**Chapter 4:**

**AD & DA Conversion**

**In the Time and Frequency Domains**

# Overview of AD-DA Chain

Anti-Aliasing-

Filter

ADC

**DSP**

DAC

**mit ZOH**\*

LPF

fcut < Fs/2

Sampling &

Quantisation

(& coding)

**ADC**

Digital

Signal

Processing

**DAC**

often ZOH

(zero order holder)

Reconstruction

or Anti-Imaging

Filter

LPF

fcut < Fs/2

****

***Figure 4 : A continuous sinusoidal time signal with fundamental frequency 1kHz, is sampled with Fs = 8KHz. Visualization of the periodic spectrum of the discrete signal, containing the original, plus the image spectra.***

****

**Chapter 3:**

**Fourier Transformation**

**Signals in Frequency Domain**

Definition as limit case of the Fourier Series



Fourier Transformation (1a)



Inverse Fourier Transformation (1b)



Pair of associated functions in time and frequency domain

 (2)

The complex valued spectrum density is often split into amplitude and phase components:

 (3)

### Some FT Reference Signals

|  |  |  |  |
| --- | --- | --- | --- |
| **Sketch Time-D** | **Equation Time-D** | **Equation Freq-D** | **Sketch Freq-D** |
|  | δ(t) | 1 |  |
|  | 1 | δ(f) |  |
|  | A.rect( t / τ) | A. τ .sinc(f.τ) |  |
|  | A. τ .sinc(t.τ) | A.rect( f / τ) |  |
|  | ej2πf0t | δ(f-f0) |  |
|  | cos(2πf0t) | 1/2 . [δ(f-f0) + δ(f+f0)] |  |

# Properties of Fourier Transformation

Table 3-1 below gives an overview of some properties of the Fourier transformation. For a complete list of properties and the proof of them please refer to the bibliography reference [1].

|  |  |  |
| --- | --- | --- |
| **Property** | **Time Domain** | **Frequency Domain** |
| Linearity or superposition |  |  |
| Time-Shift |  |  |
| Time-Scaling or Time-Bandwidth Product |  |  |
| Duality or Symmetry between time and frequency domain |  |  |
| Frequency-Shift |  |  |
| Derivation in time domain |  |  |
|  |  |
| Convolution vs Multiplication |  |  |
|  |  |
| Symmetry in the frequency domain |  | Amplitude  is even  Phase  is odd |
| Symmetry in the time domain | and even | is purely real |
| and odd | is purely imaginary |
| Parseval Theorem  or Signal Energy |  |  |

Table 3-1 Properties of the Fourier Transformation

**Chapter 2:**

**Fourier Series and DFT**

**Periodic Signals in the Frequency Domain**

|  |
| --- |
| (4a)  with  (4b) |

# Reference Signal: Periodic Square

x(t)

A

-T0 0 +T0 +2.T0 t

τ

**…**

**…**

Figure 2‑3 Periodic square pulse function with duty cycle ( τ / T0 ).100%

So that the coefficients ck can be rewritten as:

|  |
| --- |
| (6) |

# Properties of Fourier Series

|  |  |
| --- | --- |
| Property | Observation |
| Symmetry | Because  the amplitude spectrum is always an even function, and the phase spectrum an odd function |
| Discrete Spectrum | Functions which are periodic in the time domain are discrete in the frequency domain (because they can be represented with Fourier series). |
| DC-Offset | Adding a constant value (DC-offset) to a time function only affects its c0 coefficient. |
| Time-Shift | Shifting a time function only affects the phase spectrum (phase of ck).  (7) |
| Parseval Theorem | The power of a periodic function can be calculated as the sum of the power of its harmonics.  (8) |

Table 2‑1 Selected Properties of the Fourier Series

# Numerical Approximation with DFT

DFT

N-Points

time

domain

N-Points

frequency

domain

Ts

N.Ts

t (s)

fstep

N.fstep

f (Hz)

Fs

Fs/2

Fs (N-1)/N

t (s)

fmin= fstep=1/(N.Ts) = Fs /N

fstep