CSci 423 Homework 4

Due: 12:30 pm, Thursday, 10/10/19
Daniel Quiroga

1. (1, 6, 2 points) Determine whether or not the following languages are regular. Justify your answers with proofs.

   (a) $L_1 = \{0^n1^n \mid n \geq 0\} \cup \{0^n1^m \mid m,n \geq 0\}$
   The first language is a subset to the second language. Therefore the union would depend on the second. The second language can be written as $0*1*$ in RE therefore the language is regular.

   (b) $L_2 = \{0^n1^n \mid n \geq 0\} \cup \{0^n1^{n+2} \mid n \geq 0\}$
   From the class notes, we know that the left side is non-regular. The second one can be dispproved using the pumping lemma
   Assume the language is a RL, Then PL apples to the language.
   There exist a constant P
   select s = 0^P1^{P+2} which is an element of the language and has length greater than (or equal to P).
   = $0..0(p\ length)1..1(P\ +\ 2\ length)$ = xyz
   where $|y| > 0, |xy| <= p, xy^iz \in L\ for\ any\ i >= 0$
   say xy is in the first block of O if we set i = 1000..0 (infinite), then there will be a string that has more Os than 1s and thus would be a contradiction. meaning that the second language is non-regular.
   since we know that there will always be a definite amount of 0^n and the union would accept either 1^n or 1^{n+2} amount of 1s. we can say that the union of these two languages will also be non-regular since whichever case is chosen the language can be disproven using the pumping lemma either from class or above.

   (c) $L_3 = \{0^m1^n \mid 0 \leq m \leq 50 \text{ and } n \geq 100\}$
   L_3 can be written as $(\varepsilon\ U\ 0\ U\ 00\ U\ 000\ U..U\ 0^{50})1^{100}(1)^*$ there it is regular since it can be written as a RE.

   Collaborators: Ethan Young, Will Elliot and Yang Zhang

2. (3, 6 points) Consider the following similarly defined languages.

   (a) Let $B = \{1^kw \mid w \in \{0,1\}^*$ and w contains at least k 1s, for any $k \geq 1\}$. Prove that B is a regular language by giving its simple regular expression.
   $1^k(0^*1)^k(0U1)*$

   (b) Let $C = \{1^kw \mid w \in \{0,1\}^*$ and w contains at most k 1s, for any $k \geq 1\}$. Prove by the pumping lemma that C is not regular.
   Assume the language is a RL, Then PL apples to the language.
   There exist a constant P, where P = k -1
   select s = 1^Pw which is an element of the language and has length greater than (or equal to P).
   = $1..1\ (P\ length)w\ (at\ most\ has\ P\ 1s)$ = xyz
   where $|y| > 0, |xy| <= p, xy^iz \in L\ for\ any\ i >= 0$

consider y = 1 and z = w. If we choose i = 3. The string xy^3z would yield a string that would have more 1s in the first block of 1s and the string in z would never be able to match that same number of 1s therefore the string would not be able to be accepted by the language and thus is a contradiction to the pumping lemma.

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3. (6 points) Let $\Sigma = \{0,1,+,=\}$ and

$$ADD = \{x = y+z \mid x,y,z \text{ are binary numbers and } x \text{ is the sum of } y \text{ and } z\}.$$

For example, $110 = 01+101 \in ADD$. Prove by the pumping lemma that ADD is not regular. (Hint: Select a string, where $x = z$ and $y = 0$.)

Assume the language is a RL, Then PL apples to the language.
There exist a constant P
select s = $1^P = 0+1^P$ which is an element of the language and has length greater than (or equal to P).
= $1..1(plength) = 0+1..1(Plength)$ = xyz
where $|y| > 0, |xy| <= p, xy^iz \in L\ for\ any\ i >= 0$

say $y = 1^+$ and we set $i = 0$ then the string xy^0z would be $1^{P'} = 0+1^P \notin ADD$ (P' < P) A contradiction

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4. (6 points) Prove by the pumping lemma that $B = \{vgv \mid v,g \in \{0,1\}^+\}$ is not regular. Note that $\{0,1\}^+ = \{0,1\}^* - \varepsilon$. (Hint: Select $s = v1v$ for some simple string v.)
Assume the language is a RL, Then PL apples to the language.
There exist a constant P
select s = 0^p10^p which is an element of the language and has length greater than (or equal to P).
= $0..0(plength)10..0(Plength)$ = xyz
where $|y| > 0, |xy| <= p, xy^iz \in L\ for\ any\ i >= 0$
consider $x = 0^{P-k}y = 0^k$ say we set i = 0 then we would have xy^0z would be $0^{P'}10^P \notin ADD$ (P' = P - k < P) A contradiction

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