Laboratory 3c: **Fourier Transformation Applied to Systems**

In this laboratory we use Fourier Transformation properties to obtain the description of a linear time invariant system (here a passive electrical circuit) in the frequency domain. This description is called: the frequency response of the system. Furthermore, you will carry out measurements and generate plots to experiment the meaning of this frequency response and other system responses to test signals.

Exercise 1 *System Frequency Response*

(a) As a warm up solve exercises 3-4 and 3-5 from the SiSy Script Chapter 3.
There you are asked to solve the following questions:

I. Apply the Fourier transformation to the following equation $i_C(t) = C \cdot \frac{dv_C(t)}{dt}$ and determine the complex impedance of a capacitor, defined as:

$$Z_C(f) = \frac{V_C(f)}{I_C(f)} \qquad \text{or} \qquad Z_C(\omega) = \frac{V_C(\omega)}{I_C(\omega)}$$

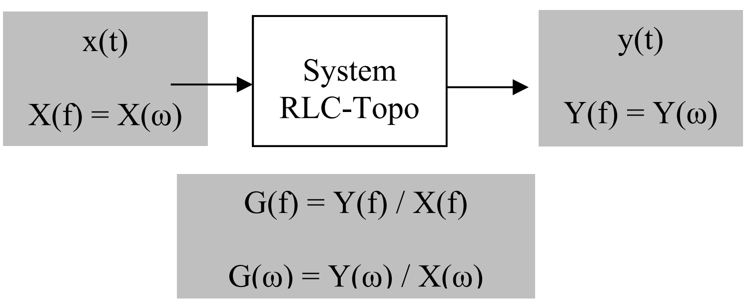
II. Repeat the same procedure for an inductor, with basic equation: $v_L(t) = L \cdot \frac{di_L(t)}{dt}$ and complex impedance:

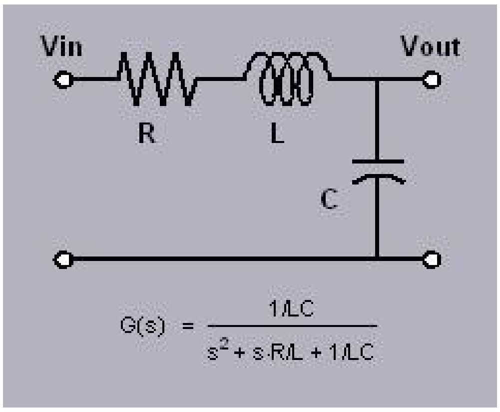
$$Z_L(f) = \frac{V_L(f)}{I_L(f)} \qquad \text{or} \qquad Z_L(\omega) = \frac{V_L(\omega)}{I_L(\omega)}$$

III. Calculate the differential equation describing the following passive RLC topology, and apply the Fourier Transformation to these equation to calculate the corresponding frequency response defined as:

$$G(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{Y(f)}{X(f)} \qquad \text{or} \qquad G(f) = \frac{Y(f)}{X(f)}$$

with Y(ω) and X(ω), being the Fourier Transform of the output and input signals of the system.





Observation:
The frequency response is also called Transfer Function, when you use (s) instead of (jω) .

This is actually equivalent to carry out the previous calculation with the Laplace Transform instead of the Fourier Transform.

For our purposes in this semester, we can set s=jω , and check your result with the equation beside.

IV. Experiment now a second alternative to obtain the frequency response Y(ω) / X(ω), with a calculation directly in the frequency domain.
Use the complex impedances from (i) and (ii), and apply the principle of a voltage divider.

Exercise 2 *LTI-System Description and Responses in Matlab*

Consider the example below, showing how one can describe a 1st order RC passive low pass filter (LPF) in the frequency domain in Matlab, and plot its frequency response (Bode diagram):

$$G(\omega) = \frac{1}{j\omega\tau + 1}$$

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w = logspace(-3,3,100);     % Define frequency vector:e.g. from 10-3 to 10+3
G_w = 1./(j*w*tau + 1);     % Define frequency response

% Plot amplitude in dB and phase in deg
subplot(2,1,1),semilogx(w,20*log10(abs(G_w))), grid on
subplot(2,1,2),semilogx(w,180/pi*angle(G_w)),grid on
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There is also another method to describe a continuous LTI system in Matlab, which is very practical. It requires only that you describe your continuous LTI system, or more specifically its frequency response with the following notation:

$$G(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{b_L(j\omega)^L + b_{L-1}(j\omega)^{L-1} + \ldots + b_1(j\omega) + b_0}{a_N(j\omega)^N + a_{N-1}(j\omega)^{N-1} + \ldots + a_1(j\omega) + a_0}$$

$num = [b_L \quad b_{L-1} \quad \ldots \quad b_1 \quad b_0]$ % *num : coefficients of the numerator (Zähler)*

$den = [a_N \quad a_{N-1} \quad \ldots \quad a_1 \quad a_0]$ % *den : coefficietns of the denominator (Nenner)*

$sys = tf(num, den)$ % *tf : transfer function (Übertragungsfunktion)*

$bode(sys), grid\ on$

With this notation, the 1st order system above would be described as:

num = [1];

den = [tau 1];

sys_1ord = tf (num, den);

(a) Describe the LPF RLC topologies from exercise 1 with this Matlab LTI-system notation and generate the corresponding responses:

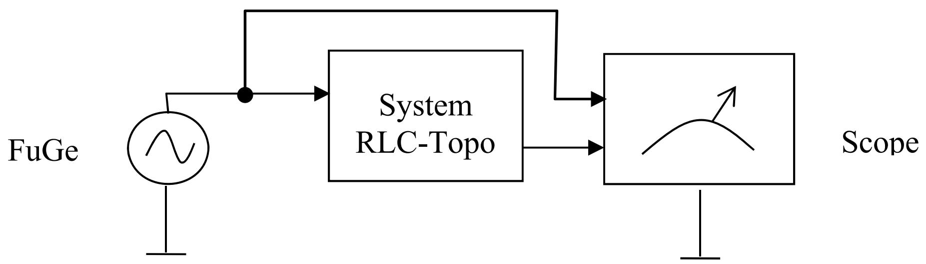
- Step response: command ***step(sys)*** in Matlab
- Impulse response: command ***impulse(sys)*** in Matlab
- Bode diagrams: command ***bode(sys)*** in Matlab

Use the following numeric values:

- series topologies: R= 560Ω ; C= 1nF; L in the range 3.3mH – 100mH;

(b) Calculate the natural frequency $\omega_0 = 1/\text{sqrt}(LC)$ of the RLC LPF.

(c) Mount now in the Hirschmann-Board the serial LPF topology and set as input signal a sinus wave with amplitude1Veff. Check the output value for its amplitude and phase shift with respect to the input signal. Compare your response for 3 different frequency values: ($\omega_1 = \omega_0 /10$; $\omega_2 = \omega_0$; $\omega_3 = 10.\omega_0$) and compare to the bode diagram from item (a).



Fill out the table below to help on the calculation and comparison:

Input signal (with a constant amplitude for all measurements)
amplitude value (linear) = amplitude (dB) =

Input Signal	Output signal				
frequency (Hz)	frequency (Hz)	amplitude linear (V)	amplitude (dB)	time-shift (s)	phase-shift (°)

(d) Change now the input signal to a square pulse with a long period, such that you can observe in the output, a curve which basically corresponds to the step response. Compare this output to the result of the command ***step(sys)*** in Matlab.

(e) Change the value of the resistor (for example take 56Ω instead of 560Ω) and verify and describe the influence on the step response.

(f) How can you change the input signal to get in the output a curve which approximates the impulse response? Compare this output to the result of the Matlab command ***impulse(sys)***.