Nonlinear F-16 Model Description

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Nomenclature

State variables:

V_T	=	total velocity, m/s
α,	=	angle of attack, rad
β	=	angle of sideslip, rad
ϕ, θ, ψ	=	Euler angles, rad
q_0, q_1, q_2, q_3	=	quaternion components
p, q, r	=	body-axis roll, pitch and yaw rates, rad/s
x_E, y_E, z_E	=	aircraft position w.r.t. reference point, m
pow	=	power setting, %

Control variables:

δ_{th}	=	throttle setting, $0-1$
δ_e	=	elevator deflection, rad
δ_a	=	aileron deflection, rad
δ_r	=	rudder deflection, rad
δ_{LEF}	=	leading edge flap deflection, rad

Additional parameters

b	=	reference wing span, m
$\bar{c}$	=	mean aerodynamic chord, m
C_*	=	Aerodynamic coefficient of *
F_T	=	thrust force, N
g_1, g_2, g_3	=	gravity components, m/s^2
H_{eng}	=	engine angular momentum, $kg.m^2/s$
I_x, I_y, I_z	=	moments of inertia $kg.m^2$
I_{xz}	=	product moment of inertia $kg.m^2$
$\bar{L}, \bar{M}, \bar{N}$	=	rolling, pitching and yawing moments, Nm
m	=	mass, kg
M	=	Mach number
p_s	=	static pressure, Pa
$\bar{q}$	=	dynamic pressure, Pa
S	=	reference wing surface area, m^2
T	=	Temperature, K
$T_{s/b}$	=	Rotation matrix body-fixed to stability-axes reference frame
$T_{w/b}$	=	Rotation matrix body-fixed to wind-axes reference frame
u, v, w	=	body-axis velocity components, m/s
$\bar{X}, \bar{Y}, \bar{Z}$	=	axial, lateral and normal force components, N
x_{cg}	=	center of gravity location, m
x_{cg_r}	=	reference center of gravity location, m
γ	=	flight path angle, rad
ρ	=	air density, kg/m^3

1 Aircraft Dynamics

In this section the equations of motion for the F-16 model are derived, this derivation is based on [1, 2, 3]. A very thorough discussion on flight dynamics can be found in the course notes [4].

1.1 Reference Frames

The reference frames used in this thesis are the earth-fixed reference frame F_E, used as the inertial frame and the vehicle carried local earth reference frame F_O with its origin fixed in the center of gravity of the aircraft which is assumed to have the same orientation as F_E; the wind-axes reference frame F_W, obtained from F_O by three successive rotations of χ, γ and μ; the stability-axes reference frame F_S, obtained from F_W by a rotation of $-\beta$; and finally the body-fixed reference frame F_B, obtained from F_S by a rotation of α as is also indicated in Figure 1. The body-fixed reference frame F_B can also be obtained directly from

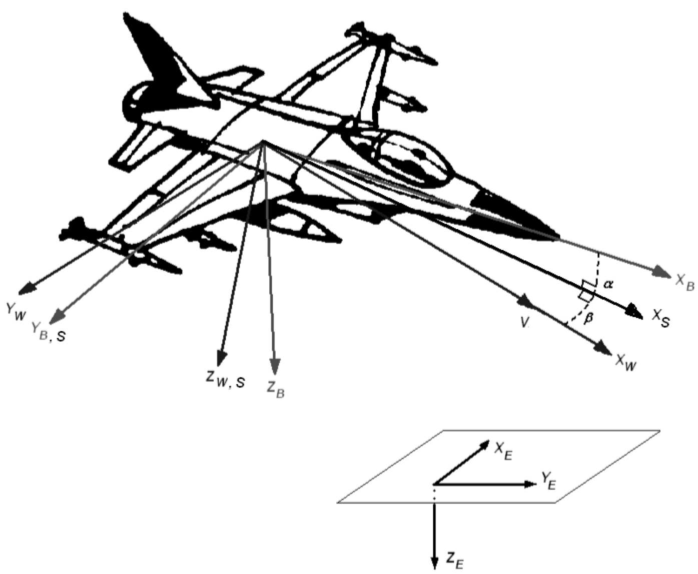


Figure 1: Aircraft reference frames: earth-fixed reference frame F_E, body-fixed reference frame F_B, stability-axes reference frame F_S and wind-axes reference frame F_W

F_O by three successive rotations of yaw angle ψ, pitch angle θ and roll angle ϕ. All reference frames are right-handed and orthogonal. In the earth-fixed reference frame the z_E-axis points to the center of the earth, the x_E-axis points in some arbitrary direction, e.g. the north, and the y_E-axis is perpendicular to the x_E-axis.

The transformation matrices from F_B to F_S and from F_B to F_W are defined as

$$T_{s/b} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}, \quad T_{w/b} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}.$$

1.2 Aircraft Variables

A number of assumptions has to be made, before proceeding with the derivation of the equations of motion:

1. *The aircraft is a rigid-body*, which means that any two points on or within the airframe remain fixed with respect to each other. This assumption is quite valid for a small fighter aircraft.

2. *The earth is flat and non-rotating and regarded as an inertial reference.* This assumption is valid when dealing with control design of aircraft, but not when analyzing inertial guidance systems.

3. *The mass is constant during the time interval over which the motion is considered*, the fuel consumption is neglected during this time-interval. This assumption is necessary to apply Newton's motion laws.

4. *The mass distribution of the aircraft is symmetric relative to the X_BOZ_B-plane*, this implies that the products of inertia I_{yz} and I_{xy} are equal to zero. This assumption is valid for most aircraft.

Under the above assumptions the motion of the aircraft has six degrees of freedom (rotation and translation in three dimensions). The aircraft dynamics can be described by its position, orientation, velocity and angular velocity over time. $\mathbf{p_E} = (x_E, y_E, z_E)^T$ is the position vector expressed in an earth-fixed coordinate system. $\mathbf{V}$ is the velocity vector given by $\mathbf{V} = (u, v, w)^T$, where u is the longitudinal velocity, v the lateral velocity and w the normal velocity. The orientation vector is given by $\mathbf{\Phi} = (\phi, \theta, \psi)^T$, where ϕ is the roll angle, θ the pitch angle and ψ the yaw angle, and the angular velocity vector is given by $\boldsymbol{\omega} = (p, q, r)^T$, where p,q and r are the roll, pitch and yaw angular velocities, respectively. Various components of the aircraft motions are illustrated in Figure 2.
The relation between the attitude vector $\mathbf{\Phi}$ and the angular velocity vector $\boldsymbol{\omega}$ is given as

$$\dot{\mathbf{\Phi}} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \boldsymbol{\omega} \tag{1}$$

Defining V_T as the total velocity and using Figure 2, the following relations can be derived:

$$\begin{aligned} V_T &= \sqrt{u^2 + v^2 + w^2} \\ \alpha &= \arctan\frac{w}{u} \\ \beta &= \arcsin\frac{v}{V_T} \end{aligned} \tag{2}$$

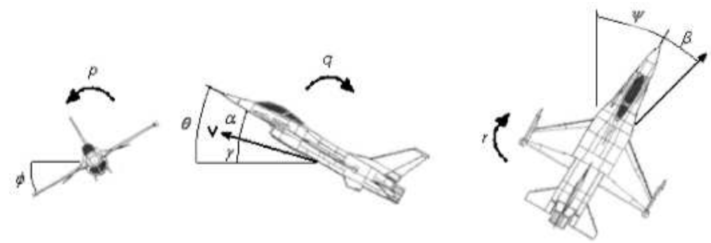


Figure 2: Aircraft orientation angles ϕ, θ and ψ, aerodynamic angles α and β, and the angular rates p, q and r. All angles and rates are defined positive in the figure.

Furthermore, when $\beta = \phi = 0$, the flight path angle γ can be defined as

$$\gamma = \theta - \alpha \tag{3}$$

1.3 Equations of Motion for a Rigid Body Aircraft

The equations of motion for the aircraft can be derived from Newton's Second Law of motion, which states that the summation of all external forces acting on a body must be equal to the time rate of change of its momentum, and the summation of the external moments acting on a body must be equal to the time rate of change of its angular momentum. In the inertial, Earth-fixed reference frame F_E, Newton's Second Law can be expressed by two vector equations [4]

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V})\Big]_E \tag{4}$$

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}\Big]_E \tag{5}$$

where $\mathbf{F}$ represents the sum of all externally applied forces, m is the mass of the aircraft, $\mathbf{M}$ represents the sum of all applied torques and $\mathbf{H}$ is the angular momentum.

1.3.1 Force Equation

First, to further evaluate the force equation (4) it is necessary to obtain an expression for the time rate of change of the velocity vector with respect to earth. This process is complicated by the fact that the velocity vector may be rotating while it is changing in magnitude. Using the equation of Coriolis in appendix A of [1] results in

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V})\Big]_B + \boldsymbol{\omega} \times m\mathbf{V}, \tag{6}$$

where$\boldsymbol{\omega}$ is the total angular velocity of the aircraft with respect to the earth (inertial reference frame). Expressing the vectors as the sum of their components with respect to the body-fixed reference frame F_B gives

$$\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w \tag{7}$$
$$\boldsymbol{\omega} = \mathbf{i}p + \mathbf{j}q + \mathbf{k}r \tag{8}$$

where **i**,**j** and **k** are unit vectors along the aircraft's x_B, y_B and z_B axes, respectively. Expanding (6) using (7), (8) results in

$$\begin{aligned} F_x &= m(\dot{u} + qw - rv) \\ F_y &= m(\dot{v} + ru - pw) \\ F_z &= m(\dot{w} + pv - qu) \end{aligned} \tag{9}$$

where the external forces F_x, F_y and F_z depend on the weight vector $\mathbf{W}$, the aerodynamic force vector $\mathbf{R}$ and the thrust vector $\mathbf{E}$. It is assumed the thrust produced by the engine, F_T, acts parallel to the aircraft's x_B-axis. Hence,

$$\begin{aligned} E_x &= F_T \\ E_y &= 0 \\ E_z &= 0 \end{aligned} \tag{10}$$

The components of $\mathbf{W}$ and $\mathbf{R}$ along the body-axes are

$$\begin{aligned} W_x &= -mg\sin\theta \\ W_y &= mg\sin\phi\cos\theta \\ W_z &= mg\cos\phi\cos\theta \end{aligned} \tag{11}$$

and

$$\begin{aligned} R_x &= \bar{X} \\ R_y &= \bar{Y} \\ R_z &= \bar{Z} \end{aligned} \tag{12}$$

where g is the gravity constant. The size of the aerodynamic forces $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ is determined by the amount of air diverted by the aircraft in different directions. The amount of air diverted by the aircraft mainly depends on the following factors:

- the total velocity V_T (or Mach number M) and density of the airflow ρ,
- the geometry of the aircraft: wing area S, wing span b and mean aerodynamic chord $\bar{c}$,
- the orientation of the aircraft relative to the airflow: angle of attack α and side slip angle β,
- the control surface deflections δ,
- the angular rates p, q, r,

There are other variables such as the time derivatives of the aerodynamic angles that also play a role, but these effects are less prominent, since it is assumed that the aircraft is a rigid body. This motivates the standard way of modeling the aerodynamic force:

$$\begin{aligned}\bar{X} &= \bar{q}SC_{X_T}(\alpha, \beta, p, q, r, \delta, ...)\\ \bar{Y} &= \bar{q}SC_{Y_T}(\alpha, \beta, p, q, r, \delta, ...)\\ \bar{Z} &= \bar{q}SC_{Z_T}(\alpha, \beta, p, q, r, \delta, ...)\end{aligned} \tag{13}$$

where $\bar{q} = \frac{1}{2}\rho V_T^2$ is the aerodynamic pressure. The air density ρ is calculated according to the International Standard Atmosphere (ISA) as given in Section B. The coefficients C_{X_T}, C_{Y_T} and C_{Z_T} are usually obtained from (virtual) wind tunnel data and flight tests. Combining equations (11) and (12) and the thrust components (10) with (9), results in the complete body-axes force equation:

$$\begin{aligned}\bar{X} + F_T - mg\sin\theta &= m(\dot{u} + qw - rv)\\ \bar{Y} + mg\sin\phi\cos\theta &= m(\dot{v} + ru - pw)\\ \bar{Z} + mg\cos\phi\sin\theta &= m(\dot{w} + pv - qu)\end{aligned} \tag{14}$$

1.3.2 Moment Equation

To obtain the equations for angular motion, consider again Equation (5). The time rate of change of $\mathbf{H}$ is required and since $\mathbf{H}$ can change in magnitude and direction, (5) can be written as

$$\mathbf{M} = \left.\frac{d\mathbf{H}}{dt}\right]_B + \boldsymbol{\omega} \times \mathbf{H} \tag{15}$$

In the body-fixed reference frame, under the rigid body and constant mass assumptions, the angular momentum $\mathbf{H}$ can be expressed as

$$\mathbf{H} = I\boldsymbol{\omega} \tag{16}$$

where, under the symmetrical aircraft assumption, the inertia matrix is defined as

$$I = \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \tag{17}$$

Expanding (15) using (16) results in

$$\begin{aligned}M_x &= \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz}\\ M_y &= \dot{q}I_y + pq(I_x - I_z) + (p^2 - r^2)I_{xz}\\ M_z &= \dot{r}I_z - \dot{p}I_{xz} + pq(I_y - I_x) + qrI_{xz}.\end{aligned} \tag{18}$$

The external moments M_x, M_y and M_z are those due to aerodynamics and engine angular momentum. As a result the aerodynamic moments are

$$
\begin{aligned}
M_x &= \bar{L} \\
M_y &= \bar{M} - rH_{eng} \\
M_z &= \bar{N} + qH_{eng}
\end{aligned} \tag{19}
$$

where $\bar{L}$,$\bar{M}$ and $\bar{N}$ are the aerodynamic moments and H_{eng} is the engine angular momentum. Note that the engine angular momentem is assumed to be parallel to the body x-axis of the aircraft. The aerodynamic moments can be expressed in a similar way as the aerodynamic forces in Equation (13):

$$
\begin{aligned}
\bar{L} &= \bar{q}SbC_{l_T}(\alpha, \beta, p, q, r, \delta, ...) \\
\bar{M} &= \bar{q}S\bar{c}C_{m_T}(\alpha, \beta, p, q, r, \delta, ...) \\
\bar{N} &= \bar{q}SbC_{n_T}(\alpha, \beta, p, q, r, \delta, ...)
\end{aligned} \tag{20}
$$

Combining (18) and (19), the complete body-axis moment equation is formed as

$$
\begin{aligned}
\bar{L} &= \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz} \\
\bar{M} - rH_{eng} &= \dot{q}I_y + pq(I_x - I_z) + (p^2 - r^2)I_{xz} \\
\bar{N} + qH_{eng} &= \dot{r}I_z - \dot{p}I_{xz} + pq(I_y - I_x) + qrI_{xz}.
\end{aligned} \tag{21}
$$

1.4 Gathering the Equations of Motion

1.4.1 Euler Angles

The equations of motion derived in the previous sections are now collected and written as a system of twelve scalar first order differential equations.

$$\dot{u} = rv - qw - g\sin\theta + \frac{1}{m}(\bar{X} + F_T) \tag{22}$$

$$\dot{v} = pw - ru + g\sin\phi\cos\theta + \frac{1}{m}\bar{Y} \tag{23}$$

$$\dot{w} = qu - pv + g\cos\phi\cos\theta + \frac{1}{m}\bar{Z} \tag{24}$$

$$\dot{p} = (c_1 r + c_2 p)q + c_3\bar{L} + c_4(\bar{N} + qH_{eng}) \tag{25}$$

$$\dot{q} = c_5 pr - c_6(p^2 - r^2) + c_7(\bar{M} - rH_{eng}) \tag{26}$$

$$\dot{r} = (c_8 p - c_2 r)q + c_4\bar{L} + c_9(\bar{N} + qH_{eng}) \tag{27}$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi) \tag{28}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{29}$$

$$\dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta} \tag{30}$$

$$
\begin{aligned}
\dot{x}_E &= u\cos\psi\cos\theta + v(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) \\
&+ w(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) && (31) \\
\dot{y}_E &= u\sin\psi\cos\theta + v(\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi) \\
&+ w(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) && (32) \\
\dot{z}_E &= -u\sin\theta + v\cos\theta\sin\phi + w\cos\theta\cos\phi && (33)
\end{aligned}
$$

where

$$
\begin{array}{rcl}
\Gamma c_1 = (I_y - I_z)I_z - I_{xz}^2 & \Gamma c_4 = I_{xz} & c_7 = \dfrac{1}{I_y} \\
\Gamma c_2 = (I_x - I_y + I_z)I_{xz} & c_5 = \frac{I_z - I_x}{I_y} & \Gamma c_8 = I_x(I_x - I_y) + I_{xz}^2 \\
\Gamma c_3 = I_z & c_6 = \frac{I_{xz}}{I_y} & \Gamma c_9 = I_x
\end{array}
$$

with $\Gamma = I_x I_z - I_{xz}^2$.

1.4.2 Quaternions

The above equations of motion make use of Euler angle approach for the orientation model. The disadvantage of the Euler angle method is that the differential equations for $\dot{p}$ and $\dot{r}$ become singular when pitch angle θ passes through $\pm\frac{\pi}{2}$. To avoid these singularities quaternions are used for the aircraft orientation presentation. A detailed explanation about quaternions and their properties can be found in [3]. With the quaternions presentation the aircraft system representation exists of 13 scalar first order differential equations:

$$
\begin{aligned}
\dot{u} &= rv - qw + \frac{1}{m}\left(\bar{X} + F_T\right) + 2(q_1q_3 - q_0q_2)g && (34) \\
\dot{v} &= pw - ru + \frac{1}{m}\bar{Y} + 2(q_2q_3 + q_0q_1)g && (35) \\
\dot{w} &= qu - pv + \frac{1}{m}\bar{Z} + (q_0^2 - q_1^2 - q_2^2 + q_3^2)g && (36)
\end{aligned}
$$

$$
\begin{aligned}
\dot{p} &= (c_1r + c_2p)q + c_3\bar{L} + c_4(\bar{N} + qH_{eng}) && (37) \\
\dot{q} &= c_5pr - c_6(p^2 - r^2) + c_7(\bar{M} - rH_{eng}) && (38) \\
\dot{r} &= (c_8p - c_2r)q + c_4\bar{L} + c_9(\bar{N} + qH_{eng}) && (39)
\end{aligned}
$$

$$
\dot{\mathbf{q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{40}
$$

$$
\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{41}
$$

where

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \pm \begin{bmatrix} \cos\phi/2\cos\theta/2\cos\psi/2 + \sin\phi/2\sin\theta/2\sin\psi/2 \\ \sin\phi/2\cos\theta/2\cos\psi/2 - \cos\phi/2\sin\theta/2\sin\psi/2 \\ \cos\phi/2\sin\theta/2\cos\psi/2 + \sin\phi/2\cos\theta/2\sin\psi/2 \\ \cos\phi/2\cos\theta/2\sin\psi/2 - \sin\phi/2\sin\theta/2\cos\psi/2 \end{bmatrix}$$

Using (40) to describe the attitude dynamics means that the four differential equations are integrated as if all quaternion components were independent. Therefore, the normalization condition $|q| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1$ and the derivative constraint $q_0\dot{q}_0 + q_1\dot{q}_1 + q_2\dot{q}_2 + q_3\dot{q}_3 = 0$ may not be satisfied after performing an integration step due to numerical round-off errors. After each integration step the constraint may be re-established by subtracting the discrepancy from the quaternion derivatives. The corrected quaternion dynamics are [6]

$$\dot{\mathbf{q}}' = \dot{\mathbf{q}} - \delta\mathbf{q}, \tag{42}$$

where $\delta = q_0\dot{q}_0 + q_1\dot{q}_1 + q_2\dot{q}_2 + q_3\dot{q}_3$.

1.4.3 Wind Axes Force Equations

For control design it is more convenient to transform the force equations (34)-(36) to the wind-axes reference frame. Taking the derivative of Eq. (2) results in [3]

$$\begin{aligned} \dot{V}_T &= \frac{1}{m}\left(-D + F_T\cos\alpha\cos\beta + mg_1\right) && (43) \\ \dot{\alpha} &= q - (p\cos\alpha + r\sin\alpha)\tan\beta - \frac{1}{mV_T\cos\beta}\left(L + F_T\sin\alpha - mg_3\right) && (44) \\ \dot{\beta} &= p\sin\alpha - r\cos\alpha + \frac{1}{mV_T}\left(Y - F_T\cos\alpha\sin\beta + mg_2\right) && (45) \end{aligned}$$

where the drag force D, the side force Y and the lift force L are defined as

$$\begin{aligned} D &= -\bar{X}\cos\alpha\cos\beta - \bar{Y}\sin\beta - \bar{Z}\sin\alpha\cos\beta \\ Y &= -\bar{X}\cos\alpha\sin\beta + \bar{Y}\cos\beta - \bar{Z}\sin\alpha\sin\beta \\ L &= \bar{X}\sin\alpha - \bar{Z}\cos\alpha \end{aligned}$$

and the gravity components as

$$\begin{aligned} g_1 &= g\left(-\cos\alpha\cos\beta\sin\theta + \sin\beta\sin\phi\cos\theta + \sin\alpha\cos\beta\cos\phi\cos\theta\right) \\ g_2 &= g\left(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta\right) \\ g_3 &= g\left(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta\right) \end{aligned}$$

2 Control Variables and Engine Modeling

The F-16 model allows for control over thrust, elevator, ailerons, and rudder. The thrust is measured in Newtons. All deflections are defined positive in the conventional way, i.e. positive thrust causes an increase in acceleration along the x_B-axis, a positive elevator deflection results in a decrease in pitch rate, a positive aileron deflection gives a decrease in roll rate and a positive rudder deflection decreases the yaw rate. The F-16 also has a leading edge flap, which helps to fly the aircraft at high angles of attack. The deflection of the leading edge flap δ_{LEF} is not controlled directly by the pilot, but is governed by the following transfer function dependent on angle of attack α and static and dynamic pressures:

$$\delta_{LEF} = 1.38\frac{2s + 7.25}{s + 7.25}\alpha - 9.05\frac{\bar{q}}{p_s} + 1.45$$

The differential elevator deflection, trailing edge flap, landing gear and speed brakes are not included in the model, since no data is publicly available. The control surfaces of the F-16 are driven by servo-controlled actuators to produce the deflections commanded by the flight control system. The actuators of the control surfaces are modeled as a first-order low-pass filters with certain gain and saturation limits in range and deflection rate. These limits can be found in Table 1. The gains of the actuators are 1/0.136 for the leading edge flap and 1/0.0495 for the other control surfaces. The maximum values and units for all control variables are given in Table 1.

Table 1: The control input units and maximum values

Control	units	MIN.	MAX.	rate limit
Elevator	deg	-25	25	$\pm$ 60 deg/s
Ailerons	deg	-21.5	21.5	$\pm$ 80 deg/s
Rudder	deg	-30	30	$\pm$ 120 deg/s
Leading edge flap	deg	0	25	$\pm$ 25 deg/s

The Lockheed-Martin F-16A is powered by an after-burning turbofan jet engine, which is modeled taking into account throttle gearing and engine power level lag. The thrust response is modeled with a first order lag, where the lag time constant is a function of the current engine power level and the commanded power. The commanded power level to the throttle position is a linear relationship apart from a change in slope when the military power level is reached at 0.77 throttle setting [5]:

$$P_c(\delta_{th}) = \begin{cases} 64.94\delta_{th} & \text{if} \quad \delta_{th} \leq 0.77 \\ 217.38\delta_{th} - 117.38 & \text{if} \quad \delta_{th} > 0.77 \end{cases} . \tag{46}$$

Note that the throttle position is limited to the range $0 \leq \delta_{th} \leq 1$. The derivative of the actual power level P_a is given by [5]

$$\dot{P}_a = \frac{1}{\tau_{eng}}\left(P_c - P_a\right), \tag{47}$$

where

$$P_c = \begin{cases} P_c & \text{if} \quad P_c \geq 50 \text{ and } P_a \geq 50 \\ 60 & \text{if} \quad P_c \leq 50 \text{ and } P_a < 50 \\ 40 & \text{if} \quad P_c < 50 \text{ and } P_a \geq 50 \\ P_c & \text{if} \quad P_c < 50 \text{ and } P_a < 50 \end{cases}$$

$$\frac{1}{\tau_{eng}} = \begin{cases} 5.0 & \text{if} \quad P_c \geq 50 \text{ and } P_a \geq 50 \\ \frac{1}{\tau_{eng}}^* & \text{if} \quad P_c \leq 50 \text{ and } P_a < 50 \\ 5.0 & \text{if} \quad P_c < 50 \text{ and } P_a \geq 50 \\ \frac{1}{\tau_{eng}}^* & \text{if} \quad P_c < 50 \text{ and } P_a < 50 \end{cases}$$

$$\frac{1}{\tau_{eng}}^* = \begin{cases} 1.0 & \text{if} \quad (P_c - P_a) \leq 25 \\ 0.1 & \text{if} \quad (P_c - P_a) \geq 50 \\ 1.9 - 0.036\,(P_c - P_a) & \text{if} \quad 25 < (P_c - P_a) < 50 \end{cases} .$$

The engine thrust data is available in a tabular form as a function of actual power, altitude and Mach number over the ranges $0 \geq h \geq 15240$ m and $0 \geq M \geq 1$ for idle, military and maximum power settings [5]. The thrust is computed as

$$F_T = \begin{cases} T_{idle} + (T_{mil} - T_{idle})\,\frac{P_a}{50} & \text{if} \quad P_a < 50 \\ T_{mil} + (T_{max} - T_{mil})\,\frac{P_a - 50}{50} & \text{if} \quad P_a \geq 50 \end{cases} . \tag{48}$$

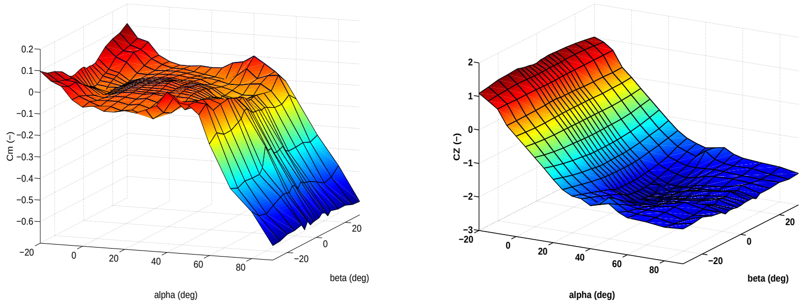
The engine angular momentum is assumed to be acting along the x_B-axis with a constant value of 216.9 kg.m^2/s.

3 Geometry and Aerodynamic Data

The relevant geometry data of the F-16A can be found in Table 2. The aerodynamic data of the F-16 model have been derived from low-speed static and dynamic (force oscillation) wind-tunnel tests conducted with sub-scale models in wind-tunnel facilities at the NASA Ames and Langley Research Centers [5]. The aerodynamic data in [5] is given in tabular form and is valid for the following subsonic flight envelope:

- $-20 \leq \alpha \leq 90$ degrees;
- $-30 \leq \beta \leq 30$ degrees.

Two examples of the aerodynamic data for the F-16 model can be found in Figure 3. The pitch moment coefficient C_m and the C_Z both depend on three variables: angle of attack, sideslip angle and elevator deflection.
The various aerodynamic contributions to a given force or moment coefficient



(a) C_m for $\delta_e = 0$ (b) C_Z for $\delta_e = 0$

Figure 3: Two examples of the aerodynamic coefficient data for the F-16 obtained from wind-tunnel tests.

are summed as follows.
For the X-axis force coefficient C_{X_T}:

$$\begin{aligned} C_{X_T} &= C_X(\alpha, \beta, \delta_e) + \delta C_{X_{LEF}} \Big(1 - \frac{\delta_{LEF}}{25}\Big) \\ &+ \frac{q\bar{c}}{2V_T}\Big[C_{X_q}(\alpha) + \delta C_{X_{q_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \end{aligned}$$

where

$$\delta C_{X_{LEF}} = C_{X_{LEF}}(\alpha, \beta) - C_X(\alpha, \beta, \delta_e = 0^o)$$

For the Y-axis force coefficient C_{Y_T}:

$$\begin{aligned} C_{Y_T} &= C_Y(\alpha, \beta) + \delta C_{Y_{LEF}} \Big(1 - \frac{\delta_{LEF}}{25}\Big) \\ &+ \Big[\delta C_{Y_{\delta_a}} + \delta C_{Y_{\delta_{a_{LEF}}}} \Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big]\Big(\frac{\delta_a}{20}\Big) \\ &+ \delta C_{Y_{\delta_r}} \Big(\frac{\delta_r}{30}\Big) + \frac{rb}{2V_T}\Big[C_{Y_r}(\alpha) + \delta C_{Y_{r_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \\ &+ \frac{pb}{2V_T}\Big[C_{Y_p}(\alpha) + \delta C_{Y_{p_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \end{aligned}$$

where

$$\begin{aligned} \delta C_{Y_{LEF}} &= C_{Y_{LEF}}(\alpha, \beta) - C_Y(\alpha, \beta) \\ \delta C_{Y_{\delta_a}} &= C_{Y_{\delta_a}}(\alpha, \beta) - C_Y(\alpha, \beta) \\ \delta C_{Y_{\delta_{a_{LEF}}}} &= C_{Y_{\delta_{a_{LEF}}}}(\alpha, \beta) - C_{Y_{LEF}}(\alpha, \beta) - \delta C_{Y_{\delta_a}} \\ \delta C_{Y_{\delta_r}} &= C_{Y_{\delta_r}}(\alpha, \beta) - C_Y(\alpha, \beta) \end{aligned}$$

For the Z-axis force coefficient C_{Z_T}:

$$\begin{aligned} C_{Z_T} &= C_Z(\alpha, \beta, \delta_e) + \delta C_{Z_{LEF}}\Big(1 - \frac{\delta_{LEF}}{25}\Big) \\ &+ \frac{q\bar{c}}{2V_T}\Big[C_{Z_q}(\alpha) + \delta C_{Z_{q_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \end{aligned}$$

where

$$\delta C_{Z_{LEF}} = C_{Z_{LEF}}(\alpha, \beta) - C_Z(\alpha, \beta, \delta_e = 0^o)$$

For the rolling-moment coefficient C_{l_T}:

$$\begin{aligned} C_{l_T} &= C_l(\alpha, \beta, \delta_e) + \delta C_{l_{LEF}}\Big(1 - \frac{\delta_{LEF}}{25}\Big) \\ &+ \Big[\delta C_{l_{\delta_a}} + \delta C_{l_{\delta_{a_{LEF}}}}\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big]\Big(\frac{\delta_a}{20}\Big) \\ &+ \delta C_{l_{\delta_r}}\Big(\frac{\delta_r}{30}\Big) + \frac{rb}{2V_T}\Big[C_{l_r}(\alpha) + \delta C_{l_{r_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \\ &+ \frac{pb}{2V_T}\Big[C_{l_p}(\alpha) + \delta C_{l_{p_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] + \delta C_{l_\beta}(\alpha)\beta \end{aligned}$$

where

$$\begin{aligned} \delta C_{l_{LEF}} &= C_{l_{LEF}}(\alpha, \beta) - C_l(\alpha, \beta, \delta_e = 0^o) \\ \delta C_{l_{\delta_a}} &= C_{l_{\delta_a}}(\alpha, \beta) - C_l(\alpha, \beta, \delta_e = 0^o) \\ \delta C_{l_{\delta_{a_{LEF}}}} &= C_{l_{\delta_{a_{LEF}}}}(\alpha, \beta) - C_{l_{LEF}}(\alpha, \beta) - \delta C_{l_{\delta_a}} \\ \delta C_{l_{\delta_r}} &= C_{l_{\delta_r}}(\alpha, \beta) - C_l(\alpha, \beta, \delta_e = 0^o) \end{aligned}$$

For the pitching-moment coefficient C_{m_T}:

$$\begin{aligned} C_{m_T} &= C_m(\alpha, \beta, \delta_e) + C_{Z_T}\Big[x_{cg_r} - x_{cg}\Big] + \delta C_{m_{LEF}}\Big(1 - \frac{\delta_{LEF}}{25}\Big) \\ &+ \frac{q\bar{c}}{2V_T}\Big[C_{m_q}(\alpha) + \delta C_{m_{q_{LEF}}}(\alpha)\Big]\Big(1 - \frac{\delta_{LEF}}{25}\Big) + \delta C_m(\alpha) + \delta C_{m_{ds}}(\alpha, \delta_e) \end{aligned}$$

where

$$\delta C_{m_{LEF}} = C_{m_{LEF}}(\alpha, \beta) - C_m(\alpha, \beta, \delta_e = 0^o)$$

For the yawing-moment coefficient C_{n_T}:

$$\begin{aligned} C_{n_T} &= C_n(\alpha, \beta, \delta_e) + \delta C_{n_{LEF}}\Big(1 - \frac{\delta_{LEF}}{25}\Big) - C_{Y_T}\Big[x_{cg_r} - x_{cg}\Big]\frac{\bar{c}}{b} \\ &+ \Big[\delta C_{n_{\delta_a}} + \delta C_{n_{\delta_{a_{LEF}}}}\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big]\Big(\frac{\delta_a}{20}\Big) \\ &+ \delta C_{n_{\delta_r}}\Big(\frac{\delta_r}{30}\Big) + \frac{rb}{2V_T}\Big[C_{n_r}(\alpha) + \delta C_{n_{r_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \\ &+ \frac{pb}{2V_T}\Big[C_{n_p}(\alpha) + \delta C_{n_{p_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] + \delta C_{n_\beta}(\alpha)\beta \end{aligned}$$

where

$$
\begin{aligned}
\delta C_{n_{LEF}} &= C_{n_{LEF}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_e = 0^o) \\
\delta C_{n_{\delta_a}} &= C_{n_{\delta_a}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_e = 0^o) \\
\delta C_{n_{\delta_{a_{LEF}}}} &= C_{n_{\delta_{a_{LEF}}}}(\alpha, \beta) - C_{n_{LEF}}(\alpha, \beta) - \delta C_{n_{\delta_a}} \\
\delta C_{n_{\delta_r}} &= C_{n_{\delta_r}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_e = 0^o)
\end{aligned}
$$

4 MATLAB/Simulink script files

Descriptions are included in the files. Most files are based on or copied from the F-16 models by R. S. Russell [7] and Y. Huo. The S-functions

```
F_16dyn.c
```

and

```
F_16dynam.c
```

both contain the aircraft dynamics, they only differ in the use of quaternions vs. Euler angles. the

```
runF16model.m
```

script file allows the user to trim the aircraft model for steady-state flight conditions.

Other files:

```
tgear.m
trim_fun.m

lofi_f16_aerodata.c
hifi_f16_aerodata.c
engine_model.c
ISA_atmos.c
mexndinterp.c
```

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A F-16 Geometry

Table 2: F-16 parameters.

Parameter	Symbol	Value
aircraft mass (kg)	m	9295.44
wing span (m)	b	9.144
wing area (m^2)	S	27.87
mean aerodynamic chord (m)	$\bar{c}$	3.45
roll moment of inertia (kg.m^2)	I_x	12874.8
pitch moment of inertia (kg.m^2)	I_y	75673.6
yaw moment of inertia (kg.m^2)	I_z	85552.1
product moment of inertia (kg.m^2)	I_{xz}	1331.4
product moment of inertia (kg.m^2)	I_{xy}	0.0
product moment of inertia (kg.m^2)	I_{yz}	0.0
c.g. location (m)	x_{cg}	$0.3\bar{c}$
reference c.g. location (m)	x_{cgr}	$0.35\bar{c}$
engine angular momentum (kg.m^2/s)	H_{eng}	216.9

B ISA Atmospheric Model

For the atmospheric data an approximation of the International Standard Atmosphere (ISA) is used [4].

$$
\begin{aligned}
T &= \begin{cases} T_0 + \lambda h & \text{if} \quad h \leq 11000 \\ T_{(h=11000)} & \text{if} \quad h > 11000 \end{cases} \\
p &= \begin{cases} p_0 \left(1 + \lambda \frac{h}{T_0}\right)^{-\frac{g_0}{R\lambda}} & \text{if} \quad h \leq 11000 \\ p_{(h=11000)} e^{-\frac{g}{RT_{(h=11000)}}(h-11000)} & \text{if} \quad h > 11000 \end{cases} \\
\rho &= \frac{p}{RT} \\
a &= \sqrt{\gamma R T}
\end{aligned}
$$

where $T_0 = 288.15$ K is the temperature at sea level, $p_0 = 101325$ N/m^2 the pressure at sea level, $R = 287.05$ J/kg.K the gas constant of air, $g_0 = 9.80665$ m/s^2 the gravity constant at sea level, $\lambda = dT/dh = -0.0065$ K/m the temperature gradient and $\gamma = 1.41$ the isentropic expansion factor for air. Given the aircraft's altitude (h in meters) it returns the current temperature (T in Kelvin), the current air pressure (p in N/m^2), the current air density (ρ in kg/m^3) and the speed of sound (a in m/s).