

CSci 423 Homework 5

Due: 12:30 pm, Thursday, 10/17/19

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1. (2, 2, 2, 2 points) Let $L = \{a^n b^n \mid n \geq 0\}$.

(a) Show that L is context-free.

$$S \rightarrow aSb \mid \epsilon$$

(b) Show that L^2 is context-free.

$$S \rightarrow AA$$

$$A \rightarrow aAb \mid \epsilon$$

(c) Show that L^k is context-free for any fixed $k > 1$.

$$S \rightarrow AA \dots A_k$$

$$A \rightarrow aAb \mid \epsilon$$

(d) Show that L^* is context-free.

$$S \rightarrow SS \mid S \mid A$$

$$A \rightarrow aAb \mid \epsilon$$

*just a quick question, but would i be able to simplify the first line to $S \rightarrow SS \mid A$ *(thanks!)

2. (3 points) Give a simple description for the language generated by the following grammar G .

$$S \rightarrow aSb \mid bY \mid Ya$$

$$Y \rightarrow bY \mid aY \mid \epsilon$$

Solution:

$$(k \geq 1)a^k(b(a \cup b)^* \cup (a \cup b)^*a)b^k \cup b(a \cup b)^* \cup (a \cup b)^*a$$

3. (3, 4, 3 points) Consider the language $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

(a) Define F as a union of three languages, depending of the value/range of i , i.e., $i = 0$, $i = 1$, and $i \geq 2$.

$$F = \{a^i b^j c^k \mid i = 0, j, k \geq 0\} \cup \{a^i b^n c^n \mid i = 1, n \geq 0 (n = j = k)\} \cup \{a^i b^j c^k \mid i \geq 2, j, k \geq 0\}$$

(b) Prove that F is not regular by closure properties.

$$F_1 = \{a^i b^j c^k \mid i, j, k \geq 0\}$$

$$F_2 = \{a^i b^j c^k \mid i = 1, j = k \geq 0\}$$

we assume that F_1 and F_2 are both regular languages therefore the intersection must be regular.

$$F = F_1 \cap F_2 = \{a^1 b^n c^n \mid n \geq 0 (n = j = k)\}$$

we have proved that $b^n c^n$ is not regular therefore we have a contradiction. F is not regular.

(c) Construct a context-free grammar for F , based on your answer to (a).

$$S \rightarrow S_1 \mid S_2 \mid S_3$$

$$S_1 \rightarrow bS_1|S_1c|\epsilon$$

$$S_2 \rightarrow aA$$

$$A \rightarrow bAc|\epsilon$$

$$S_3 \rightarrow aaB$$

$$B \rightarrow aB|C$$

$$C \rightarrow bC|Cc|\epsilon$$

4. (3, 3, 3 points) Give a simple CFG for each of the following languages, where $i, j, k \geq 0$.

(a) $A = \{a^i b^j c^k \in \{a, b, c\}^* \mid i \neq j \text{ or } j \neq k\}$.

$$S \rightarrow S_1|S_2$$

$$S_1 \rightarrow A_1|B_1 \text{ (the case where } i \text{ is not equal to } j\text{)}$$

$$A_1 \rightarrow aA_1b|C_1E_1$$

$$C_1 \rightarrow aC_1|a$$

$$B_1 \rightarrow aB_1b|D_1E_1$$

$$D_1 \rightarrow bD_1|b$$

$$E_1 \rightarrow cE_1|\epsilon$$

$$S_2 \rightarrow E_2A_2|E_2B_2 \text{ (the case where } j \text{ is not equal to } k\text{)}$$

$$A_2 \rightarrow bA_2c|C_2$$

$$C_2 \rightarrow bC_2|b$$

$$B_2 \rightarrow bB_2c|D_2$$

$$D_2 \rightarrow cD_2|c$$

$$E_2 \rightarrow aE_2|\epsilon$$

(b) $B = \{a^i b^j c^k \in \{a, b, c\}^* \mid |i - j| = k\}$

reword this into two situations: $i = k + j$ $j = k + i$

$$S \rightarrow S_1|S_2$$

$$S_1 \rightarrow aS_1c|A$$

$$A \rightarrow aAb|\epsilon$$

$$S_2 \rightarrow AB$$

$$A \rightarrow aAb|\epsilon$$

$$B \rightarrow bBc|\epsilon$$

(c) $C = \{a^i b^j c^k \in \{a, b, c\}^* \mid i + j = k\}$.

$$S \rightarrow aSc|A$$

$$A \rightarrow bAc|\epsilon$$