List 1:
Signals und Systems

Solution:

Exercise 1

a) As single cosine function:

$$x(t) = A_1 \cdot \cos(\omega_1 t + \varphi_1) = 6 \cdot \cos(4\pi t - 2\pi/5)$$

b) As sum of sine and cosine functions:

$$x(t) = a_1 \cdot \cos(\omega_1 t) + b_1 \cdot \cos(\omega_1 t) = 6 \cdot [\cos(2\pi/5) \cdot \cos(4\pi t) + \sin(2\pi/5) \cdot \sin(4\pi t)]$$

Because: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

c) As real part of a complex exponential function:

$$x(t) = \mathrm{Re}\left\{c_1 \cdot e^{j(\omega_1 t + \varphi_1)}\right\} = \mathrm{Re}\{c_1 \cdot \exp[j(\omega_1 t + \varphi_1)]\} = \mathrm{Re}\{6 \cdot \exp[j(4\pi t - 2\pi/5)]\}$$

d) As the sum of 2 complex exponential functions

$$x(t) = A \cdot \left\{\frac{exp[j(\omega t + \varphi)] + exp[-j(\omega t + \varphi)]}{2}\right\} = \frac{A}{2} \cdot \{exp[j(\omega t + \varphi)] + exp[-j(\omega t + \varphi)]\}$$

$$= 3 \cdot \{exp[j(4\pi t - 2\pi/5)] + exp[-j(4\pi t - 2\pi/5)]\}$$

Exercise 2

a) $$x(t) = 3 \cdot \left\{\frac{exp\left[j\left(6\pi t + \frac{\pi}{2}\right)\right] + exp\left[-j\left(6\pi t + \frac{\pi}{2}\right)\right]}{2}\right\} \times 2 \cdot \left\{\frac{exp\left[j\left(6\pi t + \frac{\pi}{4}\right)\right] + exp\left[-j\left(6\pi t + \frac{\pi}{4}\right)\right]}{2}\right\} =$$

$$x(t) = \frac{6}{4} \cdot \left\{exp\left[j\left(12\pi t + \frac{3\pi}{4}\right)\right] + exp\left[j\left(\frac{\pi}{4}\right)\right] + exp\left[-j\left(\frac{\pi}{4}\right)\right] + exp\left[-j\left(12\pi t + \frac{3\pi}{4}\right)\right]\right\} =$$

$$x(t) = 3 \cdot \left\{\frac{\sqrt{2}}{2} + cos\left(12\pi t + \frac{3\pi}{4}\right)\right\}$$

Checking with a plot in Matlab

```
clear all, close all
t = -2 :0.01 :2 ;
x1_t = 3*cos(6*pi*t+pi/2);
x2_t = 2*cos(6*pi*t+pi/4);
x3_t = x1_t .* x2_t;
figure()
plot(t,x1_t,'b',t,x2_t,'r',t,x3_t,'m')
```

b) $$x(t) = A \cdot \left\{\frac{exp[j(\omega t+\theta)] + exp[-j(\omega t+\theta)]}{2}\right\} = A \cdot \left\{\frac{e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}}{2}\right\}$$

⇨ $$\frac{dx(t)}{dt} = \frac{A}{2} \cdot \left\{(j\omega) \cdot e^{j(\omega t+\theta)} - (j\omega) \cdot e^{-j(\omega t+\theta)}\right\} = (j\omega) \cdot \frac{A}{2} \cdot \{2 \cdot j \cdot \sin(\omega t + \theta)\}$$

⇨ $$\frac{dx(t)}{dt} = -A\omega \cdot \sin(\omega t + \theta)$$

Exercise 3

a) $x(t) = [\varepsilon(t-1) - \varepsilon(t-5)] \cdot \sin(4\pi t + \theta)$

b) $x(t) = \sum_{n=-\infty}^{+\infty} A \cdot \delta(t - nT_s)$

c) Signal (a) is not periodic (cause sinus time limited), is an energy signal and not symmetric.
Signal (b) is periodic, symmetric (even) and a power signal.

d) sig_d is a random and discrete signal, with average value tending to 0 (use mean fct), and standard deviation tending to 1 (use std fct). Try also the histfit function.

Experimenting with a plot in Matlab

```
clear all, close all
sig_d = randn(1,1000);
figure(),histfit(sig_d)
sig_e = rand(1,1000);
figure(),histfit(sig_e)
```

What are the differences between the *randn()* and the *rand()* functions ?

Exercise 4

a) x(t-2) : same shape with a shift of 2 to the right
b) x(2.t) : similar shape, compressed by a factor 2 (non-zero values between 0…2)
c) x(t/2) : similar Form, extended by a factor 2 (non-zero values between 0…8)
d) x(-t) : mirrored about the y-axis (vertical axis where t=0) (non-zero values between -4...0)

Exercise 5

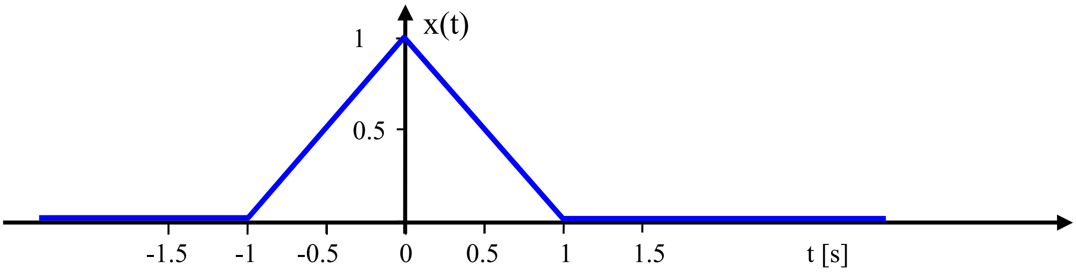
a) x[n-2] : same shape with a shift of 2 to the right
b) x[2.n] : similar shape, compressed by a factor 2 (non-zero values between 1…2)
c) x[-n] : mirrored about the y-axis (vertical axis where n=0) (non-zero values between -4...-1)
d) x[-n+2] : mirrored about the y-axis and shifted by 2 to the right
(non-zero values between -2...+1)
e)

$$x[n] = 1\cdot\delta[n-1] + 2\cdot\delta[n-2] + 3\cdot\delta[n-3] + 3\cdot\delta[n-4]$$

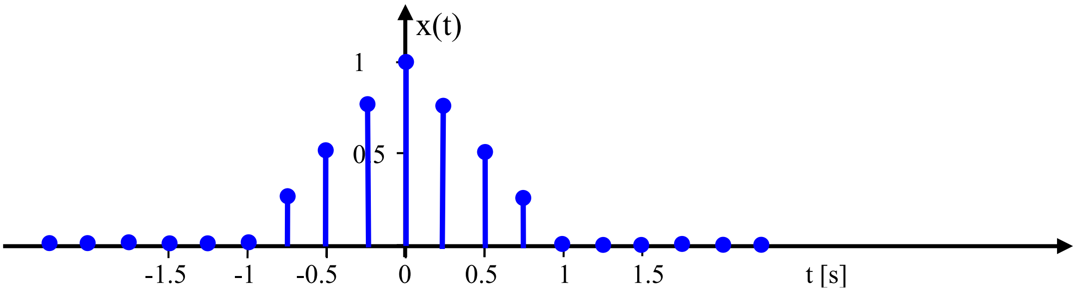
Oder $x[n] = \sum_{k=-\infty}^{+\infty} g[k]\cdot\delta[n-k]$ mit $k,n \in Z \quad und \quad g[k] = \begin{cases} 0 & k<0 \\ 1 & k=1 \\ 2 & k=2 \\ 3 & 3\le k\le 4 \\ 0 & k>4 \end{cases}$

Exercise 6 *Sampling and Discrete Signals.*

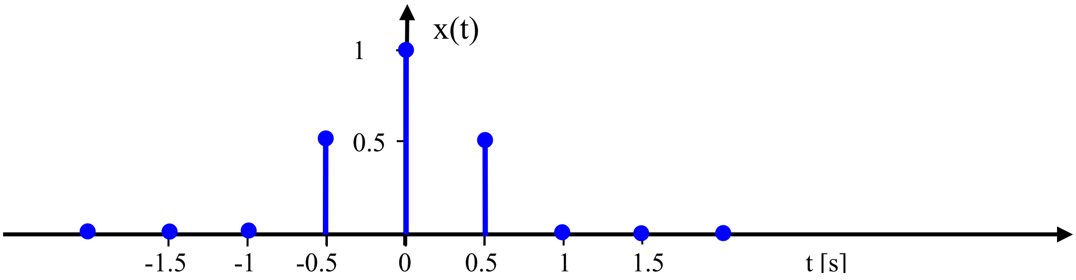
a) x(t)



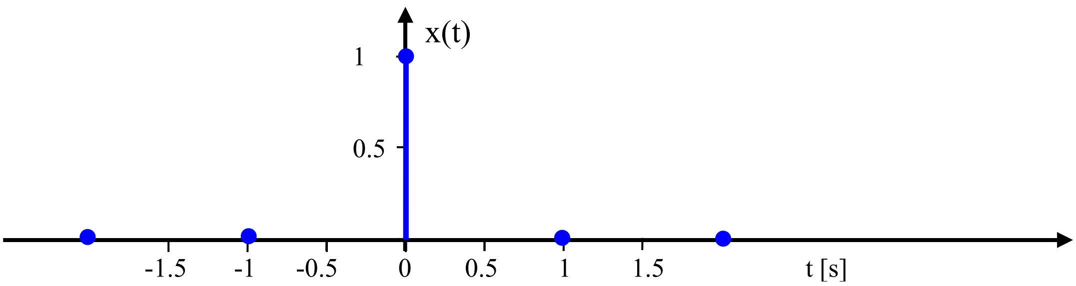
b) $T_s = 0.25$ s



c) $T_s = 0.5$ s



d) $T_s = 1$ s



Exercise 7

a) $y(t) = 0.2 \cdot x(t) - 1.5$: no
b) $y(t) = x(t) + \int x(t)dt$: yes
c) $y(t) = 0.4 \cdot x(t) + 0.2 \cdot \dot{x}(t)$: yes
d) $y(t) = 0.4 \cdot x(t) + 0.2 \cdot x^2(t)$: no

In order to test check the superposition principle (required for a linear system)

$$\begin{cases} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \\ \\ x_1(t) + x_2(t) \to y_1(t) + y_2(t) \end{cases}$$

Exercise 8

a) Simplified LTI (only amplitude effect)

$$y_3(t) = 2 \cdot \sin\left(2\pi \cdot 100t + \frac{\pi}{4}\right)$$

$$y_4(t) = 0.8 \cdot \sin\left(2\pi \cdot 10kt + \frac{\pi}{10}\right) + 4 \cdot \cos\left(2\pi \cdot 100t + \frac{\pi}{6}\right)$$

b) LTI with amplitude and phase effect

$$y_3(t) = 2 \cdot \sin\left(2\pi \cdot 100t + \frac{\pi}{4}\right)$$

$$y_4(t) = 0.8 \cdot \sin\left(2\pi \cdot 10kt - \frac{4\pi}{10}\right) + 4 \cdot \cos\left(2\pi \cdot 100t + \frac{\pi}{6}\right)$$