引力场与非幺正变换及常数计算

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摘要　长久以来人们都试图将标准模型与引力场结合。本文通过球坐标系下度规求解爱因斯坦张量G_{ab}，找到一种特殊的数学变换形式可以同时表达黎曼曲率R和Lagrange量。通过对希格斯场的计算，可以使引力场参与到物质场的变换当中。这种变换与规范变换兼容，可以加入到标准模型中，不同的是它可以是非幺正的。本文的研究有助与我们更好的理解引力与物质以及空间的关系，并解释引力常数与其他常数的数值关系，从而验证理论的证确性。

关键词　度规　黎曼曲率　非幺正　规范变范　希格斯场　引力常数

MSC (2025) 主题分类

1　引言

标准模型使用SU(N)群去描述其本作用力，其要求群变换满足幺正性，即$U^\dagger U = I$。但是它不足以包含引力，本文通过引入非幺正变换来描术引力，即可以有$U^\dagger U \neq I$，计算结果与广义相对论一致。这一变换基于一个基本的数学形式，即：

$$g^{ab}\nabla_a(\frac{1}{\phi_1\phi_2}\nabla_b(\phi_1\phi_2)) = g^{ab}\nabla_a(\frac{1}{\phi_1}\nabla_b\phi_1) + g^{ab}\nabla_a(\frac{1}{\phi_2}\nabla_b\phi_2) \tag{1.1}$$

经过一定的计算和转换右边可以用来表达黎曼曲率R和Lagrange量，从而通过这一性质规律进行变换，探讨更深刻的问题。

2　黎曼曲率计算

令时空中的度规如下:

$$ds^2 = -\frac{c^2v^2}{\phi^\dagger\phi}dt^2 + \frac{\phi^\dagger\phi}{v^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{2.1}$$

由克式符张量计算公式$\Gamma^c_{ab} = \frac{1}{2}g^{cd}(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})$，我们有:

$$\Gamma^0_{00} = -\frac{\partial_t(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^0_{11} = \frac{\phi^\dagger\phi\partial_t(\phi^\dagger\phi)}{2c^2v^4}, \Gamma^1_{01} = \Gamma^1_{10} = \frac{\partial_t(\phi^\dagger\phi)}{2\phi^\dagger\phi}$$

$$\Gamma^0_{02} = \Gamma^0_{20} = -\frac{\partial_\theta(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^1_{12} = \Gamma^1_{21} = \frac{\partial_\theta(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^2_{00} = \frac{c^2v^2\partial_\theta(\phi^\dagger\phi)}{2r^2(\phi^\dagger\phi)^2}$$

$$
\begin{aligned}
&\Gamma^2_{11} = -\frac{\partial_\theta(\phi^\dagger\phi)}{2r^2v^2} \\
&\Gamma^0_{03} = \Gamma^0_{30} = -\frac{\partial_\varphi(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^1_{13} = \Gamma^1_{31} = \frac{\partial_\varphi(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^3_{00} = \frac{c^2v^2\partial_\varphi(\phi^\dagger\phi)}{2r^2\sin^2\theta(\phi^\dagger\phi)^2} \\
&\Gamma^3_{11} = \frac{-\partial_\varphi(\phi^\dagger\phi)}{2r^2v^2\sin^2\theta} \\
&\Gamma^0_{01} = \Gamma^0_{10} = -\frac{\partial_r(\phi^\dagger\phi)}{2\phi^\dagger\phi}, \Gamma^1_{00} = -\frac{c^2v^4\partial_r(\phi^\dagger\phi)}{2(\phi^\dagger\phi)^3}, \Gamma^1_{11} = \frac{\partial_r(\phi^\dagger\phi)}{2\phi^\dagger\phi} \\
&\Gamma^1_{22} = -\frac{v^2r}{\phi^\dagger\phi}, \Gamma^1_{33} = -\frac{v^2r\sin^2\theta}{\phi^\dagger\phi}, \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} \\
&\Gamma^2_{33} = -\sin\theta\cos\theta, \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}, \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta
\end{aligned}
\tag{2.2}
$$

由矢量在弯曲时空中的适配导数，我们有$g^{ab}\nabla_a\nabla_b(\eta) = g^{ab}(\partial_a\partial_b\eta - \Gamma^c{}_{ab}\partial_c\eta)$，从而有:

$$
g^{ab}\nabla_a\nabla_b\eta = -\frac{1}{c^2}\partial_t(\frac{\phi^\dagger\phi}{v^2}\partial_t\eta) + \frac{1}{r^2}\partial_r(\frac{v^2r^2}{\phi^\dagger\phi}\partial_r\eta) + \frac{\partial_\theta(\sin\theta\partial_\theta\eta)}{r^2\sin\theta} + \frac{\partial^2_\varphi\eta}{r^2\sin^2\theta}
\tag{2.3}
$$

进一步得到:

$$
\begin{aligned}
&g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi)) \\
&= -\frac{1}{(\phi^\dagger\phi)^2}(-\frac{\phi^\dagger\phi(\partial_t(\phi^\dagger\phi))^2}{c^2v^2} + \frac{v^2(\partial_r(\phi^\dagger\phi))^2}{\phi^\dagger\phi} + \frac{(\partial_\theta(\phi^\dagger\phi))^2}{r^2} + \frac{(\partial_\varphi(\phi^\dagger\phi))^2}{r^2\sin^2\theta}) \\
&+\frac{1}{(\phi^\dagger\phi)}(-\partial_t(\frac{\phi^\dagger\phi}{c^2v^2}\partial_t(\phi^\dagger\phi)) + \frac{1}{r^2}\partial_r(\frac{v^2r^2}{\phi^\dagger\phi}\partial_r(\phi^\dagger\phi)) + \frac{\partial_\theta(\sin\theta\partial_\theta(\phi^\dagger\phi))}{r^2\sin\theta} + \frac{\partial^2_\varphi(\phi^\dagger\phi)}{r^2\sin^2\theta})
\end{aligned}
\tag{2.4}
$$

又由于$R_{\mu\nu\sigma}{}^\rho = \Gamma^\rho{}_{\mu\sigma,\nu} - \Gamma^\rho{}_{\nu\sigma,\mu} + \Gamma^\lambda{}_{\sigma\mu}\Gamma^\rho{}_{\nu\lambda} - \Gamma^\lambda{}_{\sigma\nu}\Gamma^\rho{}_{\mu\lambda}$，我们有:

$$
\begin{aligned}
R &= \frac{1}{(\phi^\dagger\phi)^2}(-\frac{\phi^\dagger\phi(\partial_t(\phi^\dagger\phi))^2}{c^2v^2} - \frac{v^2(\partial_r(\phi^\dagger\phi))^2}{\phi^\dagger\phi} + \frac{(\partial_\theta(\phi^\dagger\phi))^2}{r^2} + \frac{(\partial_\varphi(\phi^\dagger\phi))^2}{r^2\sin^2\theta}) \\
&-\frac{1}{\phi^\dagger\phi}(-\frac{1}{c^2}\partial_t(\frac{\phi^\dagger\phi}{v^2}\partial_t(\phi^\dagger\phi)) - \frac{1}{r^2}\partial_r(\frac{v^2r^2}{\phi^\dagger\phi}\partial_r(\phi^\dagger\phi)) + \frac{\partial_\theta(\sin\theta\partial_\theta(\phi^\dagger\phi))}{r^2\sin\theta} + \frac{\partial^2_\varphi(\phi^\dagger\phi)}{r^2\sin^2\theta}) \\
&+\frac{2v^2\partial_r(\phi^\dagger\phi)}{(\phi^\dagger\phi)^2r} + \frac{2}{r^2}(1 - \frac{v^2}{\phi^\dagger\phi}) - \frac{1}{(\phi^\dagger\phi)^2}(\frac{(\partial_\theta(\phi^\dagger\phi))^2}{2r^2} + \frac{(\partial_\varphi(\phi^\dagger\phi))^2}{2r^2\sin^2\theta})
\end{aligned}
\tag{2.5}
$$

比较以上两个方程，令：$\phi = J(r)K(t,\theta,\varphi)$，我们可以将黎曼曲率$R$简化为:

$$
\begin{aligned}
&R = g^{ab}\nabla_a(\frac{1}{B^\dagger B}\nabla_b(B^\dagger B)) + 2\Lambda \\
&B = J/K \\
&2\Lambda = \frac{2v^2\partial_r(\phi^\dagger\phi)}{(\phi^\dagger\phi)^2r} + \frac{2}{r^2}(1 - \frac{v^2}{\phi^\dagger\phi}) - \frac{1}{(\phi^\dagger\phi)^2}(\frac{(\partial_\theta(\phi^\dagger\phi))^2}{2r^2} + \frac{(\partial_\varphi(\phi^\dagger\phi))^2}{2r^2\sin^2\theta})
\end{aligned}
\tag{2.6}
$$

由爱因斯坦张量公式$G_{ab} = R_{ab} - \frac{R}{2}g_{ab}$，我们有:

$$
\begin{aligned}
&G_{00} = \frac{c^2v^2}{\phi^\dagger\phi}\Lambda, G_{11} = -\frac{\phi^\dagger\phi}{v^2}\Lambda \\
&G_{22} = -\frac{r^2}{2}(g^{ab}\nabla_a(\frac{1}{B^\dagger B}\nabla_b(B^\dagger B)) + \frac{(\partial_\theta(K^\dagger K))^2}{2r^2(K^\dagger K)^2} - \frac{(\partial_\varphi(K^\dagger K))^2}{2r^2\sin^2\theta(K^\dagger K)^2})
\end{aligned}
$$

$$G_{33}=-\frac{r^2\sin^2\theta}{2}(g^{ab}\nabla_a(\frac{1}{B^\dagger B}\nabla_b(B^\dagger B))-\frac{(\partial_\theta(K^\dagger K))^2}{2r^2(K^\dagger K)^2}+\frac{(\partial_\varphi(K^\dagger K))^2}{2r^2\sin^2\theta(K^\dagger K)^2}) \tag{2.7}$$

对于简单情形如：$\phi=J(r)$，我们有:

$$\begin{aligned}
&2\Lambda=\frac{2v^2\partial_r(\phi^\dagger\phi)}{(\phi^\dagger\phi)^2r}+\frac{2}{r^2}(1-\frac{v^2}{\phi^\dagger\phi})=\frac{2}{r}\partial_r(1-\frac{v^2}{\phi^\dagger\phi})-(1-\frac{v^2}{\phi^\dagger\phi})\partial_r\frac{2}{r}\\
&G_{22}=-\frac{r^2}{2}g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))\\
&G_{33}=-\frac{r^2\sin^2\theta}{2}g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))\\
&G_{ij}=0, i\neq j
\end{aligned} \tag{2.8}$$

上式第一个方程可以简化为:

$$\partial_r(r(1-\frac{v^2}{\phi^\dagger\phi}))=\Lambda r^2 \tag{2.9}$$

解得有:

$$\frac{\phi^\dagger\phi}{v^2}=(1-\frac{\Lambda r^2}{3}-\frac{C}{r})^{-1} \tag{2.10}$$

这就是Schwarzschild-de Sitter度规解. 方程2.4 可以被简化为:

$$\begin{aligned}
&g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))\\
&=-\frac{\partial_t^2(\phi^\dagger\phi)}{c^2v^2}-\frac{1}{r^2}\partial_r(r^2\partial_r(\frac{v^2}{\phi^\dagger\phi}))+\frac{1}{r^2\sin\theta}\partial_\theta(\frac{\sin\theta\partial_\theta(\phi^\dagger\phi)}{\phi^\dagger\phi})+\frac{1}{r^2\sin^2\theta}\partial_\varphi(\frac{\partial_\varphi(\phi^\dagger\phi)}{\phi^\dagger\phi})
\end{aligned} \tag{2.11}$$

由方程2.10 和2.11，我们得到:

$$g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))=2\Lambda \tag{2.12}$$

上式左边可以分成两部分，一部分与引力无关；另一部分参与时空弯曲，与引力相关:

$$2F_\iota\quad=g^{ab}\nabla_a(\tfrac{1}{\iota^\dagger\iota}\nabla_b(\iota^\dagger\iota)) \tag{2.13}$$

$$2F_\phi\quad=g^{ab}\nabla_a(\tfrac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi)) \tag{2.14}$$

当$\phi=J(r)$，方程2.4 进一步简化为:

$$g^{ab}\nabla_a(\frac{1}{\phi^\dagger\phi}\nabla_b(\phi^\dagger\phi))=-\frac{1}{r^2}\partial_r^2(r^2\partial_r(\frac{v^2}{\phi^\dagger\phi})) \tag{2.15}$$

则对于物质量m有:

$$2mF_\iota=g^{ab}\nabla_a(\frac{1}{\iota^{m\dagger}\iota^m}\nabla_b(\iota^{m\dagger}\iota^m)) \tag{2.16}$$

$$2mF_\phi=-\frac{1}{r^2}\partial_r(r^2\partial_r(\frac{mv^2}{\phi^\dagger\phi})) \tag{2.17}$$

对于引力场真空静态解，方程2.17是:

$$2mF_\phi=-\frac{1}{r^2}\partial_r(r^2\partial_r(m-\frac{2Gm}{c^2r}))=-\frac{1}{r^2}\partial_r(r^2\partial_r(1-\frac{2Gm}{c^2r})) \tag{2.18}$$

方程2.16和2.18表明惯性质量与引力质量是等比的.

3 Lagrange量计算

为了得到方程2.12形式，令场延自身的任一条传播路径积分和自身梯度满足:

$$\nabla_b\psi = (\beta - \gamma\psi^\dagger\psi)\int P_a P_b \psi dx^a \tag{3.1}$$

则:

$$g^{ab}\nabla_a \frac{1}{\beta - \gamma\psi^\dagger\psi}\nabla_b\psi = g^{ab}P_a P_b\psi = \alpha\psi \tag{3.2}$$

这导致:

$$\begin{aligned}&\frac{\beta - \gamma\psi^\dagger\psi}{-2\gamma}g^{ab}\nabla_a\frac{\nabla_b(\beta - \gamma\psi^\dagger\psi)}{\beta - \gamma\psi^\dagger\psi}\\ &= \frac{1}{2}(g^{ab}\nabla_a\nabla_b(\psi^\dagger\psi) + \frac{\gamma g^{ab}\nabla_a(\psi^\dagger\psi)\nabla_b(\psi^\dagger\psi)}{\beta - \gamma\psi^\dagger\psi})\\ &= g^{ab}\nabla_a\psi^\dagger\nabla_b\psi + \alpha\psi^\dagger\psi(\beta - \gamma\psi^\dagger\psi)\end{aligned} \tag{3.3}$$

当$\gamma = 0$，方程3.1变为:

$$\nabla_b\psi = \beta\int P_a P_b\psi dx^a \tag{3.4}$$

则有:

$$\frac{1}{2}g^{ab}\nabla_a\nabla_b(\psi^\dagger\psi) = g^{ab}\nabla_a\psi^\dagger\nabla_b\psi + \alpha\beta\psi^\dagger\psi \tag{3.5}$$

方程3.4的左边反映场的对外作用状态，右边反应自身传递过程中的累积。两边都能表达同一场，因此他们需要能相互转换, 把这种规则称为匹配变换。方程3.1的1维形式如下：

$$\frac{d\psi}{ds} = (\beta - \gamma\psi^\dagger\psi)\int \alpha\psi ds \tag{3.6}$$

进一步，令:

$$\eta = \begin{pmatrix}\psi \\ \rho e^{i\theta}\end{pmatrix}, \rho = \sqrt{-\frac{\beta}{\gamma}} \tag{3.7}$$

则:

$$\beta - \gamma\psi^\dagger\psi = -\gamma\eta^\dagger\eta = -\gamma(\psi^\dagger\psi - \frac{\beta}{\gamma}) \tag{3.8}$$

$$g^{ab}\nabla_a\eta^\dagger\nabla_b\eta = g^{ab}\nabla_a\psi^\dagger\nabla_b\psi + g^{ab}\nabla_a(\rho e^{-i\theta})\nabla_b(\rho e^{i\theta}) = g^{ab}\nabla_a\psi^\dagger\nabla_b\psi \tag{3.9}$$

我们有:

$$\frac{\eta^\dagger\eta}{2}g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta)) = g^{ab}\nabla_a\eta^\dagger\nabla_b\eta - \alpha\beta\eta^\dagger\eta - \alpha\gamma(\eta^\dagger\eta)^2 \tag{3.10}$$

令变换u满足:

$$g^{ab}\nabla_a(\frac{1}{u^\dagger u}\nabla_b(u^\dagger u)) = 0 \tag{3.11}$$

当u是幺正的规范场时，式3.13中取电势矢量$\tilde{A}_a = 0$，即满足：

$$\tilde{D}_a(\tilde{\eta}) = \tilde{D}_a(u\eta) = (\nabla_a + \tilde{A}_a)(u\eta) = u(\nabla_a + u^{-1}\nabla_a u)\eta = u(\nabla_a + A_a)\eta = uD_a\eta \tag{3.12}$$

作变换: $\eta \to u\eta = \tilde{\eta}$，规范变换相当于将式3.11加到式3.10中，结果保持不变，我们有:

$$\begin{aligned}
&\frac{1}{2}\tilde{\eta}^\dagger\tilde{\eta}g^{ab}\nabla_a(\frac{1}{\tilde{\eta}^\dagger\tilde{\eta}}\nabla_b(\tilde{\eta}^\dagger\tilde{\eta}))\\
&=\frac{1}{2}\tilde{\eta}^\dagger\tilde{\eta}(g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta))+g^{ab}\nabla_a(\frac{1}{u^\dagger u}\nabla_b(u^\dagger u)))\\
&=g^{ab}\tilde{D}_a^\dagger\tilde{\eta}\tilde{D}_b\tilde{\eta}-\alpha\beta\tilde{\eta}^\dagger\tilde{\eta}-\frac{\alpha\gamma}{u^\dagger u}(\tilde{\eta}^\dagger\tilde{\eta})^2
\end{aligned} \tag{3.13}$$

利用式2.13 和2.14，可以令η 满足:

$$g^{ab}\nabla_a(\frac{1}{\eta^\dagger\eta}\nabla_b(\eta^\dagger\eta))=2(aF_\iota+bF_\phi)=2\kappa \tag{3.14}$$

于是我们得到一种希格斯场:

$$\begin{aligned}
2\mathcal{L}_h&=g^{ab}\tilde{D}_a^\dagger\tilde{\eta}\tilde{D}_b\tilde{\eta}-(\alpha\beta+\kappa)\tilde{\eta}^\dagger\tilde{\eta}-\frac{\alpha\gamma}{u^\dagger u}(\tilde{\eta}^\dagger\tilde{\eta})^2\\
&=g^{ab}\tilde{D}_a^\dagger\tilde{\eta}\tilde{D}_b\tilde{\eta}-\mu^2\tilde{\eta}^\dagger\tilde{\eta}-\frac{\lambda}{2u^\dagger u}(\tilde{\eta}^\dagger\tilde{\eta})^2=0
\end{aligned} \tag{3.15}$$

以上计算是可逆的，所以方程3.1也能从Lagrange量得到。当$u^\dagger u = I$，对应标准模型中的SU(N)群变换。而对于静态真空场我们有：

$$u^\dagger u=(1-\frac{2Gm_g}{c^2r})^{-1} \tag{3.16}$$

$$g^{ab}\nabla_a((1-\frac{2Gm_g}{c^2r})\nabla_b(1-\frac{2Gm_g}{c^2r})^{-1})=0 \tag{3.17}$$

其中m_g 是引力质量。此时希格斯真空态有：

$$v_0=\sqrt{-\frac{\mu^2u^+u}{\lambda}}=\sqrt{\frac{-\mu^2}{\lambda(1-\frac{2Gm_g}{c^2r})}} \tag{3.18}$$

这将导至基本粒子质量由m_0 变为m_p：

$$m_p=\frac{m_0}{\sqrt{1-\frac{2Gm_g}{c^2r}}}\propto v_0 \tag{3.19}$$

这一结果与相对论一致。另外$u^\dagger u$也可用Dirac矩阵进行变换。

4 消除外因子

简单计算，式3.10中可以变换为：

$$\frac{1}{2}g^{ab}\nabla_a\nabla_b\left(\eta^\dagger\eta\right)=g^{ab}\nabla_a\eta^\dagger\nabla_b\eta+(\frac{g^{ab}\nabla_a\ln(\eta^\dagger\eta)\nabla_b\ln(\eta^\dagger\eta)}{2}-\alpha\beta)\eta^\dagger\eta-\alpha\gamma(\eta^\dagger\eta)^2 \tag{4.1}$$

于是我们可以将式3.1中的外因子移动到积分内部，变为：

$$g^{ab}\nabla_b\eta=\int(\frac{g^{ab}\nabla_a\ln(\eta^\dagger\eta)\nabla_b\ln(\eta^\dagger\eta)}{2}-\alpha\beta-\alpha\gamma\eta^\dagger\eta)\eta dx^a \tag{4.2}$$

由熵和量子态的概率密度含义可知，$\nabla_a \ln(\eta^\dagger \eta)$反映了空间结构度规关联状态数的变化对场分布变化的影响。我们可以做个近似的比较计算，令状态数为W，量子数为N，体元ΔV，且有：

$$\Delta V = \sqrt{W} \propto \sqrt{\eta^\dagger \eta} \tag{4.3}$$

于是有：

$$\begin{aligned}\frac{d\eta}{ds} &= \frac{d}{ds}(\frac{N}{\Delta V}) \propto \frac{d}{ds}(\frac{N}{\sqrt{W}}) = \frac{dN}{\sqrt{W}ds} - \frac{d\ln W}{2\sqrt{W}ds} \\ &\propto \frac{dN}{ds}\eta^{-1} - \frac{d\ln(\eta^\dagger\eta)}{2ds}\eta^{-1} \propto \frac{dN}{ds}\eta^\dagger - \frac{d\ln(\eta^\dagger\eta)}{2ds}\eta^\dagger\end{aligned} \tag{4.4}$$

可以看出时空弯曲的本质与物质状态空间的变化有关。

5 倒换

令$\eta^\dagger\eta = 1/A$，$\xi = u\sqrt{A}$，$u^\dagger u = I$，代入式3.10中有：

$$-\frac{\xi^\dagger\xi}{2} g^{ab}\nabla_a(\frac{1}{\xi^\dagger\xi}\nabla_b(\xi^\dagger\xi)) = g^{ab}\nabla_a\xi^\dagger\nabla_b\xi - \alpha\beta\xi^\dagger\xi - \alpha\gamma \tag{5.1}$$

可以看到我们也可以通过这一变换来消除或产生希格斯自作用项。需要注意，由于式中质量项取负号，即右边第一项与第二项反号，故它并不是一个值为0的Lagrange量，为0的应该是式3.5。为区别，将量子力学中值为0的Lagrange量记为L。而本文的形式记为$\mathcal{L}$，称为林式量。κ为0时，希格斯粒子质量满足$m_H c^2/\hbar = \sqrt{-2\mu} = \sqrt{-2\alpha\beta}$，故式5.1上右边非常数项等于$T^2\xi^\dagger\xi/\hbar^2$，$T = -m_H c^2$是场能动张量的迹。

6 费米场外叠加

将多个场对外因子的叠加变化，称为外叠加，相应的叠加数量称为外叠加。对于费米场F_1 和F_2, 满足:

$$F_1^\dagger F_2 + F_2^\dagger F_1 = 0 \tag{6.1}$$

则有:

$$\nabla_b(F_1 + F_2) \tag{6.2}$$

$$= \beta_F(1 - \frac{\gamma_F}{\beta_F}(F_1^\dagger + F_2^\dagger)(F_1 + F_2)) \int P_a P_b (F_1 + F_2) dx^a \tag{6.3}$$

$$= \beta_F(1 - \frac{\gamma_F}{\beta_F}(F_1^\dagger F_1 + F_2^\dagger F_2)) \int P_a P_b (F_1 + F_2) dx^a \tag{6.4}$$

如果让外量为n的费米场叠加有:

$$\nabla_b \sum_{i=1}^{n} F_i = \beta_F(1 - \frac{\gamma_F}{\beta_F}\sum_{i=1}^{n}(F_i^\dagger F_i)) \int P_a P_b \sum_{i=1}^{n} F_i dx^a \tag{6.5}$$

$$\sum_{i=1}^{n} F_i = n\bar{F} \tag{6.6}$$

$$\nabla_b \bar{F} = \beta_F(1 - \frac{\gamma_F}{\beta_F} n\bar{F}^\dagger\bar{F}) \int P_a P_b \bar{F} dx^a \tag{6.7}$$

7 玻色场外叠加

与费米场类似，对于外量为n的玻色场, 有:

$$\nabla_b \bar{B} = \beta_B (1 - \frac{\gamma_B}{\beta_B} n^2 \bar{B}^\dagger \bar{B}) \int P_a P_b \bar{B} dx^a \tag{7.1}$$

对于玻色场，由式2.16、2.17、3.2、3.14和式7.1：

$$\begin{aligned} \kappa &\propto nm \\ \gamma &\propto n^2 \\ \alpha &\propto m^2 \end{aligned} \tag{7.2}$$

其中质量m由场自身能量提供，而外量n由B在空间中的效应等同的粒子数决定，将互为的同效的所有粒子合并成一组，称为它们的模组，故n也称为模组的粒数：

$$< B >= \sqrt{-\frac{\kappa_0 nm + \alpha_0 \beta_0 m^2}{2\alpha_0 \gamma_0 n^2 m^2}} \tag{7.3}$$

对于0质量玻色子，$\beta_0 \to 0$有：

$$< B > \propto \sqrt{\frac{1}{nm}} \propto \sqrt{\frac{1}{n\hbar\omega}} \tag{7.4}$$

8 常数计算

我们可以得到总林式量$\mathcal{L}$、各粒子场林式量$\mathcal{L}_i$以及黎曼曲率R间的关系：

$$\begin{aligned} 2\mathcal{L} &= \frac{1}{2} g^{ab} \nabla_a (\frac{1}{\eta^\dagger \eta} \nabla_b (\eta^\dagger \eta)) + \sum_i (\frac{1}{2} \frac{\phi_i^\dagger \phi_i}{\eta^\dagger \eta} g^{ab} \nabla_a (\frac{1}{\phi_i^\dagger \phi_i} \nabla_b (\phi_i^\dagger \phi_i))) \\ &= \frac{\eta^\dagger \eta + \sum\limits_i (\phi_i^\dagger \phi_i)}{2\eta^\dagger \eta} g^{ab} \nabla_a (\frac{1}{\eta^\dagger \eta} \nabla_b (\eta^\dagger \eta)) + \sum_i (\frac{1}{2} \frac{\phi_i^\dagger \phi_i}{\eta^\dagger \eta} g^{ab} \nabla_a (\frac{\eta^\dagger \eta}{\phi_i^\dagger \phi_i} \nabla_b (\frac{\phi_i^\dagger \phi_i}{\eta^\dagger \eta}))) \\ &= \frac{\eta^\dagger \eta + \sum\limits_i (\phi_i^\dagger \phi_i)}{2\eta^\dagger \eta} R + 2 \sum_i \mathcal{L}_i \end{aligned} \tag{8.1}$$

上式中我们将η视为观测场，ϕ_i为被η观测到的场。$\eta^\dagger \eta$正比于观测到空间点的概率，$\phi_i^\dagger \phi_i$正比与其出现在相应空间点并被η观测到概率，而$\psi_i^\dagger \psi_i = (\phi_i^\dagger \phi_i)/(\eta^\dagger \eta)$是与观测无关的分布量，反映其在空间中出现的概率分布。由于ψ_i所在模组的粒数n_j，每一个粒子应该有一个单独的可观测项，因此n_j也反映同一模组在方程中的项数。

对于状态稳定的0质量玻色子，由方程7.4，可得ψ_i所处模组粒数为：

$$n_j = \frac{C \eta^\dagger \eta}{\hbar \omega_i \phi_i^\dagger \phi_i} \tag{8.2}$$

C为常数。考虑η相对求和量很小，且也可视为一部分模组成分，可以忽略。将式8.2代入式8.1求和，因为作了部分求和，我们更换一下求和状态下标为模组下标j，化成：

$$2\tilde{\mathcal{L}} = 2\mathcal{L}/C = \sum_j \frac{1}{\hbar \omega_j} \frac{R}{2} + \sum_j (\frac{1}{2} \frac{1}{\hbar \omega_j} g^{ab} \nabla_a (\frac{\eta^\dagger \eta}{\phi_j^\dagger \phi_j} \nabla_b (\frac{\phi_j^\dagger \phi_j}{\eta^\dagger \eta})) \tag{8.3}$$

上式右边第二项ω_j^{-1}可以移进导数中，成为场的指数因指，正好与场的能量抵消。这使得右边第二项各项在平稳时空是中变得均匀一致。它们彼此可以通过能量与数目的改变实现转换，也促成了这种平衡。

体积为L^3的方盒中，利用周期边条件，模组j波长为$\lambda_j = L/n_j$，由于$k_j = 2\pi/\lambda_j$，处于$(k_j, k_j + dk_j)$范围内的k_j分立值共有$Ldk_j/(2\pi)$。于是模组密度数为：

$$\rho_0 = \lim_{L\to\infty} \frac{N}{L^3} = \frac{1}{8\pi^3}\int d^3\vec{k} = \frac{1}{2\pi^2}\int_0^{k_{max}} k^2 dk = \frac{k_{max}^3}{6\pi^2} \tag{8.4}$$

要得到k_{max}，需要在极限小的时间或空间变化下，得到最大的相位差异，也就是π，又由于黑洞边界极限，有：

$$k_{max} r_{min} = m_{max} r_{min} c/\hbar = \pi \tag{8.5}$$

$$1 - \frac{G m_{max}}{c^2 r_{min}} = 0 \tag{8.6}$$

有：

$$k_{max} = \sqrt{\frac{\pi c^3}{\hbar G}} \tag{8.7}$$

式8.4同样的可以用于对式8.3中各不同模态作进一步求和，结合式8.7，$\omega = kc$，代入普朗克能量有：

$$\mathcal{F} = \sum_j \frac{1}{\hbar\omega_j} = \frac{1}{2\pi^2}\int \frac{1}{\hbar\omega} k^2 dk = \frac{1}{2\pi^2\hbar c}\int_0^{k_{\max}} k dk = \frac{k_{\max}^2}{4\pi^2\hbar c} = \frac{c^2}{4\pi\hbar^2 G} \tag{8.8}$$

结合式5.1、式7.2和式8.3，忽略掉常数项，视为0，不难有$R = -8\pi GT/c^2$，与广义相对论一致。

9 结论和扩展讨论

通过计算，本文解决了引力常数与其他物理物常数的关系问题，从而验证引力变换的非幺正性。解释了引力场是如何通过希格斯场引起物质质量改变，进而产生引力作用的。

然而这还只是开始，从式8.3中，我们还可以看到时空机制下隐藏的关联秘密。能量通过波场ϕ_i建立的传输隧道传递，其距离与其波长λ_i相关。本来是可以直接穿透其间空间的，但是由于传播过程中，ϕ_i可以发生模态变换，改变λ_i，从而发生途经效果，并产生位置与能量的量子不确定性。当测量时，由于ϕ_i的一端被固定，另一端也就确定了下来，发生坍缩。

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