618**	7.31E-10	**0.3226**	3.04E-08	3.04E-08	**0.1578**	**0.1578**	3.04E-05	4.96E-10
q	**0.4823**	1.05E-09	**0.5103**	4.97E-08	4.97E-08	0.0036	0.0036	2.48E-07	4.57E-12
θ	0.0088	1.79E-11	0.0160	3.92E-09	3.92E-09	**0.4876**	**0.4876**	5.32E-05	1.59E-09
h	0.0012	9.70E-13	0.0020	5.77E-10	5.77E-10	**0.4962**	**0.4962**	0.0044	8.55E-10
β	1.79E-10	**0.2311**	1.16E-07	**0.3844**	**0.3844**	3.17E-11	3.17E-11	3.30E-15	7.59E-05
p	2.81E-09	**0.4259**	5.66E-08	**0.2855**	**0.2855**	7.34E-10	7.34E-10	4.73E-12	0.0031
r	3.87E-10	0.0119	9.56E-09	**0.3412**	**0.3412**	7.91E-09	7.91E-09	1.73E-10	**0.3058**
ϕ	5.01E-11	0.0237	4.84E-08	**0.3096**	**0.3096**	1.08E-08	1.08E-08	2.88E-09	**0.3570**

Simplified Model for Controller Synthesis

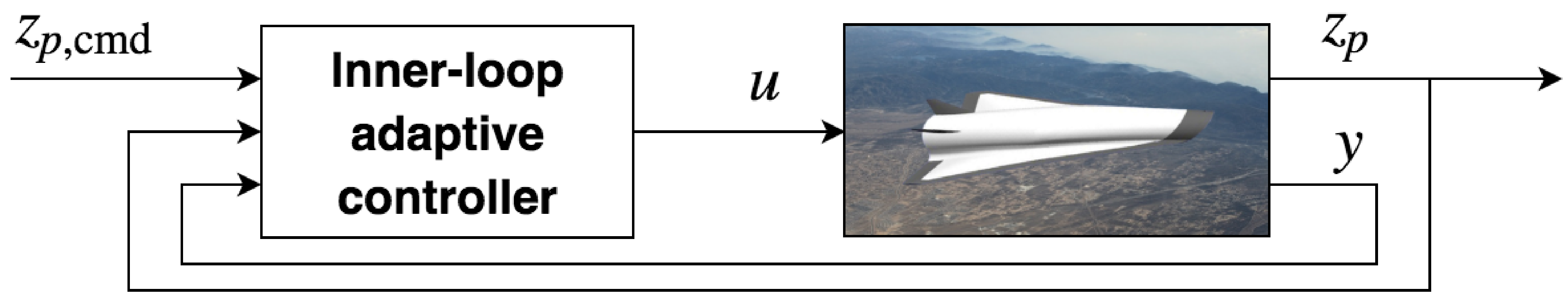
- The model exhibits decoupling between longitudinal, lateral-directional, dynamics

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
	-2.24	-4.87	1.89	1.37 ± 0.76j		0 ± 0.12j		-0.0039	-0.0272
V_T	—	—	—	—	—	0.0022	0.0022	**0.9955**	—
α	**0.3618**	—	**0.3226**	—	—	**0.1578**	**0.1578**	—	—
q	**0.4823**	—	**0.5103**	—	—	0.0036	0.0036	—	—
θ	0.0088	—	0.0160	—	—	**0.4876**	**0.4876**	—	—
h	0.0012	—	0.0020	—	—	**0.4962**	**0.4962**	0.0044	—
β	—	**0.2311**	—	**0.3844**	**0.3844**	—	—	—	—
p	—	**0.4259**	—	**0.2855**	**0.2855**	—	—	—	0.0031
r	—	0.0119	—	**0.3412**	**0.3412**	—	—	—	**0.3058**
ϕ	—	0.0237	—	**0.3096**	**0.3096**	—	—	—	**0.3570**

Why the Heirarchical Approach

- Leverage **existing knowledge** around how to design inner-loop controller
- Many systems require a “Level 2” inner-loop for **piloted vehicles**
- Structure accommodates inner-loop **command limiting**
- Control design for two small systems is easier than designing one controller for a **higher-order system**
- In practice can produce **more robust control designs**
- Facilitates using **different outer-loop** control structure with same inner-loop control law

Inner-Loop Plant Models



- Introduce the linear *uncertain* plant model

$$\dot{x}(t) = (A + B\Psi^\top)x(t) + B\Lambda u(t) + B_{\text{cmd}} z_{\text{cmd}}(t)$$
$$y(t) = Cx(t)$$

  where Λ and Ψ are unknown

- This model adds integral control on the *regulated output* and includes model uncertainty
- Some comments
  - Uncertainty enters through the columns of B
  - No direct feedthrough of the control to output

Inner-Loop Plant Models

Model Construction

- That there is no feedthrough of the control to the output is common in aerospace systems
  - Control inputs create moments and angular velocities are measured
  - One integration between input and output
  - Direct feedthrough, as occurs when measuring linear accelerations, can be accommodated using fairly trivial modification to control design to follow
- Uncertainty entering through control channels is reasonable given the uncertainty in moment coefficients in A

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ M_{\alpha} & M_{q} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ M_{\delta_e} \end{bmatrix} \delta_e$$

Inner-Loop Plant Models

Model Construction

$$\dot{x}_p(t) = A_p x_p(t) + B_p\left(\Lambda u(t) + \Psi_p^\top x_p(t)\right)$$
$$y_p(t) = C_p x_p(t)$$
$$z_p(t) = C_{pz} x_p(t) + D_{pz}\left(\Lambda u(t) + \Psi_p^\top x_p(t)\right)$$

$$A = \begin{bmatrix} A_p & 0_{n_p \times n_e} \\ -C_{pz} & 0_{n_e \times n_e} \end{bmatrix} \quad B = \begin{bmatrix} B_p \\ -D_{pz} \end{bmatrix} \quad B_{\text{cmd}} = \begin{bmatrix} 0_{n_p \times m} \\ I_{n_e \times n_e} \end{bmatrix}$$
$$C = \begin{bmatrix} C_p & 0_{\ell \times n_e} \\ 0_{n_e \times n_p} & I_{n_e \times n_e} \end{bmatrix} \quad C_z = \begin{bmatrix} C_{pz} & 0 \end{bmatrix}$$

$$\dot{x}(t) = (A + B\Psi^\top)x(t) + B\Lambda u(t) + B_{\text{cmd}} z_{\text{cmd}}(t)$$
$$y(t) = Cx(t)$$

Assumptions

$$\dot{x}(t) = (A + B\Psi^\top)x(t) + B\Lambda u(t) + B_{\text{cmd}} z_{\text{cmd}}(t)$$
$$y(t) = Cx(t)$$

- (A, B) is controllable
- (A, C) is observable
- $B,\ C,\ CB$ are full rank
- Zeros of $(A,\ B,\ C,\ 0)$ are strictly stable
- Λ is a nonsingular, diagonal matrix with entries of known sign
- $\|\Psi\| < \Psi_{\max} < \infty$ where $\Psi_{\max}$ is known

Notes on Assumptions

- Controllability and observability are standard assumptions satisfied for vehicle models such as the above
- Full rank of B, C, CB implies that inputs and outputs are not redundant, and the MIMO equivalent of relative degree one
  - One integration between aerodynamic moments and angular rates
- Strict stability (minimum phase) of zeros is straightforward to satisfy for vehicle models such as the above
- Sign of Λ known indicates no control reversal
  - Diagonal structure indicates loss of control effectiveness
- The bound $\Psi_{\max}$ need not be tight, and in practice can be easily selected
  - For example, the extent of a CG shift is bounded by the physical extents of a vehicle

Controller Synthesis

- Introduce the reference model

$$\dot{x}_m(t) = A_m x_m(t) + B_{\text{cmd}} r(t) + \boldsymbol{L}(y_m(t) - y(t))$$
$$y_m(t) = C x_m(t)$$

- Propose the following controller

$$u(t) = (K + \Theta(t))^\top x_m(t)$$
$$\dot{\Theta}(t) = -\Gamma x_m(t)(\boldsymbol{S_1} e_y(t))^\top \operatorname{sgn}(\Lambda)$$

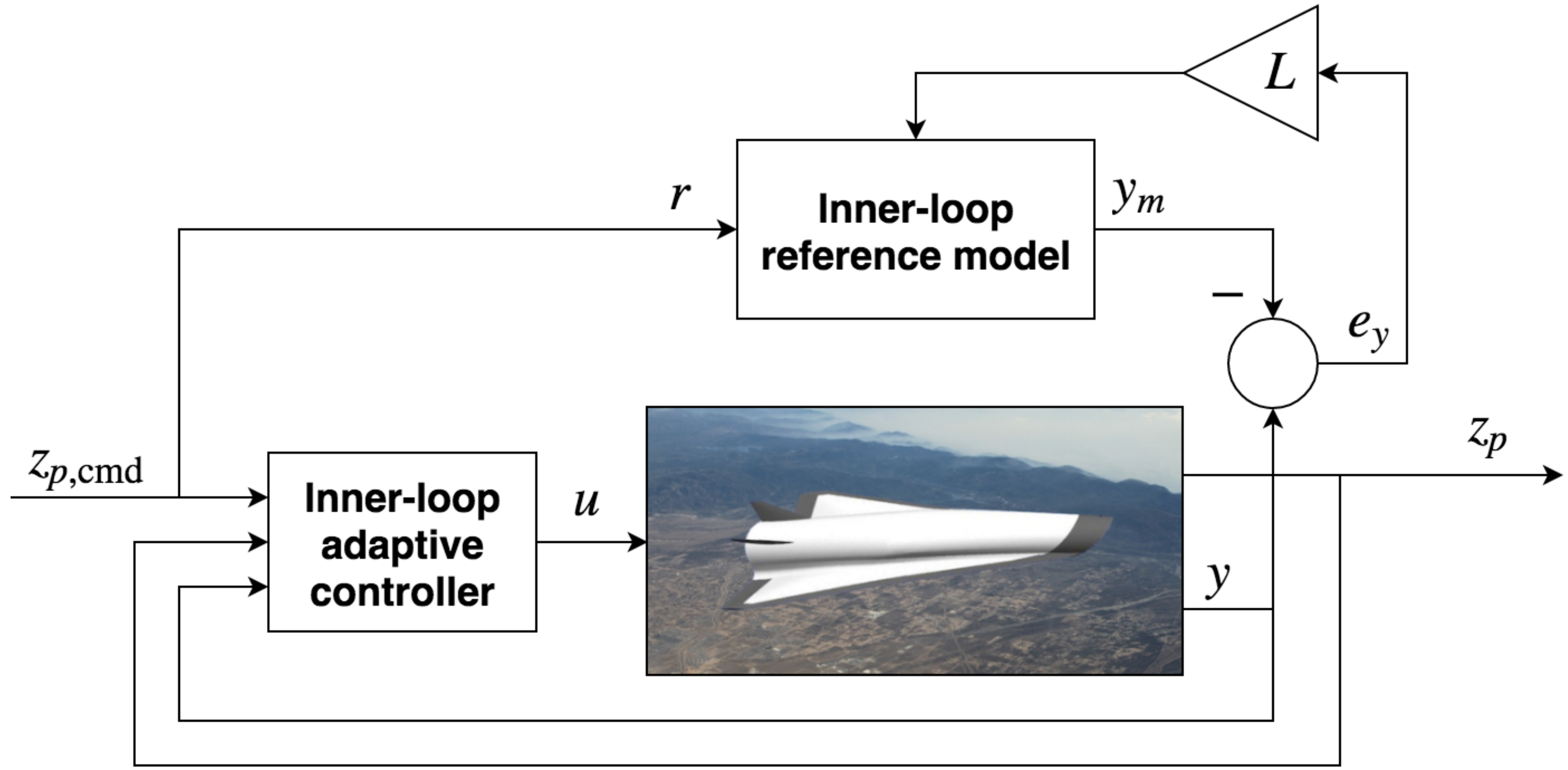
- Stability of the closed-loop system must be shown
- **The control goal is to select the control gains such that z_p tracks z_{cmd}**

Comments on the Closed-Loop System

$$
\begin{aligned}
\dot{x}(t) &= (A + B\Psi^\top)x(t) + B\Lambda u(t) + B_{\text{cmd}} z_{\text{cmd}}(t) \\
y(t) &= Cx(t) \\
\dot{x}_m(t) &= A_m x_m(t) + B_{\text{cmd}} r(t) + \boldsymbol{L}(y_m(t) - y(t)) \\
y_m(t) &= C x_m(t) \\
u(t) &= (K + \Theta(t))^\top x_m(t) \\
\dot{\Theta}(t) &= -\Gamma x_m(t)(\boldsymbol{S_1} e_y(t))^\top \operatorname{sgn}(\Lambda)
\end{aligned}
$$

- Output feedback: control depends on y only
- In the absence of uncertainty and adpatation, **architecturally this is just an LQG controller**
- Control designer must select Γ , $\boldsymbol{S_1}$, and $\boldsymbol{L}$
  - $\Gamma = \Gamma^\top > 0$
  - The control task is to select $\boldsymbol{S_1}$ and $\boldsymbol{L}$ to guarantee stability

Inner-Loop Controller Block Diagram



- In addition to needing L for closed-loop stability, it provides additional benefits for the adaptive system
  - More on this later

Overview of Stability (1)

- The error model that results from the proposed controller is

$$\begin{aligned}\dot{e}_x &= (A + \boldsymbol{L}C + B\Psi^\top)e_x + B\Lambda\tilde{\Theta}^\top x_m \\ e_y &= Ce_x \\ e_s &= \boldsymbol{S_1}e_y\end{aligned}$$

- If $\boldsymbol{S_1}$ and $\boldsymbol{L}$ can be chosen such that the error model is SPR, stability follows
- Applying the Kalman–Yakubovic Lemma, the above error dynamics are SPR if

$$\begin{aligned}&(A + \boldsymbol{L}C)^\top P + P(A + LC) + Q < 0 \\ &PB = (\boldsymbol{S_1}C)^\top\end{aligned}$$

- Q is chosen based on $\Psi_{\max}$
- $X = X^\top > 0$ is arbitrary and $B^\perp$ is an annhiliator matrix that satisfies $B^\top B^\perp = 0$
- A P satisfying the inequality exists if, and only if, $\boldsymbol{S_1}CB = (\boldsymbol{S_1}CB)^\top$

Overview of Stability (1)

- The statement of strict positive realness can be interpreted as the phase shift between the input and output being in $(-90^{\circ}, 90^{\circ})$

Overview of Stability (2)

- One choice of $\boldsymbol{S_1}$ is the generalized left-inverse of CB as follows

$$\boldsymbol{S_1} = ((CB)^\top CB)^{-1} (CB)^\top$$

- This choice of $\boldsymbol{S_1}$ ensures a P satisfying $(A + \boldsymbol{L}C)^\top P + P(A + \boldsymbol{L}C) + Q < 0$ exists given by

$$P = C^\top (CB)^{-\top} C + B^\perp X B^{\perp\top}$$

- We now have to solve a bilinear matrix inequality in $\boldsymbol{L}$ and P (or equivalently X)
- Using the Matrix Elimination Lemma, an $\boldsymbol{L}$ satisfying the inequality exists if, and only if, a P satisfies

$$M^\top (A^\top P + PA) M < -M^\top QM$$

- Using P above gives the following, where the existence of $X > 0$ is guaranteed

$$(NAM)^\top XNM + (NM)^\top X(NAM) < -M^\top QM$$

Overview of Stability (2)

- Note: M represents a particular annhilator $C^{\top\perp}$ and N a particular $B^{\perp\top}$ such that $NB = 0$ and $CM = 0$.

Overview of Stability (3)

- **The problem of finding $\boldsymbol{L}$ and X that satisfy a BMI is now reduced to finding X (whose existence is guaranteed) satisfying an LMI**

- The solutions X are given analytically thus specifying P

$$P = C^\top (CB)^{-\top} C + B^\perp X B^{\perp\top}$$

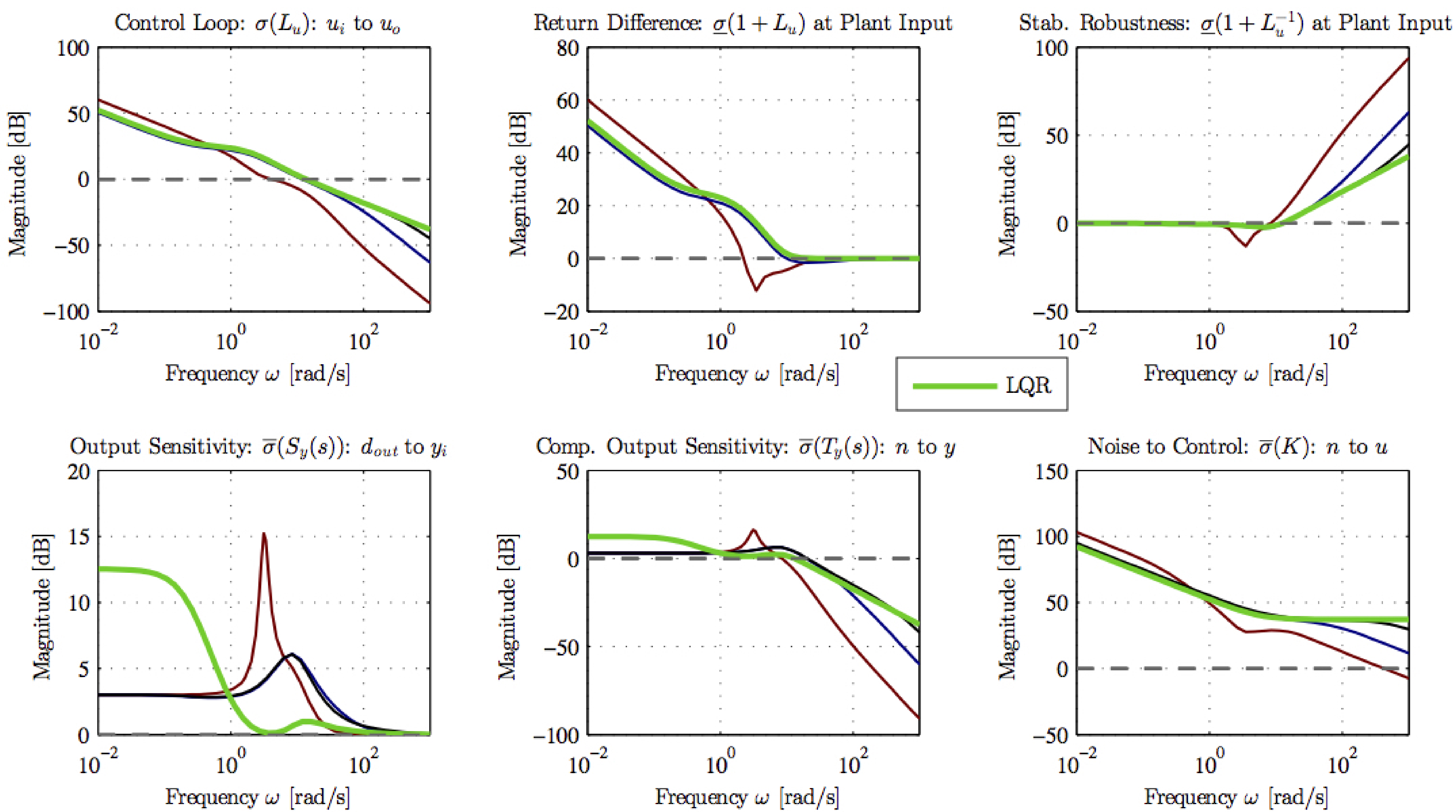
- This P then reduces the following to a feasible LMI in $\boldsymbol{L}$

$$(A + \boldsymbol{L}C)^\top P + P(A + \boldsymbol{L}C) + Q < 0$$

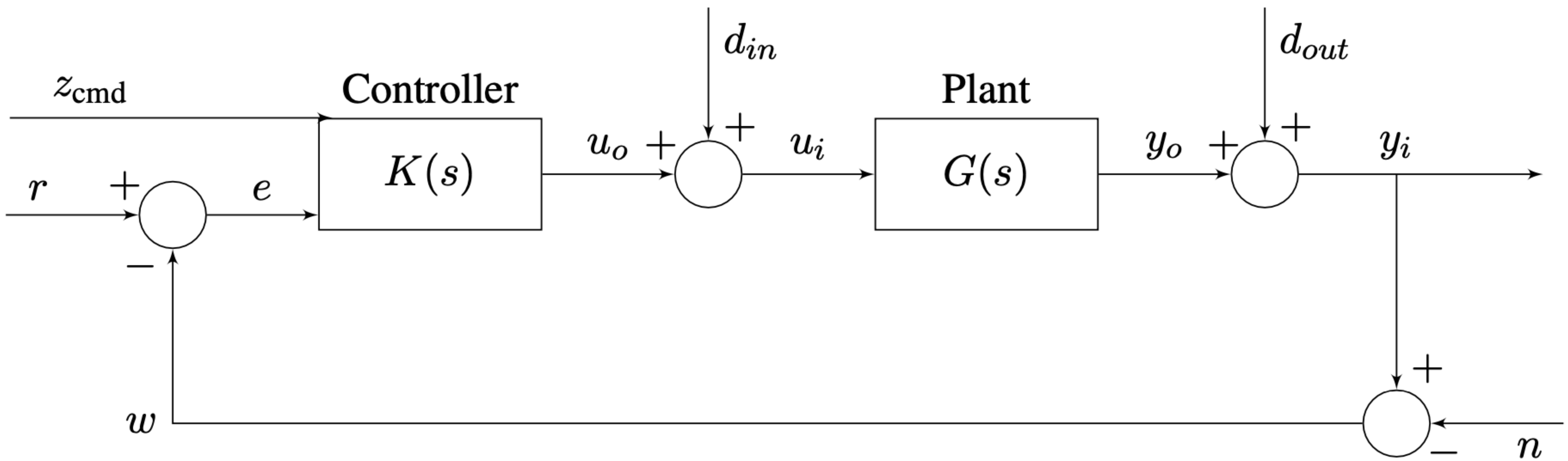
- **This control synthesis process involves a few steps of matrix algebra, and provides a guaranteed-stable closed-loop system with sufficient degrees of freedom that can be leveraged to improve the robustness proprties of the baseline controller**

Robustness Properties

- The choice of S_1 and L also affects robustness properties of the underlying LQG-like controller

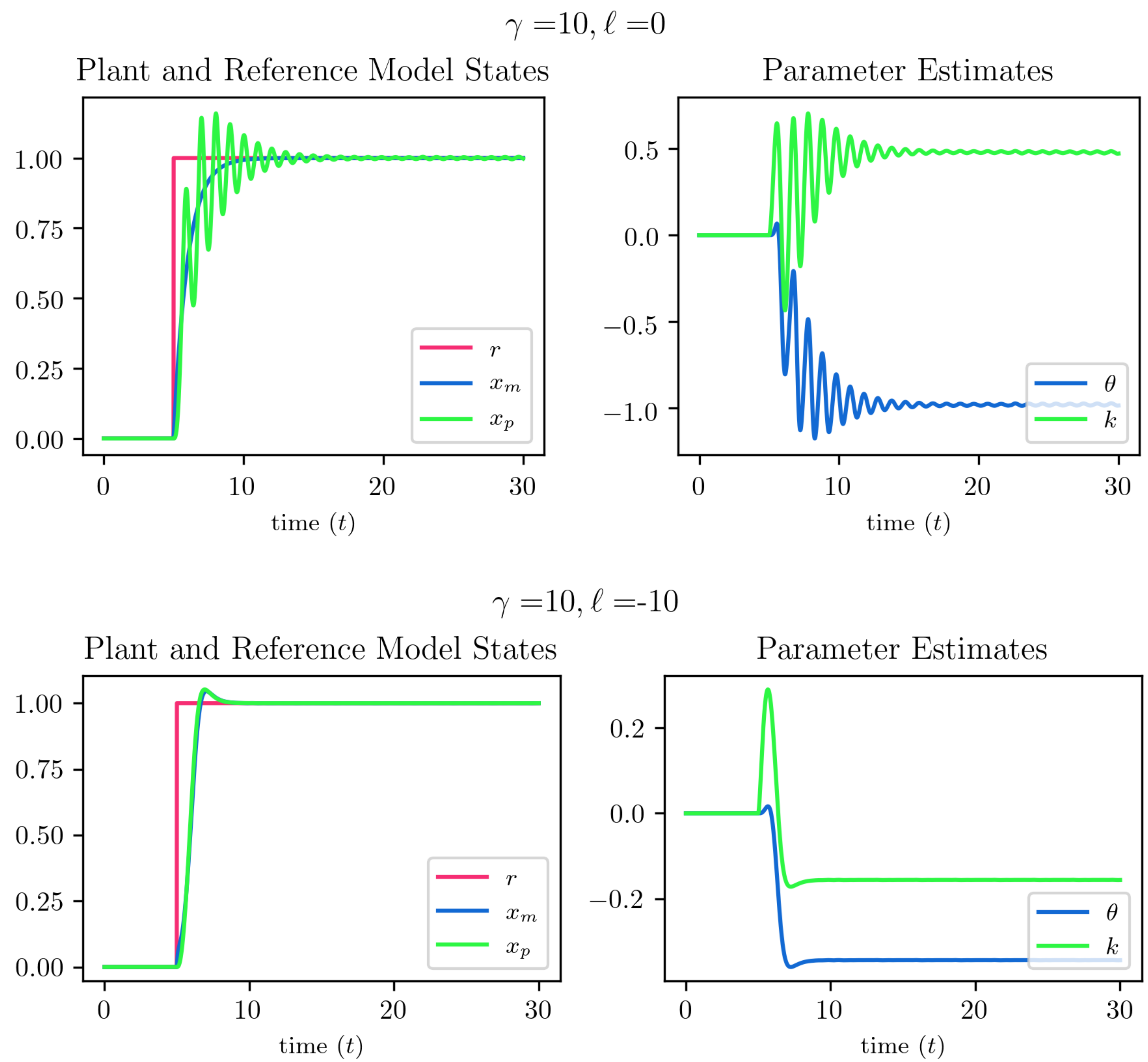


Robustness Properties



Comments on Closed-Loop Reference Model

- The closed-loop reference model provides beneficial properties, especially with regards to transient behavior
- The tuning of L through the many available degrees of freedom enables good transients to be achieved
- CRM also applicable in the case of state feedback



Inner-Loop Controllers

- This control synthesis process is repeated for each of the three inner-loop subsystems
- These combined controllers facilitate command tracking of velocity V_T, pitch rate q, and roll rate p
- The inner-loop control subsystems can be analyzed, and the controller performance on the 6-DOF nonlinear model evaluated, demonstrating the capabilities of this method

Inner-Loop Contributions and Future Work

- The adaptive output feedback method provides a stable controller with **additional available degrees of freedom** over existing approaches to achieve desirable properties for both the baseline *and* adaptive system
  - In practice, good controllers were able to be selected using heuristics for the available degrees of freedom
- The mapping between the degrees of freedom in the feasible LMI, and the solutions $\boldsymbol{L}$ should be investigated, along with analytical solutions for $\boldsymbol{L}$
- The robustness properties of the underlying baseline system should be investigated further
  - In particular, **how can these additional degrees of freedom be leveraged to produce the best controller**

Inner-Loop Contributions and Future Work

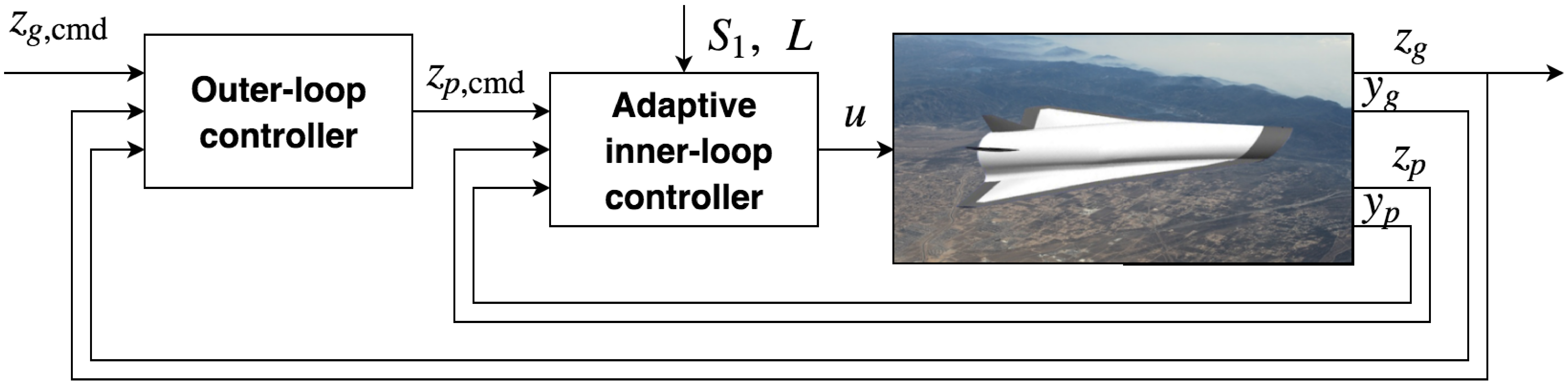
- Existing solutions to this adaptive output feedback problem involved adding fictitous *inputs* to the system in a process called “squaring up” resulting in SPR error dynamics
- Solutions were parameterized by a single scalar, as in LQG
- These fictitious inputs were then removed in controller synthesis
- The result was a stable solution, but with far fewer degrees of freedom to be used for tuning the properties of the combined classical and adaptive system

Outer-Loop Controller

- While the inner-loop controllers satisfied the control goal and facilitated tracking of the vehicle's angular rates, *suitable angular rate commands needed to be specified such that the vehicle traversed some desired trajectory*

- Such commands, generated by an *outer-loop controller*, were often done relying on sufficient timescale separation and without guaranteeing stability of the closed-loop system

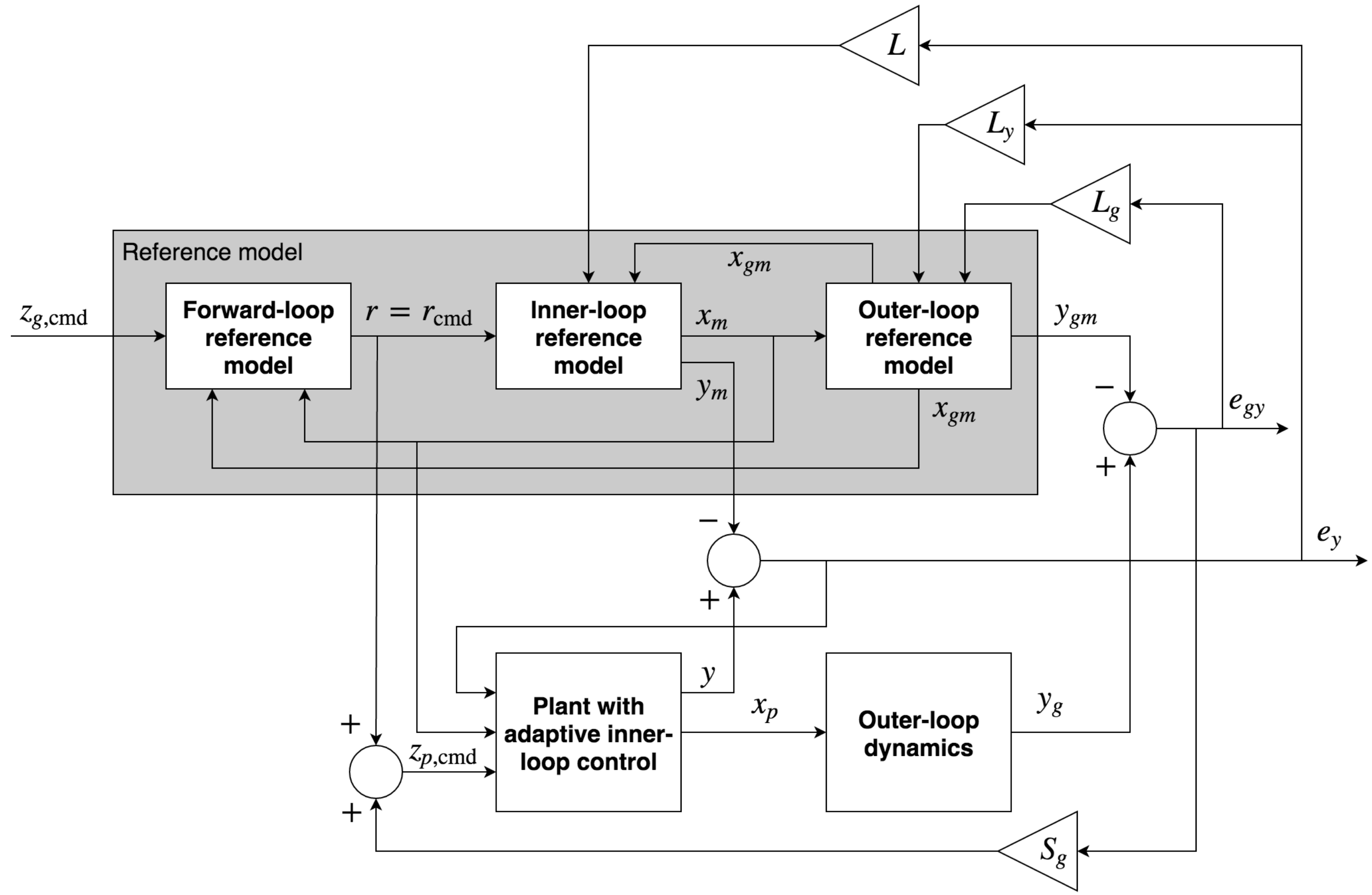
Outer-Loop Controller Block Diagram (1)



- When the outer-loop dynamics are considered, an outer-loop controller can be designed *around the inner-loop controller* as shown
- This requires the selection of some additional feedback elements $\boldsymbol{L_y}$, $\boldsymbol{L_g}$, and $\boldsymbol{S_g}$
- These feedback gains are easily determined as solutions to some feasible LMIs

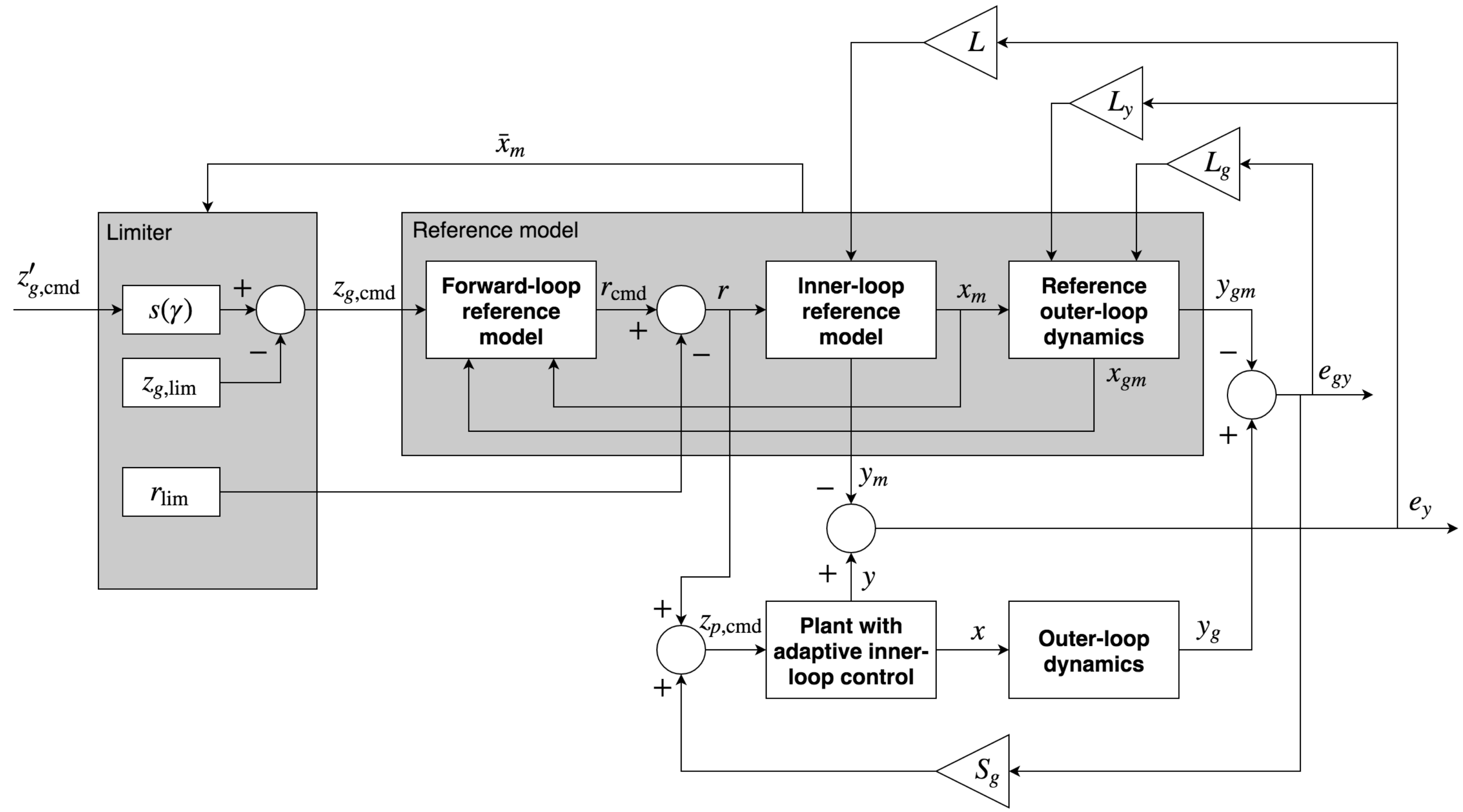
Outer-Loop Controller Block Diagram (2)

- S_g eliminates error from outer-loop coupling
- L_y modifies the outer-loop reference model due to uncertainty
- L_g provides stability of the outer-loop reference model
- Proof of stability not provided here

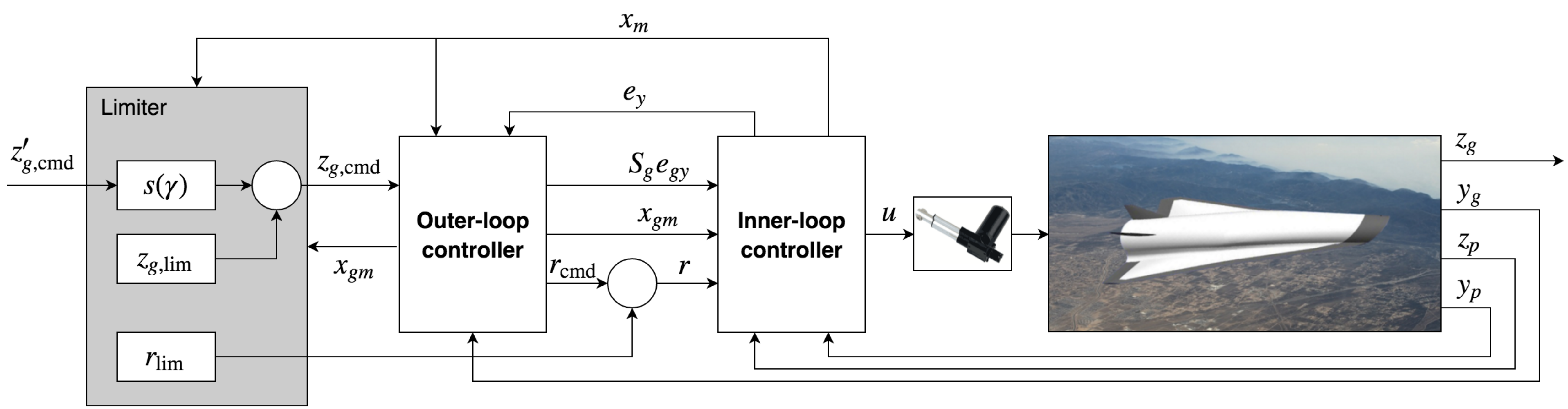


Outer-Loop Controller with Limiter

- This architecture exposes the inner-loop commands (e.g. V_T , p , q) allowing them to be limited by the outer-loop controller
- The limiter is a function of the reference model state, allowing unmeasurable states to be implicitly limited
- Stability with the limiter is guaranteed
- Proof of stability not provided here



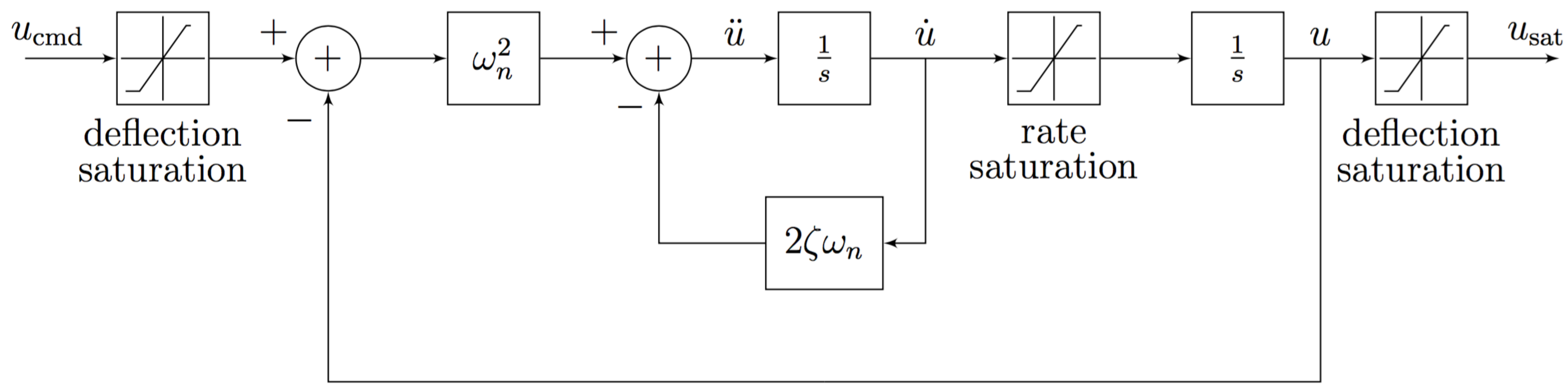
Simulation Results (1)



- Actuator dynamics were included in the simulation model
- Controllers were simulated in discrete time at 100 Hz
- Sensor noise and dynamics, and additional input delays were investigated as well

Simulation Results (1)

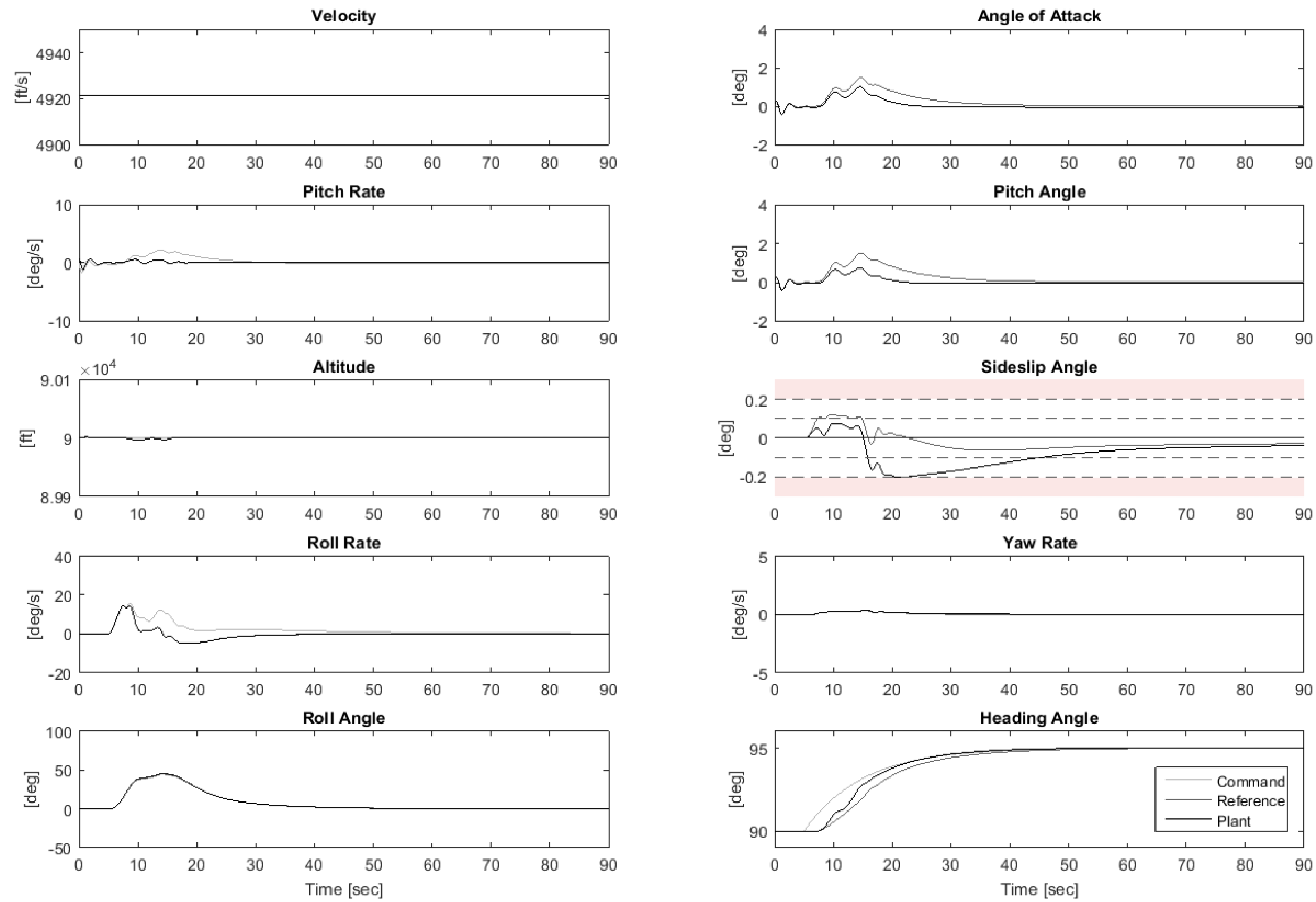
Actuator Dynamics



Parameter	Unit	Value
Surface deflection limit	[deg]	−30 to 30
Surface rate limit	[deg/s]	−100 to 100
Damping ratio ζ		0.7
Natural frequency ω_n	[rad/s]	150

Simulation Results (2)

- This plot is a represantive response to a 5 degree heading change at Mach 5 at an altitude of 80,000 feet
- Control effectiveness was reduced to 20%
- The center-of-gravity was shifted 8 inches rearward
- The rolling moment coefficient was reduced to 10% of the nominal value
- The command limiter was activated at 0.1 degrees of (estimated) sideslip



Summary of Contributions

- The inner-loop adaptive output feedback method provides a stable controller with **additional available degrees of freedom** over existing approaches to achieve desirable properties for both the baseline *and* adaptive system
- The outer-loop control design guarantees stability of the closed-loop system with adaptive inner loop
- The control architecture **accommodates state constraints** even on unmeasurable states

Section 3: Why Blue Origin

Why Blue Origin

- **Company**
  - Mission & Culture
  - Making type 2 decisions quickly
  - Conviction, tenacity, humility
- **Role**
  - Value to Blue Origin
  - Proven hands-on, empathetic leader
  - Opportunity to leverage my abilities to solve hard problems