(i) We would like now to use the same approach to model a coupled pendulum. Open the Simulink model ***coupled_pendulum.slx*** and apply “backwards engineering” to read out the two differential equations describing the coupled pendulum.

$$\ddot{\theta}_A(t) = \left(\frac{1}{m l_1^2}\right) \cdot \left[K l_2^2 \cdot \theta_B(t) - C_D \cdot \dot{\theta}_A(t) - \left(k l_2^2 + m g l_1\right) \theta_A(t) \right]$$

$$\ddot{\theta}_B(t) = \left(\frac{1}{m l_1^2}\right) \cdot \left[K \cdot l_2^2 \cdot \theta_A(t) - C_D \cdot \dot{\theta}_B(t) - \left(k \cdot l_2^2 + m g l_1\right) \theta_B(t) \right]$$

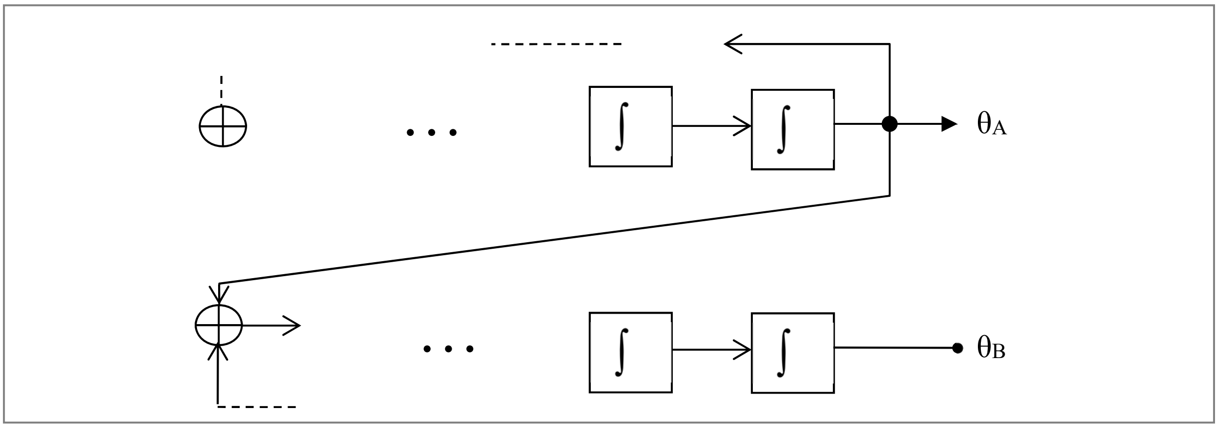

Figure 2 Block Diagram coupled pendulum

(j) If you isolate $\theta_B(t)$ in the 1st equation, and replace $\theta_B(t)$, $\dot{\theta}_B(t)$ and $\ddot{\theta}_B(t)$ in the 2nd equation, you get a single differential equation describing the complete coupled pendulum. Determine which order would this differential equation have (but without carrying out the replacement calculation).
How does this order relate to the number of integrator boxes in the model?
Differential equation of 4th order => 4 integrator boxes in the block diagram

(k) If you apply the same method of testing an exponential solution for the coupled pendulum, which solution (values for s) do you expect to find?
Expect 2 pairs of complex conjugated roots. (s = -sigma +/- omega)

How do these values relate to the response of the pendulum that you can observe in the time domain?
Open and execute the Matlab script Trial_4_roots.m and explain the relationship between the 4 roots, applied to the exponential solution and the resulting graphic.
The omega values relate to oscillation frequency, and to sinus-shaped envelope.
Plus sigma value relates to the decay of the envelope amplitude.

(l) Make a copy of your previous Matlab script, and name it ***runSimulink _coupled_pendulum.m*** . Define the required constants and add the call for the Simulink simulation of the coupled pendulum.

For the spring used to connect the two pendula you can use:

```
k = 2.5;              % Spring constant in [N/m]
l1 = 0.7;             % Length Pendel Rod in [m]
l2 = 0.4;             % Torque Arm Spring in [m]
```