CSci 423 Homework 7

Due: 12:30 pm, Thursday, 11/7/2019
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1. (2, 2 points) True or false? No explanation needed.

   (a) A language is Turing-recognizable if and only if some nondeterministic Turing machine recognizes it.
   This is True

   (b) A language is Turing-decidable if and only if some nondeterministic Turing machine decides it.
   This is True

2. (2, 2, 4 points) Let $\Sigma = \{0, 1\}$. For each of the following δ functions, describe the corresponding language $L(M)$.

   (a) $\delta(q_0, 0) = (q_0, B, R)$, $\delta(q_0, 1) = (q_1, B, R)$, $\delta(q_1, 1) = (q_1, B, R)$, and $\delta(q_1, B) = (q_{accept}, B, R)$.
   $L(M) = 0^n 1^m \; n \geq 0 \; m \geq 1$

   (b) $\delta(q_0, 0) = (q_1, 1, R)$, $\delta(q_1, 1) = (q_2, 0, L)$,, $\delta(q_2, 1) = (q_0, 1, R)$, and $\delta(q_1, B) = (q_{accept}, B, R)$.
   $L(M) = 0 \; 1^n \;\; n \geq 0$

   (c) $\delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\}$, $\delta(q_1, 1) = \{(q_2, 0, L)\}$, $\delta(q_2, 1) = \{(q_0, 1, R)\}$, and $\delta(q_1, B) = \{(q_{accept}, B, R)\}$. (Note: This is a nondeterministic TM.)
   $L(M) = 0(0 \cup 1)^*$

3. (8 points) Give the implementation-level description of a Turing machine that **decides** the following language

$$L = \{w \in \{0, 1\}^* \mid w \text{ contains twice as many 0s as 1s}\}$$

TM = on input = w = twice as many 0's as 1's
step 1: If sees a B, accept
step 2: move right until sees a 1 or B
if B, move left – if sees a 0, reject else: accept
elif sees 1:

(a) mark with an X

(b) move left until at the front

(c) move right until 0, mark with Y

(d) move right until next 0, mark with Y

else (when there is only 0s in the string): reject (might be already included in the first if but just in case)
step 3: go back to the beginning of the string, move right until X
goto step 2

4. (5, 5 points) Prove the following closure properties of TDLs.

(a) If L_1 and L_2 are Turing-decidable, so is L_1L_2.
For decidable languages L_1 and L_2, let M_1 and M_2 decide them, respectively. Design M that decides L_1L_2
TM M = on input w
For every way to split $w = w_1w_2$
$\rightarrow$ Run M_1 on w_1 M_2 on w_2
$\rightarrow$ If both accpet, accept
$\rightarrow$ else: continue with the next iteration of the w_1w_2 combo
If none are accepted after loop, reject

Since there is a turing machine that describes L_1L_2 , then L_1L_2 is turing-decidable

(b) If L_1 and L_2 are Turing-decidable, so is $L_1 \cap L_2$
For decidable languages L_1 and L_2, let M_1 and M_2 decide them, respectively. Design M that decides $L_1 \cap L_2$
TM M = on input w

run M_1 on w
If M_1 accepts:
$\rightarrow$ Run M_2 on w
$\rightarrow$ If M_2 accepts, accept w
$\rightarrow$ else: reject w
else: reject w

Since there is a turing machine that describes $L_1 \cap L_2$, then $L_1 \cap L_2$ is turing-decidable