Laboratory 3c:

**Fourier Transformation   
Applied to Systems**

In this laboratory we use Fourier Transformation properties to obtain the description of a linear time invariant system (here a passive electrical circuit) in the frequency domain. This description is called: the frequency response of the system. Furthermore, you will carry out measurements and generate plots to experiment the meaning of this frequency response and other system responses to test signals.

**Exercise 1** *System Frequency Response*

1. As a warm up solve exercises 3-4 and 3-5 from the SiSy Script Chapter 3.

There you are asked to solve the following questions:

* 1. Apply the Fourier transformation to the following equation    
      and determine the complex impedance of a capacitor, defined as:

 or 

* 1. Repeat the same procedure for an inductor, with basic equation:    
     and complex impedance:

 or 

* 1. Calculate the differential equation describing the following passive RLC topology, and apply the Fourier Transformation to these equation to calculate the corresponding frequency response defined as:

or

with Y(ω) and X(ω), being the Fourier Transform of the output and input signals of the system.

x(t)

X(f) = X(ω)

System

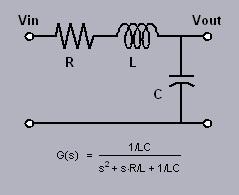
RLC-Topo

y(t)

Y(f) = Y(ω)

G(f) = Y(f) / X(f)

G(ω) = Y(ω) / X(ω)



Observation:

The frequency response is also called Transfer Function, when you use (s) instead of (jω) .

This is actually equivalent to carry out the previous calculation with the Laplace Transform instead of the Fourier Transform.

For our purposes in this semester, we can set s=jω , and check your result with the equation beside.

* 1. Experiment now a second alternative to obtain the frequency response   
     Y(ω) / X(ω), with a calculation directly in the frequency domain.

Use the complex impedances from (i) and (ii), and apply the principle of a voltage divider.

**Exercise 2** *LTI-System Description and Responses in Matlab*

Consider the example below, showing how one can describe a 1st order RC passive low pass filter (LPF) in the frequency domain in Matlab, and plot its frequency response (Bode diagram):

w = logspace(-3,3,100); % Define frequency vector:e.g. from 10-3 to 10+3

G\_w = 1./(j\*w\*tau + 1); % Define frequency response

% Plot amplitude in dB and phase in deg

subplot(2,1,1),semilogx(w,20\*log10(abs(G\_w))), grid on

subplot(2,1,2),semilogx(w,180/pi\*angle(G\_w)),grid on

There is also another method to describe a continuous LTI system in Matlab, which is very practical. It requires only that you describe your continuous LTI system, or more specifically its frequency response with the following notation:





*% num : coefficients of the numerator (Zähler)*

*% den : coefficietns of the denominator (Nenner)*

*% tf : transfer function (Übertragungsfunktion)*

With this notation, the 1st order system above would be described as:

*num = [1];*

*den = [tau 1];*

*sys\_1ord = tf ( num, den);*

1. Describe the LPF RLC topologies from exercise 1 with this Matlab LTI-system notation and generate the corresponding responses:

* Step response: command ***step(sys)*** in Matlab
* Impulse response: command ***impulse(sys)*** in Matlab
* Bode diagrams: command ***bode(sys)*** in Matlab

Use the following numeric values:

* series topologies: R= 560Ω ; C= 1nF; L in the range 3.3mH – 100mH;

1. Calculate the natural frequency ω0 = 1/sqrt(LC) of the RLC LPF.
2. Mount now in the Hirschmann-Board the serial LPF topology and set as input signal a sinus wave with amplitude1Veff. Check the output value for its amplitude and phase shift with respect to the input signal. Compare your response for 3 different frequency values:   
   (ω1 = ω0 /10 ; ω2 = ω0 ; ω3 = 10.ω0 ) and compare to the bode diagram from item (a).

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Scope

System

RLC-Topo

Fill out the table below to help on the calculation and comparison:

Input signal (with a constant amplitude for all measurements)

amplitude value (linear) = amplitude (dB) =

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Input Signal | Output signal | | | | |
| frequency  (Hz) | frequency  (Hz) | amplitude  linear  (V) | amplitude  (dB) | time-shift  (s) | phase-shift  (°) |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

1. Change now the input signal to a square pulse with a long period, such that you can observe in the output, a curve which basically corresponds to the step response. Compare this output to the result of the command ***step(sys)*** in Matlab.
2. Change the value of the resistor (for example take 56Ω instead of 560Ω) and verify and describe the influence on the step response.
3. How can you change the input signal to get in the output a curve which approximates the impulse response? Compare this output to the result of the Matlab command ***impulse(sys)***.