CSci 423 Homework 9

Due:12:30 pm, Tuesday, 11/26
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Collaborators:

1. (1, 1, 1, 3 points) In class, we learned that A_D is non-TR, A_{TM} and $HALT_{TM}$ are TR but non-TD. What can you say about their complements? Circle the correct answers below.

   (a) $\overline{A}_D$ is (i) TD; (ii) TR but non-TD; (iii) non-TR. ANSWER: ii

   (b) $\overline{A}_{TM}$ is (i) TD; (ii) TR but non-TD; (iii) non-TR. ANSWER: iii

   (c) $\overline{HALT}_{TM}$ is (i) TD; (ii) TR but non-TD; (iii) non-TR. ANSWER: iii

   In addition, justify your answer to (a) by giving a proof.

2. (6 points) Prove that $ES_{TM} = \{< M > \mid M \text{ accepts } \varepsilon\}$ is non-TD. (Hint: Reduce from A_{TM}.)
   Assume ES_{TM} is TD. Then there is a Turing Machine R that decides ES_{TM}.
   So TM R given a Turing Machine M it will accept if $\varepsilon \in L(M)$ or reject otherwise. We will try to define a Turing Machine S that decides A_{TM} (reduction sign) with this information.
   Define TM M' = on input x $\rightarrow$ Run M on w.
   $L(M') = \sum^* if\ w \in L(M) |\ \emptyset\ otherwise$
   M' accpets ε iff $w \in L(M)$
   Then we have a turning machine S that would decide $A_{TM} \rightarrow$ CONTRADICTION!
   ES_{TM} is Turing-undecidable.

3. (6 points) Let $T = \{< M > \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Recall that w^R is the reverse of w. Prove that T is non-TD.

   Hints: (1) Reduce from A_{TM}. (2) For any M and w, can you define a TM M_1 such that $L(M_1) = \{01, 10\}$ if M accepts w and $L(M_1) = \{01\}$ if M does not accept w?

4. (6 points) Prove that it is undecidable whether $L(M_1) \subseteq L(M_2)$ for any given TMs M_1 and M_2. (Hint: Reduce from EQ_{TM}.)
   In order to prove equality or inequality using subsets we need two principles to hold: for equality we need $L(M_1) \subseteq L(M_2)$ and $L(M_2) \subseteq L(M_1)$ to both come out true. If one of them fails than we have inequality.
   We first assume that the subset problem is TD. Then there must be a turning machine that decides it.
   We design a Turning Machine E that will accept the input $< M_1, M_2 >$: accept if the first input is a subset of the second, reject otherwise. We will try to define a Turning machine S that decides EQ_{TM}. Since this subset is a smaller part of the entire equality theorem.
   Define E = on input x $\rightarrow$ run input $< M_1, M_2 >$ if it accepts, then run $< M_2, M_1 >$ if it accepts then have TM S accept, if any steps rejects have TM S reject. E accpets on both of the inputs iff $L(M_1) = L(M_2)$ iff S accepts $< M_1, M_2 >$.
   We have designed a TM S that decides $EQ_{TM} \rightarrow$ CONTRADICTION!
   The problem is Turing-undecidable

5. (6 points) Prove that the question “Does $L(M)$ contain any string of length 5” is undecidable. (Hint: Reduce from A_{TM}.) We assume that the question is decidable. Then there must be TM R that decides the question So TM R give another TM M will accept if the length of the string is exactly 5, reject otherwise. We will try to define TM S that decides A_{TM} with this information.
   Define TM M’ = on input x $\rightarrow$ Run M on w.
   $L(M') = \sum^* if\ w \in L(M) |\ \emptyset\ otherwise$
   M’ accepts $woflength5$ iff $w \in L(M)$
   Then this would mean that TM S would be able to decide $A_{TM} \rightarrow$ CONTRADICITION!
   The question is undecidable.