TOP|FLIGHT

Web Resource Guide

Science

www.howstuffworks.com
www.nasa.gov
www.webelements.com - Interactive Periodic Table
www.unitconverters.net - Unit conversion
www.innerbody.com - Interactive Anatomy

Foreign Language

www.learnalanguage.com
www.how-to-learn-any-language.com
Translators: **translate.google.com, translatereference.com**

History

www.history.com
www.americaslibrary.gov
www.hyperhistory.com
www.house.gov
www.senate.gov
www.whitehouse.gov

For Fun!

Sports & Games

www.espn.com
www.sikids.com
www.olympic.org
www.pogo.com
www.games.yahoo.com

Outdoor & Travel

www.greatoutdoors.com
www.smallstep.gov
www.nps.gov
www.lonelyplanet.com
www.letsgo.com

Style & Fashion

www.teenvogue.com
www.instyle.com
www.style.com

Music & Movies

www.rollingstone.com
www.billboard
www.imdb.com
www.purevolume.com

Math

www.mathisfun.com
www.coolmath.com
www.math.com

August 29th

Operators: $A \cup B$: union

$AB = \{xy \mid x \in A \text{ and } y \in B\}$: concatent

$A^* = \{x_1 x_2 \ldots x_k \mid \text{all } k \geq 0 \text{ and } x_i \in A \text{ for } i = 1, 2, \ldots k\}$

diff $A - B = A \cap \overline{B} \Rightarrow \overline{A} = \Sigma^* - A$

$A^* = \{x_1 \ldots x_k \mid k \geq 0 \text{ and } x_i \in A \ \forall_i = 1 \ldots k\}$

$\varepsilon \in A^*$

eg $A = \{01, 10\}$

$A^* = \{\underbrace{\varepsilon}_{k=0}, \underbrace{01, 10}_{k=1}, \underbrace{0110, 1001, 0101, 1010}_{k=2}, \ldots\}$

$A = \{\varepsilon\} \Rightarrow A^* = ? \ \{\varepsilon\}$

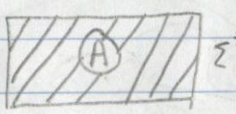
$A = \emptyset \Rightarrow A^* = \{\varepsilon\}$

power set

$A^* = A^0 \cup A^1 \cup A^2 \cup \ldots$

complement

$\overline{A}$:



$\Sigma = \{0, 1\}$

Prove that # of primes is infinite. Prove by contradiction.

Assume # of primes is finite: $p_1 < p_2 < p_3 < \ldots p_k$

Define $P = \underbrace{\prod_{i=1}^{k} p_i + 1}_{p \text{ is a prime!}} > p_k$ contradition

$\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$ by induction.

proof: basis: $n = 1$

left $= 1^2 = 1$

right $= \frac{1}{6} 1(1+1)(2(1)+1) = 1$ } left = right

Hypothesis Assume true when $n = k$ or

$\sum_{i=1}^{k} i^2 = \frac{1}{6} k(k+1)(2k+1)$

induction $n = k+1$

left $\sum_{i=1}^{k+1} i^2 = \underline{1^2 + 2^2 \ldots + k^2} + (k+1)^2 = \underline{\sum_{i=1}^{k} i^2} + (k+1)^2$

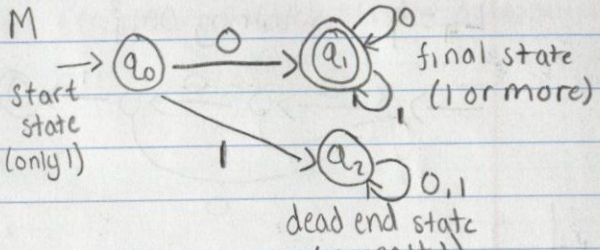
$\frac{1}{6} k(k+1)(2k+1) + (k+1)^2$

right $= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$

$= \frac{1}{6}(k+1)(k+2)(2k+3)$

September 3rd

DFA : M



| 0 | 1 | 0 | 1 | $\rightarrow$ *check to make at final state by end / accepted

| 1 | 0 | 0 | rejected string

(Binary alphabet) $\Sigma = \{0, 1\}$

arcs = # nodes * $|\Sigma|$

$L(M) = \{w \in \{0,1\}^* \mid w \text{ starts w/ } 0\}$

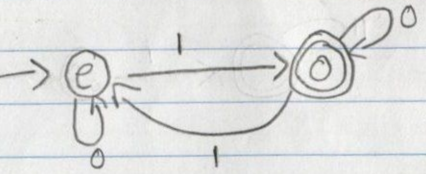
$\delta = Q \times \Sigma \rightarrow Q$

Table :

δ	0	1
$\rightarrow q_0$	q_1	q_2
* q_1	q_1	q_1
q_2	q_2	q_2

Example 1:

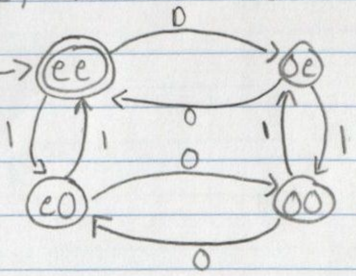
$L_1 = \{w \in \{0,1\}^* \mid w \text{ has odd number of 1's}\}$



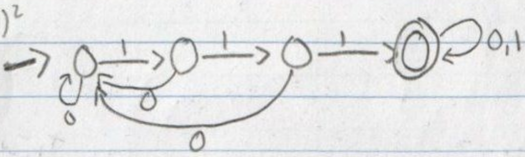
Example 2:

$L_2 = \{w \in \{0,1\}^* \mid w \text{ has even \# of 0's and 1's}\}$

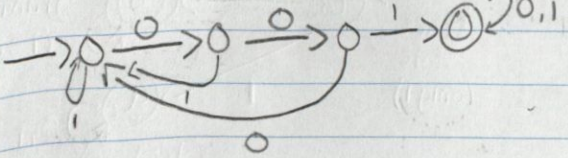
ε, 11, 01101001 $\in L_2$



$L_3 = \{w \in \{0,1\}^* \mid w \text{ contains substring } 111\}$

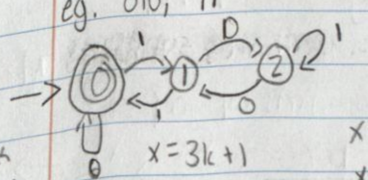


$L_4 = \{ \ldots$ substring $001 \}$



$L_5 = \{ w \in \{0,1\}^* \mid w$ is a string w/ a numerical value that is a multiple of $3 \}$

eg. 010, 11

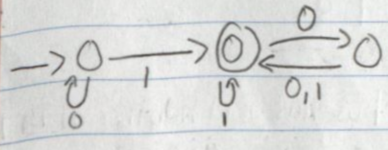


$x = 3k$
$x0 = 6k$
$x1 = 6k+1$

$x = 3k+1$
$x0 = 2(3k+1) = 6k+2$
$x1 = 6k+3$

$x = 3k+2$
$x0 = 6k+4 = 6k+3+1$
$x1 = 6k+5 = 6k+3+2$

DFA

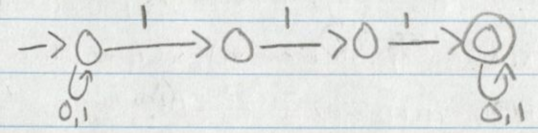


At least a 1
After the last 1 there are even num of 0's

1	1
11	3
100	4
1001	9
111	7
11001	25
1111	15

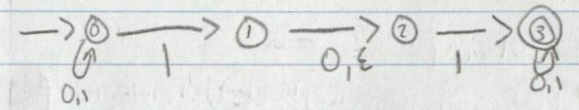
September 5th

$L_1 = \{ w \in \{0,1\}^* \mid w$ has a substring $111 \}$



NFA

$L_2 = \{ w \in \{0,1\}^* \mid w$ contains substring 101 or $11 \}$

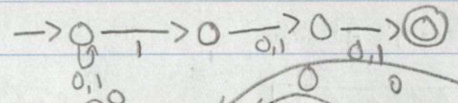


NFA

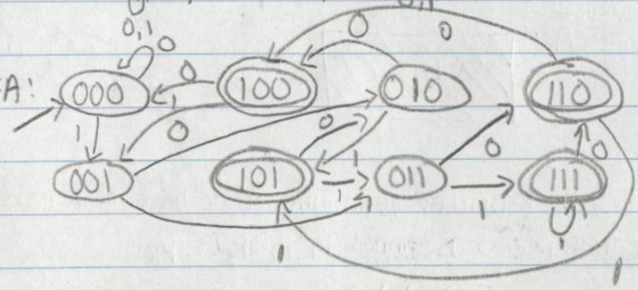
$L_3 = \{ w \in \{0,1\}^* \mid w$ has a 1 in the 3rd position to the right end$\}$

eg. ...1 _ _

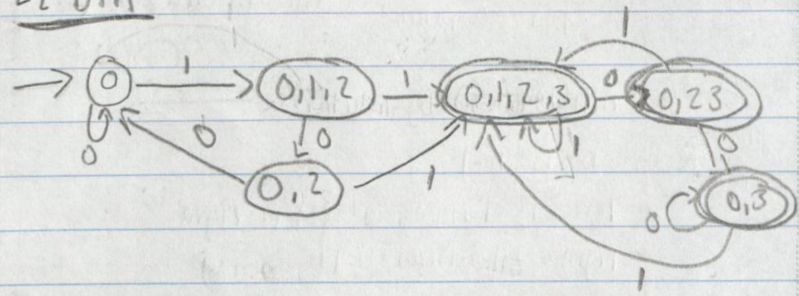
NFA:



DFA:



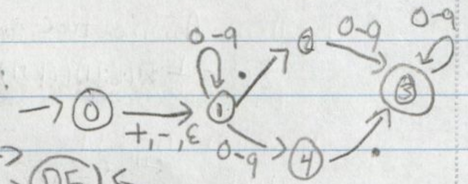
L_2 DFA



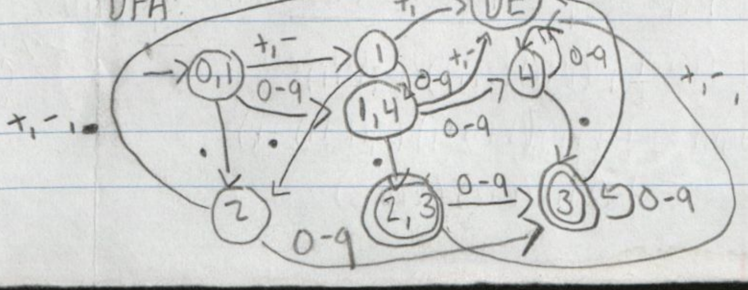
Decimal numbers:

$\Sigma = \{ \cdot, +, -, 0\text{-}9 \}$

NFA:

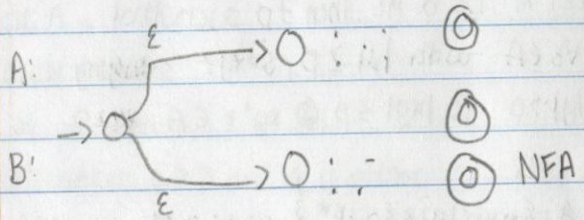


DFA:



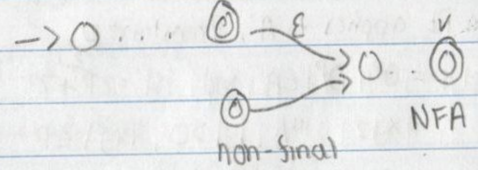
September 10th

Closure property of RL's



$A \cup B$ is regular

$A \cdot B$



$A \cap B = \overline{\bar{A} \cup \bar{B}}$

Example: Prove $A = \{w \in \{a,b\}^* \mid w$ is of odd length and even number of a's$\}$ is regular

A_1 - contains strings satisfy 1

A_2 - " " " 2

$A = A_1 \cap A_2$

$1 \cup 10^* \Rightarrow 10^*$

$(0^* 1^*)^* \Rightarrow (0 \cup 1)^*$

$((0\cup1)(0\cup1)^*)^* \Rightarrow (0\cup1)^*$ ✓

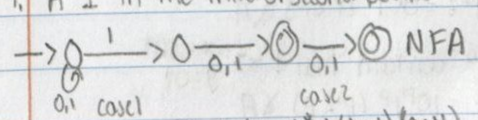
*- choose none or however many times

1. {w has no substring 10}: $0^* 1^*$

0, 01, 00, 001, 000, 0001, 0000

011, 0111, 01111

2. {w has even # of 1's}: $(0^* 1 0^* 1 0^*)^* \cup 0^*$
3. {odd length}: $(0\cup1)((0\cup1)(0\cup1))^*$
4. A 1 in the third or second position to right end:



case1 case2

$(0\cup1)^* 1 (0\cup1) \cup (0\cup1)^* 1 (0\cup1)(0\cup1)$

-10 -11 -100 -101 -110 -111

$= (0\cup1)^* 1 (0\cup1)(\varepsilon \cup 0 \cup 1)$

$\begin{cases} R_1 = 0(0\cup1)^* \\ R_2 = (0\cup1)^* 1 \end{cases} \Rightarrow L(R_1) \cap L(R_2) = \{01, 011, 001, 0011, 0111, 0001\}$

$= \boxed{0(0\cup1)^* 1}$

$\begin{cases} R_1 = (0\cup1)1(0\cup1)^* \\ R_2 = 1(0\cup1)^* \end{cases} \Rightarrow L(R_1) \cap L(R_2) = 11(0\cup1)^*$

$\begin{cases} R_1 = (01)^*(0\cup\cancel{1}) \\ R_2 = (0\cup1)(01)^* \end{cases} \Rightarrow L(R_1) \cap L(R_2) = 0$

$D = \{w \in \{0,1\}^* \mid w$ has equal # of substrings 01 and 10$\}$

$\varepsilon \in D$ $0101 \notin D$ $010 \in D$

$0|11|000|11|0|1 \notin D$

$00|11|000 \in D$ +: means block of whatever

$RE_D = 0^+(1^+0^+)^* \cup 1^+(0^+1^+)^* \cup \varepsilon$

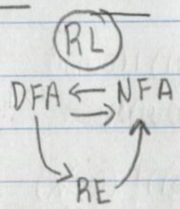
$E = \{w \in \{0,1\}^* \mid$ in w each 1 is imediately preceded by a 0 and followed by a 0$\}$ 101

$\boxed{RE = ?}$

September 12th

001 000 | 0000 | 00 | 10

0^+1 0^+1 0^+ 1 0^+ 10^+



$$RE = \begin{cases} (0^+1)^* 0^+ \cup \varepsilon \\ 0^+(10^+)^* \cup \varepsilon \ ✓ \end{cases}$$

$L = \{0^n 1^n \mid n \geq 0\}$

PL: Let A be a RL. Then $\exists P$ such that $\forall s \in A$ w/ $|s| \geq P$, $s = xyz$ satisfying ① $|y| > 0$ ② $|xy| \leq P$ ③ $xy^iz \in A$ $\forall i \geq 0$

① $B = \{0^n 1^n \mid n \geq 0\}$ is non-R

Pf: Assume B is a RL

Then PL applies to B $\exists$ constant P

Select $s = 0^P 1^P \in B$ and $|s| = 2P \geq P$

$= \underbrace{0 \ldots 0}_{P}\underbrace{1 \ldots 1}_{P} = xyz$ w/ $|y| > 0$ $|xy| \leq P$

Consider y. $y = 0^+$

Fix $i = 0$ $xy^0z = 0^{P-*}1^P \notin B$

contradiction to PL $\therefore$ B is non-R!

September 24th

PL for RLs

Let A be a RL. Then $\exists p$ such that $\forall s \in A$ with $|s| \geq p$, $s = xyz$ satisfying ① $|y| > 0$ ② $|xy| \leq p$ ③ $xy^iz \in A$ $\forall i \geq 0$

② $A = \{ww \mid w \in \{0,1\}^*\}$ is non-R

Pf: Assume A is a RL

Then PL applies to A, $\exists$ constant p

Select $s = 0^P 1 0^P 1 \in A$ and $|s| = 2P + 2 \geq P$

$= xyz$ w/ $|y| > 0$, $|xy| \leq P$

$= \underbrace{0 \ldots 0}_{P} 1 \underbrace{0 \ldots 0}_{P} 1$

x, y are in the 1st block of 0's

$y = 0^+$

Fix $i = 0$, $xy^iz = xy^0z = xz \notin A$

$= 0^{P'}10^P1$ $(P' < P)$

contradiction

③ $A = \{10^n 1^n \mid n \geq 0\}$ is non R

Pf: Assume A is a RL

The PL applies

Let p the constant in the PL

Select $s = 10^P1^P \in A$, $|s| = 2P+1 \geq P$

$= xyz$ w/ $|y| > 0$, $|xy| \leq P$

$= 1\underbrace{0 \ldots 0}_{P}\underbrace{1 \ldots 1}_{P}$

consider y

✓case1: y contains the first 1 $\Rightarrow$ $x = \varepsilon$, $y = 10^*$

Fix $i = 0$ $xy^0z = 0^{P'}1^P$ $(P' < P)$ $\notin A$

case2: y doesn't contain that 1 $\Rightarrow$ $x = 10^*$, $y = 0^+$

Fix $i = 0$ $xy^0z = 10^{P'}1^P$ $(P' < P)$ $\notin A$

Contradiction! A is non-R!

④ $A = \{1^r \mid r \text{ is a prime}\}$ is non R

Pf: Assume A is a RL

Then PL applies to A

Let p be the constant

select $s = 1^q \in A$ where $q \geq p$ and q is prime

$|s| = q \geq p$

By PL, $s = xyz = \underbrace{1 \ldots 1}_{q}$ so that

let $x = 1^{l_1}, y = 1^{l_2}, z = 1^{q - l_1 - l_2}$

$l_1 \geq 0 \quad l_2 > 0 \quad l_1 + l_2 \leq p$

By PL, $\forall i \geq 0 \quad xy^iz = 1^{l_1 + il_2 + q - (l_1 + l_2)}$ ↑A

$\forall i \geq 0 \quad \cancel{l_1} + il_2 + q - (\cancel{l_1} + l_2)$

$= l_2(i-1) + q$ is a prime

Fix $i = q+1 \qquad l_2(q) + q = q(l_2 + 1)$ is not prime!

$xy^{q+1}z \notin A$

September 26th

⑤ $A = \{(01)^a 0^b \mid a > b \geq 0\}$ is non-R

Pf: Assume A is a RL

Then the PL applies to A.

∃ constant p.

Select $s = (01)^p 0^{p-1} \in A$; $|s| = 2p + p - 1 = 3p - 1$

$= \underbrace{0101\ldots01}_{2p}\ \underbrace{0\ldots0}_{p-1} = xyz$ w/ $y \neq \epsilon, |xy| \leq p$

consider y:

case 1 $|y|$ is even

$y = 0101\ldots01$
$y = 1010\ldots10$ $\Big\rangle\ i = 0 \quad xy^0z = xz \notin A$

case 2 $|y|$ is odd

$y = 01\cdots10 \quad y = 0 \qquad i = 2 \quad xy^2z = xyyz \notin A$

$y = 10\cdots101 \quad y = 1$

contradiction

⑥ $A = \{1^{n^2} \mid n \geq 1\}$ is non-R

Assume A is a RL

Then the PL applies to A

∃ constant p

Select $s = 1^{p^2} \in A$ w/ $|s| = p^2 \geq p$

$= xyz$ w/ $|y| > 0$, $|xy| \leq p$

Let $x = 1^{l_1}, y = 1^{l_2}, z = 1^{p^2 - l_1 - l_2}$ w/ $l_2 > 0$, $l_1 + l_2 \leq p$

By PL, $\forall i \geq 0 \quad xy^iz \in A$

$1^{\cancel{l_1} + il_2 + p^2 - \cancel{l_1} - l_2} = 1^{p^2 + (i-1)l_2}$

$i = 2 \quad xy^2z = 1^{p^2 + l_2} \notin A$

$p^2 < \underbrace{p^2 + l_2}_{\text{not a perfect square}} \leq p^2 + p < p^2 + 2p + 1 = (p+1)^2$

contradiction!

How to use CPs to prove a non-RL

① $C=\{w\in\{0,1\}^* \mid w$ has an equal # of 0's & 1's$\}$ is non-R

Pf: Assume C is a RL

Let $A=\{0^*1^*\}$ Clearly A is a RL

$A\cap C=\{0^n1^n\}$

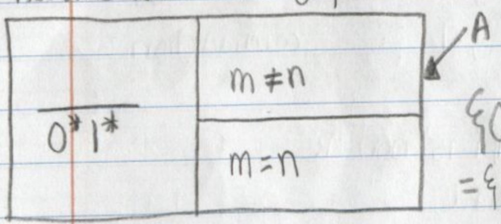
Since both A and C are RLs, by CP under intersection, $A\cap C$ must be a RL

$\{0^n1^n\}$

$\{0^n1^n\}$ is a RL, contradiction!

② $A=\{0^m1^n \mid m\neq n\}$ is non-R

Pf: Assume A is a RL 0^*1^*



$\{0^*1^*\}-A$
$=\{0^n1^n\}$

$\Sigma^*=\{0,1\}^*$

$\underbrace{\{0^*1^*\}}_{RL}-\underbrace{A}_{R}=\underbrace{\{0^n1^n\}}_{\text{RL by CP}}$

By Closure Property under difference $\{0^n1^n\}$ must be R.
Contradiction!

③ $A=\{a^mb^nc^{m+n} \mid m,n\geq 0\}$ is non-R

homomorphism:

$h:\Sigma\to\Sigma'^*$ e.g. $\Sigma=\{a,b,c\}$, $h(a)=0$, $h(b)=0$, $h(c)=1$

$\{a,b,c\}$ $\{0,1\}$

$w=aabccc$, $h(w)=000111$

$h(A)=\{h(w) \mid \forall w\in A\}$

Pf: Assume A is R.

Define h to be $h(a)=h(b)=0$, $h(c)=1$

$h(A)=\{0^{m+n}1^{m+n}\}=\{0^n1^n\}$ is R by CP But 0^n1^n is not RL. Contradiction!

October 1st

Midterm:

① DFA/NFA NFA → DFA

② R expression/languages

③ pumping lemma prove not regular

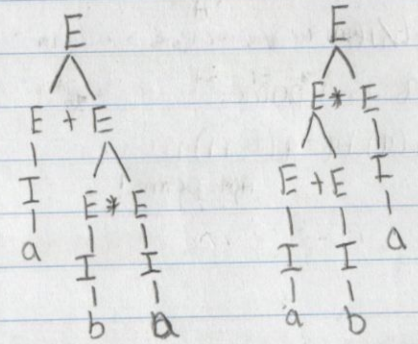
④ concepts/def closure prop.
- prove lang is/is not regular ((contradiction)

1 cheat sheet (both sides)

$E\to E+E \mid E*E \mid (E) \mid I$ a, b, 0, 1, +, *, (,)

$I\to Ia \mid Ib \mid I0 \mid I1 \mid a,b$

a+b*a



$E\to E+E$
$\to I+E$
$\to a+E$
$\to a+E*E$
$\to a+I*E$
$\to a+b*E$
$\to a+b*I$
$\to a+b*a$

ambiguous

$L=\{wgw^R \mid$ ① $w,g\in\{0,1\}^*$ ② $w,g\in\{0,1\}^+$ ③ $w\in\Sigma^+, g\in\Sigma^*$ ④ $w\in\Sigma^*, g\in\Sigma^+\}$

① $S\to 0S0 \mid 1S1 \mid G$
$G\to 0G \mid 1G \mid \varepsilon$

② $S\to 0S0 \mid 1S1 \mid 0G0 \mid 1G1$
$G\to 0G \mid 1G \mid 0 \mid 1$

Ex: $L=\{a^mb^nc^{m+n} \mid m,n\geq 0\}$

(CFG? using only two variables)

① October 3rd

$\{a^m b^n c^{m+n} \mid m,n \geq 0\}$ #study this one

$\Downarrow$

$a^m b^n c^n c^m$ $S \rightarrow aSc \mid T$

$T \rightarrow bSc \mid \varepsilon$

② $\{a^m b^m c^n d^n \mid m,n \geq 0\} \cup \{a^m b^n c^n d^m \mid m,n \geq 0\}$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow AB$ $S_2 \rightarrow aS_2d \mid C$

$A \rightarrow aAb \mid \varepsilon$ $C \rightarrow bCc \mid \varepsilon$

$B \rightarrow cBd \mid \varepsilon$

③ $\{0^m 1^n \mid m \neq n\} = \{0^m 1^n \mid m < n\} \cup \{0^m 1^n \mid m > n\}$

$0^m 1^{n-m} 1^m$ $0^n 0^{m-n} 1^n$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow 0S_11 \mid A$ $S_2 \rightarrow 0S_21 \mid B$

$A \rightarrow 1A \mid 1$ $B \rightarrow 0B \mid 0$

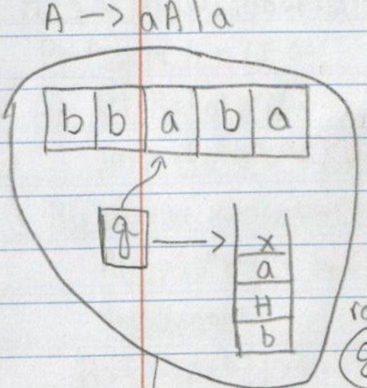
④ $\{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$

$j<i$ i $i<j<2i$ $2i$ $j>i$

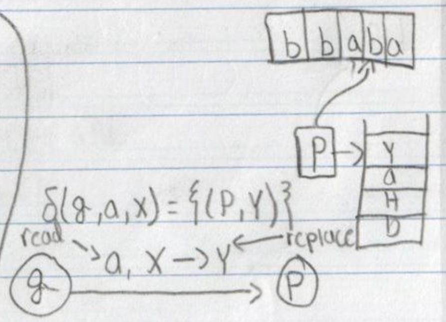
$S \rightarrow S_1 \mid S_2 \mid S_3$

$S_1 \rightarrow aS_1b \mid A$ $S_2 \rightarrow \ldots$ $S_3 \rightarrow \ldots$

$A \rightarrow aA \mid a$



$(q, aba, XaHb)$

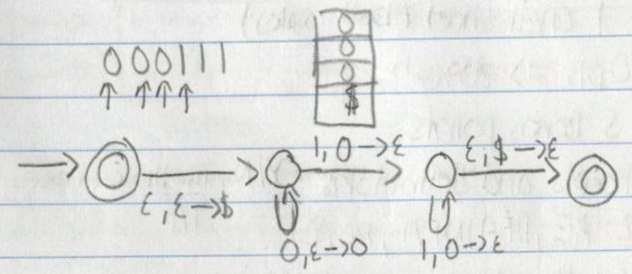


$\delta(q,a,x) = \{(P,Y)\}$

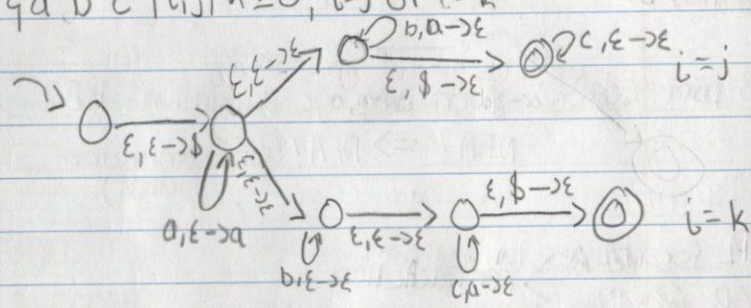
read $\rightarrow a, X \rightarrow Y \leftarrow$ replace

$q \xrightarrow{a,\, X \rightarrow Y} P$

$\{0^n 1^n \mid n \geq 0\}$



$\{a^i b^j c^k \mid i,j,k \geq 0,\ i=j \text{ or } i=k\}$



October 8th

Midterm: Thurs 12:30 - 1:50

1 cheat sheet (both sides)

40 pts => 20%

5 bonus points

Basic concepts and definitions Y/N, multiple choice T/F

RL: RE, DFA, NFA, PL, CP

CFG: Parse trees, derivations

(5 bonus points) PDA:

3 part Question: given language description RE? NFA? => DFA? (subset construction method)

PL for RLs
CP for RLs > contradiction

-

given L => CFG?

---

① $L = \{a^n b^n c^n \mid n \geq 0\}$ is non-CF

Pf: Assume L is CF

Then the PL for CFL applies to L

Let P be the constant in the PL

Select $s = a^P b^P c^P \in L$ and $|s| = 3P > P$

$= uvxyz$

$= \underbrace{a \ldots a}_{P} \underbrace{b \ldots b}_{P} \underbrace{c \ldots c}_{P}$

$= uvxyz$ w/ $|vy| > 0$, $|vxy| \leq P$

case 1. vxy contains only one type of symbols

$i=0$, $uv^0xy^0z = \begin{cases} a^{P'}b^Pc^P \ (P' < P) \\ a^Pb^{P'}c^P \\ a^Pb^Pc^{P'} \end{cases} \notin L$

case 2 vxy contains two types of symbols

$i=0$, uxz has fewer P symbols in one or two blocks $uxz \notin L$

contradiction in both cases. L is non-CF

October 17th

$D = \{ww \mid w \in \{0,1\}^*\}$

$A = \{0^i 1^j \mid j = i^2\}$

② $D = \{ww \mid w \in \{0,1\}^*\}$ is non CF

Assume D is CF. PL applies

$\exists p$ select $s = 0^p 1 0^p 1 \in D$

$= uvxyz$ w/ $|vy| > 0$, $|vxy| \leq p$

$= \underbrace{0..0}_{p} 1 \underbrace{0..0}_{p} 1$

Observe vxy can contain at most one 1 since $vxy \leq p$

case 1: vxy doesn't contain any 1

vxy is either in the 1st block or 2nd block of 0s

$i=0$, $uv^0xy^0z = \begin{cases} 0^{p'}10^p1 \ (p' < p) \\ 0^p10^{p'}1 \end{cases} \notin D$

case 2: vxy contains one 1

subcase 1: $1 \notin x \Rightarrow 1 \in v$ or $1 \in y$

$i=0$ uv^0xy^0z has only one 1 $\notin D$

subcase 2: $1 \in x$

$\underbrace{0 \ldots 0}_{u} \underbrace{0 \ldots 0}_{v} \underset{x}{1} \underbrace{0 \ldots 0}_{y} \underbrace{0 \ldots 01}_{z}$

make $|v| = |y| = 0 \ldots 0$

$uv^ixy^iz \in D$

select $s = 0^p1^p0^p1^p \in D$

$i=0$

October 17th

③ $A = \{0^i 1^j \mid j = i^2\}$ is non CF

select $s = 0^p 1^{p^2} = uvxyz$ w/ $|vy| > 0$, $|vxy| \le p$

$uxy = 0^+, 0^+1^+, 1^+$

case 3 $uxy = 0^+1^+$

subcase 3.1 v or y contains both 0 and 1

(ie v or y $= 0^+1^+$)

$i = 2$ $uv^2xy^2z \notin A$

subcase 3.2 v and y each contain up to only one type (i.e. $v = 0^{|v|}$ $y = 1^{|y|}$)

Consider $uv^ixy^iz = 0^{p+(i-1)|v|} 1^{p^2+(i-1)|y|}$

$i = 0$ or 2

$i = 2$ $(p+|v|)^2 \overset{?}{=} p^2 + |y|$

$\cancel{p^2} + 2p|v| + |v|^2 \overset{?}{=} \cancel{p^2} + |y|$

$\underset{\text{left}}{2p|v| + |v|^2} \overset{?}{=} \underset{\text{right}}{|y|}$

① $|y| = 0$ and $|v| \ne 0$ left > right

② $|y| \ne 0$ and $|v| = 0$ left < right

③ $|y|, |v| \ne 0$ left $= 2p|v| + |v|^2 > p \ge |uxy| \ge \underset{\text{right}}{|y|}$

left > right

$uv^2xy^2z \notin A$

$A = \{w \mid n_a(w) = n_b(w) = n_c(w)\}$ is non CF

Pf: Assume A is CF ⇐

Let $B = \{a^* b^* c^*\}$

$A \cap B = \{a^n b^n c^n\}$

By CP under intersection w/ a RL

$A \cap B = \{a^n b^n c^n\}$ is CF

Impossible!

$B = \{a^i b^j c^k \mid j > k + i\}$ is CF

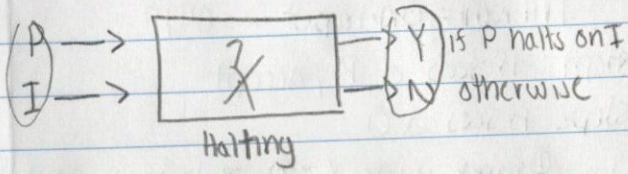
$a^i b^{i+k+\Delta} c^k \Rightarrow a^i b^{i+\Delta} b^k c^k$

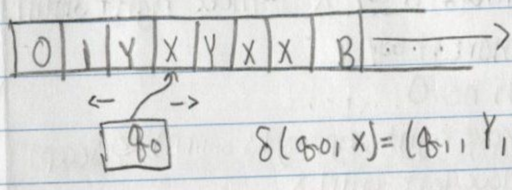
$\Rightarrow \underbrace{a^i b^i}_{CF} \underbrace{b^+}_{\substack{R \\ (CF)}} \underbrace{b^k c^k}_{CF}$

B is a concat of 3 CFLs

B must be CF

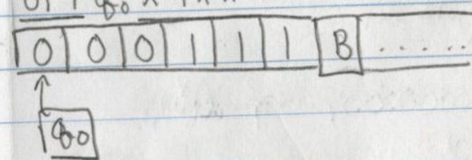
October 22nd





$\delta(q_0, X) = (q_1, Y, R)$

$01Y\,q_0\,XYXX$

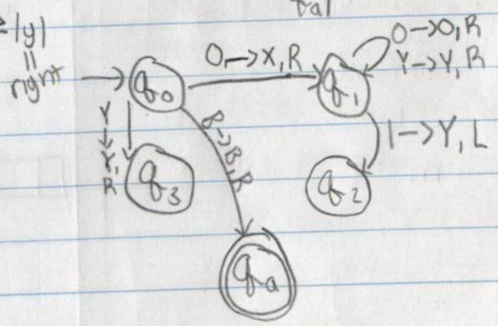


$\underline{q_0 000111}$

Design TM M s.t. $L(M) = \{0^n 1^n \mid n \ge 0\}$

e.g. 000111
X00111
XX0YY1
XXXYYY

δ	0	1	X	Y	B
q_0	(q_1, X, R) ①			(q_3, Y, R) ④	$(q\ B\ R)$ ⓪
q_1	$(q_1, 0, R)$ ②	(q_2, Y, L) ④		(q_1, Y, R) ③	
q_2	$(q_1, 0, L)$ ⑥		(q_0, X, R) ⑦	(q_2, Y, L) ⑧	
q_3				$(q_3, Y R)$ ⑨	(q_a, B, R) ⑩
q_a					



October 24th

$A = \{0^n 1^n \mid n \geq 0\}$ $\delta = ?$

TM M = On input $w = 0^n 1^n$

step1: If sees a B, accept

step2: If sees a 0

① mark it w/ X ② move right until ③ mark it w/ Y

If no 0

① move right (skipping P's until B) ② accept

step3: Move left until X
move right (one square)
goto step 2

$B = \{0^n \mid n = 2^i, i \geq 0\}$

$= \{0, 00, 0000, 00000000, \dots\}$ 0^*

TM M = On input $w \in 0^*$

1. sweep L->R, crossing out every other 0 w/ X
2. In step1, if there is more than one 0 and # ~~of~~ 0's is odd, reject

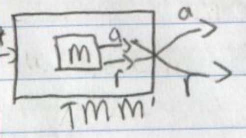
2'. In step 1, there is a single 0, accept

3. Return to left end
4. Go to step1

If L is TDL, so is $\bar{L}$

$\forall w \in \Sigma^* \rightarrow \boxed{TM\,M}$ ->a if $w \in L$ ->r otherwise

$\forall w \in \Sigma^* \rightarrow$



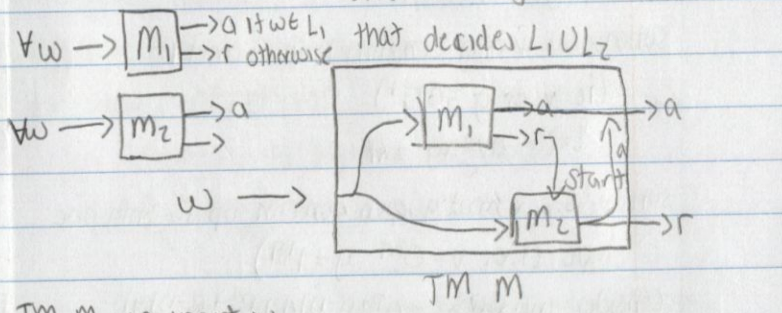
October 29th

Closure properties for TDLs and TRL

1. Union for TDLs

L_1 and L_2 are TDLs => $L_1 \cup L_2$ is also TDL

Wish to design M that decides $L_1 \cup L_2$



TM M = on input w

Run M_1 on w

If M_1 accepts, accept

else run M_2 on w

If M_2 accepts, accept

else reject

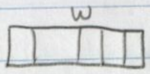
2. Star for TDLs

L is a TDL $\overset{?}{=}>$ L^* is a TRL

$\exists$TM M $\exists$NTM N

$w \in L \rightarrow \boxed{M} \rightarrow a$ $w \in L^* \rightarrow \boxed{N} \rightarrow a$

NTM N = On input $w \in L^*$



- Nondeterministically generate a partition of w into $w_1, w_2, \dots, w_z$
- Run M on $w_1 \dots w_k$
- If M accepts all, accept

October 31st

Diagonalization (proof technique)

A set S is countable / countably infinite

If $\exists$ 1-1 between S and $\mathbb{N}$

Example: $\mathbb{R}^+$ is uncountable

Assume $\mathbb{R}^+$ is countable

$\exists$ 1-1 w/ $\mathbb{N}$, ie. $\mathbb{R}^+ = \{r_1, r_2, r_3, \dots\}$

r_1	3.1415926
r_2	55.5555555
r_3	0.12345 00
r_4	4.000 00 00
$\vdots$	

Make a new $r = 0.\bar{1}\bar{5}\bar{3}\bar{0}$

$\Downarrow$

$r = 0.9418\dots$

$\neq r_k \ \forall_k$

$r \in \mathbb{R}^+$

contradiction

$\therefore \mathbb{R}^+$ is uncountable!

Example: Q: set of all rational number

$\forall x \in Q, \exists p, q, x = p/q$

p \ q	1	2	3	4 ...
1	1/1	1/2	1/3	1/4 ...
2	2/1	2/2	2/3	2/4 ...
3	3/1	3/2	3/3	3/4 ...
4	4/1	4/2	4/3	4/4 ...
$\vdots$				

Example

$F = \{f : \mathbb{N} \to \mathbb{N}\}$ is uncountable

Pf: Assume F is countable, ie $F = \{f_1, \dots, f_k\}$

	1	2	3	4 ...
f_1	$f_1(1)$	$f_1(2)$	$f_1(3)$	$f_1(4)$
f_2	$f_2(1)$	$f_2(2)$	$f_2(3)$	$f_2(4)$
f_3	$f_3(1)$	$f_3(2)$	$f_3(3)$	$f_3(4)$
f_4	$f_4(1)$	$f_4(2)$	$f_4(3)$	$f_4(4)$

Define new function $f(n) = f_n(n) + 1$

$\forall n, f(n) \neq f_n(n)$, since $f(n) \in F$, $\exists k$ s.t. $f(n) = f_k(n)$

$f(n) \overset{①}{=} f_n(n) + 1 \overset{②}{=} f_k(n)$

let $n = k$ $\quad \therefore f_k(k) + 1 = f_k(k)$ impossible!

$\therefore F$ is uncountable

$\delta(q_1, X_2) = (q_3, X_1, D_1)$

Example 1: $A_{DFA} = \{\langle B, w\rangle \mid w \in L(B)\}$ is TD

P_{DEC} is solvable

Input: A DFA B and a string w

Question: $w \in L(B)$?

Define TM m = On input $\langle B, w\rangle$

1. Simulate B on input w
2. If B ends in $q \in F$, accept
3. else, reject

Example 2: $E_{DFA} = \{\langle B\rangle \mid L(B) = \emptyset\}$ is TD

P_{DEC}

Input: a DFA B

Question: is $L(B)$ empty?

TM m = on input $\langle B\rangle$

1. create the state diagram G for B
2. Use DFS to generate all simple paths from q_0 to any $q \in F$
3. If no paths is found, accept else reject

November 5th

$\varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots$
$M_1\ M_2\ M_3\ M_4 \ldots$

$A_D = \{ w_i \mid w_i \notin L(M_i) \}$

A_D is non-TR

Pf: Assume A_D is TR

$\exists$ TM M that accepts A_D

ie $L(M) = A_D$

$M = M_i$ for some i.

	w_1	w_2	w_3	w_4
M_1	1	0	0	...
M_2	1	0	1	...
M_3	1	0	1	...
M_4	⋮	⋮		0
⋮ $L(M_i)$				

$w_i \in A_D$ iff $w_i \notin L(M_i)$ (by def of A_D)

$w_i \in A_D$ iff $w_i \in L(M_i)$ (by $L(M_i) = A_D$)

Contradiction! So A_D is non-TR

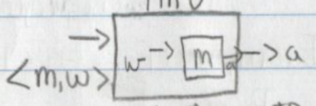
TRL but not TDL

$A_{TM} = \{ \langle M, w \rangle \mid w \in L(M) \}$

Input: TM M and string w

Question: Does M accept w?

TMU

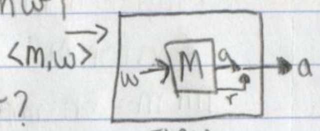


So A_{TM} is TR

$HALT_{TM} = \{ \langle M, w \rangle \mid M \text{ halts on } w \}$

Input: TM M and string w

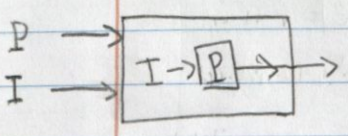
Question Does M halt on w?

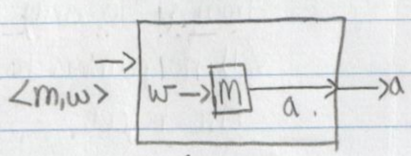


TMU

Computer

TMU ~~(e.g. $0^n 1^n$)~~





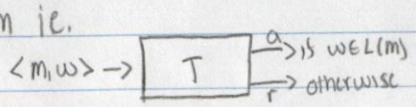
So

A_{TM} is non-TD

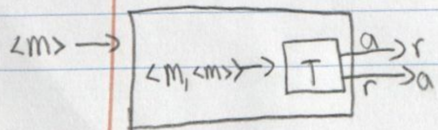
$A_{TM} = \{ \langle M, w \rangle \mid w \in L(M) \}$

Pf: Assume A_{TM} is TD

$\exists$ TM T that decides A_{TM} ie.

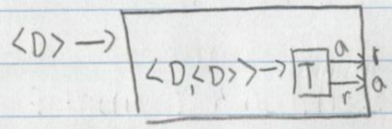


Construct TM D



*reverse them

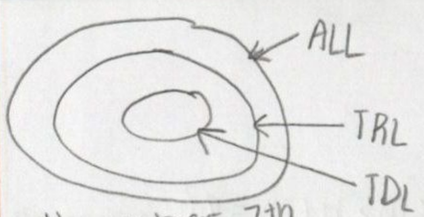
Feed $\langle D \rangle$ to D



D accepts $\langle D \rangle$ iff T rejects $\langle D, \langle D \rangle \rangle$ iff

$\langle D \rangle \notin L(D)$ iff

D rejects $\langle D \rangle$ Contradiction!



November 7th

$A_D = \{w_i \mid w_i \notin L(M_i)\}$ is non-TR

$A_{TM} = \{\langle M, w\rangle \mid w \in L(M)\}$ is TR but non-TD

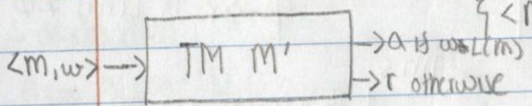
$HALT_{TM} = \{\langle M, w\rangle \mid M \text{ halts on } w\}$ is TR but non-TD

Show that A_{TM} is non-TD

Pf: Assume A_{TM} is TD,

Then $\overline{A_{TM}}$ is TD by CP of TDLs

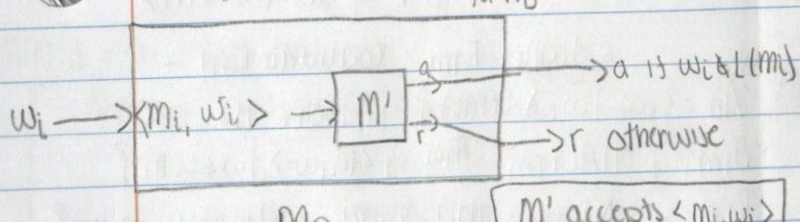
$\exists$ TM M' that decides $\overline{A_{TM}} = \{\langle M, w\rangle \mid w \notin L(M)\}$



Wish to show A_D is TD, therefore a contradiction

Wish to define a TM M_D for A_D

construction



M' accepts $\langle M_i, w_i\rangle$

iff

$w_i \notin L(M_i)$

iff

$w_i \in A_D$

iff

M_D accepts w_i

so M_D decides A_D

A_D is TD

Impossible!

Reduction: $A_D \propto \overline{A_{TM}}$

November 12th

$A_D = \{w_i \mid w_i \notin L(M_i)\}$ is non-TR

$A_{TM} = \{\langle M, w\rangle \mid w \in L(M)\}$ is TR but non-TD

$HALT_{TM} = \{\langle M, w\rangle \mid M \text{ halts on } w\}$ is TR but non-TD

A_D: Input: w_i

Question: Is w_i not accepted by M_i?

A_{TM}: Input: TM M and string w

Question: Does M accept w?

$HALT_{TM}$: Input: TM M and string w

Question: Does M halt on w?

TM $\Leftrightarrow$ alg. (b/c) decidability $\Leftrightarrow$ solvable

language $\Leftrightarrow$ DEC problem

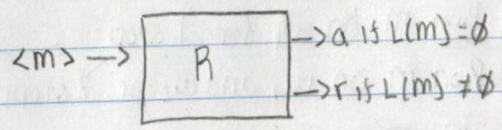
$E_{TM} = \{\langle M\rangle \mid L(M) = \emptyset\}$

$NE_{TM} = \{\langle M\rangle \mid L(M) \neq \emptyset\}$

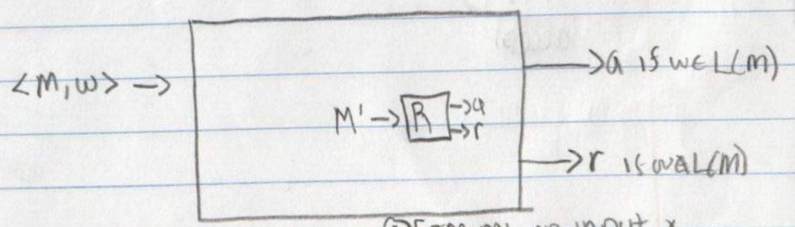
$E_{TM} = \{\langle M\rangle \mid L(M) = \emptyset\}$ is non-TD

Pf: Assume E_{TM} is TD

$\exists$ TM R s.t. R accepts $\langle M\rangle$ iff $L(M) = \emptyset$



? wish to use R to construct a TM S that decides A_{TM} (ie $A_{TM} \propto E_{TM}$)



wish to define TM M' s.t. $L(M') = \emptyset$ imply $w \in L(M)$

(1) TM M' = on input x

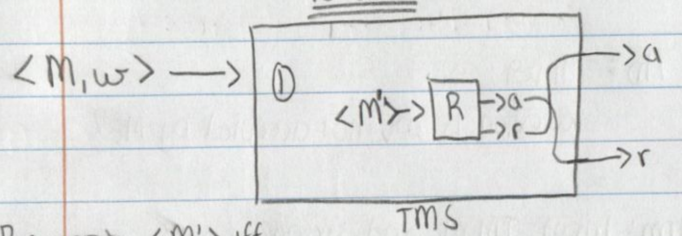
if $x \neq w$: reject

else $x = w$

run M on w

$L(M')$
$= \begin{cases} \{w\} & \text{if } w \in L(M) \\ \emptyset & \text{if } w \notin L(M) \end{cases}$

re-draw



R accepts $\langle M' \rangle$ iff
$L(M') = \emptyset$ iff
$w \notin L(M)$ iff
S rejects $\langle M, w \rangle$

so TMS decides A_{TM}
but A_{TM} is non-TD
so contradiction!
so E_{TM} is non-TD

$NE_{TM} = \{ \langle M \rangle \mid L(M) \neq \emptyset \}$ TD ✗ TR ✓ non-TD ✓

NE_{TM} is TR

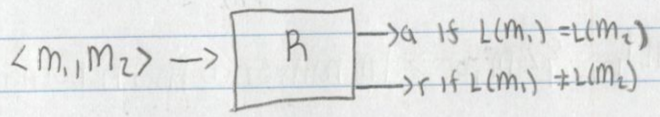
NTM N = On input $\langle M \rangle \in NE_{TM}$
① Guess a string w
② Run M on w
③ If M accepts, accept

DTM H = On input $\langle M \rangle \in NE_{TM}$
// $w_1, w_2, w_3, \ldots$
Run M on w_1 for 1 step
Run M on w_1 and w_2 for 2 steps
. . . . . .
Run M on $w_1 \ldots w_k$ for . . . .
. . . .
accept

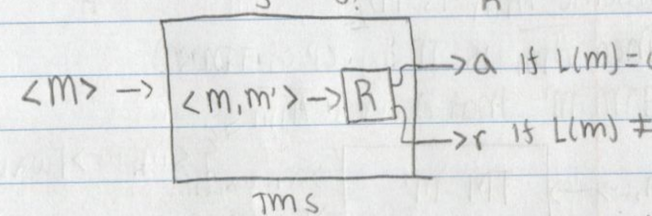
$E_{TM} = \{ \langle M \rangle \mid L(M) = \emptyset \}$

$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid L(M_1) = L(M_2) \}$ is non-TD

Pf: Assume EQ_{TM} is TD
$\exists$ TM R that accepts $\langle M_1, M_2 \rangle$



Wish to prove that $\underbrace{E_{TM}}_{S?} \propto \underbrace{EQ_{TM}}_{R}$?



TM M' = on input x
reject
$L(M') = \emptyset$

R accepts $\langle M, M' \rangle$ iff
$L(M) = L(M')$ iff
$L(M) = \emptyset$ iff
S accepts $\langle M \rangle$

S decides E_{TM} contradiction!

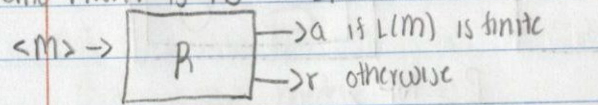
Q4 on HW
✓ $A_D = \{ w_i \mid w_i \notin L(M_i) \}$ is non-TD
✓ A_{TM}, ✓ $HALT_{TM}$ $A_{TM} = \{ \langle M, w \rangle \mid w \in L(M) \}$
✓ E_{TM}, ✓ NE_{TM} $HALT = \{ \langle M, w \rangle \mid M \text{ halts on } w \}$
✓ EQ_{TM} $E_{TM} = \{ \langle M \rangle \mid L(M) = \emptyset \}$
~~EMPTY~~ $FINITE = \{ \langle M \rangle \mid L(M) \text{ is finite} \}$
$EMPTY\_STRING = \{ \langle M \rangle \mid \varepsilon \in L(M) \}$
$ALL = \{ \langle M \rangle \mid L(M) = \Sigma^* \}$

PCP: Emil Post

November 14th

FINITE = { <M> | L(M) is finite } is non-TD

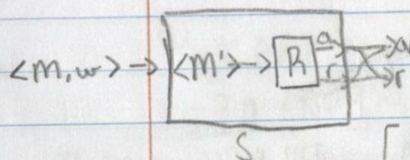
Pf: Assume FINITE is TD => ∃TM R that decides it

=> 

Try to prove $A_{TM} \propto$ FINITE

Try to use R to construct TM S that decides A_{TM}

wish to define M' s.t.



L(M') is FINITE iff w ∈ L(M)

TM M' = on input x
- Ignore x
- Run M on w

So S decides A_{TM}

But A_{TM} is non-TD

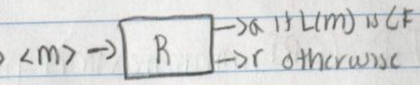
Contradiction!

So FINITE is non-TD!

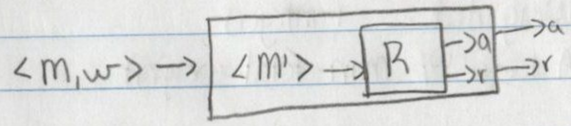
$L(M') = \begin{cases} \emptyset & \text{if } w \notin L(M) \\ \Sigma^* & \text{if } w \in L(M) \end{cases}$

TM S = On input <M, w>
1. Define TM M' = On input x
   - Ignore x
   - Run M on w
2. Run R on <M'>
3. If R accepts, reject
   If R rejects, accept

CFL_{TM} = {

Assume CFL_{TM} is TD => 

Try prove $A_{TM} \propto CFL_{TM}$



wish M' s.t. L(M') is CF iff w ∈ L(M)

Define TM M' = on input x
- If x = yy for y ∈ Σ*: accept
- else run M on w

$L(M') = \begin{cases} \Sigma^* & \text{if } w \in L(M) \\ \{yy \mid y \in \Sigma^*\} & \text{if } w \notin L(M) \end{cases}$

L(M') is CF iff w ∈ L(M)

Post's Correspondance Problem (PCP)

Input: $P = \{ \frac{t_1}{b_1}, \frac{t_2}{b_2}, \ldots, \frac{t_k}{b_k} \}$

Question: Does P have a <u>match</u>?

$i_1, i_2, \ldots, i_l$

November 19th

TSP (OPT, minimization problem)

Input ✓ complete graph G
✓ Weighted : + integers
Output A tour w/ min total weight

TSP(DEC)

Instance: ✓ complete graph G
✓ weighted
? $B \geq 0$
Question: Is there a tour w/ total weight $\leq B$?

TSP(DEC) $\in$ NP

Define $NALg$ = On input (G, B)
① Guess a permutation π of nodes
O(n)
4-3-1-2-5
② Verify that π is a tour and
total weight of $\pi \leq B$
O(n)

Graph coloring (DEC) $\in$ NP
Instance: $G=(V,E)$, $B \geq 0$
Question: If there a coloring scheme w/ $\leq B$ colors?

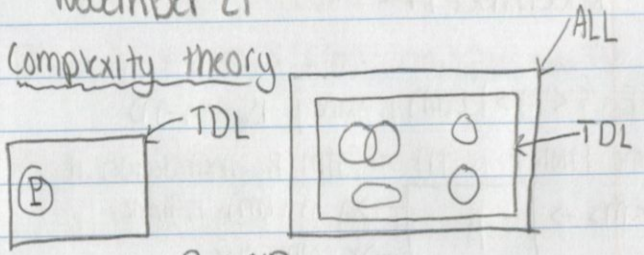
Define $NALg$ = On input (G, B)

O(|V|) ③ NTM ① Guess $f: V \to C$
O(|E|) edges O(|V|+|E|) ② Verify ① color constraints ② $|C| \leq B$
O(|V|)

poly time? ✓

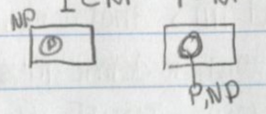
November 21st

Complexity theory



$P \subseteq NP$ ✓

$P \subset NP$ $P = NP$



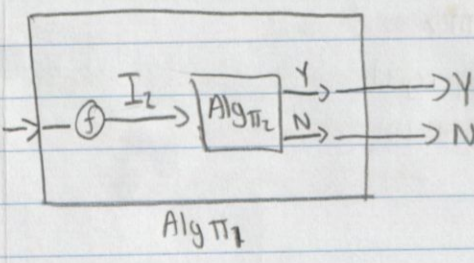
$P \overset{?}{=} NP$

NP-complete (NPC) $A \underset{\leq}{\propto} B$
TM M_A TM M_B

$A_{TM} \underset{\leq}{\propto} HALT_{TM}$

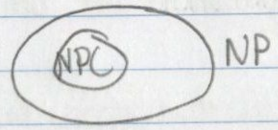
$A_D \overset{\propto}{\leq} \overline{A}_{TM} \longleftrightarrow A_{TM} \longrightarrow E_{TM} \longleftrightarrow NE_{TM}$
$HALT_{TM}$ EQ_{TM}

Polynomial reduction: $\Pi_1 \propto_p \Pi_2$
$\exists f: \{I_1\} \to \{I_2\}$ s.t.
① f can be computed in poly-time
② I_1 has a "yes" solution iff
$I_2 = f(I_2)$ has a "yes" solution



if $\Pi_1 \in P$ then $\Pi_2 \in P$

NPC



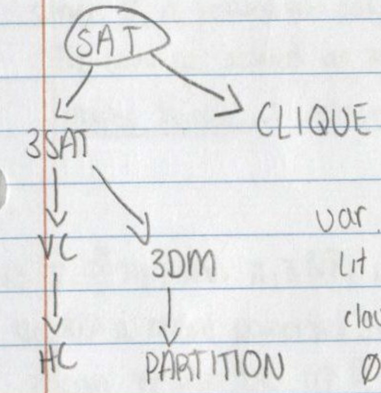
Def of class NPC: Π is in NPC iff

① Π is ~~the~~ a hardest in NP

②
- $\Pi \in NP$
- $\forall \Pi' \in NP$, $\Pi' \underset{p}{\overset{\leq}{\propto}} \Pi$

③
- $\Pi \in NP$
- $\exists \Pi' \in NPC$, $\Pi' \underset{p}{\propto} \Pi$

7 basic NPC problems



var. $x_1, \dots x_n$

Lit $x_1, \overline{x}_2$

clause: $x_1 \vee x_2 \vee x_3$

$\emptyset$ in CNF: $c_1 \wedge c_2 \wedge c_3 \wedge c$

SAT $\in$ NP

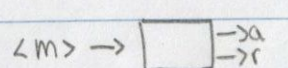
HW9

1a) $\overline{A}_D$ is non-TD but $\boxed{TR}$

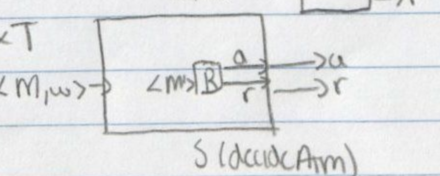
TMU ✓

3. $T = \{\langle m \rangle \mid m \text{ accepts } w^R, w\}$

TD by TM R



$A_{TM} \propto T$



Define TM m' = on input x

If x = 01, accept

If x = 10, run m on w

If x ≠ 01 and x ≠ 10, reject

$L(m') = \begin{cases} \{01, 10\} & w \in L(m) \\ \{0, 1\} & w \notin L(m) \end{cases}$

November 26th

SAT (satisfiability) DEC

Instance: $\emptyset$ in CNF

Question: Is $\emptyset$ satisfiable?

- If $\Pi_1 \in NPC$, $\Pi_2 \in NP$, and $\Pi_1 \underset{p}{\propto} \Pi_2$ then $\Pi_2 \in NPC$

If $\exists \Pi \in NPC$ and $\Pi \in P$, then $P = NP$. If $NPC \cap P \neq \emptyset$ then $P = NP$

- If $\exists \Pi \in NPC$ and $\Pi \notin P$, then $P \neq NP$

If $NPC - P \neq \emptyset$, then $P \neq NP$

CLIQUE

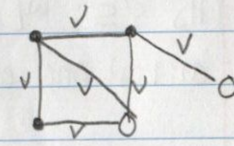
Instance: $G = (V, E)$, $B \geq 0$

Question: Is there a clique w/ size $\geq B$?

Vertex cover (VC)

Instance: $G = (V, E)$, $B \geq 0$

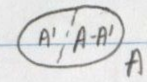
Question: Is there a VC of size $\leq B$?



PARTITION

Instance: $A = \{a_1, \dots, a_n\}$

Question: ? $\exists A' \subseteq A$ s.t. $\sum_{a_i \in A'} a_i = \sum_{a_i \in A - A'} a_i$?



3DM

Instance: X, Y, Z of some size of q

$M = \{(x_1, y_2, z_4), (x_1, y_3, z_1), (\dots)\}$

Question: Is there a matching? ✓

$M' \subseteq M$

December 3rd

define P w/ deterministic Turing Machine

define NP w/ non - " " "

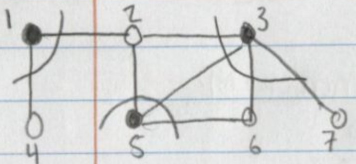
PARTITION $\propto$ KNAPSACK

P is entirely in NP, but not vice versa

$P \subseteq NP$

VC: Instance: $G = (V, E)$, $B \geq 0$

Question: ? $\exists$ a VC called $V' \subseteq V$ of size $\leq B$?



HS: Hitting Set

Instance: set S and collection C of subsets of S $K \geq 0$

Question: Does S contain a HS S' of size $\leq k$?

(HS $S' \subseteq S$ w/ $|S'| \leq k$ s.t. S' contains at least one element from each subsets in C)

$VC \propto_p HS$

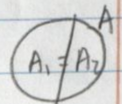
$G = (V, E)$ $S = \{1, 2, 3, 4, 5, 6, 7\}$

B $C = \{(1,2)(1,4), \ldots \}$

$K = 3$
$\overset{\shortparallel}{B}$

PARTITION: Instance: $A = \{a_1, \ldots, a_n\}$

Question: ? $\exists$ $A' \subseteq A$ s.t. $\sum_{A'} a_i = \frac{1}{2} \sum_{A} a_i$



Knapsack: Instance: $U = \{u_1, \ldots, u_n\}$, W, B

$\forall u_i$, $w(u_i)$, $v(u_i)$

Question: ? $\exists$ $U' \subseteq U$ s.t. $\sum_{u_i \in U'} w(u_i) \leq W$

and $\sum_{u_i \in U'} v(u_i) \geq B$?

Final exam: Thu. Dec 12 small1 @9

Format: 2 cheat sheets, both sides

Content: Automata Theory: RL, DFA, NFA, RE Closure Properties (the list), PL, CFG (PDA?) (20 pts)

Computative Theory: TM, NTM, TD, $\bar{L}$ halt, TRL, CP (diff bt the 2), C-T thesis | A_D A_{TM} $HALT_{TM}$ (≤), reduction (10 pts) (30 pts) concepts, undecidability

Complex Theory: P, NP, NPC, poly. reduction (30 pts) + NP (4 pts)

- 4 keywords: NTM, Guess, verify, poly. time

writing: 6ts

π_2 is at least as hard as π_1

eg If $\pi_1 \underset{p}{\propto} \pi_2$ and $\pi_1 \in P$, then $\pi_2 \in P$?

since π_1 is solved in poly time we know π_2 can be solved at least in poly time or maybe harder. ∴ we cannot say π_2 is in P

eg If $\pi_1 \overset{\leq}{\underset{p}{\propto}} \pi_2$, and $\pi_1 \notin NP$, is $\pi_1 \in P$?

No, NP is more powerful so since it is non-NP it can never be in P.

~10 bonus pts: PARTITION $\underset{p}{\propto}$ KNAPSACK

$A = \{a_1, \dots, a_n\}$ $U = \{u_1, \dots, u_n\}$

$\forall u_i$, $w(u_i) = ?$

$v(u_i) = ?$

s.t. A (A_1 / A_2) W, $B = ?$

$U' \leq U$?