Zürcher Hochschule
für Angewandte Wissenschaften

School of
Engineering

HS18, dqtm

SiSy Semester Exam

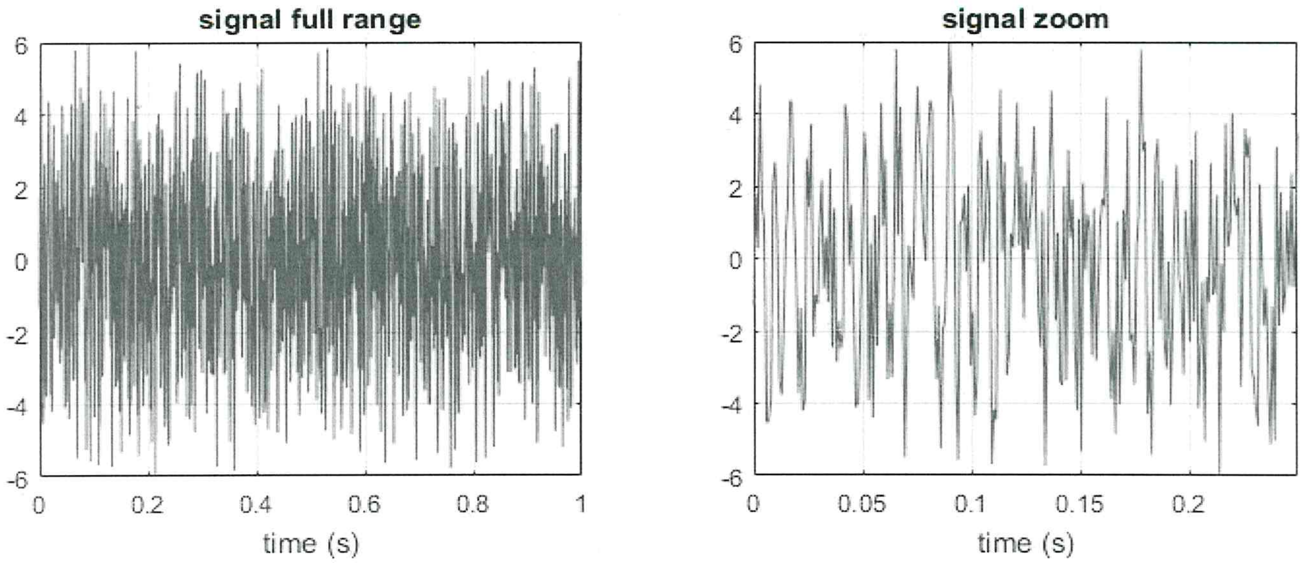
Duration: 90 Minutes Open book exam, without calculator. Your calculations and solution approach need to be readable and comprehensible in order to get the full points. Please write your final results in the reserved gray fields and use the provided spaces for the sketches. Do not forget to label your axes.

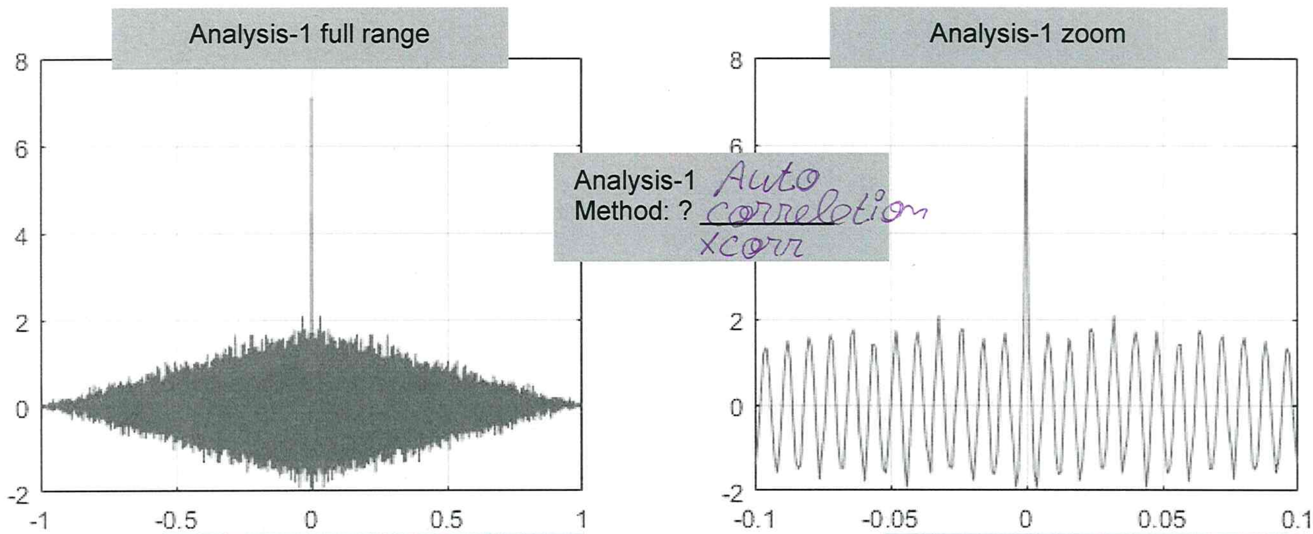
Name: Marina Q.T.									Class: teacher	
1:	2:	3:	4:	5:					Points:	Grade:

Sample solution

Exercise 1 *Signal Analysis* *[6+6+6 = 18 Points]*

A noisy input signal s(t) is analysed using 2 methods: FFT and auto-correlation. The plots generated in these analyses are shown below.



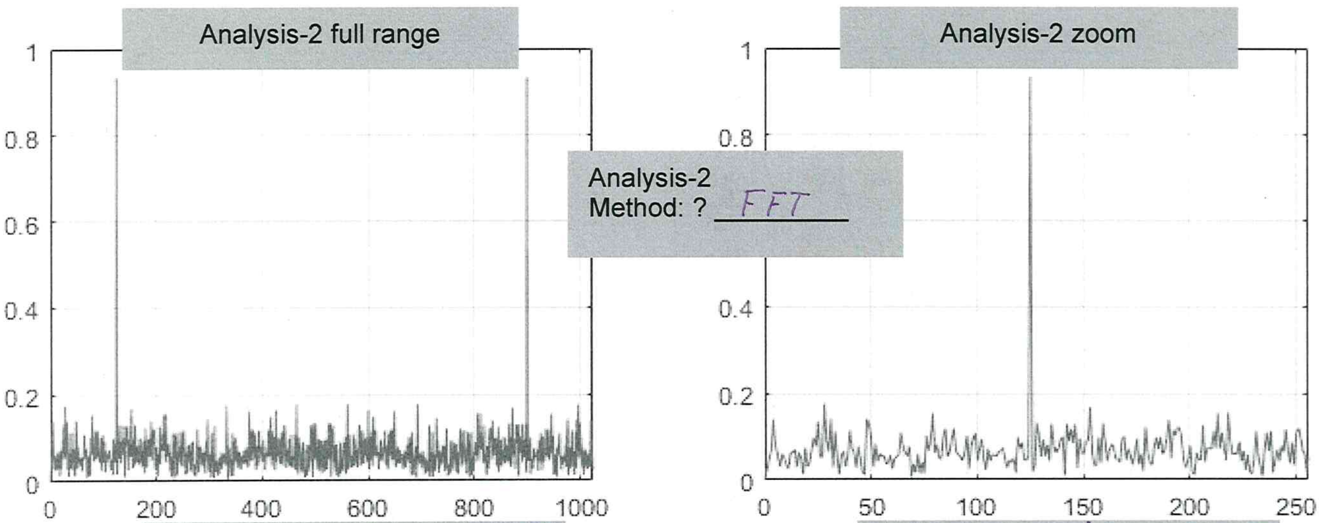


2x1,5

Analysis-1 Method: ? Auto correlation xcorr

variable (unit):? time [s]

variable (unit):? time [s]



2x1,5

Analysis-2 Method: ? FFT

variable (unit):? freq [Hz]

variable (unit):? freq [Hz]

missing units -0,5

(A) Identify in the previous page, which plots belong to which analysis method. Then, make a supposition about the variables and units used for the horizontal axes in the plots, and write your assumptions in the reserved grey fields along the axes. $\boxed{2\times 3P = 6P}$

(B) Which characteristics of the signal s(t) can you identify and confirm with the outputs of these two analyses? $\boxed{3\times 2P = 6P}$

- Input s(t) contains white noise + 1-sinusoidal tone
- Noise looks like white noise because: peak at xcorr at 0, and wide spread in spectrum.
- Sinusoidal tone has $f_0 \approx 125\,Hz$ and $A_1 = 2.(0{,}95) = 1{,}9$

(C) Complete the extract of code below, used to generate the plots on the previous page $\boxed{2\times 3P = 6P}$

Obs: Missing normalisation -0,5

```
% PARAMETERS
% Suppose N, aux (index vector), and Fs are already defined, then …
t = Ts*aux;
f = (Fs/N)*aux;
% Suppose s_t is the measured noisy signal, which can be used below
S_f = (1/N) * fft(s_t);              % compute the coresponding spectrum
s_x = (1/N)*xcorr(s_t, s_t);         % compute the coresponding autocorrelation
aux_long = -(N-1): 1 : (N-1);        % long index-vector to plot autocorrelation
t_long = Ts*aux_long;
```

Obs: missing abs( ) -0,5
exchanged -0,5
units of zoom

zoom

```
figure()
subplot(321),plot(t,s_t),…
subplot(322),plot(t, s_t),…, xlim([0 t(end)/4]), …
subplot(323),plot(t_long, s_x),…
subplot(324),plot(t_long, s_x),…, xlim([-0.1 +0.1])
subplot(325),plot(f, abs(S_f)),…
subplot(326),plot(f, abs(S_f)),…, xlim([0, f(end)/4])
```

T_1: 18P

Exercise 2 *Fourier Series with Complex Coefficients* *[2+4+4+4+5 = 19 Points]*

The complex Fourier coefficients of y(t) and a plot of y(t) in the time domain are given below:

Obs.: in order to identify these coefficients as related to y(t), we call them cky.

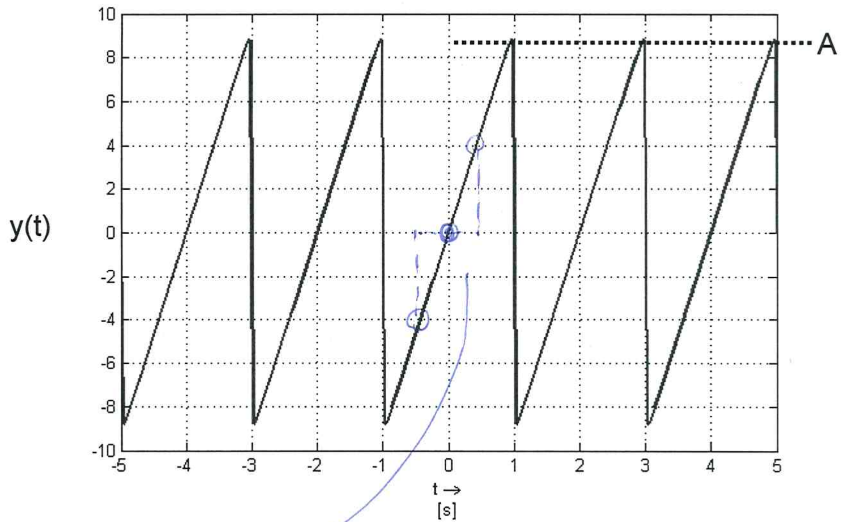
$$c_{ky} = j.\frac{A}{k\pi}.\cos(k\pi)$$

für $k \in N^+$

with

$$c_{-ky} = (c_{ky})^*$$

and $A = 3\pi$



(A) Explain how the symmetry observed in y(t) can be confirmed by the expression of the corresponding c_{ky} coefficients.

$\boxed{2P}$

$y(t_1) = -y(-t_1)$ for $\forall\, t_1$ ⇒ y(t) is odd !

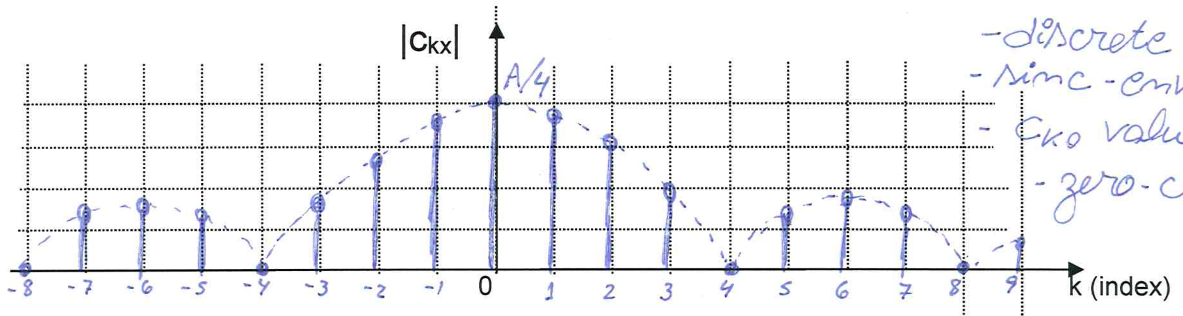
This implies c_{ky} purely imaginary (confirmed w/ expression above)

(B) The complex Fourier coefficients of another time signal, c_{kx} for x(t) , are given below. Draw a sketch of the amplitude spectrum of x(t) in the axis below.

$$c_{kx} = A.\frac{\tau}{T_0}.\operatorname{sinc}\left(k.\frac{\tau}{T_0}\right) \qquad \text{with} \qquad \tau = \frac{T_0}{4}$$

$\boxed{4x1P}$

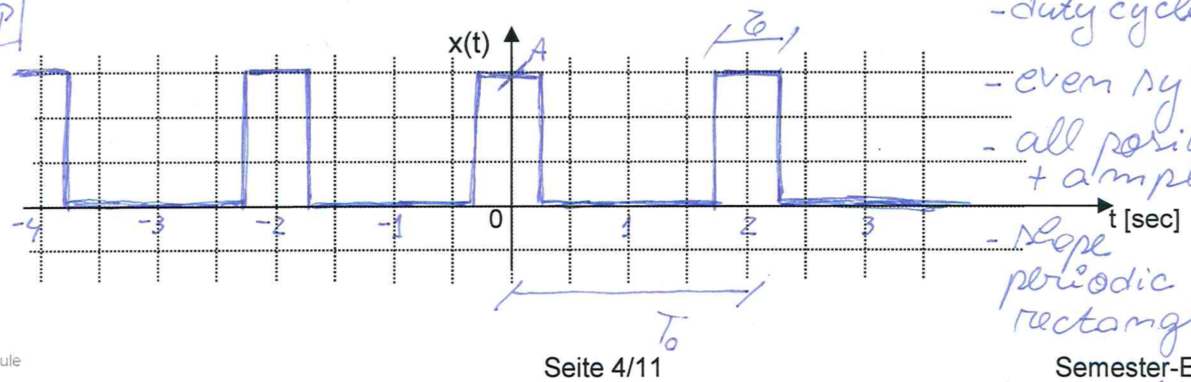
- discrete
- sinc-envelope
- c_{k0} value
- zero-crossings



(C) Draw a sketch of x(t) in the time domain. Explain, how did you find out the characteristics of x(t).
Hint: compare to a reference signal discussed in lectures and labs.

$\boxed{4x1P}$

- duty cycle 25%
- even symmetry
- all positive + amplitude
- slope periodic rectangle
- $T_0 = 2$ (as y(t))

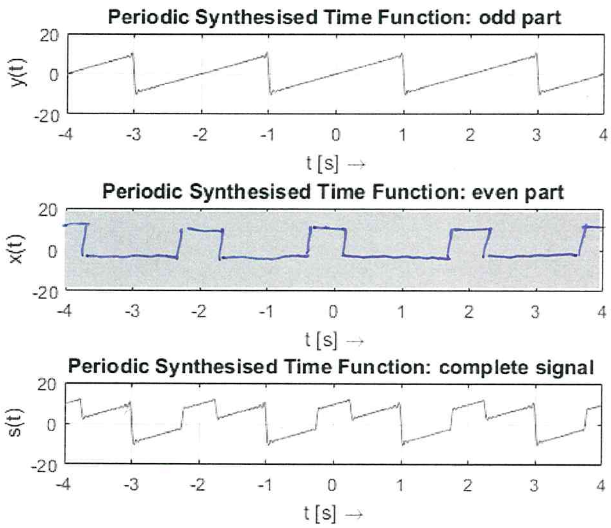


(D) The signals y(t) and x(t) are summed up to generate a new signal s(t). Determine the c_{ks} coefficients of s(t) and complete the Matlab code below, which implement the synthesis of s(t).

$s(t) = y(t) + x(t)$

$\boxed{4x\,1P = 4P}$

```
% Fourier Synthesis to check coeffs
% ========================================
% PARAMETERS & TIME-VECTOR
T0 = 2; w0 = 2*pi/T0;
t = -2*T0:T0/200:2*T0;
A= 3*pi;
tau = T0/4;

% Number of harmonics and DC-Offset
Kmax = 30;

c0x = A * tau / T0 ;

x_t = c0x * ones(1,length(t));   % initialise even part with DC-value

y_t = zeros(1,length(t));        % initialise odd part with zero

for k = -Kmax:1:Kmax

    if k~=0                      % jump zero, cause cky expression not valid

      cky = j*(A/(k*pi))*cos(k*pi);     % odd part

      y_t = y_t + exp(j*k*w0*t) * cky ;

      ckx = (A*tau/T0)*sinc(k*tau/T0);        % even part

      x_t = x_t + exp(j*k*w0*t)*Ckx ;

    end
end

% combine even+odd parts for complete signal
s_t = x_t + y_t;
...
figure() ...
subplot(311),plot(t,y_t),...
subplot(312),plot(t,x_t),...
subplot(313),plot(t,s_t),...
```



T_2: 19P

(E) Suppose the signal s(t) passes now through an ideal low pass filter, and gets band limited. The cut frequency of the ideal LPF equals to the 1st zero crossing of the spectrum of x(t) (first lobe of the sinc-shape). Explain how would you calculate the percentage of the power of the original s(t) signal, which gets through the LPF.

$\boxed{2+2+1 = 5P}$

We can calculate the total power in the time domain:

$$P_{s\_total} = \frac{1}{T_0}\int_{T_0} [s(t)]^2\,dt$$

and the power of the band limited version in the freq domain

$$P_{s\_limBW} = \sum_{k=-4}^{+4} |(C_{ky} + C_{kx})^2|$$

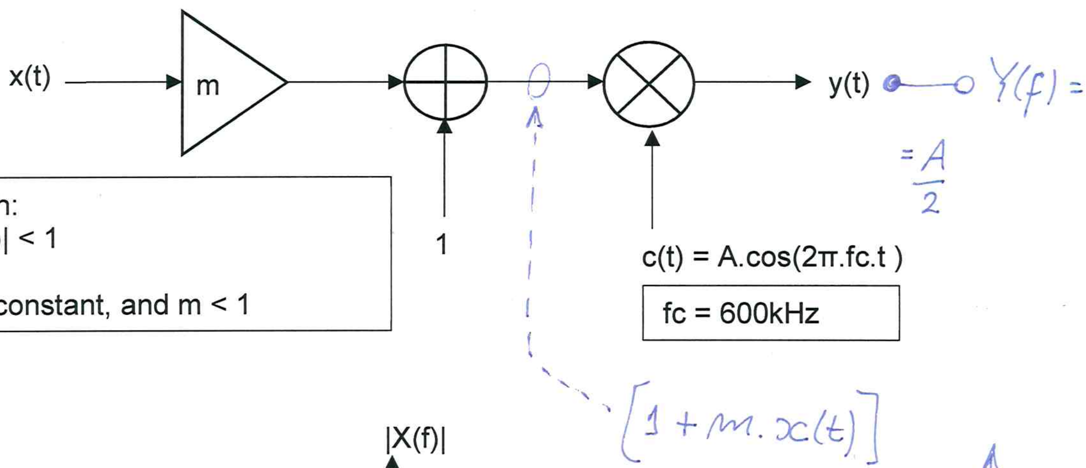
And then $\frac{P_{s\_limBW}}{P_{s\_total}}$ = % of power coming thru

Exercise 3 *Signal Processing Chain* *[7+2+6 = 15 Points]*

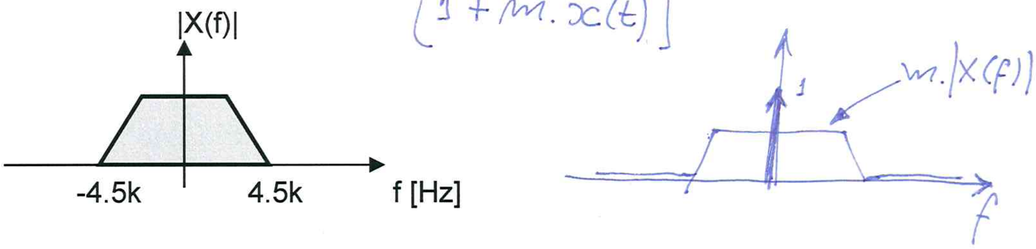
(A) The block diagram below shows a signal processing chain. Assume an input signal x(t), with the corresponding amplitude spectrum abs(X(f)) shown below.

Then, complete the sketches and comments required (marked in grey).

$\boxed{2+3+2=7P}$

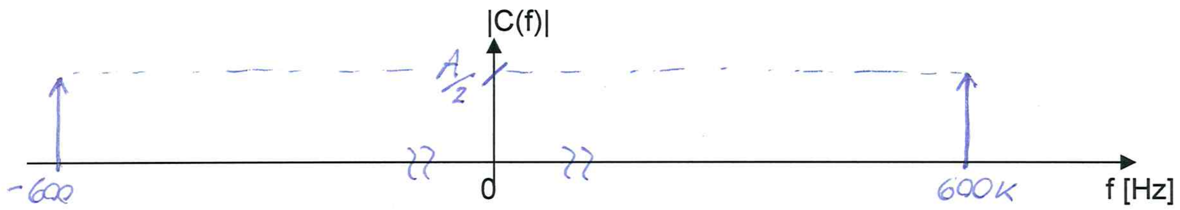


Given:

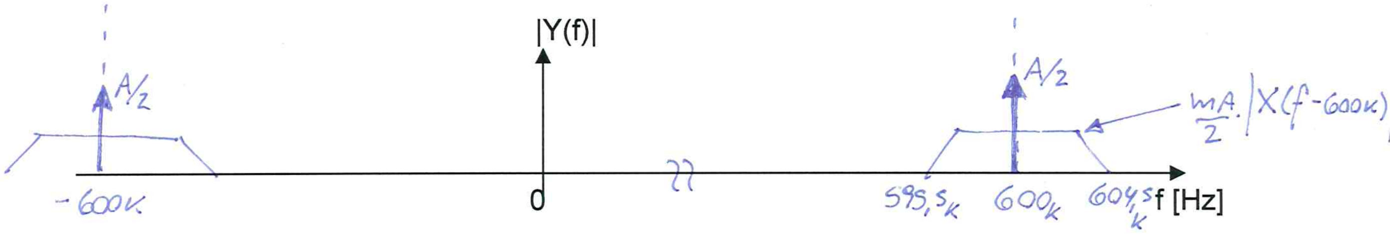


Asked:

2P



3P



Related Fourier Transformation Property: (name + mathematical expression)

2P Frequency - Shift & Linearity

$x(t).\exp(j2\pi f_0 t) \circ\!\!-\!\!\bullet\, X(f-f_0)$ $A.x_1(t) + B.x_2(t) \circ\!\!-\!\!\bullet\, A.X_1(f) + B.X_2(f)$

(B) What is the common denomination for the signal processing chain shown in item (a)? Which commercial application used it for a long time (from about 1920 until 2015 in many countries around the world) ?

→ Amplitude Modulation $\boxed{2P}$

→ AM - Radio

(C) In another signal processing application, you want to add a reverberation effect to an audio data, which reproduces the acustic characteristics of a cathedral. Explain how you can do that. Which measurements and which processing steps are required? Add equations and/or a block diagram to illustrate your answer.

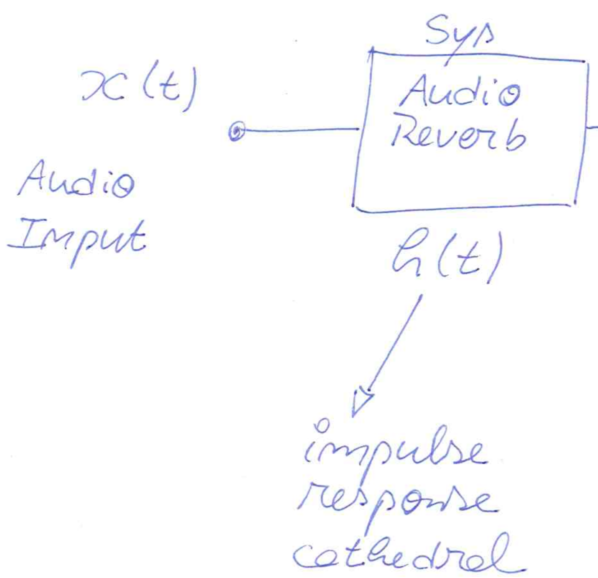
$\boxed{6P}$
- convolution
- w/ imp. resp of cathedral
- schema
- expl. how to get imp. resp
- discrete approx conv
- compare F_s ...

Steps:

(1) Measure impulse response of cathedral.
For example w/ short-"pong" & measure response for about 1-2 sec.

(2) Check that both impulse response and input audio data have same F_s.
If not, interpolate to adopt.

(3) Convolve both signals, and you have the desired output.
A longer time vector (equals length of sum of both lengths) may be helpful for plot.



$$y(t) = x(t) * h(t)$$

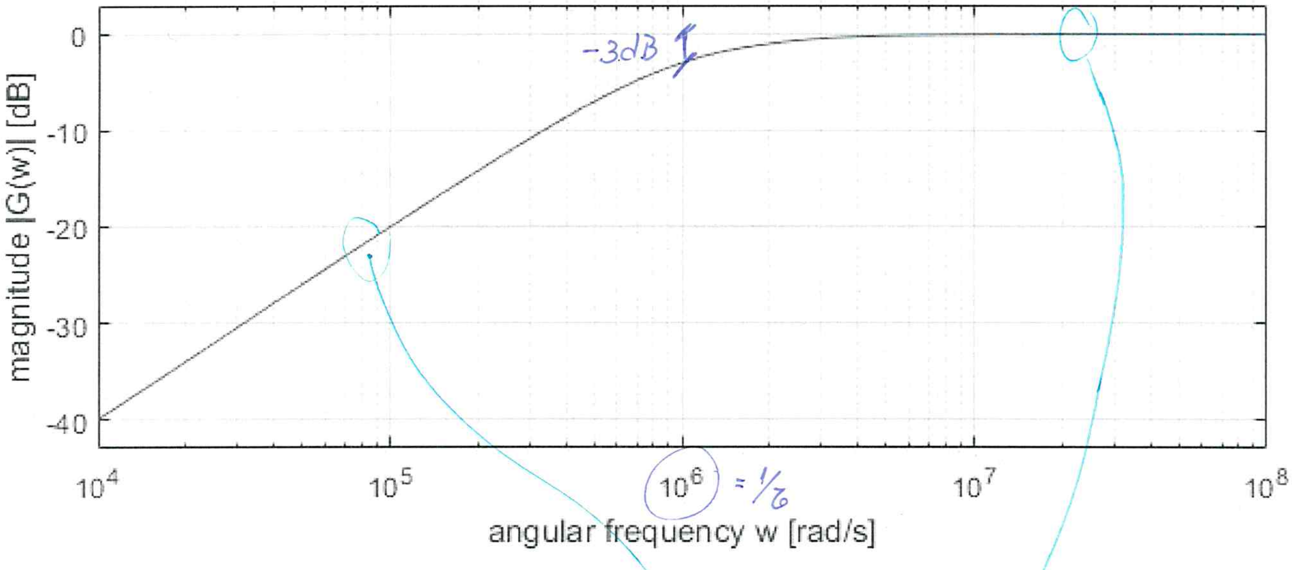
$$y(t) = \int_{-\infty}^{+\infty} x(\lambda) \cdot h(t-\lambda)\, d\lambda$$

$\Downarrow$ numerical approx

$$y[n] = \sum_{k=0}^{2N-1} x[k] \cdot h[n-k] \cdot T_s$$

Exercise 4 *Frequency response of an electrical LTI System* *[4+3+6+6+3=22 points]*

The amplitude part of the Bode Diagram of a frequency response G(ω) is given below:



(a) Determine the equation describing G(ω) .
Hint: The term in the numerator describes a line with constant slope of +20dB/decade, and the term in the denominator describes a curve with a corner and slopes of 0dB/decade before the corner, and -20dB/decade after the corner point.

4P

$$G(\omega) = \frac{j\omega\tau}{j\omega\tau + 1}$$

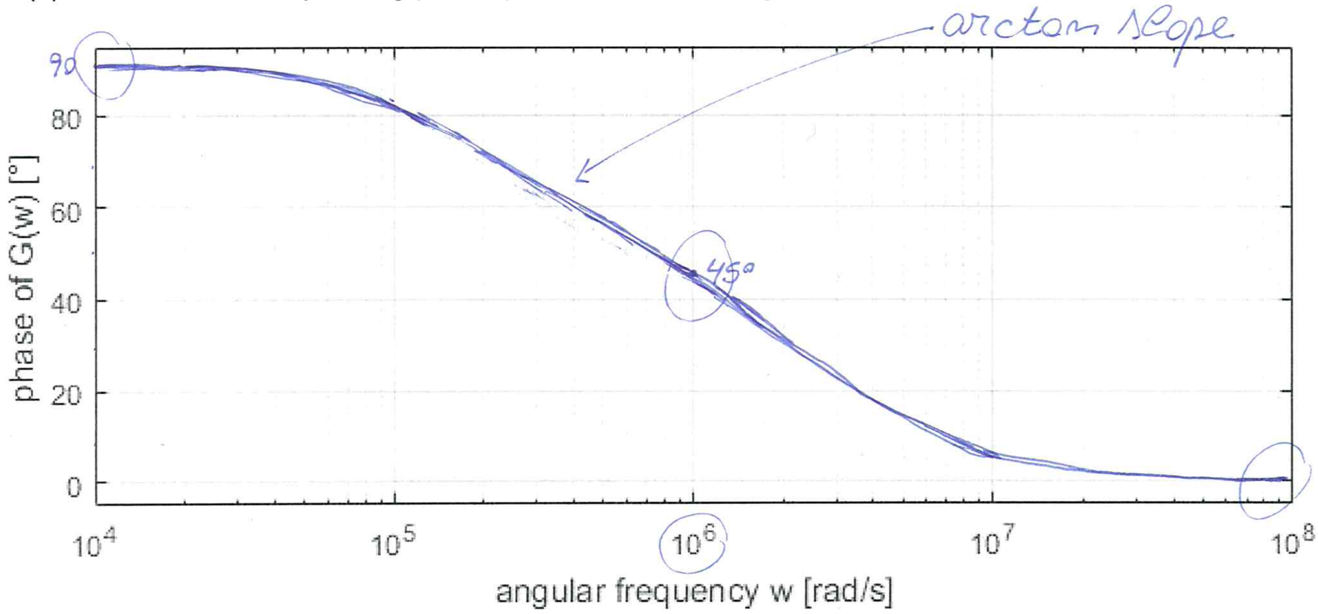
$\omega >> \frac{1}{\tau} \Rightarrow G(\omega) \approx \frac{j\omega\tau}{j\omega\tau} = 1 = 0dB$

$\omega << \frac{1}{\tau} \Rightarrow G(\omega) \approx j\omega\tau \Rightarrow +20dB/decade$

$\frac{1}{\tau} = 10^6 \Rightarrow \tau = 10^{-6} = 1\mu s$

(b) Sketch the corresponding phase part of the Bode diagram.

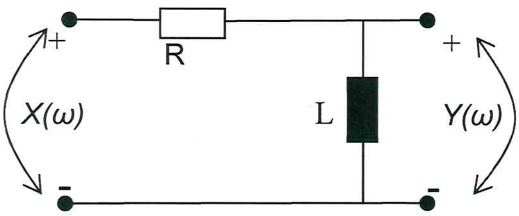
3P



6P

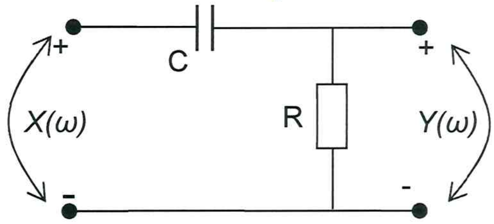
(c) Which of the following circuits can be used to implement the frequency response of item (a)? Justify your response by calculating the frequency response of both circuits using the method of the complex impedances.

(A)



(B)

$Z_C = \frac{1}{j\omega C}$



$Z_L = j\omega L$

$$\frac{Y(\omega)}{X(\omega)} = \frac{Z_L}{Z_R + Z_L} = \frac{j\omega L}{R + j\omega L}$$

$$G(\omega) = \frac{j\omega L/R}{j\omega L/R + 1}$$

possible w/ $\tau = L/R$

$$\frac{Y(\omega)}{X(\omega)} = \frac{Z_R}{Z_C + Z_R} = \frac{R}{\frac{1}{j\omega C} + R} =$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{j\omega RC}{1 + j\omega RC}$$

$$G(\omega) = \frac{j\omega RC}{j\omega RC + 1}$$; possible w/ $\tau = RC$

Both are possible!

6P

(d) Using R=1k Ω, determine the corresponding value for L and/or C .

$$\tau = 10^{-6} = \frac{L}{R}\Big|_{R=10^3} \Rightarrow L = 10^{-6} \cdot 10^3 = 10^{-3} = 1mH$$

$$\tau = 10^{-6} = R.C\Big|_{R=10^3} \Rightarrow C = 10^{-6}/10^3 = 10^{-9} = 1nF$$

3P

(e) Determine the response(*) of a system with the frequency response G(ω) above, for an input signal $x(t) = \cos(10^6 . t\ )$. Justify your answer with a short sentence. → The frequency response at $\omega = \omega_0$ gives us the changes in amplitude and phase for corresponding harmonic input signal

$$G(\omega)\Big|_{\omega = 10^6 rad/s} \Rightarrow \begin{cases} |G(10^6)| = 1/\sqrt{2} = -3dB \\ \angle G(10^6) = \pi/4 = 45° \end{cases}$$

$$\boxed{y(t) = \left(1/\sqrt{2}\right) \cdot \cos\left(10^6 t + \pi/4\right)}$$

(*)Only the stationary part of the response is expected

T4: 22P

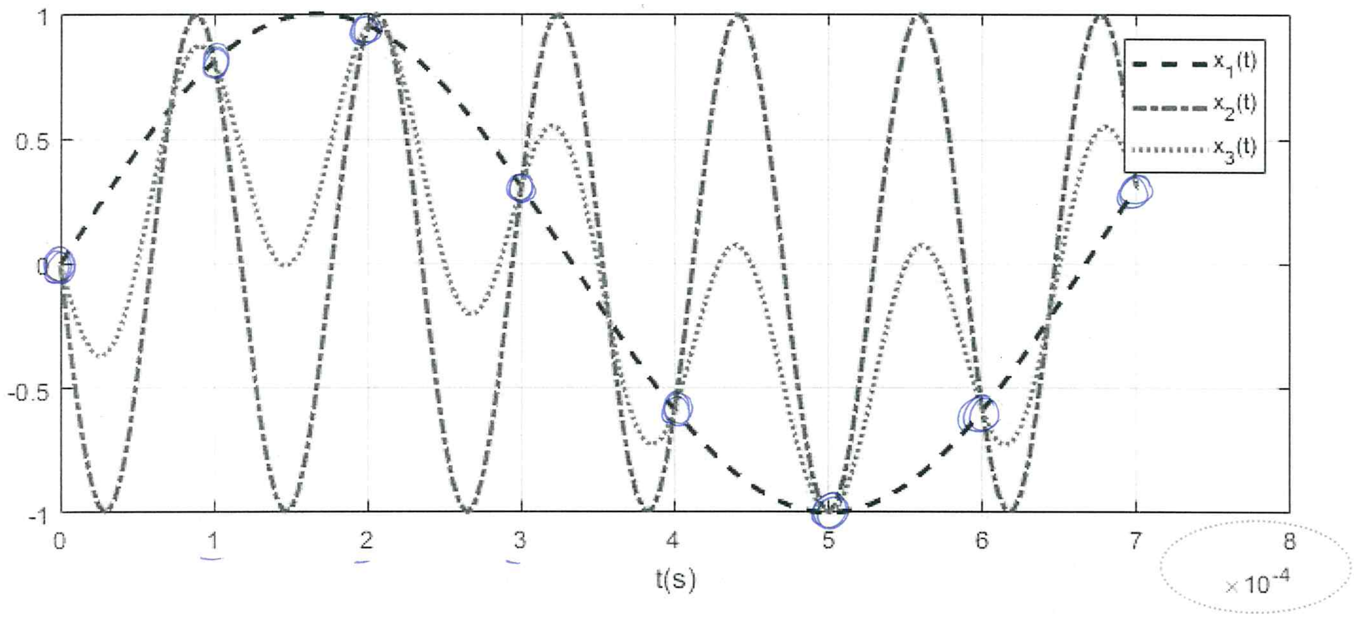
Exercise 5 *ADC & Aliasing & FFT* *[3+4+8 = 15 Points]*

The equation of three continuous time signals $x_1(t)$, $x_2(t)$, and $x_3(t)$ and the corresponding plots are given below:

$$x_1(t) = \sin(2\pi\, f_1.t) \quad with \quad f_1 = 1.5\ kHz$$

$$x_2(t) = -\sin(2\pi\, f_2.t) \quad with \quad f_2 = 8.5\ kHz$$

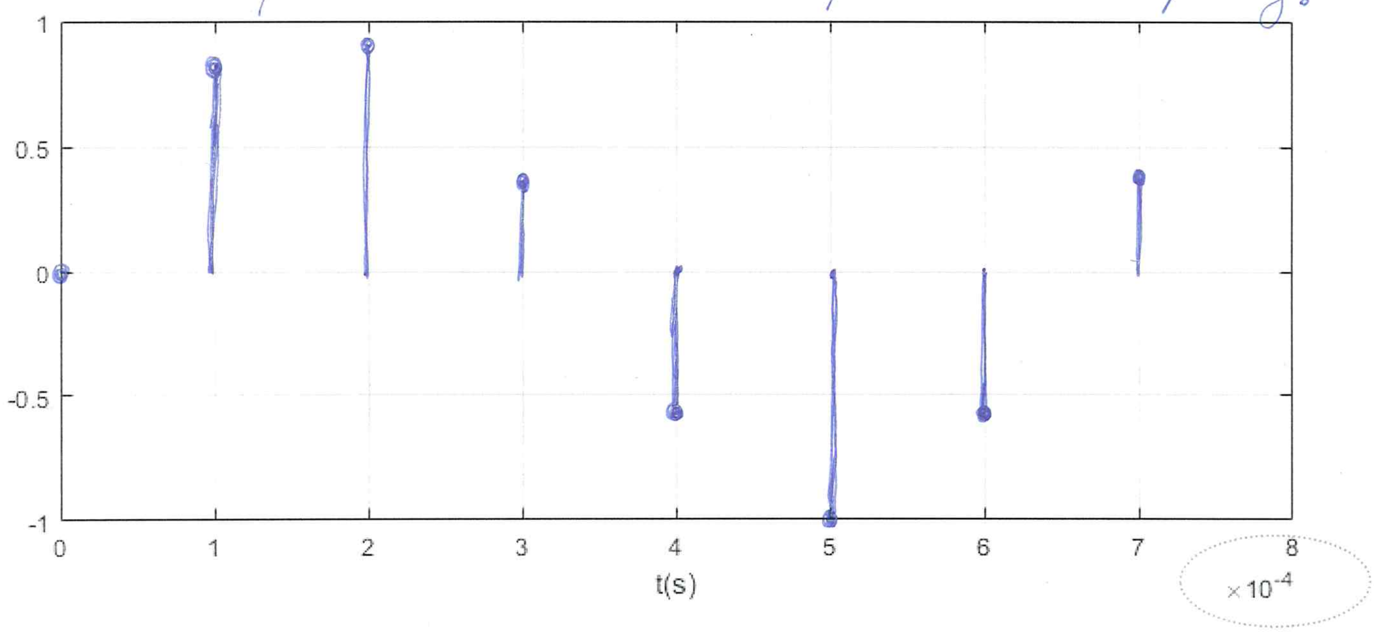
$$x_3(t) = (0.5).x_1(t) + (0.5).x_2(t)$$



(a) Each of the three signals $x_1(t)$, $x_2(t)$, and $x_3(t)$ are converted by an ADC with sampling frequency Fs = 10kHz. Prepare a sketch of the corresponding discrete outputs $x_1[n]$, $x_2[n]$, and $x_3[n]$. Hint: Identify in the plot above the sampling points, starting at t=0s.

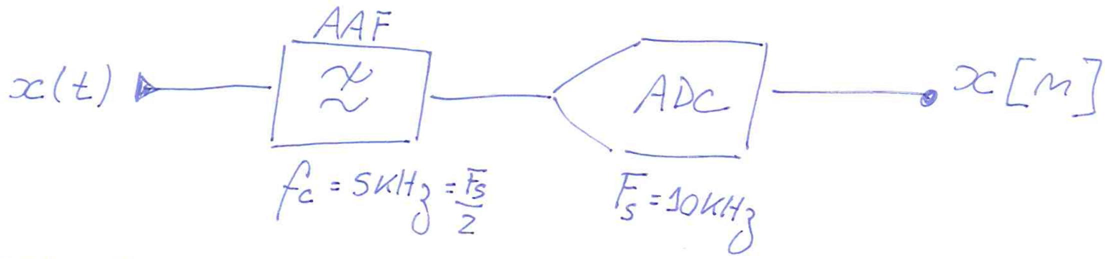
3P

All 3 functions look alike after the sampling!



(b) What would change in the discrete outputs $x_1[n]$, $x_2[n]$, and $x_3[n]$, if one added an ideal anti-aliasing filter before the ADC?
Justify your answer, explaining the characteristics (filter type and cut-off frequency) of the anti-aliasing filter, and its effect on the input signals.

4P



The harmonics w/ frequency above 5kHz would be attenuated. Therefore: $x_1[n]$ unchanged; $x_2[n]$ disappeared (strongly attenuated); $x_3[n]$ only lower harmonic

(c) Suppose the three signals $x_1(t)$, $x_2(t)$, and $x_3(t)$ are now converted by an ADC with a higher sampling frequency of Fs= 32kHz. And the discrete outputs $x_1[n]$, $x_2[n]$, and $x_3[n]$ are recorded in a DSP, which calculates the corresponding spectra $X_1[k]$, $X_2[k]$, and $X_3[k]$ using the FFT algorithm. The DSP records the sampled signals in data blocks of 64 values.
Answer the questions below, indicating the expected characteristics of the spectra $X_1[k]$, $X_2[k]$, and $X_3[k]$

8P

- What is the frequency range represented in the spectra $X_1[k]$, $X_2[k]$, and $X_3[k]$?

1,5

$$f \in [0 ; F_s) = [0 ; 32kHz)$$

- What is the frequency step (or frequency resolution) in the spectra $X_1[k]$, $X_2[k]$, and $X_3[k]$?

1,5

$$\Delta f = f_{step} = \frac{F_s}{N} = \frac{32K}{64} = 0{,}5kHz = 500Hz$$

- For which frequencies (and corresponding k index) do you expect to find non-zero X[k] coefficients? Fill your answers in the table.

5

Spectrum	Non-Zero Coeffs for f = ?	Non-Zero Coeffs for k = ?
$X_1[k]$	$f = \pm 1{,}5kHz$ but w/ FFT $f = +1{,}5kHz$; $30{,}5k = 32 - 1{,}5k$ Hz	$k = \pm 3$; because $k \cdot \Delta f = \pm f_0$ (or $k = +3$; $+61 = +64-3$ w/ FFT)
$X_2[k]$	$f = \pm 8{,}5kHz$; but w/ FFT $f = +8{,}5kHz$; $23{,}5kHz$	$k = \pm 17$; but w/ FFT $k = +17$; $+47$
$X_3[k]$	$f = \pm 1{,}5kHz$ and $\pm 8{,}5kHz$ but w/ FFT $f = +1{,}5kHz$ & $+30{,}5kHz$ & $+8{,}5kHz$ & $23{,}5kHz$	$k = +3$; $+61$; $+17$; $+47$