$$ds^2 = -\frac{c^2 v^2}{\phi^+\phi} dt^2 + \frac{\phi^+\phi}{v^2} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right)$$

$$\Gamma^0_{00} = \sum_i \frac{1}{2} g^{0i} \left(g_{i0,0} + g_{0i,0} - g_{00,i} \right) = \frac{1}{2} g^{00} g_{00,0} \ *$$

$$\Gamma^0_{01} = \sum_i \frac{1}{2} g^{0i} \left(g_{i0,1} + g_{1i,0} - g_{01,i} \right) = \frac{1}{2} g^{00} g_{00,1}$$

$$\Gamma^0_{02} = \sum_i \frac{1}{2} g^{0i} \left(g_{i0,2} + g_{2i,0} - g_{02,i} \right) = \frac{1}{2} g^{00} g_{00,2} \ \#$$

$$\Gamma^0_{03} = \sum_i \frac{1}{2} g^{0i} \left(g_{i0,3} + g_{3i,0} - g_{03,i} \right) = \frac{1}{2} g^{00} g_{00,3} \ \#$$

$$\Gamma^0_{10} = \sum_i \frac{1}{2} g^{0i} \left(g_{i1,0} + g_{0i,1} - g_{10,i} \right) = \frac{1}{2} g^{00} g_{00,1}$$

$$\Gamma^0_{11} = \sum_i \frac{1}{2} g^{0i} \left(g_{i1,1} + g_{1i,1} - g_{11,i} \right) = -\frac{1}{2} g^{00} g_{11,0} \ *$$

$$\Gamma^0_{12} = \sum_i \frac{1}{2} g^{0i} \left(g_{i1,2} + g_{2i,1} - g_{12,i} \right) = 0$$

$$\Gamma^0_{13} = \sum_i \frac{1}{2} g^{0i} \left(g_{i1,3} + g_{3i,1} - g_{13,i} \right) = 0$$

$$\Gamma^0_{20} = \sum_i \frac{1}{2} g^{0i} \left(g_{i2,0} + g_{0i,2} - g_{20,i} \right) = \frac{1}{2} g^{00} g_{00,2} \ \#$$

$$\Gamma^0_{21} = \sum_i \frac{1}{2} g^{0i} \left(g_{i2,1} + g_{1i,2} - g_{21,i} \right) = 0$$

$$\Gamma^0_{22} = \sum_i \frac{1}{2} g^{0i} \left(g_{i2,2} + g_{2i,2} - g_{22,i} \right) = 0$$

$$\Gamma^0_{23} = \sum_i \frac{1}{2} g^{0i} \left(g_{i2,3} + g_{3i,2} - g_{23,i} \right) = 0$$

$$\Gamma^0_{30} = \sum_i \frac{1}{2} g^{0i} \left(g_{i3,0} + g_{0i,3} - g_{30,i} \right) = \frac{1}{2} g^{00} g_{00,3} \ \#$$

$$\Gamma^0_{31} = \sum_i \frac{1}{2} g^{0i} \left(g_{i3,1} + g_{1i,3} - g_{31,i} \right) = 0$$

$$\Gamma^0_{32} = \sum_i \frac{1}{2} g^{0i} \left(g_{i3,2} + g_{2i,3} - g_{32,i} \right) = 0$$

$$\Gamma^0_{33} = \sum_i \frac{1}{2} g^{0i} \left(g_{i3,3} + g_{3i,3} - g_{33,i} \right) = 0$$

$$\Gamma^1_{00} = \sum_i \frac{1}{2} g^{1i}\left(g_{i0,0} + g_{0i,0} - g_{00,i}\right) = \frac{1}{2} g^{11} g_{00,1}$$

$$\Gamma^1_{01} = \sum_i \frac{1}{2} g^{1i}\left(g_{i0,1} + g_{1i,0} - g_{01,i}\right) = \frac{1}{2} g^{11} g_{11,0} \;*$$

$$\Gamma^1_{02} = \sum_i \frac{1}{2} g^{1i}\left(g_{i0,2} + g_{2i,0} - g_{02,i}\right) = 0$$

$$\Gamma^1_{03} = \sum_i \frac{1}{2} g^{1i}\left(g_{i0,3} + g_{3i,0} - g_{03,i}\right) = 0$$

$$\Gamma^1_{10} = \sum_i \frac{1}{2} g^{1i}\left(g_{i1,0} + g_{0i,1} - g_{10,i}\right) = \frac{1}{2} g^{11} g_{11,0} \;*$$

$$\Gamma^1_{11} = \sum_i \frac{1}{2} g^{1i}\left(g_{i1,1} + g_{1i,1} - g_{11,i}\right) = \frac{1}{2} g^{11} g_{11,1}$$

$$\Gamma^1_{12} = \sum_i \frac{1}{2} g^{1i}\left(g_{i1,2} + g_{2i,1} - g_{12,i}\right) = \frac{1}{2} g^{11} g_{11,2} \;\#$$

$$\Gamma^1_{13} = \sum_i \frac{1}{2} g^{1i}\left(g_{i1,3} + g_{3i,1} - g_{13,i}\right) = \frac{1}{2} g^{11} g_{11,3} \;\#$$

$$\Gamma^1_{20} = \sum_i \frac{1}{2} g^{1i}\left(g_{i2,0} + g_{0i,2} - g_{20,i}\right) = 0$$

$$\Gamma^1_{21} = \sum_i \frac{1}{2} g^{1i}\left(g_{i2,1} + g_{1i,2} - g_{21,i}\right) = \frac{1}{2} g^{11} g_{11,2} \;\#$$

$$\Gamma^1_{22} = \sum_i \frac{1}{2} g^{1i}\left(g_{i2,2} + g_{2i,2} - g_{22,i}\right) = -\frac{1}{2} g^{11} g_{22,1}$$

$$\Gamma^1_{23} = \sum_i \frac{1}{2} g^{1i}\left(g_{i2,3} + g_{3i,2} - g_{23,i}\right) = 0$$

$$\Gamma^1_{30} = \sum_i \frac{1}{2} g^{1i}\left(g_{i3,0} + g_{0i,3} - g_{30,i}\right) = 0$$

$$\Gamma^1_{31} = \sum_i \frac{1}{2} g^{1i}\left(g_{i3,1} + g_{1i,3} - g_{31,i}\right) = \frac{1}{2} g^{11} g_{11,3} \;\#$$

$$\Gamma^1_{32} = \sum_i \frac{1}{2} g^{1i}\left(g_{i3,2} + g_{2i,3} - g_{32,i}\right) = 0$$

$$\Gamma^1_{33} = \sum_i \frac{1}{2} g^{1i}\left(g_{i3,3} + g_{3i,3} - g_{33,i}\right) = -\frac{1}{2} g^{11} g_{33,1}$$

$$\Gamma^2_{00} = \sum_i \frac{1}{2} g^{2i} \left(g_{i0,0} + g_{0i,0} - g_{00,i} \right) = -\frac{1}{2} g^{22} g_{00,2} \ \#$$

$$\Gamma^2_{01} = \sum_i \frac{1}{2} g^{2i} \left(g_{i0,1} + g_{1i,0} - g_{01,i} \right) = 0$$

$$\Gamma^2_{02} = \sum_i \frac{1}{2} g^{2i} \left(g_{i0,2} + g_{2i,0} - g_{02,i} \right) = 0$$

$$\Gamma^2_{03} = \sum_i \frac{1}{2} g^{2i} \left(g_{i0,3} + g_{3i,0} - g_{03,i} \right) = 0$$

$$\Gamma^2_{10} = \sum_i \frac{1}{2} g^{2i} \left(g_{i1,0} + g_{0i,1} - g_{10,i} \right) = 0$$

$$\Gamma^2_{11} = \sum_i \frac{1}{2} g^{2i} \left(g_{i1,1} + g_{1i,1} - g_{11,i} \right) = -\frac{1}{2} g^{22} g_{11,2} \ \#$$

$$\Gamma^2_{12} = \sum_i \frac{1}{2} g^{2i} \left(g_{i1,2} + g_{2i,1} - g_{12,i} \right) = \frac{1}{2} g^{22} g_{22,1}$$

$$\Gamma^2_{13} = \sum_i \frac{1}{2} g^{2i} \left(g_{i1,3} + g_{3i,1} - g_{13,i} \right) = 0$$

$$\Gamma^2_{20} = \sum_i \frac{1}{2} g^{2i} \left(g_{i2,0} + g_{0i,2} - g_{20,i} \right) = 0$$

$$\Gamma^2_{21} = \sum_i \frac{1}{2} g^{2i} \left(g_{i2,1} + g_{1i,2} - g_{21,i} \right) = \frac{1}{2} g^{22} g_{22,1}$$

$$\Gamma^2_{22} = \sum_i \frac{1}{2} g^{2i} \left(g_{i2,2} + g_{2i,2} - g_{22,i} \right) = 0$$

$$\Gamma^2_{23} = \sum_i \frac{1}{2} g^{2i} \left(g_{i2,3} + g_{3i,2} - g_{23,i} \right) = 0$$

$$\Gamma^2_{30} = \sum_i \frac{1}{2} g^{2i} \left(g_{i3,0} + g_{0i,3} - g_{30,i} \right) = 0$$

$$\Gamma^2_{31} = \sum_i \frac{1}{2} g^{2i} \left(g_{i3,1} + g_{1i,3} - g_{31,i} \right) = 0$$

$$\Gamma^2_{32} = \sum_i \frac{1}{2} g^{2i} \left(g_{i3,2} + g_{2i,3} - g_{32,i} \right) = 0$$

$$\Gamma^2_{33} = \sum_i \frac{1}{2} g^{2i} \left(g_{i3,3} + g_{3i,3} - g_{33,i} \right) = -\frac{1}{2} g^{22} g_{33,2}$$

$$\Gamma^3_{00} = \sum_i \frac{1}{2} g^{3i} \left(g_{i0,0} + g_{0i,0} - g_{00,i} \right) = -\frac{1}{2} g^{33} g_{00,3} \ \#$$

$$\Gamma^3_{01} = \sum_i \frac{1}{2} g^{3i} \left(g_{i0,1} + g_{1i,0} - g_{01,i} \right) = 0$$

$$\Gamma^3_{02} = \sum_i \frac{1}{2} g^{3i} \left(g_{i0,2} + g_{2i,0} - g_{02,i} \right) = 0$$

$$\Gamma^3_{03} = \sum_i \frac{1}{2} g^{3i} \left(g_{i0,3} + g_{3i,0} - g_{03,i} \right) = 0$$

$$\Gamma^3_{10} = \sum_i \frac{1}{2} g^{3i} \left(g_{i1,0} + g_{0i,1} - g_{10,i} \right) = 0$$

$$\Gamma^3_{11} = \sum_i \frac{1}{2} g^{3i} \left(g_{i1,1} + g_{1i,1} - g_{11,i} \right) = -\frac{1}{2} g^{33} g_{11,3} \ \#$$

$$\Gamma^3_{12} = \sum_i \frac{1}{2} g^{3i} \left(g_{i1,2} + g_{2i,1} - g_{12,i} \right) = 0$$

$$\Gamma^3_{13} = \sum_i \frac{1}{2} g^{3i} \left(g_{i1,3} + g_{3i,1} - g_{13,i} \right) = \frac{1}{2} g^{33} g_{33,1}$$

$$\Gamma^3_{20} = \sum_i \frac{1}{2} g^{3i} \left(g_{i2,0} + g_{0i,2} - g_{20,i} \right) = 0$$

$$\Gamma^3_{21} = \sum_i \frac{1}{2} g^{3i} \left(g_{i2,1} + g_{1i,2} - g_{21,i} \right) = 0$$

$$\Gamma^3_{22} = \sum_i \frac{1}{2} g^{3i} \left(g_{i2,2} + g_{2i,2} - g_{22,i} \right) = -\frac{1}{2} g^{33} g_{22,3} = 0$$

$$\Gamma^3_{23} = \sum_i \frac{1}{2} g^{3i} \left(g_{i2,3} + g_{3i,2} - g_{23,i} \right) = \frac{1}{2} g^{33} g_{33,2}$$

$$\Gamma^3_{30} = \sum_i \frac{1}{2} g^{3i} \left(g_{i3,0} + g_{0i,3} - g_{30,i} \right) = 0$$

$$\Gamma^3_{31} = \sum_i \frac{1}{2} g^{3i} \left(g_{i3,1} + g_{1i,3} - g_{31,i} \right) = \frac{1}{2} g^{33} g_{33,1}$$

$$\Gamma^3_{32} = \sum_i \frac{1}{2} g^{3i} \left(g_{i3,2} + g_{2i,3} - g_{32,i} \right) = \frac{1}{2} g^{33} g_{33,2}$$

$$\Gamma^3_{33} = \sum_i \frac{1}{2} g^{3i} \left(g_{i3,3} + g_{3i,3} - g_{33,i} \right) = 0$$

Summary:

$$\Gamma^0_{00} = \frac{1}{2} g^{00} g_{00,0} = \frac{1}{2} \frac{\phi^+\phi}{c^2 v^2} \partial_t \left(\frac{c^2 v^2}{\phi^+\phi} \right) = -\frac{\partial_t \left(\phi^+\phi \right)}{2\phi^+\phi}$$

$$\Gamma^0_{11} = -\frac{1}{2} g^{00} g_{11,0} = \frac{1}{2} \frac{\phi^+\phi}{c^2 v^2} \partial_t \left(\frac{\phi^+\phi}{v^2} \right) = \frac{\phi^+\phi \partial_t \left(\phi^+\phi \right)}{2c^2 v^4}$$

$$\Gamma^1_{01}=\Gamma^1_{10}=\frac{1}{2}g^{11}g_{11,0}=\frac{1}{2}\frac{v^2}{\phi^+\phi}\partial_t\left(\frac{\phi^+\phi}{v^2}\right)=\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^0_{02}=\Gamma^0_{20}=\frac{1}{2}g^{00}g_{00,2}=\frac{1}{2}\frac{\phi^+\phi}{c^2v^2}\partial_\theta\left(\frac{c^2v^2}{\phi^+\phi}\right)=-\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^1_{12}=\Gamma^1_{21}=\frac{1}{2}g^{11}g_{11,2}=\frac{1}{2}\frac{v^2}{\phi^+\phi}\partial_\theta\frac{\phi^+\phi}{v^2}=\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^2_{00}=-\frac{1}{2}g^{22}g_{00,2}=-\frac{1}{2r^2}\partial_\theta\left(\frac{c^2v^2}{\phi^+\phi}\right)=\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}$$

$$\Gamma^2_{11}=-\frac{1}{2}g^{22}g_{11,2}=-\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}$$

$$\Gamma^0_{03}=\Gamma^0_{30}=\frac{1}{2}g^{00}g_{00,3}=-\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^1_{13}=\Gamma^1_{31}=\frac{1}{2}g^{11}g_{11,3}=\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^3_{00}=-\frac{1}{2}g^{33}g_{00,3}=-\frac{1}{2r^2\sin^2\theta}\partial_\varphi\left(\frac{c^2v^2}{\phi^+\phi}\right)=\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}$$

$$\Gamma^3_{11}=-\frac{1}{2}g^{33}g_{11,3}=-\frac{1}{2r^2\sin^2\theta}\partial_\varphi\left(\frac{\phi^+\phi}{v^2}\right)=\frac{-\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}$$

$$\Gamma^0_{01}=\Gamma^0_{10}=\frac{1}{2}g^{00}g_{00,1}=\frac{1}{2}\frac{\phi^+\phi}{c^2v^2}\partial_r\left(\frac{c^2v^2}{\phi^+\phi}\right)=\frac{1}{2}\phi^+\phi\partial_r\left(\frac{1}{\phi^+\phi}\right)=-\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^1_{00}=-\frac{1}{2}g^{11}g_{00,1}=-\frac{1}{2}\frac{v^2}{\phi^+\phi}\partial_r\left(-\frac{c^2v^2}{\phi^+\phi}\right)=-\frac{1}{2}\frac{v^2}{\phi^+\phi}\frac{c^2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}=-\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}$$

$$\Gamma^1_{11}=\frac{1}{2}g^{11}g_{11,1}=\frac{1}{2}\frac{v^2}{\phi^+\phi}\partial_r\left(\frac{\phi^+\phi}{v^2}\right)=\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$\Gamma^1_{22}=-\frac{1}{2}g^{11}g_{22,1}=-\frac{1}{2}\frac{v^2}{\phi^+\phi}\partial_r\left(r^2\right)=-\frac{v^2r}{\phi^+\phi}$$

$$\Gamma^1_{33}=-\frac{1}{2}g^{11}g_{33,1}=-\frac{1}{2}\frac{v^2}{\phi^+\phi}\partial_r\left(r^2\sin^2\theta\right)=-\frac{v^2r\sin^2\theta}{\phi^+\phi}$$

$$\Gamma^2_{12}=\Gamma^2_{21}=\frac{1}{2}g^{22}g_{22,1}=\frac{1}{2r^2}\partial_r r^2=\frac{1}{r}$$

$$\Gamma^2_{33} = -\frac{1}{2} g^{22} g_{33,2} = -\frac{1}{2r^2}\partial_\theta\left(r^2 \sin^2\theta\right) = -\sin\theta\cos\theta$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{2} g^{33} g_{33,1} = \frac{\partial_r\left(r^2\sin^2\theta\right)}{2r^2\sin^2\theta} = \frac{1}{r}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \frac{1}{2} g^{33} g_{33,2} = \frac{\partial_\theta\left(r^2\sin^2\theta\right)}{2r^2\sin^2\theta} = \cot\theta$$

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$$R_{\mu\nu\sigma}{}^{\rho} = \Gamma^{\rho}{}_{\mu\sigma,\nu} - \Gamma^{\rho}{}_{\nu\sigma,\mu} + \Gamma^{\lambda}{}_{\sigma\mu}\Gamma^{\rho}{}_{\nu\lambda} - \Gamma^{\lambda}{}_{\sigma\nu}\Gamma^{\rho}{}_{\mu\lambda}$$

$$R_{010}{}^{0} = \Gamma^0{}_{00,1} - \Gamma^0{}_{10,0} + \Gamma^1{}_{00}\Gamma^0{}_{11} - \Gamma^1{}_{01}\Gamma^0{}_{01} = \frac{\left(\partial_t\partial_r - \partial_r\partial_t\right)\left(\phi^+\phi\right)}{2\phi^+\phi} = 0$$

$$R_{010}{}^{1} = \Gamma^1{}_{00,1} - \Gamma^1{}_{10,0} + \Gamma^{\lambda}{}_{00}\Gamma^1{}_{1\lambda} - \Gamma^{\lambda}{}_{01}\Gamma^1{}_{0\lambda}$$

$$= \Gamma^1{}_{00,1} - \Gamma^1{}_{10,0} + \Gamma^0{}_{00}\Gamma^1{}_{10} + \Gamma^1{}_{00}\Gamma^1{}_{11} - \Gamma^0{}_{01}\Gamma^1{}_{00} - \Gamma^1{}_{01}\Gamma^1{}_{01} +^{\#} \Gamma^2{}_{00}\Gamma^1{}_{12} + \Gamma^3{}_{00}\Gamma^1{}_{13}$$

$$= \partial_r\left(-\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\right) - \partial_t\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$-\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$-\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} +^{\#} \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{c^2v^4\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^4} - \frac{c^2v^4\partial_r^2\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{\partial_t^2\left(\phi^+\phi\right)}{2\phi^+\phi} +^{\#} \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4r^2\sin^2\theta\left(\phi^+\phi\right)^3}$$

$$R_{010}{}^{2}=\Gamma^{2}_{00,1}-\Gamma^{0}_{01}\Gamma^{2}_{00}+\Gamma^{1}_{00}\Gamma^{2}_{11}+\Gamma^{2}_{00}\Gamma^{2}_{12}$$

$$=\partial_r\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}+\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}$$

$$=-\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{r^3\left(\phi^+\phi\right)^2}+\frac{c^2v^2}{2r^2}\partial_r\frac{\partial_\theta\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{c^2v^2\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3}+\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}$$

$$=-\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}+\frac{c^2v^2}{2r^2}\partial_r\frac{1}{\left(\phi^+\phi\right)^2}\partial_\theta\left(\phi^+\phi\right)+\frac{c^2v^2\partial_r\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{c^2v^2\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3}$$

$$=-\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}-\frac{c^2v^2\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\left(\phi^+\phi\right)^3}+\frac{c^2v^2\partial_r\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{c^2v^2\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3}$$

$$=-\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}-\frac{c^2v^2\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3}+\frac{c^2v^2\partial_r\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}$$

$$R_{010}{}^{3}=\Gamma^{3}_{00,1}-\Gamma^{0}_{01}\Gamma^{3}_{00}+\Gamma^{1}_{00}\Gamma^{3}_{11}+\Gamma^{3}_{00}\Gamma^{3}_{13}$$

$$=\partial_r\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}+\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}+\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^3\sin^2\theta\left(\phi^+\phi\right)^2}$$

$$=\frac{1}{\sin^2\theta}\left(\partial_r\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\theta}+\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}\right)$$

$$=\frac{1}{\sin^2\theta}\left(-\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^3\left(\phi^+\phi\right)^2}-\frac{c^2v^2\partial_r\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3}+\frac{c^2v^2\partial_r\partial_\varphi\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}\right)$$

$$R_{011}{}^{0}=\Gamma^{0}_{01,1}-\Gamma^{0}_{11,0}+\Gamma^{0}_{10}\Gamma^{0}_{10}-\Gamma^{0}_{11}\Gamma^{0}_{00}+\Gamma^{1}_{10}\Gamma^{0}_{11}-\Gamma^{1}_{11}\Gamma^{0}_{01}-\Gamma^{2}_{11}\Gamma^{0}_{02}-\Gamma^{3}_{11}\Gamma^{0}_{03}$$

$$=-\partial_r\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}-\partial_t\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$+\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$=\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}-\frac{\partial_r^2\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2c^2v^4}-\frac{\phi^+\phi\partial_t^2\left(\phi^+\phi\right)}{2c^2v^4}+\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{4c^2v^4}$$

$$+\frac{\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{4c^2v^4}+\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2v^2\phi^+\phi}-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4r^2v^2\sin^2\theta\phi^+\phi}$$

$$=\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}-\frac{\partial_r^2\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\phi^+\phi\partial_t^2\left(\phi^+\phi\right)}{2c^2v^4}-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2v^2\phi^+\phi}-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4r^2v^2\sin^2\theta\phi^+\phi}$$

$$R_{011}{}^{1}=\Gamma^{1}_{01,1}-\Gamma^{1}_{11,0}+\Gamma^{0}_{10}\Gamma^{1}_{10}-\Gamma^{0}_{11}\Gamma^{1}_{00}$$

$$=\partial_r\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}-\partial_t\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}$$

$$=-\frac{\partial_r\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\frac{\partial_r\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_t\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_t\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$

$$-\frac{\partial_r\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial_t\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}=0$$

$$R_{011}{}^{2}=-\Gamma^{2}_{11,0}-\Gamma^{0}_{11}\Gamma^{2}_{00}+\Gamma^{1}_{10}\Gamma^{2}_{11}$$

$$=\partial_t\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}-\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}$$

$$=\frac{\partial_t\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}-\frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r^2v^2}$$

$$R_{011}{}^{3}=-\Gamma^{3}_{11,0}-\Gamma^{0}_{11}\Gamma^{3}_{00}+\Gamma^{1}_{10}\Gamma^{3}_{11}$$

$$=\partial_t\frac{\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}+\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{-\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}$$

$$=\frac{\partial_t\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}-\frac{\partial_t\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2v^2r^2\sin^2\theta\left(\phi^+\phi\right)}$$

$$R_{012}{}^{0}=\Gamma^{0}_{02,1}+\Gamma^{0}_{20}\Gamma^{0}_{10}-\Gamma^{1}_{21}\Gamma^{0}_{01}-\Gamma^{2}_{21}\Gamma^{0}_{02}$$

$$=-\partial_r\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{1}{r}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$=\frac{\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$R_{012}{}^{1}=-\Gamma^{1}_{12,0}+\Gamma^{0}_{20}\Gamma^{1}_{10}-\Gamma^{1}_{21}\Gamma^{1}_{01}$$

$$=-\partial_t\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$=-\frac{\partial_t\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}-\frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}=-\frac{\partial_t\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{013}{}^{0} = \Gamma^{0}_{03,1} + \Gamma^{0}_{30}\Gamma^{0}_{10} - \Gamma^{1}_{31}\Gamma^{0}_{01} - \Gamma^{3}_{31}\Gamma^{0}_{03}$$

$$= -\partial_r \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{1}{r}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2} + \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$R_{013}{}^{1} = -\Gamma^{1}_{13,0} + \Gamma^{0}_{30}\Gamma^{1}_{10} - \Gamma^{1}_{31}\Gamma^{1}_{01}$$

$$= -\partial_t \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= -\frac{\partial_t\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{\partial_t\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} - \frac{\partial_t\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} = -\frac{\partial_t\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{020}{}^{0} = \Gamma^{0}_{00,2} - \Gamma^{0}_{20,0}$$

$$= -\partial_\theta \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \partial_t \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{\partial_\theta\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} - \frac{\partial_\theta\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} + \frac{\partial_t\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} = 0$$

$$R_{020}{}^{1} = \Gamma^{1}_{00,2} - \Gamma^{0}_{02}\Gamma^{1}_{00} + \Gamma^{1}_{00}\Gamma^{1}_{21} + \Gamma^{2}_{00}\Gamma^{1}_{22}$$

$$= -\partial_\theta \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}\frac{v^2r}{\phi^+\phi}$$

$$= \frac{3c^2v^4\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^4} - \frac{c^2v^4\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^4} - \frac{c^2v^4\partial_\theta\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$= \frac{c^2v^4\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^4} - \frac{c^2v^4\partial_\theta\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$R_{020}{}^{2} = \Gamma^{2}_{00,2} - \Gamma^{0}_{02}\Gamma^{2}_{00} + \Gamma^{1}_{00}\Gamma^{2}_{21}$$

$$= \partial_\theta \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$= -\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\theta^2\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} + \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\left(\phi^+\phi\right)^3} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$= -\frac{3c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\theta^2\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$R_{020}{}^{3} = \Gamma^{3}_{00,2} - \Gamma^{0}_{02}\Gamma^{3}_{00} + \Gamma^{3}_{00}\Gamma^{3}_{23}$$

$$= \partial_\theta \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2} + \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}\cot\theta$$

$$= -\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\sin\theta}{r^2\sin^3\theta\left(\phi^+\phi\right)^2} - \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\sin^2\theta\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}$$

$$+\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\sin^2\theta\left(\phi^+\phi\right)^3} + \frac{c^2v^2\cos\theta\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^3\theta\left(\phi^+\phi\right)^2}$$

$$= -\frac{\cos\theta c^2v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^3\theta\left(\phi^+\phi\right)^2} - \frac{3c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\sin^2\theta\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}$$

$$R_{021}{}^{0} = \Gamma^{0}_{01,2} + \Gamma^{0}_{10}\Gamma^{0}_{20} - \Gamma^{1}_{12}\Gamma^{0}_{01} - \Gamma^{2}_{12}\Gamma^{0}_{02}$$

$$= -\partial_\theta\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{1}{r}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} - \frac{\partial_\theta\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$= -\frac{\partial_\theta\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_r\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$R_{021}{}^{1} = \Gamma^{1}_{01,2} - \Gamma^{1}_{21,0}$$

$$= \partial_\theta\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} - \partial_t\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= -\frac{\partial_\theta\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} + \frac{\partial_\theta\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\theta\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} - \frac{\partial_\theta\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= 0$$

$$R_{021}{}^{2} = \Gamma^{1}_{10}\Gamma^{2}_{21} = \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$R_{022}{}^{0} = \Gamma^{0}_{02,2} + \Gamma^{0}_{20}\Gamma^{0}_{20} - \Gamma^{1}_{22}\Gamma^{0}_{01}$$

$$= -\partial_\theta \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{v^2 r}{\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} - \frac{\partial_\theta^2\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2} - \frac{v^2 r\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$

$$= \frac{3\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2} - \frac{\partial_\theta^2\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{v^2 r\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$

$$R_{022}{}^{1} = -\Gamma^{1}_{22,0} - \Gamma^{1}_{22}\Gamma^{1}_{01}$$

$$= \partial_t \frac{v\ r}{\phi^+\phi} + \frac{v^2 r}{\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= -\frac{v^2 r\partial_t\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2} + \frac{v^2 r\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} = -\frac{v^2 r\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$

$$R_{023}{}^{0} = \Gamma^{0}_{03,2} + \Gamma^{0}_{30}\Gamma^{0}_{20} - \Gamma^{3}_{32}\Gamma^{0}_{03}$$

$$= -\partial_\theta \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} + \cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{3\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2} - \frac{\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} + \cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{030}{}^{0} = \Gamma^{0}_{00,3} - \Gamma^{0}_{30,0}$$

$$= -\partial_\varphi \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \partial_t \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$= \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} - \frac{\partial_\varphi\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_t\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} = 0$$

$$R_{030}{}^{1} = \Gamma^{1}_{00,3} - \Gamma^{0}_{03}\Gamma^{1} + \Gamma^{1}\ \Gamma^{1} + \Gamma^{3}\ \Gamma^{1}{}_{3}$$

$$= -\partial_\varphi \frac{c^2 v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{c^2 v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{c^2 v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{c^2 v^2\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}\frac{v^2 r\sin^2\theta}{\phi^+\phi}$$

$$= \frac{3c^2 v^4\partial_r\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^4} - \frac{c^2 v^4\partial_\varphi\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{c^2 v^4\partial_r\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^4} - \frac{c^2 v^4\partial_\varphi\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$= \frac{c^2 v^4\partial_r\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^4} - \frac{c^2 v^4\partial_\varphi\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{c^2 v^4\partial_\varphi\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3}$$

$$R_{030}{}^{2}=\Gamma^{2}_{00,3}-\Gamma^{0}_{03}\Gamma^{2}_{00}+\Gamma^{3}_{00}\Gamma^{2}_{33}$$

$$=\partial_{\varphi}\frac{c^{2}v^{2}\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{c^{2}v^{2}\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}-\frac{c^{2}v^{2}\sin\theta\cos\theta\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{2}}$$

$$=-\frac{c^{2}v^{2}\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{r^{2}\left(\phi^{+}\phi\right)^{3}}+\frac{c^{2}v^{2}\partial_{\varphi}\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}+\frac{c^{2}v^{2}\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4r^{2}\left(\phi^{+}\phi\right)^{3}}-\frac{c^{2}v^{2}\cot\theta\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}$$

$$=-\frac{3c^{2}v^{2}\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4r^{2}\left(\phi^{+}\phi\right)^{3}}+\frac{c^{2}v^{2}\partial_{\varphi}\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}-\frac{c^{2}v^{2}\cot\theta\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}$$

$$R_{030}{}^{3}=\Gamma^{3}_{00,3}-\Gamma^{0}_{03}\Gamma^{3}_{00}+\Gamma^{1}_{00}\Gamma^{3}_{31}+\Gamma^{2}_{00}\Gamma^{3}_{32}$$

$$=\partial_{\varphi}\frac{c\ v\ \partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{2}}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{c\ v\ \partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{2}}-\frac{c\ v\ \partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{3}r}+\frac{c^{2}v\ \cot\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}$$

$$=\frac{-c^{2}v^{2}\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{3}}+\frac{c^{2}v^{2}\partial_{\varphi}^{2}\left(\phi^{+}\phi\right)}{2r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{2}}+\frac{c^{2}v^{2}\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{3}}-\frac{c^{2}v^{4}\partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{3}r}+\frac{c^{2}v^{2}\cot\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}$$

$$=\frac{-3c^{2}v^{2}\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{3}}+\frac{c^{2}v^{2}\partial_{\varphi}^{2}\left(\phi^{+}\phi\right)}{2r^{2}\sin^{2}\theta\left(\phi^{+}\phi\right)^{2}}-\frac{c^{2}v^{4}\partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{3}r}+\frac{c^{2}v^{2}\cot\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}\left(\phi^{+}\phi\right)^{2}}$$

$$R_{031}{}^{0}=\Gamma^{0}_{01,3}+\Gamma^{0}_{10}\Gamma^{0}_{30}-\Gamma^{1}_{13}\Gamma^{0}_{01}-\Gamma^{3}_{13}\Gamma^{0}_{03}$$

$$=-\partial_{\varphi}\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{1}{r}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{\partial_{r}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}-\frac{\partial_{\varphi}\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{r}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$=\frac{\partial_{r}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{\left(\phi^{+}\phi\right)^{2}}-\frac{\partial_{\varphi}\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{031}{}^{1}=\Gamma^{1}_{01,3}-\Gamma^{1}_{31,0}=\partial_{\varphi}\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\partial_{t}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}=0$$

$$R_{031}{}^{3}=\Gamma^{1}_{10}\Gamma^{3}_{31}=\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{032}{}^{0}=\Gamma^{0}_{02,3}+\Gamma^{0}_{20}\Gamma^{0}_{30}-\Gamma^{3}_{23}\Gamma^{0}_{03}$$

$$=-\partial_{\varphi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\cot\theta\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}+\frac{\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}}+\cot\theta\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{3\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}}+\cot\theta\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$R_{033}{}^{0}\quad {}^{0}_{03,3}\quad {}^{0}_{30}\,{}^{0}_{30}\quad {}^{1}_{33}\,{}^{0}_{01}\quad \Gamma^{2}_{33}\,{}^{0}_{02}$$

$$=-\partial_{\varphi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\sin\theta\cos\theta\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}-\frac{\partial^{2}_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}}-\frac{v^{2}r\sin^{2}\theta\partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}\quad\sin\theta\cos\theta\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{3\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}}-\frac{\partial^{2}_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\frac{v^{2}r\sin^{2}\theta\partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}-\sin\theta\cos\theta\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$R_{033}{}^{1}=-\Gamma^{1}_{33,0}-\Gamma^{1}_{33}\Gamma^{1}_{01}$$

$$=\partial_{t}\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}+\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=-\frac{v^{2}r\sin^{2}\theta\partial_{t}\left(\phi^{+}\phi\right)}{\left(\phi^{+}\phi\right)^{2}}+\frac{v^{2}r\sin^{2}\theta\partial_{t}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}=-\frac{v^{2}r\sin^{2}\theta\partial_{t}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}$$

$$R_{100}{}^{0}=\Gamma^{0}_{10,0}-\Gamma^{0}_{00,1}+\Gamma^{1}_{01}\Gamma^{0}_{01}-\Gamma^{1}_{00}\Gamma^{0}_{11}=-R_{010}{}^{0}=0$$

$$R_{100}{}^{1}=\Gamma^{1}_{10,0}-\Gamma^{1}_{00,1}+\Gamma^{0}_{01}\Gamma^{1}_{00}-\Gamma^{0}_{00}\Gamma^{1}_{10}+\Gamma^{1}_{01}\Gamma^{1}_{01}-\Gamma^{1}_{00}\Gamma^{1}_{11}-\Gamma^{2}_{00}\Gamma^{1}_{12}-\Gamma^{3}_{00}\Gamma^{1}_{13}=-R_{010}{}^{1}$$

$$R_{100}{}^{2}=-\Gamma^{2}_{00,1}+\Gamma^{0}_{01}\Gamma^{2}_{00}-\Gamma^{1}_{00}\Gamma^{2}_{11}-\Gamma^{2}_{00}\Gamma^{2}_{12}=-R_{010}{}^{2}$$

$$R_{100}{}^{3}=-\Gamma^{3}_{00,1}+\Gamma^{0}_{01}\Gamma^{3}_{00}-\Gamma^{1}_{00}\Gamma^{3}_{11}-\Gamma^{3}_{00}\Gamma^{3}_{13}=-R_{010}{}^{3}$$

$$R_{101}{}^{0}=\Gamma^{0}_{11,0}-\Gamma^{0}_{01,1}+\Gamma^{0}_{11}\Gamma^{0}_{00}-\Gamma^{0}_{10}\Gamma^{0}_{10}+\Gamma^{1}_{11}\Gamma^{0}_{01}-\Gamma^{1}_{10}\Gamma^{0}_{11}+\Gamma^{2}_{11}\Gamma^{0}_{02}+\Gamma^{3}_{11}\Gamma^{0}_{03}=-R_{011}{}^{0}$$

$$R_{101}{}^{1}=\Gamma^{1}_{11,0}-\Gamma^{1}_{01,1}+\Gamma^{0}_{11}\Gamma^{1}_{00}-\Gamma^{0}_{10}\Gamma^{1}_{10}=-R_{011}{}^{1}$$

$$R_{101}{}^{2}=\Gamma^{2}_{11,0}+\Gamma^{0}_{11}\Gamma^{2}_{00}-\Gamma^{1}_{10}\Gamma^{2}_{11}=-R_{011}{}^{2}$$

$$R_{101}{}^{3}=\Gamma^{3}_{11,0}+\Gamma^{0}_{11}\Gamma^{3}_{00}-\Gamma^{1}_{10}\Gamma^{3}_{11}=-R_{011}{}^{3}$$

$$R_{102}{}^{0}=-\Gamma^{0}_{02,1}-\Gamma^{0}_{20}\Gamma^{0}_{10}+\Gamma^{1}_{21}\Gamma^{0}_{01}+\Gamma^{2}_{21}\Gamma^{0}_{02}=-R_{012}{}^{0}$$

$$R_{102}{}^{1} = \Gamma^{1}_{12,0} - \Gamma^{0}_{20}\Gamma^{1}_{10} + \Gamma^{1}_{21}\Gamma^{1}_{01} = -R_{012}{}^{1}$$

$$R_{103}{}^{0} = -\Gamma^{0}_{03,1} - \Gamma^{0}_{30}\Gamma^{0}_{10} + \Gamma^{1}_{31}\Gamma^{0}_{01} + \Gamma^{3}_{31}\Gamma^{0}_{03} = -R_{013}{}^{0}$$

$$R_{103}{}^{1} = \Gamma^{1}_{13,0} - \Gamma^{0}_{30}\Gamma^{1}_{10} + \Gamma^{1}_{31}\Gamma^{1}_{01} = -R_{013}{}^{1}$$

$$R_{120}{}^{0} = \Gamma^{0}_{10,2} - \Gamma^{0}_{20,1} = -\partial_\theta \frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} + \partial_r \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} = 0$$

$$\begin{aligned}
R_{120}{}^{1} &= \Gamma^{1}_{10,2} - \Gamma^{0}_{02}\Gamma^{1}_{10} + \Gamma^{1}_{01}\Gamma^{1}_{21} \\
&= \partial_\theta \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} \\
&= -\frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} + \frac{\partial_\theta\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} \\
&= \frac{\partial_\theta\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}
\end{aligned}$$

$$R_{120}{}^{2} = \Gamma^{1}_{01}\Gamma^{2}_{21} = \frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$\begin{aligned}
R_{121}{}^{0} &= \Gamma^{0}_{11,2} + \Gamma^{0}_{11}\Gamma^{0}_{20} - \Gamma^{1}_{12}\Gamma^{0}_{11} \\
&= \partial_\theta \frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4} - \frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4} \\
&= \frac{\partial_\theta\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{2c^2v^4} + \frac{\phi^+\phi\partial_\theta\partial_t\left(\phi^+\phi\right)}{2c^2v^4} - \frac{\partial_t\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4c^2v^4} - \frac{\partial_\theta\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{4c^2v^4} \\
&= \frac{\phi^+\phi\partial_\theta\partial_t\left(\phi^+\phi\right)}{2c^2v^4}
\end{aligned}$$

$$\begin{aligned}
R_{121}{}^{1} &= \Gamma^{1}_{11,2} - \Gamma^{1}_{21,1} + \Gamma^{2}_{11}\Gamma^{1}_{22} - \Gamma^{2}_{12}\Gamma^{1}_{12} \\
&= \partial_\theta \frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi} - \partial_r \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}\frac{v^2r}{\phi^+\phi} - \frac{1}{r}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi} \\
&= \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r} - \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r} = 0
\end{aligned}$$

$$\begin{aligned}
R_{121}{}^{2} &= \Gamma^{2}_{11,2} - \Gamma^{2}_{21,1} + \Gamma^{1}_{11}\Gamma^{2}_{21} - \Gamma^{1}_{12}\Gamma^{2}_{11} - \Gamma^{2}_{12}\Gamma^{2}_{12} \\
&= -\partial_\theta \frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2} - \partial_r \frac{1}{r} + \frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi r} + \frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2} - \frac{1}{r^2} \\
&= -\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2r^2v^2} + \frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi r} + \frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\phi^+\phi r^2v^2}
\end{aligned}$$

$$R_{121}{}^{3}=\Gamma^{3}_{11,2}-\Gamma^{1}_{12}\Gamma^{3}_{11}+\Gamma^{3}_{11}\Gamma^{3}_{23}$$
$$=\partial_\theta\frac{-\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}+\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}-\frac{\cot\theta\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}$$
$$=\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\sin\theta}{r^2v^2\sin^3\theta}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4\phi^+\phi r^2v^2\sin^2\theta}-\frac{\cos\theta\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^3\theta}$$
$$=\frac{\partial_\varphi\left(\phi^+\phi\right)\cos\theta}{2r^2v^2\sin^3\theta}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4\phi^+\phi r^2v^2\sin^2\theta}$$

$$R_{122}{}^{0}=-\Gamma^{1}_{22}\Gamma^{0}_{11}=\frac{v^2r}{\phi^+\phi}\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)}{2c^2v^4}=\frac{r\partial_t\left(\phi^+\phi\right)}{2c^2v^2}$$

$$R_{122}{}^{1}=\Gamma^{1}_{12,2}-\Gamma^{1}_{22,1}+\Gamma^{1}_{21}\Gamma^{1}_{21}-\Gamma^{1}_{22}\Gamma^{1}_{11}+\Gamma^{2}_{21}\Gamma^{1}_{22}$$
$$=\partial_\theta\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}+\partial_r\frac{v^2r}{\phi^+\phi}+\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{v^2r}{\phi^+\phi}\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{1}{r}\frac{v^2r}{\phi^+\phi}$$
$$=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\frac{\partial^2_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{v^2r\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{v^2}{\phi^+\phi}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{v^2r\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}-\frac{v^2}{\phi^+\phi}$$
$$=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial^2_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{v^2r\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$

$$R_{122}{}^{2}=\Gamma^{1}_{21}\Gamma^{2}_{21}-\Gamma^{1}_{22}\Gamma^{2}_{11}$$
$$=\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r}-\frac{v^2r}{\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}=0$$

$$R_{122}{}^{3}=-\Gamma^{1}_{22}\Gamma^{3}_{11}=\frac{v^2r}{\phi^+\phi}\frac{-\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}=\frac{-\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi r\sin^2\theta}$$

$$R_{123}{}^{1}=\Gamma^{1}_{13,2}+\Gamma^{1}_{31}\Gamma^{1}_{21}-\Gamma^{3}_{32}\Gamma^{1}_{13}$$
$$=\partial_\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}-\cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$
$$=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\frac{\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}-\cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$
$$=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}-\cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{123}{}^{2}=\Gamma^{1}_{31}\Gamma^{2}_{21}=\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi r}$$

$$R_{130}{}^{0}=\Gamma^{0}_{10,3}-\Gamma^{0}_{30,1}=-\partial_\varphi\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi}+\partial_r\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}=0$$

$$R_{130}{}^{1}=\Gamma^{1}_{10,3}-\Gamma^{0}_{03}\Gamma^{1}_{10}+\Gamma^{1}_{01}\Gamma^{1}_{31}=\partial_{\varphi}\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=-\frac{\partial_{t}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}+\frac{\partial_{\varphi}\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{t}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}$$

$$=\frac{\partial_{\varphi}\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$R_{130}{}^{3}=\Gamma^{1}_{01}\Gamma^{3}_{31}=\frac{\partial_{t}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{131}{}^{0}=\Gamma^{0}_{11,3}+\Gamma^{0}_{11}\Gamma^{0}_{30}-\Gamma^{1}_{13}\Gamma^{0}_{11}=\partial_{\varphi}\frac{\phi^{+}\phi\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}}-\frac{\phi^{+}\phi\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\phi^{+}\phi\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}}$$

$$=\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}}+\frac{\phi^{+}\phi\partial_{\varphi}\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}}-\frac{\partial_{t}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4c^{2}v^{4}}-\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{t}\left(\phi^{+}\phi\right)}{4c^{2}v^{4}}$$

$$=\frac{\phi^{+}\phi\partial_{\varphi}\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}}$$

$$R_{131}{}^{1}=\Gamma^{1}_{11,3}-\Gamma^{1}_{31,1}+\Gamma^{3}_{11}\Gamma^{1}_{33}-\Gamma^{3}_{13}\Gamma^{1}_{13}$$

$$=\partial_{\varphi}\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\partial_{r}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}\sin^{2}\theta}\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}-\frac{1}{r}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}-\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}=0$$

$$R_{131}{}^{2}=\Gamma^{2}_{11,3}-\Gamma^{1}_{13}\Gamma^{2}_{11}+\Gamma^{3}_{11}\Gamma^{2}_{33}$$

$$=-\partial_{\varphi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}\sin^{2}\theta}\sin\theta\cos\theta$$

$$=-\frac{\partial_{\varphi}\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\theta}\left(\phi^{+}\phi\right)}{4\phi^{+}\phi r^{2}v^{2}}+\frac{\cot\theta\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}}$$

$$R_{131}{}^{3}=\Gamma^{3}_{11,3}-\Gamma^{3}_{31,1}+\Gamma^{1}_{11}\Gamma^{3}_{31}-\Gamma^{1}_{13}\Gamma^{3}_{11}+\Gamma^{2}_{11}\Gamma^{3}_{32}-\Gamma^{3}_{13}\Gamma^{3}_{13}$$

$$=\partial_{\varphi}\frac{-\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}\sin^{2}\theta}-\partial_{r}\frac{1}{r}+\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}-\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{-\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}\sin^{2}\theta}-\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}}\cot\theta-\frac{1}{r^{2}}$$

$$=\frac{-\partial_{\varphi}^{2}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}\sin^{2}\theta}+\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}+\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\phi^{+}\phi r^{2}v^{2}\sin^{2}\theta}-\frac{\cot\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}}$$

$$R_{132}{}^{1} = \Gamma^{1}_{12,3} + \Gamma^{1}_{21}\Gamma^{1}_{31} - \Gamma^{3}_{23}\Gamma^{1}_{13}$$
$$= \partial_{\varphi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} + \frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} - \cot\theta\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$
$$= -\frac{\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}} + \frac{\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}} - \frac{\cot\theta\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$
$$= -\frac{\partial_{\theta}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}} - \frac{\cot\theta\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$R_{132}{}^{3} = \Gamma^{1}_{21}\Gamma^{3}_{31} + \Gamma^{2}_{21}\Gamma^{3}_{32} - \Gamma^{3}_{23}\Gamma^{3}_{13}$$
$$= \frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r} + \frac{1}{r}\cot\theta - \cot\theta\frac{1}{r} = \frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{133}{}^{0} = -\Gamma^{1}_{33}\Gamma^{0}_{11} = \frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{\phi^{+}\phi\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{4}} = \frac{r\sin^{2}\theta\partial_{t}\left(\phi^{+}\phi\right)}{2c^{2}v^{2}}$$

$$R_{133} = \Gamma_{13,3} - \Gamma_{33,1} + \Gamma_{31}\Gamma_{31} - \Gamma_{33}\Gamma_{11} - \Gamma^{2}_{33}\Gamma_{12} + \Gamma_{31}\Gamma_{33}$$
$$= \partial_{\varphi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} + \partial_{r}\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi} + \frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} + \frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{\partial_{r}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} + \sin\theta\cos\theta\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} - \frac{1}{r}\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}$$
$$= -\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}} + \frac{\partial_{\varphi}^{2}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} + \frac{v^{2}\sin^{2}\theta}{\phi^{+}\phi} - \frac{v^{2}r\sin^{2}\theta\partial_{r}\left(\phi^{+}\phi\right)}{\left(\phi^{+}\phi\right)^{2}} + \frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}}$$
$$+ \frac{v^{2}r\sin^{2}\theta\partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}} + \sin\theta\cos\theta\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} - \frac{v^{2}\sin^{2}\theta}{\phi^{+}\phi}$$
$$= -\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)\partial_{\varphi}\left(\phi^{+}\phi\right)}{4\left(\phi^{+}\phi\right)^{2}} + \frac{\partial_{\varphi}^{2}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi} - \frac{v^{2}r\sin^{2}\theta\partial_{r}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}} + \sin\theta\cos\theta\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$R_{133}{}^{2} = -\Gamma^{1}_{33}\Gamma^{2}_{11} - \Gamma^{2}_{33}\Gamma^{2}_{12} + \Gamma^{3}_{31}\Gamma^{2}_{33}$$
$$= -\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}} + \sin\theta\cos\theta\frac{1}{r} - \frac{1}{r}\sin\theta\cos\theta$$
$$= -\frac{\sin^{2}\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{133}{}^{3} = \Gamma^{1}_{31}\Gamma^{3}_{31} - \Gamma^{1}_{33}\Gamma^{3}_{11}$$
$$= \frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r} + \frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{-\partial_{\varphi}\left(\phi^{+}\phi\right)}{2r^{2}v^{2}\sin^{2}\theta}$$
$$= 0$$

$$R_{200}{}^{0} = \Gamma^{0}_{20,0} - \Gamma^{0}_{00,2} = -R_{020}{}^{0}$$

$$R_{200}{}^{1} = -\Gamma^{1}_{00,2} + \Gamma^{0}_{02}\Gamma^{1}_{00} - \Gamma^{1}_{00}\Gamma^{1}_{21} - \Gamma^{2}_{00}\Gamma^{1}_{22} = -R_{020}{}^{1}$$

$$R_{200}{}^{2} = -\Gamma^{2}_{00,2} + \Gamma^{0}_{02}\Gamma^{2}_{00} - \Gamma^{1}_{00}\Gamma^{2}_{21} = -R_{020}{}^{2}$$

$$R_{200}{}^{3} = -\Gamma^{3}_{00,2} + \Gamma^{0}_{02}\Gamma^{3}_{00} - \Gamma^{3}_{00}\Gamma^{3}_{23} = -R_{020}{}^{3}$$

$$R_{201}{}^{0} = -\Gamma^{0}_{01,2} - \Gamma^{0}_{10}\Gamma^{0}_{20} + \Gamma^{1}_{12}\Gamma^{0}_{01} + \Gamma^{2}_{12}\Gamma^{0}_{02} = -R_{021}{}^{0}$$

$$R_{201}{}^{1} = \Gamma^{1}_{21,0} - \Gamma^{1}_{01,2} = -R_{021}{}^{1}$$

$$R_{201}{}^{2} = -\Gamma^{1}_{10}\Gamma^{2}_{21} = -R_{021}{}^{2}$$

$$R_{202}{}^{0} = -\Gamma^{0}_{02,2} - \Gamma^{0}_{20}\Gamma^{0}_{20} + \Gamma^{1}_{22}\Gamma^{0}_{01} = -R_{022}{}^{0}$$

$$R_{202}{}^{1} = \Gamma^{1}_{22,0} + \Gamma^{1}_{22}\Gamma^{1}_{01} = -R_{022}{}^{1}$$

$$R_{203}{}^{0} = -\Gamma^{0}_{03,2} - \Gamma^{0}_{30}\Gamma^{0}_{20} + \Gamma^{3}_{32}\Gamma^{0}_{03} = -R_{023}{}^{0}$$

$$R_{210}{}^{0} = \Gamma^{0}_{20,1} - \Gamma^{0}_{10,2} = -R_{120}{}^{0}$$

$$R_{210}{}^{1} = -\Gamma^{1}_{10,2} + \Gamma^{0}_{02}\Gamma^{1}_{10} - \Gamma^{1}_{01}\Gamma^{1}_{21} = -R_{120}{}^{1}$$

$$R_{210}{}^{2} = -\Gamma^{1}_{01}\Gamma^{2}_{21} = -R_{120}{}^{2}$$

$$R_{211}{}^{0} = -\Gamma^{0}_{11,2} - \Gamma^{0}_{11}\Gamma^{0}_{20} + \Gamma^{1}_{12}\Gamma^{0}_{11} = -R_{121}{}^{0}$$

$$R_{211}{}^{1} = \Gamma^{1}_{21,1} - \Gamma^{1}_{11,2} + \Gamma^{2}_{12}\Gamma^{1}_{12} - \Gamma^{2}_{11}\Gamma^{1}_{22} = -R_{121}{}^{1}$$

$$R_{211}{}^{2} = \Gamma^{2}_{21,1} - \Gamma^{2}_{11,2} + \Gamma^{1}_{12}\Gamma^{2}_{11} - \Gamma^{1}_{11}\Gamma^{2}_{21} + \Gamma^{2}_{12}\Gamma^{2}_{12} = -R_{121}{}^{2}$$

$$R_{211}{}^{3} = -\Gamma^{3}_{11,2} + \Gamma^{1}_{12}\Gamma^{3}_{11} - \Gamma^{3}_{11}\Gamma^{3}_{23} = -R_{121}{}^{3}$$

$$R_{212}{}^{0} = \Gamma^{1}_{22}\Gamma^{0}_{11} = -R_{122}{}^{0}$$

$$R_{212}{}^{1} = \Gamma^{1}_{22,1} - \Gamma^{1}_{12,2} + \Gamma^{1}_{22}\Gamma^{1}_{11} - \Gamma^{1}_{21}\Gamma^{1}_{21} - \Gamma^{2}_{21}\Gamma^{1}_{22} = -R_{122}{}^{1}$$

$$R_{212}{}^{2} = \Gamma^{1}_{22}\Gamma^{2}_{11} - \Gamma^{1}_{21}\Gamma^{2}_{21} = -R_{122}{}^{2}$$

$$R_{212}{}^{3} = \Gamma^{1}_{22}\Gamma^{3}_{11} = -R_{122}{}^{3}$$

$$R_{213}{}^{1} = -\Gamma^{1}_{13,2} - \Gamma^{1}_{31}\Gamma^{1}_{21} + \Gamma^{3}_{32}\Gamma^{1}_{13} = -R_{123}{}^{1}$$

$$R_{213}{}^{2} = -\Gamma^{1}_{31}\Gamma^{2}_{21} = -R_{123}{}^{2}$$

$$R_{230}{}^{0}=\Gamma^{0}_{20,3}-\Gamma^{0}_{30,2}=-\partial_{\varphi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\partial_{\theta}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}=0$$

$$R_{231}{}^{1}=\Gamma^{1}_{21,3}-\Gamma^{1}_{31,2}=\partial_{\varphi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\partial_{\theta}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}=0$$

$$R_{231}{}^{2}=-\Gamma^{1}_{13}\Gamma^{2}_{21}=-\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{231}{}^{3}=\Gamma^{1}_{12}\Gamma^{3}_{31}+\Gamma^{2}_{12}\Gamma^{3}_{32}-\Gamma^{3}_{13}\Gamma^{3}_{23}=\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}+\frac{\cot\theta}{}-\frac{\cot\theta}{}=\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi r}$$

$$R_{232}{}^{1}=\Gamma^{1}_{22,3}+\Gamma^{1}_{22}\Gamma^{1}_{31}=-\partial_{\varphi}\frac{v^{2}r}{\phi^{+}\phi}-\frac{v^{2}r}{\phi^{+}\phi}\frac{\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}$$

$$=\frac{v^{2}r\partial_{\varphi}\left(\phi^{+}\phi\right)}{\left(\phi^{+}\phi\right)^{2}}-\frac{v^{2}r\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}=\frac{v^{2}r\partial_{\varphi}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}$$

$$R_{232}{}^{3}=-\Gamma^{3}_{32,2}+\Gamma^{1}_{22}\Gamma^{3}_{31}-\Gamma^{3}_{23}\Gamma^{3}_{23}$$

$$=-\partial_{\theta}\cot\theta-\frac{v^{2}r}{\phi^{+}\phi r}-\cot^{2}\theta$$

$$=-\partial_{\theta}\frac{\cos\theta}{\sin\theta}-\frac{v^{2}}{\phi^{+}\phi}-\cot^{2}\theta=\frac{1}{\sin^{2}\theta}-\frac{v^{2}}{\phi^{+}\phi}-\cot^{2}\theta$$

$$=\frac{1-\cos^{2}\theta}{\sin^{2}\theta}-\frac{v^{2}}{\phi^{+}\phi}=1-\frac{v^{2}}{\phi^{+}\phi}$$

$$R_{233}{}^{1}=-\Gamma^{1}_{33,2}-\Gamma^{1}_{33}\Gamma^{1}_{21}-\Gamma^{2}_{33}\Gamma^{1}_{22}+\Gamma^{3}_{32}\Gamma^{1}_{33}$$

$$=\partial_{\theta}\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}+\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}\frac{\partial_{\theta}\left(\phi^{+}\phi\right)}{2\phi^{+}\phi}-\sin\theta\cos\theta\frac{v^{2}r}{\phi^{+}\phi}-\cot\theta\frac{v^{2}r\sin^{2}\theta}{\phi^{+}\phi}$$

$$=\frac{2v^{2}r\sin\theta\cos\theta}{\phi^{+}\phi}-\frac{v^{2}r\sin^{2}\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{\left(\phi^{+}\phi\right)^{2}}$$

$$+\frac{v^{2}r\sin^{2}\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}-\sin\theta\cos\theta\frac{v^{2}r}{\phi^{+}\phi}-\frac{v^{2}r\sin\theta\cos\theta}{\phi^{+}\phi}$$

$$=-\frac{v^{2}r\sin^{2}\theta\partial_{\theta}\left(\phi^{+}\phi\right)}{2\left(\phi^{+}\phi\right)^{2}}$$

$$R_{233}{}^{2} = -\Gamma^{2}_{33,2} - \Gamma^{1}_{33}\Gamma^{2}_{21} + \Gamma^{3}_{32}\Gamma^{2}_{33}$$
$$= \partial_\theta\left(\sin\theta\cos\theta\right) + \frac{v^2 r\sin^2\theta}{\phi^+\phi r} - \cot\theta\sin\theta\cos\theta$$
$$= \cos^2\theta - \sin^2\theta + \frac{v^2 r\sin^2\theta}{\phi^+\phi r} - \cos^2\theta$$
$$= -\sin^2\theta + \frac{v^2\sin^2\theta}{\phi^+\phi}$$

$$R_{300}{}^{0} = \Gamma^{0}_{30,0} - \Gamma^{0}_{00,3} = -R_{030}{}^{0}$$

$$R_{300}{}^{1} = -\Gamma^{1}_{00,3} + \Gamma^{0}_{03}\Gamma^{1}_{00} - \Gamma^{1}_{00}\Gamma^{1}_{31} - \Gamma^{3}_{00}\Gamma^{1}_{33} = -R_{030}{}^{1}$$

$$R_{300}{}^{2} = -\Gamma^{2}_{00,3} + \Gamma^{0}_{03}\Gamma^{2}_{00} - \Gamma^{3}_{00}\Gamma^{2}_{33} = -R_{030}{}^{2}$$

$$R_{300}{}^{3} = -\Gamma^{3}_{00,3} + \Gamma^{0}_{03}\Gamma^{3}_{00} - \Gamma^{1}_{00}\Gamma^{3}_{31} - \Gamma^{2}_{00}\Gamma^{3}_{32} = -R_{030}{}^{3}$$

$$R_{301}{}^{0} = -\Gamma^{0}_{01,3} - \Gamma^{0}_{10}\Gamma^{0}_{30} + \Gamma^{1}_{13}\Gamma^{0}_{01} + \Gamma^{3}_{13}\Gamma^{0}_{03} = -R_{031}{}^{0}$$

$$R_{301}{}^{1} = \Gamma^{1}_{31,0} - \Gamma^{1}_{01,3} = -R_{031}{}^{1}$$

$$R_{301}{}^{3} = -\Gamma^{1}_{10}\Gamma^{3}_{31} = -R_{031}{}^{3}$$

$$R_{302}{}^{0} = -\Gamma^{0}_{02,3} - \Gamma^{0}_{20}\Gamma^{0}_{30} + \Gamma^{3}_{23}\Gamma^{0}_{03} = -R_{032}{}^{0}$$

$$R_{303}{}^{0} = -\Gamma^{0}_{03,3} - \Gamma^{0}_{30}\Gamma^{0}_{30} + \Gamma^{1}_{33}\Gamma^{0}_{01} + \Gamma^{2}_{33}\Gamma^{0}_{02} = -R_{033}{}^{0}$$

$$R_{303}{}^{1} = \Gamma^{1}_{33,0} + \Gamma^{1}_{33}\Gamma^{1}_{01} = -R_{033}{}^{1}$$

$$R_{310}{}^{0} = \Gamma^{0}_{30,1} - \Gamma^{0}_{10,3} = -R_{130}{}^{0}$$

$$R_{310}{}^{1} = -\Gamma^{1}_{10,3} + \Gamma^{0}_{03}\Gamma^{1}_{10} - \Gamma^{1}_{01}\Gamma^{1}_{31} = -R_{130}{}^{1}$$

$$R_{310}{}^{3} = -\Gamma^{1}_{01}\Gamma^{3}_{31} = -R_{130}{}^{3}$$

$$R_{311}{}^{0} = -\Gamma^{0}_{11,3} - \Gamma^{0}_{11}\Gamma^{0}_{30} + \Gamma^{1}_{13}\Gamma^{0}_{11} = -R_{131}{}^{0}$$

$$R_{311}{}^{1} = \Gamma^{1}_{31,1} - \Gamma^{1}_{11,3} + \Gamma^{3}_{13}\Gamma^{1}_{13} - \Gamma^{3}_{11}\Gamma^{1}_{33} = -R_{131}{}^{1}$$

$$R_{311}{}^{2} = -\Gamma^{2}_{11,3} + \Gamma^{1}_{13}\Gamma^{2}_{11} - \Gamma^{3}_{11}\Gamma^{2}_{33} = -R_{131}{}^{2}$$

$$R_{311}{}^{3} = \Gamma^{3}_{31,1} - \Gamma^{3}_{11,3} + \Gamma^{1}_{13}\Gamma^{3}_{11} - \Gamma^{1}_{11}\Gamma^{3}_{31} - \Gamma^{2}_{11}\Gamma^{3}_{32} + \Gamma^{3}_{13}\Gamma^{3}_{13} = -R_{131}{}^{3}$$

$$R_{312}{}^{1} = -\Gamma^{1}_{12,3} - \Gamma^{1}_{21}\Gamma^{1}_{31} + \Gamma^{3}_{23}\Gamma^{1}_{13} = -R_{132}{}^{1}$$

$$R_{312}{}^{3} = -\Gamma^1_{21}\Gamma^3_{31} - \Gamma^2_{21}\Gamma^3_{32} + \Gamma^3_{23}\Gamma^3_{13} = -R_{132}{}^{3}$$

$$R_{313}{}^{0} = \Gamma^1_{33}\Gamma^0_{11} = -R_{133}{}^{0}$$

$$R_{313}{}^{1} = \Gamma^1_{33,1} - \Gamma^1_{13,3} + \Gamma^1_{33}\Gamma^1_{11} - \Gamma^1_{31}\Gamma^1_{31} + \Gamma^2_{33}\Gamma^1_{12} - \Gamma^3_{31}\Gamma^1_{33} = -R_{133}{}^{1}$$

$$R_{313}{}^{2} = \Gamma^1_{33}\Gamma^2_{11} + \Gamma^2_{33}\Gamma^2_{12} - \Gamma^3_{31}\Gamma^2_{33} = -R_{133}{}^{2}$$

$$R_{313}{}^{3} = \Gamma^1_{33}\Gamma^3_{11} - \Gamma^1_{31}\Gamma^3_{31} = -R_{133}{}^{3}$$

$$R_{320}{}^{0} = \Gamma^0_{30,2} - \Gamma^0_{20,3} = -R_{230}{}^{0}$$

$$R_{321}{}^{1} = \Gamma^1_{31,2} - \Gamma^1_{21,3} = -R_{231}{}^{1}$$

$$R_{321}{}^{2} = \Gamma^1_{13}\Gamma^2_{21} = -R_{231}{}^{2}$$

$$R_{321}{}^{3} = -\Gamma^1_{12}\Gamma^3_{31} - \Gamma^2_{12}\Gamma^3_{32} + \Gamma^3_{13}\Gamma^3_{23} = -R_{231}{}^{3}$$

$$R_{322}{}^{1} = -\Gamma^1_{22,3} - \Gamma^1_{22}\Gamma^1_{31} = -R_{232}{}^{1}$$

$$R_{322}{}^{3} = \Gamma^3_{32,2} - \Gamma^1_{22}\Gamma^3_{31} + \Gamma^3_{23}\Gamma^3_{23} = -R_{232}{}^{3}$$

$$R_{323}{}^{1} = \Gamma^1_{33,2} + \Gamma^1_{33}\Gamma^1_{21} + \Gamma^2_{33}\Gamma^1_{22} - \Gamma^3_{32}\Gamma^1_{33} = -R_{233}{}^{1}$$

$$R_{323}{}^{2} = \Gamma^2_{33,2} + \Gamma^1_{33}\Gamma^2_{21} - \Gamma^3_{32}\Gamma^2_{33} = -R_{233}{}^{2}$$

$$\begin{aligned}
R_{00} &= R_{010}{}^{1} + R_{020}{}^{2} + R_{030}{}^{3} \\
&= \frac{c^2v^4\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^4} - \frac{c^2v^4\partial_r^2\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{\partial_t^2\left(\phi^+\phi\right)}{2\phi^+\phi} + \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4r^2\sin^2\theta\left(\phi^+\phi\right)^3} \\
&- \frac{3c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\theta^2\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2r\left(\phi^+\phi\right)^3} \\
&- \frac{3c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4r^2\sin^2\theta\left(\phi^+\phi\right)^3} + \frac{c^2v^2\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3 r} + \frac{c^2v^2\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} \\
&= \frac{c^2v^4\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^4} - \frac{c^2v^4\partial_r^2\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3} - \frac{\partial_t^2\left(\phi^+\phi\right)}{2\phi^+\phi} - \frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3} - \frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^3} \\
&+ \frac{c^2v^2\partial_\theta^2\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2} - \frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^3 r} + \frac{c^2v^2\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2} + \frac{c^2v^2\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}
\end{aligned}$$

$$R_{01}=R_{011}{}^{1}+R_{021}{}^{2}+R_{031}{}^{3}=0+\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi r}=\frac{\partial_t\left(\phi^+\phi\right)}{\phi^+\phi r}$$

$$R_{02}=R_{012}{}^{1}=-\frac{\partial_t\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{03}=R_{013}{}^{1}=-\frac{\partial_t\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{10}=R_{100}{}^{0}+R_{120}{}^{2}+R_{130}{}^{3}=0+\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi r}=\frac{\partial_t\left(\phi^+\phi\right)}{\phi^+\phi r}$$

$$R_{11}=R_{101}{}^{0}+R_{121}{}^{2}+R_{131}{}^{3}$$

$$=-\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{\partial_r^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\phi^+\phi\partial_t^2\left(\phi^+\phi\right)}{2c^2v^4}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4r^2v^2\phi^+\phi}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4r^2v^2\sin^2\theta\phi^+\phi}$$

$$-\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2r^2v^2}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi r}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\phi^+\phi r^2v^2}$$

$$-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}+\frac{\partial_r\left(\phi^+\phi\right)}{2\phi^+\phi r}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4\phi^+\phi r^2v^2\sin^2\theta}-\frac{\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}$$

$$=-\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{\partial_r^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\phi^+\phi\partial_t^2\left(\phi^+\phi\right)}{2c^2v^4}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2\phi^+\phi}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta\phi^+\phi}$$

$$-\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2r^2v^2}+\frac{\partial_r\left(\phi^+\phi\right)}{\phi^+\phi r}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}-\frac{\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}$$

$$R_{12}=R_{102}{}^{0}+R_{122}{}^{2}+R_{132}{}^{3}=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}-\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r}+\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi r}=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}$$

$$R_{13}=R_{103}{}^{0}+R_{123}{}^{2}+R_{133}{}^{3}=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}-\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi r}+\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi r}+0=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}$$

$$R_{20}=R_{200}{}^{0}+R_{210}{}^{1}=-\frac{\partial_\theta\partial_t\left(\phi^+\phi\right)}{2\phi^+\phi}$$

$$R_{21}=R_{201}{}^{0}+R_{211}{}^{1}+R_{231}{}^{3}=R_{12}=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}$$

$$R_{22}=R_{202}{}^{0}+R_{212}{}^{1}+R_{232}{}^{3}$$
$$=-\frac{3\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{v^2r\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$
$$+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}-\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{v^2r\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+1-\frac{v^2}{\phi^+\phi}$$
$$=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\frac{v^2r\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+1-\frac{v^2}{\phi^+\phi}$$
$$R_{23}=R_{203}{}^{0}+R_{213}{}^{1}=-\frac{3\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}-\cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$
$$+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}-\frac{\partial_\theta\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}+\cot\theta\frac{\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$
$$R_{30}=R_{300}{}^{0}+R_{310}{}^{1}=R_{03}=-\frac{\partial_t\partial_\varphi\left(\phi^+\phi\right)}{2\phi^+\phi}$$
$$R_{31}=R_{301}{}^{0}+R_{311}{}^{1}+R_{321}{}^{2}=R_{13}=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}$$
$$R_{32}=R_{302}{}^{0}+R_{312}{}^{1}=R_{23}=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}$$
$$R_{33}=R_{303}{}^{0}+R_{313}{}^{1}+R_{323}{}^{2}$$
$$=-\frac{3\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{v^2r\sin^2\theta\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\sin\theta\cos\theta\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$
$$+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{4\left(\phi^+\phi\right)^2}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{v^2r\sin^2\theta\partial_r\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}-\sin\theta\cos\theta\frac{\partial_\theta\left(\phi^+\phi\right)}{2\phi^+\phi}$$
$$+\sin^2\theta-\frac{v^2r\sin^2\theta}{\phi^+\phi r}$$
$$=-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\frac{v^2r\sin^2\theta\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\sin^2\theta\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$R = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33}$$

$$= -\frac{\phi^+\phi}{c^2v^2}\left(\begin{array}{l} \frac{c^2v^4\partial_r(\phi^+\phi)\partial_r(\phi^+\phi)}{(\phi^+\phi)^4} - \frac{c^2v^4\partial_r^2(\phi^+\phi)}{2(\phi^+\phi)^3} - \frac{\partial_t^2(\phi^+\phi)}{2\phi^+\phi} - \frac{c^2v^2\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{2r^2(\phi^+\phi)^3} - \frac{c^2v^2\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{2r^2\sin^2\theta\,(\phi^+\phi)^3} \\ + \frac{c^2v^2\partial_\theta^2(\phi^+\phi)}{2r^2(\phi^+\phi)^2} - \frac{c^2v^4\partial_r(\phi^+\phi)}{(\phi^+\phi)^3 r} + \frac{c^2v^2\partial_\varphi^2(\phi^+\phi)}{2r^2\sin^2\theta\,(\phi^+\phi)^2} + \frac{c^2v^2\cot\theta\partial_\theta(\phi^+\phi)}{2r^2(\phi^+\phi)^2} \end{array}\right)$$

$$+\frac{v^2}{\phi^+\phi}\left(\begin{array}{l} -\frac{\partial_r(\phi^+\phi)\partial_r(\phi^+\phi)}{(\phi^+\phi)^2} + \frac{\partial_r^2(\phi^+\phi)}{2\phi^+\phi} + \frac{\phi^+\phi\partial_t^2(\phi^+\phi)}{2c^2v^4} + \frac{\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{2r^2v^2\phi^+\phi} + \frac{\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{2r^2v^2\sin^2\theta\phi^+\phi} \\ -\frac{\partial_\theta^2(\phi^+\phi)}{2r^2v^2} + \frac{\partial_r(\phi^+\phi)}{\phi^+\phi r} - \frac{\partial_\varphi^2(\phi^+\phi)}{2r^2v^2\sin^2\theta} - \frac{\cot\theta\partial_\theta(\phi^+\phi)}{2r^2v^2} \end{array}\right)$$

$$+\frac{1}{r^2}\left(-\frac{\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{2(\phi^+\phi)^2} + \frac{v^2r\partial_r(\phi^+\phi)}{(\phi^+\phi)^2} + 1 - \frac{v^2}{\phi^+\phi}\right)$$

$$+\frac{1}{r^2\sin^2\theta}\left(-\frac{\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{2(\phi^+\phi)^2} + \frac{v^2r\sin^2\theta\partial_r(\phi^+\phi)}{(\phi^+\phi)^2} + \sin^2\theta\left(1 - \frac{v^2}{\phi^+\phi}\right)\right)$$

$$= 2\left(\begin{array}{l} -\frac{v^2\partial_r(\phi^+\phi)\partial_r(\phi^+\phi)}{(\phi^+\phi)^3} + \frac{v^2\partial_r^2(\phi^+\phi)}{2(\phi^+\phi)^2} + \frac{\partial_t^2(\phi^+\phi)}{2c^2v^2} + \frac{\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{2r^2(\phi^+\phi)^2} + \frac{\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{2r^2\sin^2\theta\,(\phi^+\phi)^2} \\ -\frac{\partial_\theta^2(\phi^+\phi)}{2r^2\phi^+\phi} + \frac{v^2\partial_r(\phi^+\phi)}{(\phi^+\phi)^2 r} - \frac{\partial_\varphi^2(\phi^+\phi)}{2r^2\sin^2\theta\phi^+\phi} - \frac{\cot\theta\partial_\theta(\phi^+\phi)}{2r^2\phi^+\phi} \end{array}\right)$$

$$+\frac{2}{r^2}\left(-\frac{\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{4(\phi^+\phi)^2} - \frac{\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{4(\phi^+\phi)^2\sin^2\theta} + \frac{v^2r\partial_r(\phi^+\phi)}{(\phi^+\phi)^2} + 1 - \frac{v^2}{\phi^+\phi}\right)$$

$$= 2\left(\begin{array}{l} -\frac{v^2\partial_r(\phi^+\phi)\partial_r(\phi^+\phi)}{(\phi^+\phi)^3} + \frac{v^2\partial_r^2(\phi^+\phi)}{2(\phi^+\phi)^2} + \frac{\partial_t^2(\phi^+\phi)}{2c^2v^2} + \frac{\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{4r^2(\phi^+\phi)^2} + \frac{\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{4r^2\sin^2\theta\,(\phi^+\phi)^2} \\ -\frac{\partial_\theta^2(\phi^+\phi)}{2r^2\phi^+\phi} + \frac{v^2\partial_r(\phi^+\phi)}{(\phi^+\phi)^2 r} - \frac{\partial_\varphi^2(\phi^+\phi)}{2r^2\sin^2\theta\phi^+\phi} - \frac{\cot\theta\partial_\theta(\phi^+\phi)}{2r^2\phi^+\phi} \end{array}\right)$$

$$+\frac{2}{r^2}\left(\frac{v^2r\partial_r(\phi^+\phi)}{(\phi^+\phi)^2} + 1 - \frac{v^2}{\phi^+\phi}\right)$$

$$= 2\left(\begin{array}{l} -\frac{v^2\partial_r(\phi^+\phi)\partial_r(\phi^+\phi)}{(\phi^+\phi)^3} + \frac{v^2\partial_r^2(\phi^+\phi)}{2(\phi^+\phi)^2} + \frac{\partial_t^2(\phi^+\phi)}{2c^2v^2} + \frac{\partial_\theta(\phi^+\phi)\partial_\theta(\phi^+\phi)}{4r^2(\phi^+\phi)^2} + \frac{\partial_\varphi(\phi^+\phi)\partial_\varphi(\phi^+\phi)}{4r^2\sin^2\theta\,(\phi^+\phi)^2} \\ -\frac{\partial_\theta^2(\phi^+\phi)}{2r^2\phi^+\phi} + \frac{2v^2\partial_r(\phi^+\phi)}{(\phi^+\phi)^2 r} - \frac{\partial_\varphi^2(\phi^+\phi)}{2r^2\sin^2\theta\phi^+\phi} - \frac{\cot\theta\partial_\theta(\phi^+\phi)}{2r^2\phi^+\phi} + \frac{1}{r^2}\left(1 - \frac{v^2}{\phi^+\phi}\right) \end{array}\right)$$

$$=-\frac{2v^2\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^3}+\frac{v^2\partial_r^2\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{4v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2 r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$+\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}-\frac{\partial_\theta^2\left(\phi^+\phi\right)}{r^2\phi^+\phi}$$

$$+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}-\frac{\cot\theta\partial_\theta\left(\phi^+\phi\right)}{r^2\phi^+\phi}$$

$$=-\frac{2v^2\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^3}+\frac{v^2\partial_r^2\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{4v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2 r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$+\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}$$

$$+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}$$

In which：

$$-\frac{2v^2\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^3}+\frac{v^2\partial_r^2\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{4v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2 r}$$

$$=-\frac{1}{r^2}\partial_r\left(r^2\partial_r\left(\frac{v^2}{\phi^+\phi}\right)\right)+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{r\left(\phi^+\phi\right)^2}$$

$$=\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\left(\phi^+\phi\right)^2}\partial_r\left(\phi^+\phi\right)\right)+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{r\left(\phi^+\phi\right)^2}$$

$$=\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{1}{r^2}\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\frac{1}{\phi^+\phi}+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{r\left(\phi^+\phi\right)^2}$$

$$=\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{r\left(\phi^+\phi\right)^2}$$

$$R=\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}+\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}$$

$$-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}$$

$$+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2 r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$\frac{d^2\eta}{ds^2}=-\frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\phi^+\phi}{v^2}\frac{\partial\eta}{\partial t}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{v^2r^2}{\phi^+\phi}\frac{\partial\eta}{\partial r}\right)+\frac{1}{r^2\,\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\eta}{\partial\theta}\right)+\frac{1}{r^2\,\sin^2\theta}\frac{\partial^2\eta}{\partial\varphi^2}$$

$$g^{ab}\nabla_a\nabla_b\eta=-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\eta\right)+\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\eta\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\eta\right)}{r^2\,\sin\theta}+\frac{\partial_\varphi^2\eta}{r^2\,\sin^2\theta}$$

$$g^{ab}\nabla_a\left(\frac{1}{\eta}\nabla_b\eta\right)=-\frac{1}{\eta^2}g^{ab}\nabla_a\eta\nabla_b\eta+\frac{1}{\eta}g^{ab}\nabla_a\nabla_b\eta$$

$$=-\frac{1}{\eta^2}\left(-\frac{\phi^+\phi}{c^2v^2}\partial_t\eta\partial_t\eta+\frac{v^2}{\phi^+\phi}\partial_r\eta\partial_r\eta+\frac{\partial_\theta\eta\partial_\theta\eta}{r^2}+\frac{\partial_\varphi\eta\partial_\varphi\eta}{r^2\,\sin^2\theta}\right)$$

$$+\frac{1}{\eta}\left(-\partial_t\left(\frac{\phi^+\phi}{c^2v^2}\partial_t\eta\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\eta\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\eta\right)}{r^2\,\sin\theta}+\frac{\partial_\varphi^2\eta}{r^2\,\sin^2\theta}\right)$$

$$g^{ab}\nabla_a\left(\frac{1}{\phi^+\phi}\nabla_b\left(\phi^+\phi\right)\right)$$

$$=-\frac{1}{\left(\phi^+\phi\right)^2}\left(-\frac{\phi^+\phi}{c^2v^2}\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)+\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\,\sin^2\theta}\right)$$

$$+\frac{1}{\left(\phi^+\phi\right)}\left(-\partial_t\left(\frac{\phi^+\phi}{c^2v^2}\partial_t\left(\phi^+\phi\right)\right)+\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\,\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\,\sin^2\theta}\right)$$

$$R=\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}+\frac{1}{c^2\phi^+\phi}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)$$

$$-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}-\frac{\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2\phi^+\phi}$$

$$+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$=\frac{1}{\left(\phi^+\phi\right)^2}\left(-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2}-\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)$$

$$-\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)-\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)$$

$$+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$=\frac{1}{\left(\phi^+\phi\right)^2}\left(-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2}-\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)$$

$$-\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)-\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)$$

$$+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)-\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)$$

Let $\phi=J(r)K(t,\theta,\varphi)$

$$R=g^{ab}\nabla_a\left(\frac{1}{J^+J}\nabla_b\left(J^+J\right)\right)-g^{ab}\nabla_a\left(\frac{1}{K^+K}\nabla_b\left(K^+K\right)\right)+Q$$

We have： $=g^{ab}\nabla_a\left(\frac{K^+K}{J^+J}\nabla_b\left(\frac{J^+J}{K^+K}\right)\right)+Q=g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right)+Q$

$$B=J/K$$

$$\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}=-\frac{\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2\phi^+\phi}+\frac{1}{c^2\phi^+\phi}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)$$

$$Q=\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)-\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)$$

$$R_{00}=\frac{c^2v^4\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^4}-\frac{c^2v^4\partial_r^2\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^3}-\frac{\partial_t^2\left(\phi^+\phi\right)}{2\phi^+\phi}-\frac{c^2v^2\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^3}-\frac{c^2v^2\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^3}$$

$$+\frac{c^2v^2\partial_\theta^2\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}-\frac{c^2v^4\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^3r}+\frac{c^2v^2\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}+\frac{c^2v^2\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}$$

$$=-\frac{c^2v^2}{\phi^+\phi}\left(\begin{array}{l}\frac{1}{2}\left(-\frac{2v^2\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^3}+\frac{v^4\partial_r^2\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}\right)\\+\frac{\partial_t^2\left(\phi^+\phi\right)}{2c^2v^2}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}\\-\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2r^2\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2\sin^2\theta\phi^+\phi}-\frac{\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2\phi^+\phi}\end{array}\right)$$

$$=-\frac{c^2v^2}{2\phi^+\phi}\left(\begin{array}{l}\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)\\+\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta\left(\phi^+\phi\right)^2}\\-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}\end{array}\right)$$

$$R_{11}=-\frac{\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+\frac{\partial_r^2\left(\phi^+\phi\right)}{2\phi^+\phi}+\frac{\phi^+\phi\partial_t^2\left(\phi^+\phi\right)}{2c^2v^4}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2\phi^+\phi}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta\phi^+\phi}$$

$$-\frac{\partial_\theta^2\left(\phi^+\phi\right)}{2r^2v^2}+\frac{\partial_r\left(\phi^+\phi\right)}{\phi^+\phi r}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{2r^2v^2\sin^2\theta}-\frac{\cot\theta\partial_\theta\left(\phi^+\phi\right)}{2r^2v^2}$$

$$=\frac{\phi^+\phi}{2v^2}\left(\begin{array}{l}\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)\\+\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta\left(\phi^+\phi\right)^2}\\-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}\end{array}\right)$$

$$G_{00}=R_{00}-\frac{R}{2}g_{00}$$

$$=-\frac{c^2v^2}{2\phi^+\phi}\left(\begin{array}{l}\cancel{\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}\\ \cancel{-\frac{\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2\phi^+\phi}+\frac{1}{c^2\phi^+\phi}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta\left(\phi^+\phi\right)^2}\\ \cancel{-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}}\end{array}\right)$$

$$+\frac{c^2v^2}{2\phi^+\phi}\left(\begin{array}{l}\frac{1}{\left(\phi^+\phi\right)^2}\left(\cancel{-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2}-\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\ -\frac{1}{\phi^+\phi}\left(\cancel{-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)-\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)}+\cancel{\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}}\right)\\ +\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)-\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)\end{array}\right)$$

$$G_{00}=\frac{c^2v^2}{2\phi^+\phi}\left(-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)\right)$$

$$=\frac{c^2v^2}{2\phi^+\phi}Q=-\frac{g_{00}Q}{2}$$

$$G_{11}=R_{11}-\frac{R}{2}g_{11}$$

$$=\frac{\phi^+\phi}{2v^2}\left(\begin{array}{l}\frac{1}{r^2\phi^+\phi}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)-\frac{v^2}{\left(\phi^+\phi\right)^3}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)\\+\frac{\partial_t^2\left(\phi^+\phi\right)}{c^2v^2}+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2\left(\phi^+\phi\right)^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta\left(\phi^+\phi\right)^2}\\-\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta\phi^+\phi}-\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta\phi^+\phi}\end{array}\right)$$

$$-\frac{\phi^+\phi}{2v^2}\left(\begin{array}{l}\frac{1}{\left(\phi^+\phi\right)^2}\left(-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2}-\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\-\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)-\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)-\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)\end{array}\right)$$

$$G_{11}=-\frac{\phi^+\phi}{2v^2}\left(-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2\left(\phi^+\phi\right)^2}-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta\left(\phi^+\phi\right)^2}+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)\right)$$

$$=-\frac{g_{11}Q}{2}=-\frac{\phi^+\phi Q}{2v^2}$$

$$G_{22}=R_{22}-\frac{R}{2}g_{22}$$

$$=-\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2}+\frac{v^2r\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2}+1-\frac{v^2}{\phi^+\phi}$$

$$-\frac{r^2}{2}\left(\begin{array}{l}\frac{1}{\left(\phi^+\phi\right)^2}\left(-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2}-\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\ -\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)-\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\ +\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2r}+\frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right)-\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)\end{array}\right)$$

$$=-\frac{r^2}{2}\left(\begin{array}{l}\frac{1}{\left(\phi^+\phi\right)^2}\left(-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2}-\frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2}+\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\ -\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right)-\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\ +\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)\end{array}\right)$$

$$=-\frac{r^2}{2}\left(g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right)+\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2}-\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)\right)$$

$$=-\frac{r^2}{2}\left(g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right)+\frac{1}{\left(K^+K\right)^2}\left(\frac{\partial_\theta\left(K^+K\right)\partial_\theta\left(K^+K\right)}{2r^2}-\frac{\partial_\varphi\left(K^+K\right)\partial_\varphi\left(K^+K\right)}{2r^2\sin^2\theta}\right)\right)$$

$$=-\frac{r^2}{2}\left(g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right)+\frac{1}{\left(B^+B\right)^2}\left(\frac{\partial_\theta\left(B^+B\right)\partial_\theta\left(B^+B\right)}{2r^2}-\frac{\partial_\varphi\left(B^+B\right)\partial_\varphi\left(B^+B\right)}{2r^2\sin^2\theta}\right)\right)$$

$$G_{33} = R_{33} - \frac{R}{2} g_{33}$$

$$= -\frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2\left(\phi^+\phi\right)^2} + \frac{v^2 r\sin^2\theta\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2} + \sin^2\theta\left(1-\frac{v^2}{\phi^+\phi}\right)$$

$$-\frac{r^2\sin^2\theta}{2}\left(\begin{array}{l}
\frac{1}{\left(\phi^+\phi\right)^2}\left(\begin{array}{l}
-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2} - \frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)\\
+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2} + \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}
\end{array}\right)\\
-\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right) - \frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right) + \frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta} + \frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\
+\frac{2v^2\partial_r\left(\phi^+\phi\right)}{\left(\phi^+\phi\right)^2 r} + \frac{2}{r^2}\left(1-\frac{v^2}{\phi^+\phi}\right) - \frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2} + \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)
\end{array}\right)$$

$$= -\frac{r^2\sin^2\theta}{2}\left(\begin{array}{l}
\frac{1}{\left(\phi^+\phi\right)^2}\left(\begin{array}{l}
-\frac{\phi^+\phi\partial_t\left(\phi^+\phi\right)\partial_t\left(\phi^+\phi\right)}{c^2v^2} - \frac{v^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\partial_r\left(\phi^+\phi\right)\\
+\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{r^2} + \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{r^2\sin^2\theta}
\end{array}\right)\\
-\frac{1}{\phi^+\phi}\left(-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\left(\phi^+\phi\right)\right) - \frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\left(\phi^+\phi\right)\right) + \frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^+\phi\right)\right)}{r^2\sin\theta} + \frac{\partial_\varphi^2\left(\phi^+\phi\right)}{r^2\sin^2\theta}\right)\\
-\frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2} - \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)
\end{array}\right)$$

$$= -\frac{r^2\sin^2\theta}{2}\left(g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right) - \frac{1}{\left(\phi^+\phi\right)^2}\left(\frac{\partial_\theta\left(\phi^+\phi\right)\partial_\theta\left(\phi^+\phi\right)}{2r^2} - \frac{\partial_\varphi\left(\phi^+\phi\right)\partial_\varphi\left(\phi^+\phi\right)}{2r^2\sin^2\theta}\right)\right)$$

$$= -\frac{r^2\sin^2\theta}{2}\left(g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right) - \frac{1}{\left(K^+K\right)^2}\left(\frac{\partial_\theta\left(K^+K\right)\partial_\theta\left(K^+K\right)}{2r^2} - \frac{\partial_\varphi\left(K^+K\right)\partial_\varphi\left(K^+K\right)}{2r^2\sin^2\theta}\right)\right)$$

$$= -\frac{r^2\sin^2\theta}{2}\left(g^{ab}\nabla_a\left(\frac{1}{B^+B}\nabla_b\left(B^+B\right)\right) - \frac{1}{\left(B^+B\right)^2}\left(\frac{\partial_\theta\left(B^+B\right)\partial_\theta\left(B^+B\right)}{2r^2} - \frac{\partial_\varphi\left(B^+B\right)\partial_\varphi\left(B^+B\right)}{2r^2\sin^2\theta}\right)\right)$$

$$\nabla_b \psi = \left(\beta + \gamma \psi^+ \psi\right) \int P_a P_b \psi \, dx^a$$

$$\nabla_b \psi^+ = \left(\beta + \gamma \psi^+ \psi\right) \int P_a P_b \psi^+ \, dx^a$$

$$g^{ab} \nabla_a \frac{1}{\beta + \gamma \psi^+ \psi} \nabla_b \psi = g^{ab} P_a P_b \psi = \alpha \psi$$

$$\frac{1}{\beta + \gamma \psi^+ \psi} g^{ab} \nabla_a \nabla_b \psi - \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \psi}{\left(\beta + \gamma \psi^+ \psi\right)^2} = \alpha \psi$$

$$g^{ab} \nabla_a \nabla_b \psi = \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \psi}{\beta + \gamma \psi^+ \psi} + \alpha \psi \left(\beta + \gamma \psi^+ \psi\right)$$

$$\psi^+ g^{ab} \nabla_a \nabla_b \psi = \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \psi^+ \nabla_b \psi}{\beta + \gamma \psi^+ \psi} + \alpha \psi^+ \psi \left(\beta + \gamma \psi^+ \psi\right)$$

$$g^{ab} \nabla_a \nabla_b \psi^+ \psi = \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \psi^+ \psi}{\beta + \gamma \psi^+ \psi} + \alpha \psi^+ \psi \left(\beta + \gamma \psi^+ \psi\right)$$

$$\psi^+ g^{ab} \nabla_a \nabla_b \psi + g^{ab} \nabla_a \nabla_b \psi^+ \psi = \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \left(\psi^+ \psi\right)}{\beta + \gamma \psi^+ \psi} + 2\alpha \psi^+ \psi \left(\beta + \gamma \psi^+ \psi\right)$$

$$\begin{aligned}
&g^{ab} \nabla_a \nabla_b \left(\psi^+ \psi\right) \\
&= g^{ab} \nabla_a \left(\psi^+ \nabla_b \psi + \nabla_b \psi^+ \psi\right) \\
&= \psi^+ g^{ab} \nabla_a \nabla_b \psi + g^{ab} \nabla_a \nabla_b \psi^+ \psi + 2 g^{ab} \nabla_a \psi^+ \nabla_b \psi \\
&\Rightarrow \psi^+ g^{ab} \nabla_a \nabla_b \psi + g^{ab} \nabla_a \nabla_b \psi^+ \psi = g^{ab} \nabla_a \nabla_b \left(\psi^+ \psi\right) - 2 g^{ab} \nabla_a \psi^+ \nabla_b \psi
\end{aligned}$$

$$g^{ab} \nabla_a \nabla_b \left(\psi^+ \psi\right) - 2 g^{ab} \nabla_a \psi^+ \nabla_b \psi = \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \left(\psi^+ \psi\right)}{\beta + \gamma \psi^+ \psi} + 2\alpha \psi^+ \psi \left(\beta + \gamma \psi^+ \psi\right)$$

$$\begin{aligned}
&\frac{1}{2}\left(g^{ab} \nabla_a \nabla_b \left(\psi^+ \psi\right) - \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \left(\psi^+ \psi\right)}{\beta + \gamma \psi^+ \psi} \right) = g^{ab} \nabla_a \psi^+ \nabla_b \psi + \alpha \psi^+ \psi \left(\beta + \gamma \psi^+ \psi\right) \\
&= \frac{\beta + \gamma \psi^+ \psi}{2} \left(\frac{g^{ab} \nabla_a \nabla_b \left(\psi^+ \psi\right)}{\beta + \gamma \psi^+ \psi} - \frac{\gamma g^{ab} \nabla_a \left(\psi^+ \psi\right) \nabla_b \left(\psi^+ \psi\right)}{\left(\beta + \gamma \psi^+ \psi\right)^2} \right) \\
&= \frac{\beta + \gamma \psi^+ \psi}{2} g^{ab} \nabla_a \frac{\nabla_b \left(\psi^+ \psi\right)}{\beta + \gamma \psi^+ \psi} \\
&= \frac{\beta + \gamma \psi^+ \psi}{2\gamma} g^{ab} \nabla_a \frac{\nabla_b \left(\beta + \gamma \psi^+ \psi\right)}{\beta + \gamma \psi^+ \psi}
\end{aligned}$$

$$\frac{1}{2}\left(\beta + \gamma \phi^+ \phi\right) g^{ab} \nabla_a \left(\frac{1}{\beta + \gamma \phi^+ \phi} \nabla_a \left(\phi^+ \phi\right) \right) = g^{ab} \nabla_a \phi^+ \nabla_b \phi + \alpha \beta \phi^+ \phi + \alpha \gamma \left(\phi^+ \phi\right)^2$$

$$\eta = \begin{pmatrix} \phi \\ \sqrt{\frac{\beta}{\gamma}} e^{i\theta} \end{pmatrix}$$

$$\frac{1}{2}\eta^{+}\eta g^{ab}\nabla_a\left(\frac{1}{\gamma\eta^{+}\eta}\nabla_b\left(\gamma\eta^{+}\eta\right)\right) = g^{ab}\nabla_a\eta^{+}\nabla_b\eta - \alpha\beta\eta^{+}\eta + \alpha\gamma\left(\eta^{+}\eta\right)^2$$

$$g^{ab}\nabla_a\frac{1}{\xi^{+}}\nabla_b\frac{1}{\xi} - \alpha\beta\frac{1}{\xi^{+}\xi} + \alpha\gamma\left(\frac{1}{\xi^{+}\xi}\right)^2$$

$$= \left(\frac{1}{\xi^{+}\xi}\right)^2 g^{ab}\nabla_a\xi^{+}\nabla_b\xi - \alpha\beta\frac{1}{\xi^{+}\xi} + \alpha\gamma\left(\frac{1}{\xi^{+}\xi}\right)^2$$

$$= \left(\frac{1}{\xi^{+}\xi}\right)^2\left(g^{ab}\nabla_a\xi^{+}\nabla_b\xi - \alpha\beta\xi^{+}\xi + \alpha\gamma\right)$$

$$= \frac{1}{2}\left(\frac{1}{\xi^{+}\xi}\right) g^{ab}\nabla_a\left(\frac{\xi^{+}\xi}{\gamma}\nabla_b\left(\frac{\gamma}{\xi^{+}\xi}\right)\right)$$

$$= -\frac{1}{2}\left(\frac{1}{\xi^{+}\xi}\right) g^{ab}\nabla_a\left(\frac{1}{\gamma\xi^{+}\xi}\nabla_b\left(\gamma\xi^{+}\xi\right)\right)$$

$$g^{ab}\nabla_a\xi^{+}\nabla_b\xi - \alpha\beta\xi^{+}\xi + \alpha\gamma = -\xi^{+}\xi g^{ab}\nabla_a\left(\frac{1}{\gamma\xi^{+}\xi}\nabla_b\left(\gamma\xi^{+}\xi\right)\right)$$

$$g^{ab}\nabla_a\left(\frac{1}{\phi^{+}\phi}\nabla_b\left(\phi^{+}\phi\right)\right)$$

$$= -\frac{1}{\left(\phi^{+}\phi\right)^2}\left(-\frac{\phi^{+}\phi}{c^2v^2}\partial_t\left(\phi^{+}\phi\right)\partial_t\left(\phi^{+}\phi\right) + \frac{v^2}{\phi^{+}\phi}\partial_r\left(\phi^{+}\phi\right)\partial_r\left(\phi^{+}\phi\right) + \frac{\partial_\theta\left(\phi^{+}\phi\right)\partial_\theta\left(\phi^{+}\phi\right)}{r^2} + \frac{\partial_\varphi\left(\phi^{+}\phi\right)\partial_\varphi\left(\phi^{+}\phi\right)}{r^2\sin^2\theta}\right)$$

$$+ \frac{1}{\left(\phi^{+}\phi\right)}\left(-\partial_t\left(\frac{\phi^{+}\phi}{c^2v^2}\partial_t\left(\phi^{+}\phi\right)\right) + \frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^{+}\phi}\partial_r\left(\phi^{+}\phi\right)\right) + \frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^{+}\phi\right)\right)}{r^2\sin\theta} + \frac{\partial_\varphi^2\left(\phi^{+}\phi\right)}{r^2\sin^2\theta}\right)$$

$$= -\frac{\phi^{+}\phi}{c^2v^2}\partial_t\left(\frac{1}{\phi^{+}\phi}\right)\partial_t\left(\phi^{+}\phi\right) - \frac{1}{\left(\phi^{+}\phi\right)}\partial_t\left(\frac{\phi^{+}\phi}{c^2v^2}\partial_t\left(\phi^{+}\phi\right)\right)$$

$$+ \frac{1}{r^2}\left(\frac{v^2r^2}{\left(\phi^{+}\phi\right)}\partial_r\left(\phi^{+}\phi\right)\partial_r\left(\frac{1}{\phi^{+}\phi}\right) + \frac{1}{\left(\phi^{+}\phi\right)}\partial_r\left(\frac{v^2r^2}{\phi^{+}\phi}\partial_r\left(\phi^{+}\phi\right)\right)\right)$$

$$+ \frac{1}{r^2\sin\theta}\left(\sin\theta\partial_\theta\left(\phi^{+}\phi\right)\partial_\theta\left(\frac{1}{\phi^{+}\phi}\right) + \frac{\partial_\theta\left(\sin\theta\partial_\theta\left(\phi^{+}\phi\right)\right)}{\left(\phi^{+}\phi\right)}\right) + \frac{1}{r^2\sin^2\theta}\left(\partial_\varphi\left(\phi^{+}\phi\right)\partial_\varphi\left(\frac{1}{\phi^{+}\phi}\right) + \frac{\partial_\varphi^2\left(\phi^{+}\phi\right)}{\phi^{+}\phi}\right)$$

$$= -\frac{\partial_t^2\left(\phi^{+}\phi\right)}{c^2v^2} - \frac{1}{r^2}\partial_r\left(r^2\partial_r\left(\frac{v^2}{\phi^{+}\phi}\right)\right) + \frac{1}{r^2\sin\theta}\partial_\theta\left(\frac{\sin\theta\partial_\theta\left(\phi^{+}\phi\right)}{\phi^{+}\phi}\right) + \frac{1}{r^2\sin^2\theta}\partial_\varphi\left(\frac{\partial_\varphi\left(\phi^{+}\phi\right)}{\phi^{+}\phi}\right)$$

$$\frac{1}{2}u^+u\eta^+\eta g^{ab}\nabla_a\left(\frac{1}{\gamma\eta^+\eta}\nabla_b\left(\gamma\eta^+\eta\right)\right)=u^+ug^{ab}\nabla_a\eta^+\nabla_b\eta-\alpha\beta u^+u\eta^+\eta+\alpha\gamma u^+u\left(\eta^+\eta\right)^2$$

$$\frac{1}{2}(u\eta)^+(u\eta)g^{ab}\nabla_a\left(\frac{\nabla_b\left(\gamma(u\eta)^+(u\eta)\right)}{\gamma(u\eta)^+(u\eta)}\right)=u^+ug^{ab}\nabla_a\eta^+\nabla_b\eta-\alpha\beta(u\eta)^+(u\eta)+\alpha\gamma(u\eta)^+(u\eta)\eta^+\eta$$

$$=g^{ab}\left(\nabla_a(u\eta)-\left(u^{-1}\nabla_a u\right)u\eta\right)^+\left(\nabla_b(u\eta)-\left(u^{-1}\nabla_b u\right)u\eta\right)-\alpha\beta(u\eta)^+(u\eta)+\frac{\alpha\gamma}{u^+u}(u\eta)^+(u\eta)(u\eta)^+(u\eta)$$

$$=g^{ab}D_a^+(u\eta)D_b(u\eta)-\alpha\beta(u\eta)^+(u\eta)+\frac{\alpha\gamma}{u^+u}(u\eta)^+(u\eta)(u\eta)^+(u\eta)$$

$$\frac{1}{2}\tilde{\eta}^+\tilde{\eta}g^{ab}\nabla_a\left(\frac{\nabla_b\left(\gamma\tilde{\eta}^+\tilde{\eta}\right)}{\gamma\tilde{\eta}^+\tilde{\eta}}\right)=g^{ab}D_a^+\tilde{\eta}D_b\tilde{\eta}-\alpha\beta\tilde{\eta}^+\tilde{\eta}+\frac{\alpha\gamma}{u^+u}\left(\tilde{\eta}^+\tilde{\eta}\right)^2$$

$$g^{ab}\nabla_a\left(\frac{1}{\phi^+\phi}\nabla_b\left(\phi^+\phi\right)\right)=-\frac{1}{r^2}\partial_r\left(r^2\partial_r\left(\frac{v^2}{\phi^+\phi}\right)\right)$$

$$=-\frac{1}{r^2}\partial_r\left(r^2\partial_r\left(1-\frac{GM}{r}\right)\right)=0$$

$$\nabla_b\psi=(\beta_c-\gamma_cF^+F)\int P_aP_b\psi\,dx^a$$

$$g^{ab}\nabla_a\frac{1}{\beta_c-\gamma_cF^+F}\nabla_b\psi=g^{ab}P_aP_b\psi=\alpha\psi$$

$$\frac{1}{\beta_c-\gamma_cF^+F}g^{ab}\nabla_a\nabla_b\psi+\frac{\gamma_cg^{ab}\nabla_a\left(F^+F\right)\nabla_b\psi}{\left(\beta_c-\gamma_cF^+F\right)^2}=\alpha\psi$$

$$g^{ab}\nabla_a\nabla_b\psi=-\frac{\gamma_cg^{ab}\nabla_a\left(F^+F\right)\nabla_b\psi}{\beta_c-\gamma_cF^+F}+\alpha\psi\left(\beta_c-\gamma_cF^+F\right)$$

$$\psi^+g^{ab}\nabla_a\nabla_b\psi=-\frac{\gamma_cg^{ab}\nabla_a\left(F^+F\right)\psi^+\nabla_b\psi}{\beta_c-\gamma_cF^+F}+\alpha\psi^+\psi\left(\beta_c-\gamma_cF^+F\right)$$

$$g^{ab}\nabla_a\nabla_b\psi^+\psi=-\frac{\gamma_cg^{ab}\nabla_a\left(F^+F\right)\nabla_b\psi^+\psi}{\beta_c-\gamma_cF^+F}+\alpha\psi^+\psi\left(\beta_c-\gamma_cF^+F\right)$$

$$\psi^+g^{ab}\nabla_a\nabla_b\psi+g^{ab}\nabla_a\nabla_b\psi^+\psi=-\frac{\gamma_cg^{ab}\nabla_a\left(F^+F\right)\nabla_b\left(\psi^+\psi\right)}{\beta_c-\gamma_cF^+F}+2\alpha\psi^+\psi\left(\beta_c-\gamma_cF^+F\right)$$

$$\psi^+g^{ab}\nabla_a\nabla_b\psi+g^{ab}\nabla_a\nabla_b\psi^+\psi=g^{ab}\nabla_a\nabla_b\left(\psi^+\psi\right)-2g^{ab}\nabla_a\psi^+\nabla_b\psi$$

$$-\frac{\gamma_c g^{ab}\nabla_a\left(F^+F\right)\nabla_b\left(\psi^+\psi\right)}{\beta_c-\gamma_c F^+F}+2\alpha\psi^+\psi\left(\beta_c+\gamma_c F^+F\right)=g^{ab}\nabla_a\nabla_b\left(\psi^+\psi\right)-2g^{ab}\nabla_a\psi^+\nabla_b\psi$$

$$\frac{1}{2}\left(g^{ab}\nabla_a\nabla_b\left(\psi^+\psi\right)+\frac{\gamma_c g^{ab}\nabla_a\left(F^+F\right)\nabla_b\left(\psi^+\psi\right)}{\beta_c-\gamma_c F^+F}\right)=g^{ab}\nabla_a\psi^+\nabla_b\psi+\alpha\psi^+\psi\left(\beta_c+\gamma_c F^+F\right)$$

$$=\frac{\beta_c-\gamma_c F^+F}{2}\left(\frac{g^{ab}\nabla_a\nabla_b\left(\psi^+\psi\right)}{\beta_c-\gamma_c F^+F}+\frac{\gamma_c g^{ab}\nabla_a\left(F^+F\right)\nabla_b\left(\psi^+\psi\right)}{\left(\beta_c-\gamma_c F^+F\right)^2}\right)$$

$$=\frac{\beta_c-\gamma_c F^+F}{2}g^{ab}\nabla_a\frac{\nabla_b\left(\psi^+\psi\right)}{\beta_c-\gamma_c F^+F}$$

$$g^{ab}\nabla_a\nabla_b\eta=-\frac{1}{c^2}\partial_t\left(\frac{\phi^+\phi}{v^2}\partial_t\eta\right)+\frac{1}{r^2}\partial_r\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\eta\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\eta\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\eta}{r^2\sin^2\theta}$$

$$g^{ab}\nabla_a\left(\frac{1}{\eta}\nabla_b\eta\right)=-\frac{1}{\eta^2}g^{ab}\nabla_a\eta\nabla_b\eta+\frac{1}{\eta}g^{ab}\nabla_a\nabla_b\eta$$

$$=-\frac{1}{\eta^2}\left(-\frac{\phi^+\phi}{c^2v^2}\partial_t\eta\partial_t\eta+\frac{v^2}{\phi^+\phi}\partial_r\eta\partial_r\eta+\frac{\partial_\theta\eta\partial_\theta\eta}{r^2}+\frac{\partial_\varphi\eta\partial_\varphi\eta}{r^2\sin^2\theta}\right)$$

$$+\frac{1}{\eta}\left(-\partial_t\left(\frac{\phi^+\phi}{c^2v^2}\partial_t\eta\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{v^2r^2}{\phi^+\phi}\partial_r\eta\right)+\frac{\partial_\theta\left(\sin\theta\partial_\theta\eta\right)}{r^2\sin\theta}+\frac{\partial_\varphi^2\eta}{r^2\sin^2\theta}\right)$$

$$g^{ab}\nabla_a\left(\frac{1}{\phi^+\phi}\nabla_b\left(\eta^+\eta\right)\right)$$

$$=\frac{g^{ab}}{\phi^+\phi}\nabla_a\nabla_b\left(\eta^+\eta\right)-\frac{g^{ab}}{\left(\phi^+\phi\right)^2}\nabla_a\left(\phi^+\phi\right)\nabla_b\left(\eta^+\eta\right)$$

$$\nabla_b\psi=(\beta-\sum_i\gamma_iZ_i^+Z_i)\int P_aP_b\psi\,dx^a$$

$$\frac{\beta-\sum_i\left(\gamma_iZ_i^+Z_i\right)}{2}g^{ab}\nabla_a\frac{\nabla_b\left(\psi^+\psi\right)}{\beta-\sum_i\left(\gamma_iZ_i^+Z_i\right)}=g^{ab}\nabla_a\psi^+\nabla_b\psi+\alpha\beta\psi^+\psi+\psi^+\psi\sum_i\left(\gamma_iZ_i^+Z_i\right)$$

$$m_{zi}^2=\frac{\gamma_i}{2}g^{ab}\nabla_a\frac{\nabla_b\left(\psi^+\psi\right)}{\beta-\sum_i\left(\gamma_iZ_i^+Z_i\right)}+\psi^+\psi\gamma_i$$