+) x^{α}

- α nguyên dương, $\alpha > 0$: $x \in \mathbb{R}$
- α nguyên âm : $x \neq 0$
- $\alpha \notin$ ' : $x > 0$

$$\boxed{x^{\frac{p}{q}} = \sqrt[q]{x^{p}}} \quad (p, q > 0)$$

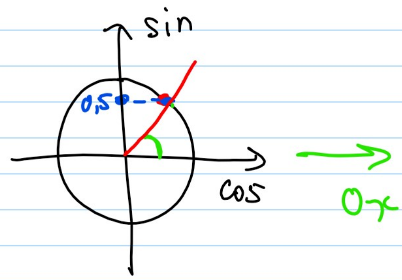
$$x^{\frac{3}{5}} = \sqrt[5]{x^{3}}$$

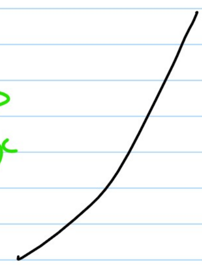
HSố cơ bản : TXĐ, TGT, Hàm chẵn lẻ | đồng biến ; nghịch biến

+) sin x ; cos x ; tan x ; cot x

+) sin x : TXĐ $\mathbb{R}$; TGT $[-1;1]$

Hàm lẻ





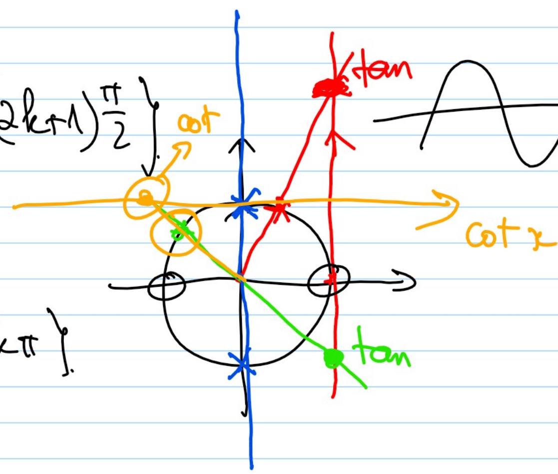
+) cos x : TXĐ $\mathbb{R}$; TGT $[-1;1]$

Hàm chẵn

+) tan x : TXĐ : $\mathbb{R} \setminus \left\{(2k+1)\frac{\pi}{2}\right\}$

TGT : $\mathbb{R}$

Hàm lẻ



+) cot x : TXĐ : $\mathbb{R} \setminus \{k\pi\}$

TGT : $\mathbb{R}$

hàm lẻ.

+) Chẵn : cos

+) lẻ : sin, tan, cot

+) arcsin x ; arccos ; arctan ; arccot x arcsin : ↗ hàm tăng $\quad [-1;1] \to \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

arccos : ↘ hàm giảm $\quad [-1;1] \to [0;\pi]$

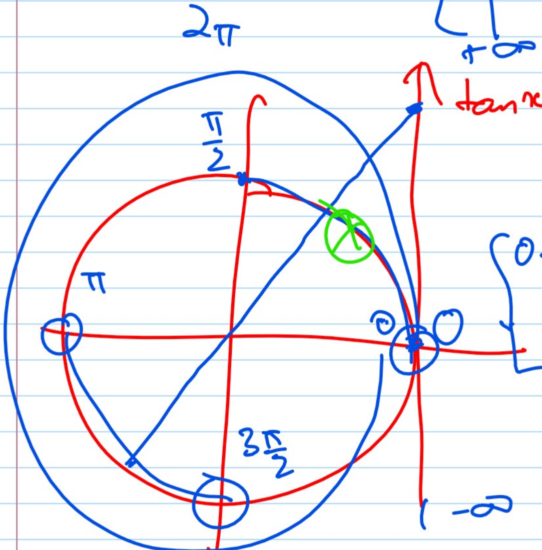
arctan : ↗ hàm tăng $\quad \mathbb{R}\setminus\{\ \} \to \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

arccot : ↘ hàm giảm $\quad \mathbb{R}\setminus\{\ \} \to (0;\pi)$

$$\frac{2x-1}{1+3x} \geq 0 \iff \left[\begin{array}{l} \begin{cases} 2x-1 > 0 \\ 1+3x > 0 \end{cases} \iff \begin{cases} x \geq \frac{1}{2} \\ x \leq -\frac{1}{3} \end{cases} \text{(vô lý)} \\ \begin{cases} 2x-1 \leq 0 \\ 1+3x \leq 0 \end{cases} \iff \begin{cases} -\frac{1}{3} \leq x \leq \frac{1}{2} \\ x \neq -\frac{1}{3} \end{cases} \end{array}\right.$$

$\to -\frac{1}{3} < x \leq \frac{1}{2}$

$\to$ TXĐ.



$$\begin{cases} 0 + k2\pi \leq x \leq \frac{\pi}{2} + k2\pi \\ \pi + k2\pi \leq x \leq \frac{3\pi}{2} + k2\pi \end{cases}$$

$|-3| \geq 3$

$3 \geq 3$ lđ

$1 \geq 1 \quad 2 \geq 1$

$2x + \sqrt{1+4x^2} > 0$

$$\begin{cases} \sinh = \dfrac{e^x - e^{-x}}{2} \\ \cosh = \dfrac{e^x + e^{-x}}{2} \end{cases}$$

$\ln(x + \sqrt{1+x^2})$ $\quad x + \sqrt{1+x^2} > 0$

$\iff x \in \mathbb{R}$

TXĐ : $\mathbb{R}$.

LĐ

$\sqrt{1+x^2} > \sqrt{x^2} = |x| \geq -x$

$\Rightarrow x + \sqrt{1+x^2} > 0$

so sánh

$f(x)$

$f(-x) = \ln(-x + \sqrt{1+x^2})$

chẵn x

$\log(a.b) = \log a + \log b$

lẻ : $f(x) = -f(-x) \iff f(x) + f(-x) = 0$ ✓

$\Leftrightarrow \ln(\cancel{x} + \sqrt{1+x^2}) + \ln(-x + \sqrt{1+x^2}) = 0$??

$\Leftrightarrow \ln(\overset{a+b}{1+x^2} - \overset{a-b}{x^2}) = \ln(1) = 0$

$\sin x + \cos x$ $\mathbb{R}$

2ĐK. $x \in TXĐ$; $-x \in TXĐ$ ✓

s^2 $f(x) = -f(-x)$.

$f(-x) = \sin(-x) + \cos(-x)$

$= -\sin x + \cos x$

$2\cos x \neq 0$

Bài 6. Cho hàm số $f(x) = \dfrac{x+1}{x-1}$ và hàm số $g(x) = \dfrac{2x}{x+2}$.

a) Tính $f(f(x)), g(g(x))$ và $f(g(x))$.

$$f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}.$$

$f(h(x)) = g(x) \rightarrow$ Tìm $h(x)$.

$$\frac{h(x)+1}{h(x)-1} = \frac{2x}{x+2} \Leftrightarrow (x+2)(h(x)+1) = 2x(h(x)-1)$$

Bài 7. Tìm hàm ngược của các hàm số sau:

a) $f(x)\colon [\pi, 2\pi] \to [-1, 1], f(x) = \cos x$

b) $y = 2\arcsin x$

c) $y = \dfrac{1-x}{1+x}$

d) $y = \sinh x$

$x = y \ldots t \ldots (y)$

$f(x) = \cos x \rightarrow \arccos(x)$

$[-1;1] \rightarrow [\pi ; 2\pi]$

b) $y = 2\arcsin x$

$\Leftrightarrow \dfrac{y}{2} = \arcsin x$

$\Leftrightarrow \sin\left(\dfrac{y}{2}\right) = \cancel{\sin}(\arcsin x)$.

$= x$

hàm

$x = \sin\left(\dfrac{y}{2}\right)$ → biến

Hàm ngược $y = \sin\left(\dfrac{x}{2}\right)$

c) $y = \dfrac{1-x}{1+x}$ → ít y

$\Leftrightarrow (1+x)y = 1-x$

$\Leftrightarrow y + xy = 1-x$.

$\Leftrightarrow y + xy + x = 1$

$\Leftrightarrow y + x(y+1) = 1$

$\Leftrightarrow x(y+1) = 1-y \Leftrightarrow x = \dfrac{1-y}{y+1}$. → Hàm ngược $y = \dfrac{1-x}{x+1}$.

$$y = \frac{10x-1}{10x+1} \Leftrightarrow y(10x+1) = 10x-1$$

$$\Leftrightarrow 10xy + y = 10x - 1$$

$$\Leftrightarrow 10xy + y - 10x = -1$$

$$\Leftrightarrow y + x(10y - 10) = -1$$

$$\Leftrightarrow x(10y-10) = -1-y$$

$$x = \frac{-1-y}{10y-10}$$

a) $u_n = n - \sqrt{n^2 - n}$

b) $u_n = \dfrac{1}{1.2} + \dfrac{1}{2.3} + \ldots + \dfrac{1}{(n-1)n}$

c) $u_n = \dfrac{\sin^2 n - \cos^3 n}{n}$

d) $u_n = \dfrac{n + \cos n}{n^2 + \sin(n^2)}$

e) $\dfrac{\sqrt{n}\cos n}{n+1}$

f) $u_n = \dfrac{n\cos(n\pi)}{n+1}$

$\lim \dfrac{1}{n}$

$$\lim\left(n - \sqrt{n^2-n}\right) = \lim \frac{n^2 - (n^2-n)}{n + \sqrt{n^2-n}} = \lim \frac{n}{n + \sqrt{n^2-n}}$$

$\sqrt{n^2-n} \sim \sqrt{n^2} = n$

$$= \lim \frac{1}{1 + \frac{\sqrt{n^2-n}}{n}} = \lim \frac{1}{1 + \sqrt{\frac{n^2-n}{n^2}}} = \lim \frac{1}{1 + \sqrt{1 - \frac{1}{n}}}$$

$\frac{1}{n} \to 0$

$$\lim \frac{1}{1+1} = \frac{1}{2}.$$

hữu hạn

$\dfrac{a}{0} \to \infty$

$\dfrac{a}{\infty} \to 0$

$\sqrt{n}$

$$\frac{1}{2\cdot 3} = \frac{1}{2} - \frac{1}{3}$$

$$\lim\left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{(n-1)n}\right)$$

$$\lim\left(1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cdots + \frac{1}{n-1} - \frac{1}{n}\right) = \lim\left(1 - \frac{1}{n}\right) = 1$$

$-1,1$ $\quad(-1,1)$

$\lim \dfrac{\sin^2 n - \cos^3 n}{n}$ → bị chặn

$0 \quad n \longrightarrow$ vô cùng $\rightarrow \dfrac{b\bar{c}}{\infty} = 0$

Ngl' kẹp $\quad f(x) \le g(x) \le h(x) \rightarrow L.$

$$\lim\left(-\frac{1}{n}\right) \le \underbrace{\frac{\sin^2 n - \cos^3 n}{n}}_{0} \le \lim \frac{1}{n}$$

$\lim\left(-\frac{1}{n}\right) \rightarrow 0$, $\quad \lim\frac{1}{n} \rightarrow 0$

a) $\lim\limits_{x\to 0}\left(\frac{1}{x}\sqrt{1+x}-\frac{1}{x}\right)$

b) $\lim\limits_{x\to+\infty}(\sqrt[3]{x^3+x^2-1}-x)$

c) $\lim\limits_{x\to-\infty}(x+\sqrt{x^2+1})$

$$= \lim_{x\to 0}\frac{\sqrt{x+1}-1}{x} = \lim_{x\to 0}\frac{(x+1)-1}{x(\sqrt{x+1}+1)} = \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)} = \lim_{x\to 0}\frac{1}{\sqrt{x+1}+1} = \frac{1}{2}.$$

$$= \lim_{x\to\infty}\frac{x^3+x^2-1-x^3}{\left(\sqrt[3]{x^3+x^2-1}\right)^2 + x\sqrt[3]{x^3+x^2-1}+x^2}$$

$$= \lim \frac{x^2-1}{\left(\sqrt[3]{x^3+x^2-1}\right)^2 + x\sqrt[3]{x^3+x^2-1}+x^2} = \frac{1}{3}.$$

$x \to \infty$

$x^2 \quad + x^2 \quad + x^2$

$\rightarrow a-b = \dfrac{a^3-b^3}{a^2+ab+b^2}$

$$= \lim \frac{1 - \frac{1}{x^2}}{\left(\sqrt[3]{1+\frac{1}{x}-\frac{1}{x^3}}\right)^2 + \sqrt[3]{1+\frac{1}{x}-\frac{1}{x^3}} + 1}$$

$\frac{1}{x^2} \rightarrow 0$, $\quad \frac{1}{x} \rightarrow 0$, $\quad \frac{1}{x^3} \rightarrow 0$

$\lim\limits_{x\to\infty}\dfrac{x^2 \rightarrow 2}{2x^2+\sqrt{x^4} \rightarrow 2}$

$$\left(\sqrt[3]{\frac{x^3+x^2-1}{x^3}}\right)^2 = \left(\sqrt[3]{\frac{x^3+x^2-1}{x^3}}\right)^2 = \left(\sqrt[3]{1+\frac{1}{x}-\frac{1}{x^3}}\right)^2$$

$-100 \quad x+1$

$$\underbrace{\lim_{x\to-\infty}\left(x+\sqrt{x^2+1}\right)}_{①}$$

$-100 \quad +\infty$

Cách 1

Đặt $x = -t \Rightarrow t \to +\infty$

$$① = \lim_{t\to+\infty}\left(-t+\sqrt{t^2+1}\right) = \lim_{t\to+\infty}\frac{t^2+1-t^2}{\sqrt{t^2+1}+t}$$

$$= \lim_{t\to+\infty}\frac{1}{\sqrt{t^2+1}+t} = 0$$