引力场与非幺正变换及常数计算

林志朋[①*]

① 安普利生物有限公司电子研发部, 厦门　361028;
E-mail: dqjl_111@163.com

摘要　长久以来人们都试图将标准模型与引力场结合。本文通过球坐标系下度规求解爱因斯坦张量G_{ab}, 找到一种特殊的数学变换形式可以同时表达黎曼曲率R和Lagrange量。通过对希格斯场的计算，可以使引力场参与到物质场的变换当中。这种变换与规范变换兼容，可以加入到标准模型中，不同的是它可以是非幺正的。本文的研究有助与我们更好的理解引力与物质以及空间的关系，并解释引力常数与其他常数的数值关系，从而验证理论的证确性。

关键词　度规　黎曼曲率　非幺正　规范变范　希格斯场　引力常数
MSC (2025) 主题分类

1　引言

标准模型使用SU(N)群去描述其本作用力，其要求群变换满足幺正性，即$U^\dagger U = I$。但是它不足以包含引力，本文通过引入非幺正变换来描术引力，即可以有$U^\dagger U \neq I$，计算结果与广义相对论一致。这一变换基于一个基本的数学形式，即：

$$g^{ab}\nabla_a(\frac{1}{\phi_1\phi_2}\nabla_b(\phi_1\phi_2)) = g^{ab}\nabla_a(\frac{1}{\phi_1}\nabla_b\phi_1) + g^{ab}\nabla_a(\frac{1}{\phi_2}\nabla_b\phi_2) \tag{1.1}$$

经过一定的计算和转换右边可以用来表达黎曼曲率R和Lagrange量，从而通过这一性质规律进行变换，探讨更深刻的问题。

2　黎曼曲率计算

令时空中的度规如下:

$$ds^2 = -\frac{c^2v^2}{\phi^\dagger\phi}dt^2 + \frac{\phi^\dagger\phi}{v^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{2.1}$$

由克式符张量计算公式$\Gamma^c_{ab} = \frac{1}{2}g^{cd}(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})$，我们有:

$$\Gamma^0_{00} = -\frac{\partial_t(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^0_{11} = \frac{\phi^\dagger\phi\partial_t(\phi^\dagger\phi)}{2c^2v^4}, \Gamma^1_{01} = \Gamma^1_{10} = \frac{\partial_t(\phi^\dagger\phi)}{2\phi^\dagger\phi}$$
$$\Gamma^0_{02} = \Gamma^0_{20} = -\frac{\partial_\theta(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^1_{12} = \Gamma^1_{21} = \frac{\partial_\theta(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^2_{00} = \frac{c^2v^2\partial_\theta(\phi^\dagger\phi)}{2r^2(\phi^\dagger\phi)^2}$$

$$
\begin{aligned}
&\Gamma_{11}^{2}=-\frac{\partial_{\theta}(\phi^{\dagger}\phi)}{2r^{2}v^{2}}\\
&\Gamma_{03}^{0}=\Gamma_{30}^{0}=-\frac{\partial_{\varphi}(\phi^{\dagger}\phi)}{2\phi^{\dagger}\phi},\Gamma_{13}^{1}=\Gamma_{31}^{1}=\frac{\partial_{\varphi}(\phi^{\dagger}\phi)}{2\phi^{\dagger}\phi},\Gamma_{00}^{3}=\frac{c^{2}v^{2}\partial_{\varphi}(\phi^{\dagger}\phi)}{2r^{2}\sin^{2}\theta{(\phi^{\dagger}\phi)}^{2}}\\
&\Gamma_{11}^{3}=\frac{-\partial_{\varphi}(\phi^{\dagger}\phi)}{2r^{2}v^{2}\sin^{2}\theta}\\
&\Gamma_{01}^{0}=\Gamma_{10}^{0}=-\frac{\partial_{r}(\phi^{\dagger}\phi)}{2\phi^{\dagger}\phi},\Gamma_{00}^{1}=-\frac{c^{2}v^{4}\partial_{r}(\phi^{\dagger}\phi)}{2{(\phi^{\dagger}\phi)}^{3}},\Gamma_{11}^{1}=\frac{\partial_{r}(\phi^{\dagger}\phi)}{2\phi^{\dagger}\phi}\\
&\Gamma_{22}^{1}=-\frac{v^{2}r}{\phi^{\dagger}\phi},\Gamma_{33}^{1}=-\frac{v^{2}r\sin^{2}\theta}{\phi^{\dagger}\phi},\Gamma_{12}^{2}=\Gamma_{21}^{2}=\frac{1}{r}\\
&\Gamma_{33}^{2}=-\sin\theta\cos\theta,\Gamma_{13}^{3}=\Gamma_{31}^{3}=\frac{1}{r},\Gamma_{23}^{3}=\Gamma_{32}^{3}=\cot\theta
\end{aligned}
\tag{2.2}
$$

由矢量在弯曲时空中的适配导数，我们有$g^{ab}\nabla_a\nabla_b(\eta)=g^{ab}(\partial_a\partial_b\eta-\Gamma^c{}_{ab}\partial_c\eta)$，从而有:

$$
g^{ab}\nabla_a\nabla_b\eta=-\frac{1}{c^2}\partial_t(\frac{\phi^{\dagger}\phi}{v^2}\partial_t\eta)+\frac{1}{r^2}\partial_r(\frac{v^2r^2}{\phi^{\dagger}\phi}\partial_r\eta)+\frac{\partial_\theta(\sin\theta\partial_\theta\eta)}{r^2\sin\theta}+\frac{\partial_\varphi^2\eta}{r^2\sin^2\theta}
\tag{2.3}
$$

进一步得到:

$$
\begin{aligned}
&g^{ab}\nabla_a(\frac{1}{\phi^{\dagger}\phi}\nabla_b(\phi^{\dagger}\phi))\\
&=-\frac{1}{{(\phi^{\dagger}\phi)}^2}(-\frac{\phi^{\dagger}\phi(\partial_t(\phi^{\dagger}\phi))^2}{c^2v^2}+\frac{v^2(\partial_r(\phi^{\dagger}\phi))^2}{\phi^{\dagger}\phi}+\frac{(\partial_\theta(\phi^{\dagger}\phi))^2}{r^2}+\frac{(\partial_\varphi(\phi^{\dagger}\phi))^2}{r^2\sin^2\theta})\\
&+\frac{1}{(\phi^{\dagger}\phi)}(-\partial_t(\frac{\phi^{\dagger}\phi}{c^2v^2}\partial_t(\phi^{\dagger}\phi))+\frac{1}{r^2}\partial_r(\frac{v^2r^2}{\phi^{\dagger}\phi}\partial_r(\phi^{\dagger}\phi))+\frac{\partial_\theta(\sin\theta\partial_\theta(\phi^{\dagger}\phi))}{r^2\sin\theta}+\frac{\partial_\varphi^2(\phi^{\dagger}\phi)}{r^2\sin^2\theta})
\end{aligned}
\tag{2.4}
$$

又由于$R_{\mu\nu\sigma}{}^{\rho}=\Gamma^{\rho}{}_{\mu\sigma,\nu}-\Gamma^{\rho}{}_{\nu\sigma,\mu}+\Gamma^{\lambda}{}_{\sigma\mu}\Gamma^{\rho}{}_{\nu\lambda}-\Gamma^{\lambda}{}_{\sigma\nu}\Gamma^{\rho}{}_{\mu\lambda}$，我们有:

$$
\begin{aligned}
R=&\frac{1}{{(\phi^{\dagger}\phi)}^2}(-\frac{\phi^{\dagger}\phi{(\partial_t(\phi^{\dagger}\phi))}^2}{c^2v^2}-\frac{v^2{(\partial_r(\phi^{\dagger}\phi))}^2}{\phi^{\dagger}\phi}+\frac{{(\partial_\theta(\phi^{\dagger}\phi))}^2}{r^2}+\frac{{(\partial_\varphi(\phi^{\dagger}\phi))}^2}{r^2\sin^2\theta})\\
&-\frac{1}{\phi^{\dagger}\phi}(-\frac{1}{c^2}\partial_t(\frac{\phi^{\dagger}\phi}{v^2}\partial_t(\phi^{\dagger}\phi))-\frac{1}{r^2}\partial_r(\frac{v^2r^2}{\phi^{\dagger}\phi}\partial_r(\phi^{\dagger}\phi))+\frac{\partial_\theta(\sin\theta\partial_\theta(\phi^{\dagger}\phi))}{r^2\sin\theta}+\frac{\partial_\varphi^2(\phi^{\dagger}\phi)}{r^2\sin^2\theta})\\
&+\frac{2v^2\partial_r(\phi^{\dagger}\phi)}{{(\phi^{\dagger}\phi)}^2r}+\frac{2}{r^2}(1-\frac{v^2}{\phi^{\dagger}\phi})-\frac{1}{{(\phi^{\dagger}\phi)}^2}(\frac{{(\partial_\theta(\phi^{\dagger}\phi))}^2}{2r^2}+\frac{{(\partial_\varphi(\phi^{\dagger}\phi))}^2}{2r^2\sin^2\theta})
\end{aligned}
\tag{2.5}
$$

比较以上两个方程，令：$\phi=J(r)K(t,\theta,\varphi)$，我们可以将黎曼曲率$R$简化为:

$$
\begin{aligned}
&R=g^{ab}\nabla_a(\frac{1}{B^{\dagger}B}\nabla_b(B^{\dagger}B))+2\Lambda\\
&B=J/K\\
&2\Lambda=\frac{2v^2\partial_r(\phi^{\dagger}\phi)}{{(\phi^{\dagger}\phi)}^2r}+\frac{2}{r^2}(1-\frac{v^2}{\phi^{\dagger}\phi})-\frac{1}{{(\phi^{\dagger}\phi)}^2}(\frac{{(\partial_\theta(\phi^{\dagger}\phi))}^2}{2r^2}+\frac{{(\partial_\varphi(\phi^{\dagger}\phi))}^2}{2r^2\sin^2\theta})
\end{aligned}
\tag{2.6}
$$

由爱因斯坦张量公式$G_{ab}=R_{ab}-\frac{R}{2}g_{ab}$，我们有:

$$
\begin{aligned}
&G_{00}=\frac{c^2v^2}{\phi^{\dagger}\phi}\Lambda,G_{11}=-\frac{\phi^{\dagger}\phi}{v^2}\Lambda\\
&G_{22}=-\frac{r^2}{2}(g^{ab}\nabla_a(\frac{1}{B^{\dagger}B}\nabla_b(B^{\dagger}B))+\frac{{(\partial_\theta(K^{\dagger}K))}^2}{2r^2{(K^{\dagger}K)}^2}-\frac{{(\partial_\varphi(K^{\dagger}K))}^2}{2r^2\sin^2\theta{(K^{\dagger}K)}^2})
\end{aligned}
$$

$$G_{33}=-\frac{r^2\sin^2\theta}{2}(g^{ab}\nabla_a(\frac{1}{B^\dagger B}\nabla_b(B^\dagger B))-\frac{(\partial_\theta(K^\dagger K))^2}{2r^2(K^\dagger K)^2}+\frac{(\partial_\varphi(K^\dagger K))^2}{2r^2\sin^2\theta(K^\dagger K)^2}) \tag{2.7}$$

对于简单情形如：$\phi=J(r)$，我们有:

$$\begin{aligned}
&2\Lambda=\frac{2v^2\partial_r(\phi^\dagger\phi)}{(\phi^\dagger\phi)^2r}+\frac{2}{r^2}(1-\frac{v^2}{\phi^\dagger\phi})=\frac{2}{r}\partial_r(1-\frac{v^2}{\phi^\dagger\phi})-(1-\frac{v^2}{\phi^\dagger\phi})\partial_r\frac{2}{r}\\
&G_{22}=-\frac{r^2}{2}g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))\\
&G_{33}=-\frac{r^2\sin^2\theta}{2}g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))\\
&G_{ij}=0,i\neq j
\end{aligned} \tag{2.8}$$

上式第一个方程可以简化为:

$$\partial_r(r(1-\frac{v^2}{\phi^\dagger\phi}))=\Lambda r^2 \tag{2.9}$$

解得有:

$$\frac{\phi^\dagger\phi}{v^2}=(1-\frac{\Lambda r^2}{3}-\frac{C}{r})^{-1} \tag{2.10}$$

这就是Schwarzschild-de Sitter度规解. 方程2.4 可以被简化为:

$$\begin{aligned}
&g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))\\
&=-\frac{\partial_t^2(\phi^\dagger\phi)}{c^2v^2}-\frac{1}{r^2}\partial_r(r^2\partial_r(\frac{v^2}{\phi^\dagger\phi}))+\frac{1}{r^2\sin\theta}\partial_\theta(\frac{\sin\theta\partial_\theta(\phi^\dagger\phi)}{\phi^\dagger\phi})+\frac{1}{r^2\sin^2\theta}\partial_\varphi(\frac{\partial_\varphi(\phi^\dagger\phi)}{\phi^\dagger\phi})
\end{aligned} \tag{2.11}$$

由方程2.10 和2.11，我们得到:

$$g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))=2\Lambda \tag{2.12}$$

上式左边可以分成两部分，一部分与引力无关；另一部分参与时空弯曲，与引力相关:

$$2F_\iota \quad =g^{ab}\nabla_a(\tfrac{1}{\iota^\dagger\iota}\nabla_b(\iota^\dagger\iota)) \tag{2.13}$$

$$2F_\phi \quad =g^{ab}\nabla_a(\tfrac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi)) \tag{2.14}$$

当$\phi=J(r)$，方程2.4 进一步简化为:

$$g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))=-\frac{1}{r^2}\partial_r^2(r^2\partial_r(\frac{v^2}{\phi^\dagger\phi})) \tag{2.15}$$

则对于物质量m有:

$$2mF_\iota=g^{ab}\nabla_a(\frac{1}{\iota^{m\dagger}\iota^m}\nabla_b(\iota^{m\dagger}\iota^m)) \tag{2.16}$$

$$2mF_\phi=-\frac{1}{r^2}\partial_r(r^2\partial_r(\frac{mv^2}{\phi^\dagger\phi})) \tag{2.17}$$

对于引力场真空静态解，方程2.17是:

$$2mF_\phi=-\frac{1}{r^2}\partial_r(r^2\partial_r(m-\frac{2Gm}{c^2r}))=-\frac{1}{r^2}\partial_r(r^2\partial_r(1-\frac{2Gm}{c^2r})) \tag{2.18}$$

方程2.16和2.18表明惯性质量与引力质量是等比的.

3 Lagrange量计算

为了得到方程2.12形式，令场延自身的任一条传播路径积分和自身梯度满足:

$$\nabla_b\psi = (\beta - \gamma\psi^\dagger\psi)\int P_a P_b \psi dx^a \tag{3.1}$$

则:

$$g^{ab}\nabla_a \frac{1}{\beta - \gamma\psi^\dagger\psi}\nabla_b\psi = g^{ab}P_a P_b\psi = \alpha\psi \tag{3.2}$$

这导致:

$$\begin{aligned}&\frac{\beta - \gamma\psi^\dagger\psi}{-2\gamma} g^{ab}\nabla_a \frac{\nabla_b(\beta - \gamma\psi^\dagger\psi)}{\beta - \gamma\psi^\dagger\psi}\\ &= \frac{1}{2}(g^{ab}\nabla_a\nabla_b(\psi^\dagger\psi) + \frac{\gamma g^{ab}\nabla_a(\psi^\dagger\psi)\nabla_b(\psi^\dagger\psi)}{\beta - \gamma\psi^\dagger\psi})\\ &= g^{ab}\nabla_a\psi^\dagger\nabla_b\psi + \alpha\psi^\dagger\psi(\beta - \gamma\psi^\dagger\psi)\end{aligned} \tag{3.3}$$

当$\gamma = 0$，方程3.1变为:

$$\nabla_b\psi = \beta\int P_a P_b\psi dx^a \tag{3.4}$$

则有:

$$\frac{1}{2}g^{ab}\nabla_a\nabla_b(\psi^\dagger\psi) = g^{ab}\nabla_a\psi^\dagger\nabla_b\psi + \alpha\beta\psi^\dagger\psi \tag{3.5}$$

方程3.4的左边反映场的对外作用状态，右边反应自身传递过程中的累积。两边都能表达同一场，因此他们需要能相互转换, 把这种规则称为匹配变换。方程3.1的1维形式如下：

$$\frac{d\psi}{ds} = (\beta - \gamma\psi^\dagger\psi)\int \alpha\psi ds \tag{3.6}$$

进一步，令:

$$\eta = \begin{pmatrix}\psi \\ \rho e^{i\theta}\end{pmatrix}, \rho = \sqrt{-\frac{\beta}{\gamma}} \tag{3.7}$$

则:

$$\beta - \gamma\psi^\dagger\psi = -\gamma\eta^\dagger\eta = -\gamma(\psi^\dagger\psi - \frac{\beta}{\gamma}) \tag{3.8}$$

$$g^{ab}\nabla_a\eta^\dagger\nabla_b\eta = g^{ab}\nabla_a\psi^\dagger\nabla_b\psi + g^{ab}\nabla_a(\rho e^{-i\theta})\nabla_b(\rho e^{i\theta}) = g^{ab}\nabla_a\psi^\dagger\nabla_b\psi \tag{3.9}$$

我们有:

$$\frac{\eta^\dagger\eta}{2}g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta)) = g^{ab}\nabla_a\eta^\dagger\nabla_b\eta - \alpha\beta\eta^\dagger\eta - \alpha\gamma(\eta^\dagger\eta)^2 \tag{3.10}$$

令变换u满足:

$$g^{ab}\nabla_a(\frac{1}{u^\dagger u}\nabla_b(u^\dagger u)) = 0 \tag{3.11}$$

当u是幺正的规范场时，式3.13中取电势矢量$\tilde{A}_a = 0$，即满足：

$$\tilde{D}_a(\tilde{\eta}) = \tilde{D}_a(u\eta) = (\nabla_a + \tilde{A}_a)(u\eta) = u(\nabla_a + u^{-1}\nabla_a u)\eta = u(\nabla_a + A_a)\eta = uD_a\eta \tag{3.12}$$

作变换: $\eta \to u\eta = \tilde{\eta}$，规范变换相当于将式3.11加到式3.10中，结果保持不变，我们有:

$$\begin{aligned}
&\frac{1}{2}\tilde{\eta}^\dagger\tilde{\eta}g^{ab}\nabla_a(\frac{1}{\tilde{\eta}^\dagger\tilde{\eta}}\nabla_b(\tilde{\eta}^\dagger\tilde{\eta})) \\
&= \frac{1}{2}\tilde{\eta}^\dagger\tilde{\eta}(g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta)) + g^{ab}\nabla_a(\frac{1}{u^\dagger u}\nabla_b(u^\dagger u))) \\
&= g^{ab}\tilde{D}_a^\dagger\tilde{\eta}\tilde{D}_b\tilde{\eta} - \alpha\beta\tilde{\eta}^\dagger\tilde{\eta} - \frac{\alpha\gamma}{u^\dagger u}(\tilde{\eta}^\dagger\tilde{\eta})^2
\end{aligned} \tag{3.13}$$

利用式2.13 和2.14，可以令η 满足:

$$g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta)) = 2(aF_\iota + bF_\phi) = 2\kappa \tag{3.14}$$

于是我们得到一种希格斯场:

$$\begin{aligned}
2\mathcal{L}_h &= g^{ab}\tilde{D}_a^\dagger\tilde{\eta}\tilde{D}_b\tilde{\eta} - (\alpha\beta+\kappa)\tilde{\eta}^\dagger\tilde{\eta} - \frac{\alpha\gamma}{u^\dagger u}(\tilde{\eta}^\dagger\tilde{\eta})^2 \\
&= g^{ab}\tilde{D}_a^\dagger\tilde{\eta}\tilde{D}_b\tilde{\eta} - \mu^2\tilde{\eta}^\dagger\tilde{\eta} - \frac{\lambda}{2u^\dagger u}(\tilde{\eta}^\dagger\tilde{\eta})^2 = 0
\end{aligned} \tag{3.15}$$

以上计算是可逆的，所以方程3.1也能从Lagrange量得到。当$u^\dagger u = I$，对应标准模型中的SU(N)群变换。而对于静态真空场我们有：

$$u^\dagger u = (1 - \frac{2Gm_g}{c^2 r})^{-1} \tag{3.16}$$

$$g^{ab}\nabla_a((1 - \frac{2Gm_g}{c^2 r})\nabla_b(1 - \frac{2Gm_g}{c^2 r})^{-1}) = 0 \tag{3.17}$$

其中m_g 是引力质量。此时希格斯真空态有：

$$v_0 = \sqrt{-\frac{\mu^2 u^+ u}{\lambda}} = \sqrt{\frac{-\mu^2}{\lambda(1 - \frac{2Gm_g}{c^2 r})}} \tag{3.18}$$

这将导至基本粒子质量由m_0 变为m_p：

$$m_p = \frac{m_0}{\sqrt{1 - \frac{2Gm_g}{c^2 r}}} \propto v_0 \tag{3.19}$$

这一结果与相对论一致。

4 消除外因子

简单计算，式3.10中可以变换为：

$$\frac{1}{2}g^{ab}\nabla_a\nabla_b\left(\eta^\dagger\eta\right) = g^{ab}\nabla_a\eta^\dagger\nabla_b\eta + (\frac{g^{ab}\nabla_a\ln(\eta^\dagger\eta)\nabla_b\ln(\eta^\dagger\eta)}{2} - \alpha\beta)\eta^\dagger\eta - \alpha\gamma(\eta^\dagger\eta)^2 \tag{4.1}$$

于是我们可以将式3.1中的外因子移动到积分内部，变为：

$$g^{ab}\nabla_b\eta = \int(\frac{g^{ab}\nabla_a\ln(\eta^\dagger\eta)\nabla_b\ln(\eta^\dagger\eta)}{2} - \alpha\beta - \alpha\gamma\eta^\dagger\eta)\eta dx^a \tag{4.2}$$

由熵和量子态的概率密度含义可知，$\nabla_a \ln(\eta^\dagger\eta)$反映了空间结构度规关联状态数的变化对场分布变化的影响。我们可以做个近似的比较计算，令状态数为W，量子数为N，体元ΔV，且有：

$$\Delta V = \sqrt{W} \propto \sqrt{\eta^\dagger\eta} \tag{4.3}$$

于是有：

$$\begin{aligned}\frac{d\eta}{ds} &= \frac{d}{ds}(\frac{N}{\Delta V}) \propto \frac{d}{ds}(\frac{N}{\sqrt{W}}) = \frac{dN}{\sqrt{W}ds} - \frac{d\ln W}{2\sqrt{W}ds}\\ &\propto \frac{dN}{ds}\eta^{-1} - \frac{d\ln(\eta^\dagger\eta)}{2ds}\eta^{-1} \propto \frac{dN}{ds}\eta^\dagger - \frac{d\ln(\eta^\dagger\eta)}{2ds}\eta^\dagger\end{aligned} \tag{4.4}$$

可以看出时空弯曲的本质与物质状态空间的变化有关。

5 倒换

令$\eta^\dagger\eta = 1/A$，$\xi = u\sqrt{A}$，$u^\dagger u = I$，代入式3.10中有：

$$-\frac{\xi^\dagger\xi}{2}g^{ab}\nabla_a(\frac{1}{\xi^\dagger\xi}\nabla_b(\xi^\dagger\xi)) = g^{ab}\nabla_a\xi^\dagger\nabla_b\xi - \alpha\beta\xi^\dagger\xi - \alpha\gamma \tag{5.1}$$

可以看到我们也可以通过这一变换来消除或产生希格斯自作用项。

6 常数计算

当式3.10中$\alpha = 0$时可以用来描述静质量为0的粒子场，如电磁场。我们可以得到总Lagrange量$\mathcal{L}$、各粒子场Lagrange量$\mathcal{L}_i$以及黎曼曲率R间的关系：

$$\begin{aligned}2\mathcal{L} &= \frac{1}{2}g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta)) + \sum_i(\frac{1}{2}\frac{\phi_i^\dagger\phi_i}{\eta^\dagger\eta}g^{ab}\nabla_a(\frac{1}{\phi_i^\dagger\phi_i}\nabla_b(\phi_i^\dagger\phi_i)))\\ &= \frac{\eta^\dagger\eta + \sum\limits_i(\phi_i^\dagger\phi_i)}{2\eta^\dagger\eta}g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta)) + \sum_i(\frac{1}{2}\frac{\phi_i^\dagger\phi_i}{\eta^\dagger\eta}g^{ab}\nabla_a(\frac{\eta^\dagger\eta}{\phi_i^\dagger\phi_i}\nabla_b(\frac{\phi_i^\dagger\phi_i}{\eta^\dagger\eta})))\\ &= \frac{\eta^\dagger\eta + \sum\limits_i(\phi_i^\dagger\phi_i)}{2\eta^\dagger\eta}R + 2\sum_i\mathcal{L}_i\end{aligned} \tag{6.1}$$

由广义相对论方程和电磁场能动张量，结合式3.10可以得到：

$$\mathcal{F} = \frac{\eta^\dagger\eta + \sum\limits_i(\phi_i^\dagger\phi_i)}{\eta^\dagger\eta} = \frac{c^2}{4\pi\hbar^2 G} \tag{6.2}$$

上述结果也可以通过累计真空态下的ϕ_i场实现，类似电磁场。由于ϕ_i场与η场一样有相同的β和γ系数，只是$\alpha = 0$。因此它们形成的各模态平均累积幅值与η相同，由真空态v_0决定。ϕ_i的能量密度为$\hbar\omega_i\phi_i^\dagger\phi_i$，对比电磁场幅值，可得其幅值为$\sqrt{\hbar\omega_i\phi_i^\dagger\phi_i}$。由于电磁场幅值线性可加，则参与振荡的线元量为：

$$n_l = \frac{v_0}{\sqrt{\hbar\omega_i\phi_i^\dagger\phi_i}} = \sqrt{\frac{\eta\dagger\eta}{\hbar\omega_i\phi_i^\dagger\phi_i}} \tag{6.3}$$

振荡可以在垂直于传播方向的圆面上，沿任一方向进行。则同一传播方向上，角频率同为ω_i的面元量为：

$$n_s = \pi {n_l}^2 = \frac{\pi\eta^\dagger\eta}{\hbar\omega_i\phi_i^\dagger\phi_i} \tag{6.4}$$

考虑η相对求和量很小，且也可视为一部分模态成分，可以忽略。将式6.4代入式6.1求和，因为作了部分求和，我们更换一下求和下标为模态下标j，化成：

$$2\mathcal{L} = \sum_j \frac{\pi}{\hbar\omega_j}\frac{R}{2} + \sum_j (\frac{1}{2}\frac{\pi}{\hbar\omega_j} g^{ab}\nabla_a(\frac{\eta^\dagger\eta}{\phi_j^\dagger\phi_j}\nabla_b(\frac{\phi_j^\dagger\phi_j}{\eta^\dagger\eta})) \tag{6.5}$$

上式右边第二项ω_j^{-1}可以移进导数中，成为场的指数因指，正好与场的能量抵消。这使得右边第二项各项在平稳时空是中变得均匀一致。它们彼此可以通过能量与数目的改变实现转换，也促成了这种平衡。

体积为L^3的方盒中，利用周期边条件，模态j波长为$\lambda_j = L/n_j$，由于$k_j = 2\pi/\lambda_j$，处于$(k_j, k_j + dk_j)$范围内的k_j分立值共有$Ldk_j/(2\pi)$。于是模态密度数为：

$$\rho_0 = \lim_{L\to\infty}\frac{N}{L^3} = \frac{1}{8\pi^3}\int d^3\vec{k} = \frac{1}{2\pi^2}\int_0^{k_{\max}} k^2 dk = \frac{k_{\max}^3}{6\pi^2} \tag{6.6}$$

其中$k_{\max}$可由普朗克能量得到，即$k_{\max} = E_p/(\hbar c)$。普朗克能量满足：

$$\begin{aligned} E_p &= \sqrt{\frac{\hbar c^5}{G}} \\ 1 - \frac{GE_p}{c^4 r_p} &= 0 \\ r_p E_p &= \hbar c \end{aligned} \tag{6.7}$$

式6.6同样的可以用于对式6.5中各不同模态作进一步求和，结合式6.2，$\omega = kc$，代入普朗克能量有：

$$\mathcal{F} = \frac{1}{2\pi^2}\int \frac{\pi}{\hbar\omega}k^2 dk = \frac{1}{2\pi\hbar c}\int_0^{k_{\max}} kdk = \frac{k_{\max}^2}{4\pi\hbar c} = \frac{c^2}{4\pi\hbar^2 G} \tag{6.8}$$

结果与式6.2一致，也暗示理论证确。

7 结论和扩展讨论

通过计算，本文解决了引力常数与其他物理物常数的关系问题，从而验证引力变换的非幺正性。解释了引力场是如何通过希格斯场引起物质质量改变，进而产生引力作用的。

然而这还只是开始，从式6.5中，我们还可以看到时空机制下隐藏的关联秘密。能量通过波场ϕ_i建立的传输隧道传递，其距离与其波长λ_i相关。本来是可以直接穿透其间空间的，但是由于传播过程中，ϕ_i可以发生模态变换，改变λ_i，从而发生途经效果，并产生位置与能量的量子不确定性。当测量时，由于ϕ_i的一端被固定，另一端也就确定了下来，发生坍缩。

参考文献

1 梁灿彬，周彬-微分几何入门与广义相对.上册[M] .2ed.北京：北京师范大学出版社，2006, 55–82：222–225：253–258：397–398

2 刘辽，赵峥-广义相对论[M] .2ed.北京：高等教育出版社，2004, 96–104：148–152

3 沈一兵-整体微分几何初步[M]. 3ed.北京：高等教育出版社，2009, 1–23：

4 何宝鹏，熊钰庆-量子场论导论[M]. 广州：华南理工大学出版社，1990, 65–67：287–315

5 苏汝铿-量子力学[M]. 3ed.北京：高等教育出版社, 5–8：410–413

6 郭硕鸿，黄迺本，李志兵，林琼桂-电动力学[M]. 3ed.北京：高等教育出版社, 5–8：153–185：188–234：279