CSci 423 Homework 1

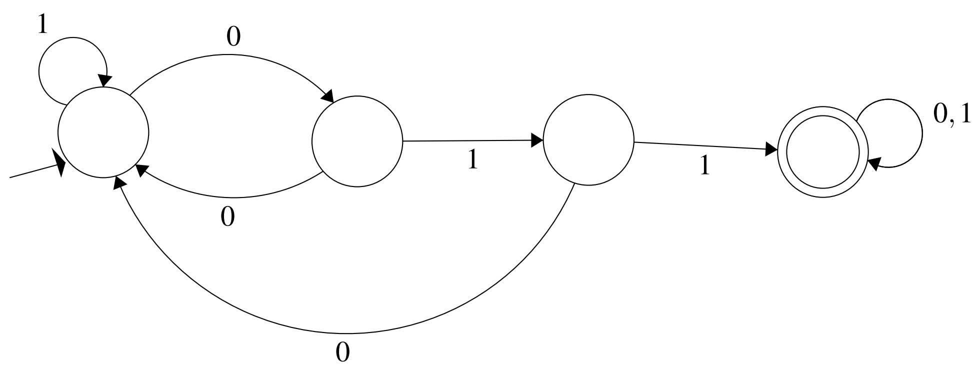
Due: 12:30 pm in class, Thursday, 9/12/19
Daniel Quiroga

1. (5 points) Prove by induction that $2^n \geq n^2$, for integer $n \geq 4$.
   Base case: $n = 4 ==> 2^4 \geq 4^2$ check
   Induction step: Let $k \geq 4$ be given is true for $n = k$. Then
   $2^{k+1} > (k+1)^2 = k^2 + 2k + 1$
   $2^{k+1} = 2^k \times 2^1$ I assume that 2^k is $> k^2$
   $2^k \times 2^1 > k^2 \times 2 = 2k^2 > k^2 + 2k + 1$
   Thus, holds for $n = k + 1$, and the proof of the induction step is complete.
   Conclusion: By the principle of mathematical induction, it follows that $2^n \geq n^2$, is true for $n \geq 4$

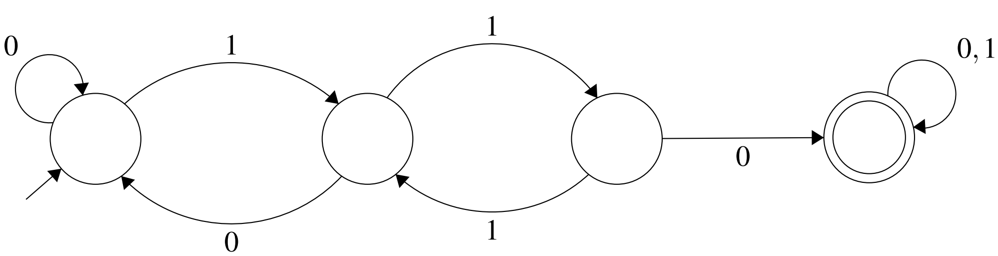
   Collaborators: NONE

2. (3, 3, 3 points) In class, we studied a DFA that accepts strings with 111 as a substring. Here, give DFAs (in state diagrams, and each with no more than four states) that accept the following languages over the alphabet $\{0, 1\}$:

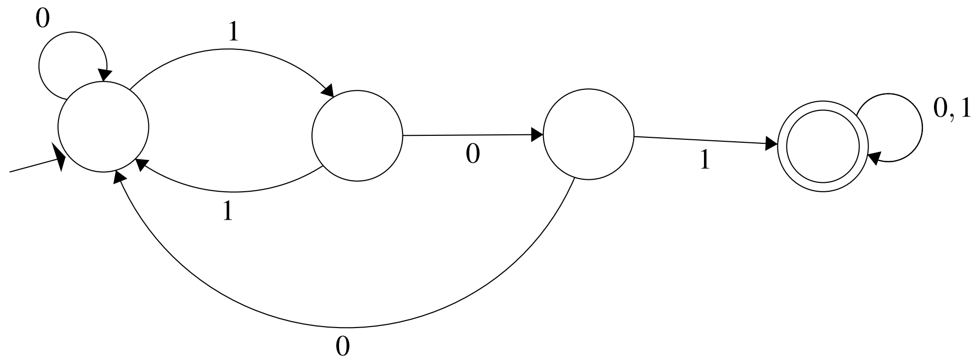
   (a) The set of strings with 011 as a substring



   (b) The set of strings with 110 as a substring



   (c) The set of strings with 101 as a substring

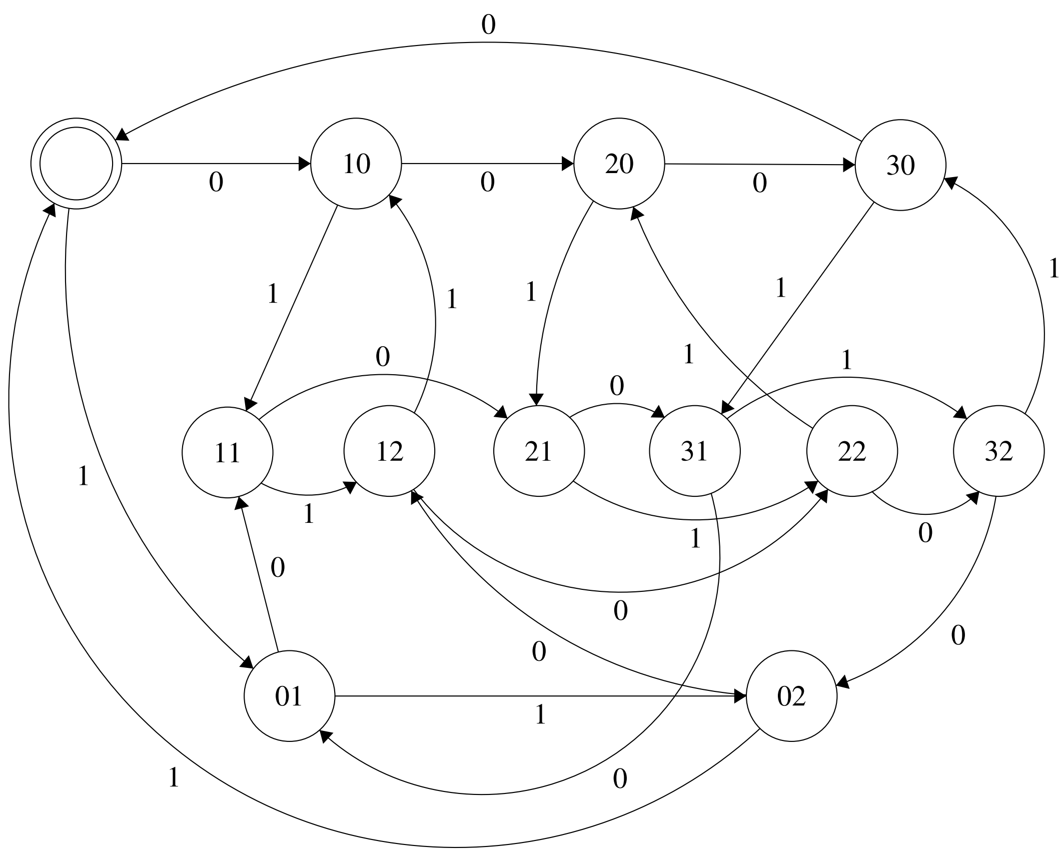


Collaborators: NONE

3. (5 points) Give the state diagram of a DFA with no more than 12 states that accepts the language containing all strings in $\{0,1\}^*$ such that in each string the number of 0s is divisible by 4 and the number of 1s is divisible by 3.

   Daniel Quiroga ¡dquiroga@email.wm.edu¿ 9:42 PM (0 minutes ago)

   to me



4. (3 points) Assume language A is accepted by DFA M. Describe a simple method in just one short sentence to construct a DFA $\overline{M}$ that accepts $\overline{A}$.

   $\bar{M} = \{v \| v \notin A\}$ would give you a DFA $\bar{M}$ that accepts all of $\bar{A}$

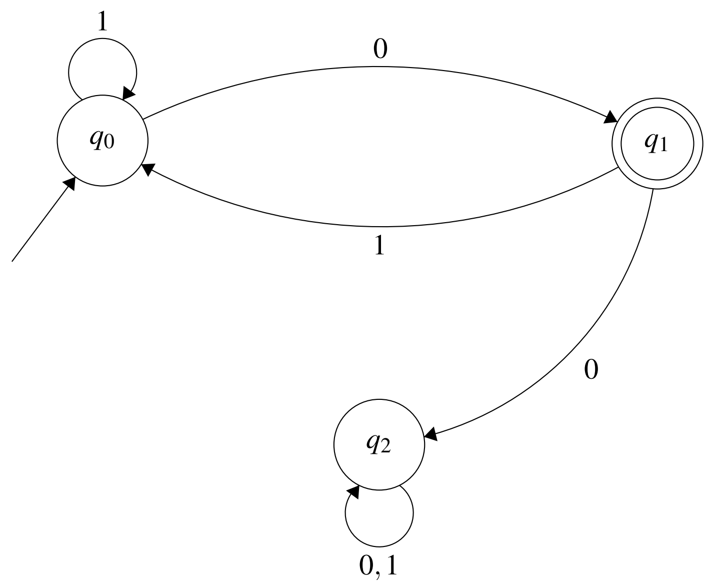
Collaborators: Yang Zhang

5. (4, 4 points) For the following DFAs given in the transition table format, draw their state diagrams and then describe precisely and concisely the languages accepted by the DFAs.

(a)

	0	1
$\rightarrow q_0$	q_1	q_0
$*q_1$	q_2	q_0
q_2	q_2	q_2

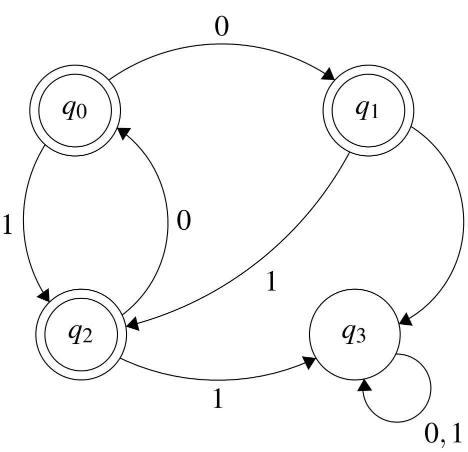
$L_1 = \{w \in \{0,1\}^* | w \text{ ends with a 0 and doesn't have substring } 00\}$



(b)

	0	1
$\rightarrow *q_0$	q_1	q_2
$*q_1$	q_3	q_2
$*q_2$	q_1	q_3
q_3	q_3	q_3

$L_2 = \{w \in \{0,1\}^* | w \text{ doesn't contain substrings } 00 \text{ or } 11\}$



Collaborators: Yang Zhang