**Laboratory 2B:**

Fourier Series and Line Spectrum

In this laboratory you practice the concept of the Fourier Series and the corresponding Line Spectrum, which describes periodic signals in the frequency domain. We use the Fourier Series notation with the complex coefficients ck and the associated Double Sided Spectrum.

Let us consider the example below:

|  |  |
| --- | --- |
| **Time Domain** | -T0 0 T0 t[s]  x(t)  A |
| **Transformation**  Fourier Series  with Complex Coefficients | **Fourier Analysis: from time to frequency domain**  (find out the harmonics composing a signal) |
| **Fourier Synthesis: from frequency to time domain**  (sum up the harmonics components building up a signal ) |
| **Frequency Domain**  Line Spectrum  Amp(ck)  Phase(ck) |  |

The ck coefficients for the example above (periodic sawtooth signal), are calculated step-by-step in the list of exercises **sisy-en-material\EXERCISES\SiSy\_exer2\_fouseri.docx** (exercise-7), and are equal to:

Calculating the ck coefficients for x(t), means doing the **Fourier Analysis** of x(t) and determining the corresponding Double Sided Spectrum of x(t).

Now, in order to check the expression that you calculated for the ck coefficients, you can carry out the **Fourier Synthesis**, and build up the time signal, by adding up its harmonic components. Let us do that in a Matlab script.

% Define Constants and time vector

T0 = 2; w0 = 2\*pi/T0; A = 1;

t = -2\*T0:T0/100:2\*T0;

Kmax = 5; % Max number of harmonics

c0 = A/2; % DC-Offset (average value)

% since c0 has a different expression than the other ck, will take c0  
% outside of the loop, and combine with the initialisation of x(t)

x\_t = c0\*ones(1,length(t)); % initialise x(t) with DC-content

for k = -Kmax:1:Kmax

if k ~= 0

ck = j\*A/(k\*2\*pi);

x\_t = x\_t + ck\*exp(j\*k\*w0\*t);

end

end

figure(), plot(t,x\_t), grid on

xlabel('t [s] \rightarrow'),ylabel('x(t)')

title('Periodic Synthesised Time Function')

disp('Because of warning, check how big mag(imag(x\_t))')

max(imag(x\_t))

The for-loop above implements the sum of the **Fourier Synthesis** equation. This loop could also be implemented as:



…

for k = 1:Kmax

ck = j\*A/(k\*2\*pi);

x\_t = x\_t + ck\*exp(j\*k\*w0\*t) + conj(ck)\*exp(-j\*k\*w0\*t) ;

end

…

Or also like this, using the relationship of the ck coefficients with the Ak and φk coefficients of the single sided spectrum.

…

for k = 1:Kmax

ck = j\*A/(k\*2\*pi);

x\_t = x\_t + 2\*abs(ck)\*cos(k\*w0\*t + phase(ck));

end

…

Now it is your turn to experiment the Fourier Analysis and the Fourier Synthesis with another periodic signal. Follow the instructions below.

# Fourier Series Coefficients: calculate and plot

1. Calculate the complex Fourier coefficients of an odd periodic square with amplitude varying between +A and -A, period T0 and duty cycle 50%.

-T0 0 +T0 t

x(t)

A

-A

Please use the ck integral definition for your calculation:



1. Do the values of ck depend on T0 ?

Is it possible to have a simpler expression for all odd, and all even ck coefficients?

1. Check what are the numerical values of ck for k within [-5 ; +5] .

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| ck |  |  |  |  |  |  |  |  |  |  |  |

1. Are the non-zero ck coefficients pure real or pure complex numbers? What does this mean compared to the real coefficients ak and bk ?

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1. Check the values of your ck coefficient with a Fourier Synthesis in a Matlab script.
2. Prepare a plot of the corresponding Double Sided Spectrum in Matlab. You can use k (index of harmonics) or frequency in Hz for the horizontal axis.

*Hint*: you can define your f vector as f = k\*f0 , where k is the index vector for the ck coefficients and f0 is the fundamental frequency of the periodic signal x(t).

# Numerical Approximation for Fourier Series with FFT

1. Start a new script in Matlab, and insert the code shown below:

% SiSy : LAB-2B Exercise-2

% =============================

clear all, close all, clc;

% DEFINE PARAMETERS & VECTORS

T0 = 2; % period of the signal in time domain

N = 2^8; % number of points for the FFT

aux = 0:1:N-1; % auxiliary index vector

% Select the sampling period, to have exactly N-points/period (ideal situation)

tstep = 1\*T0/N; % resolution in the time domain

t = tstep\*aux; % time vector

Fs = (1/tstep); % sampling frequency

fstep = Fs/N; % resolution in the frequency domain

f = fstep\*aux; % frequency vector

% DEFINE TIME-FUNCTION AND CALCULATE NUMERICAL APPROX OF SPECTRUM

x\_t = square(2\*pi\*t/T0,50); % periodic square with 50% duty cycle and period T0

c\_k = (1/N)\*fft(x\_t); % normalised FFT = approx(ck coefficients)

scrsz = get(groot,'ScreenSize');

figure('Position',[1 0.6\*scrsz(4) 0.8\*scrsz(3) 0.3\*scrsz(4)])

subplot(121),plot(t,x\_t),grid on,xlabel('t (sec)'),ylabel('x(t)')

subplot(122),stem(aux,abs(c\_k)),grid on,xlabel('k'),ylabel('abs(c\_k)')

pause(2)

subplot(121),ylim([-1.2 1.2]),title('Time Domain')

subplot(122),xlim([0 20]),title('Freq Domain')

1. Execute the code and analyse the following points:
   1. How is the periodic square function x(t) defined?
   2. With how many points per period?
   3. Which function is used to calculate the numerical approximation of the ck coefficients?
   4. Which ck coefficients are visible after the zoom (with xlim() function) ?
   5. What is plotted: amplitude-part or phase-part of the spectrum ?
2. Vary the duty cycle of the periodic square x(t) (e.g. 50%, 25%, 20%, 10%). What happens with the corresponding spectra? Find out what are the zero-crossings of the amplitude spectrum for the different values of the duty cycle. For example for:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Duty cycle | 50% or | 25% or | 20% | 10% |
| Zero-crossings  ck = 0 for … | ck = 0 for  k = 2\*n  with n ϵ Z  ck ≠ 0 for  k = 1 ; 3 ; 5 ; 7 … |  |  |  |

1. Change x(t) for a periodic ramp (use in Matlab the function sawtooth() ) . Which changes can you observe in the spectrum?

x\_t = sawtooth(2\*pi\*t/T0); % periodic sawtooth with period T0