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FINE 452 Applied Quantitative Finance Assignment 2

Question 1

Dear investors,

1997 was an interesting year to say the least. LTCM delivered record breaking returns to our investors, but not all was well in the world. The East Asian crisis last summer taught institutional investors some valuable lessons. While some learned, others decided to move away from some business activities. We at LTCM decided to learn and introduce a new product offering - we will be selling structured products comprised of long-dated options.

Banks use these structured products to construct Guaranteed Investment Certificates (GICs) and other products for their clients. In order to earn profits, banks need long-dated call options which is our value proposition, allowing us to charge a premium and share the profits. This product allows investors to purchase 'insurance' for volatility on long-dated options, creating value due to the flexibility it offers. Given the risks associated with selling long-dated options and the events that unfolded in Asia last year, we at LTCM believe that using a delta hedging strategy is paramount to safeguarding the capital of our investors.

Delta is the relation between the value of the option and the underlying asset. Our delta hedging strategy works by allowing us to manage the risk we take on from selling (shorting) long-dated options. Since volatility has a big effect on long-dated options, we will attempt to hedge ourselves against shocks in volatility. To achieve close to zero delta, we go long index futures by buying a certain amount of the index. This purchase is partially financed by borrowing a certain amount. We use the universally accepted Black-Scholes model to calculate the number of index futures we buy and the amount that we borrow to finance this purchase.

One potential risk of the strategy is volatility. Delta hedging is only a linear approximation and does not mitigate all volatility, thus we are still at risk of unusually large swings in volatility. However, sudden changes in volatility increase the value of the calls, and put us at risk of paying out. What makes it hard to implement this strategy is that volatility is hard to predict. This risk is magnified by the long time to maturity of our product. This means there is more time for something unexpected to occur. When volatility works as we expect, this longer time give us the option to charge a higher option premium and make more money, but it also means we put ourselves at greater risk if something goes wrong. Further, there is a time component to delta hedging. The delta is constantly changing, and rebalancing the portfolio to purchase or sell stocks as necessary to meet the delta's requirements incurs transaction costs. Thus, a perfect hedge is impossible to achieve but we are confident that our new product offering will generate high returns for our investors and our delta hedging strategy will keep your capital safe.

Question 2

The profit we would be able to book up front is $33,908,450 on 100,000 calls with a strike of 1000, or $339.08 per call.

Question 3

We can put in a delta hedge by buying the index, partially financed by borrowing based on the Black-Scholes model. The likely impact of this business line will depend on how the S&P behaves, as well as the volatility in that timeframe. If the volatility of the S&P 500 is stable throughout the next 9 months, delta hedging would be effective. The portfolio profit and loss would be attributed solely to the markup. However, if the volatility changes drastically, our strategy collapses, as it is outside the main assumptions of the BSM model.

In reality, until July 1998, the volatility of the S&P500 was relatively stable and low and the delta hedging strategy was effectively protecting LTCM from incurring losses as the S&P 500 went up. However, when volatility spiked, the limitations of the Black-Scholes equation were exploited and the model could not recover. Specifically, the S&P 500 went down tremendously; the profit made by the decreasing option value was exceeded by the loss in the LTCM hedge portfolio very quickly. Such shocks to profitability can undermine the viability of the business line as a whole, resulting in bankruptcy. This demonstrates the fragility of delta hedging and LTCM's strategy in the face of large market shocks.

Question 4

The hedge PL and portfolio both lost value in August/September 1998. The delta and S&P500 both experienced sharp decreases which result in increased volatility (see Figure I). Consequently, we can conclude that volatility was the main factor that drove down returns in this time period. In general, LTCM's delta hedging strategy performs best when there are no major swings in volatility.

To test the effects of volatility and interest rates, we decided to run simulations by holding each variable constant. Given constant volatility of 0.2, the portfolio value is constant, and the only profit is the markup (see Figure II). With constant interest rates of 5%, not much changed, meaning that volatility is the main driver. Rebalancing is another potential driver, however we did not account for that in our model. Rebalancing more often could mean tracking the client P/L even better.

Figure I: Plots of actual data

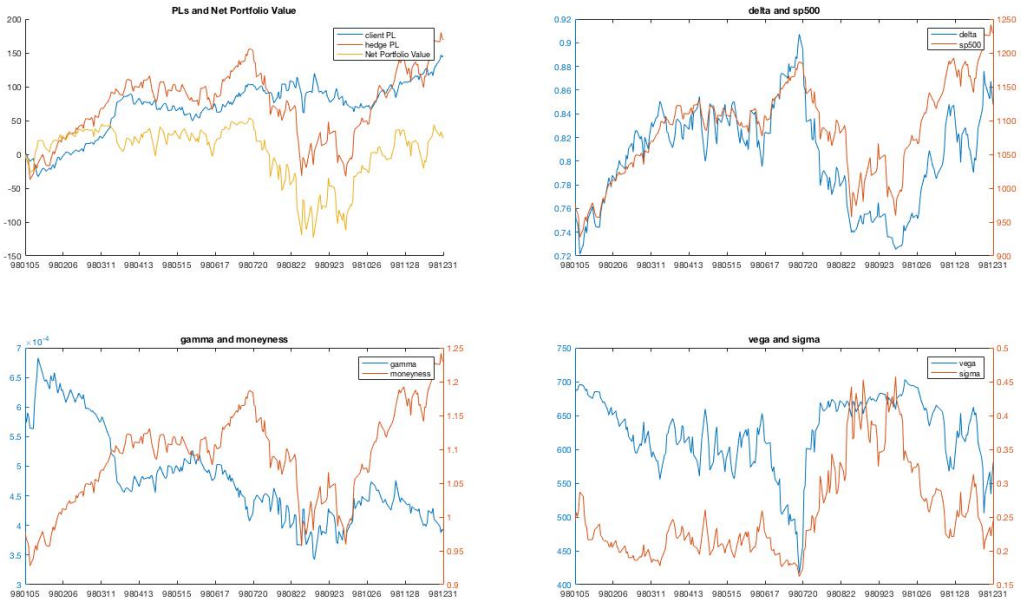

Figure II: Simulation when a constant volatility of 0.2 is assumed